KNOWLEDGE BASED RESERVOIR COMPUTING

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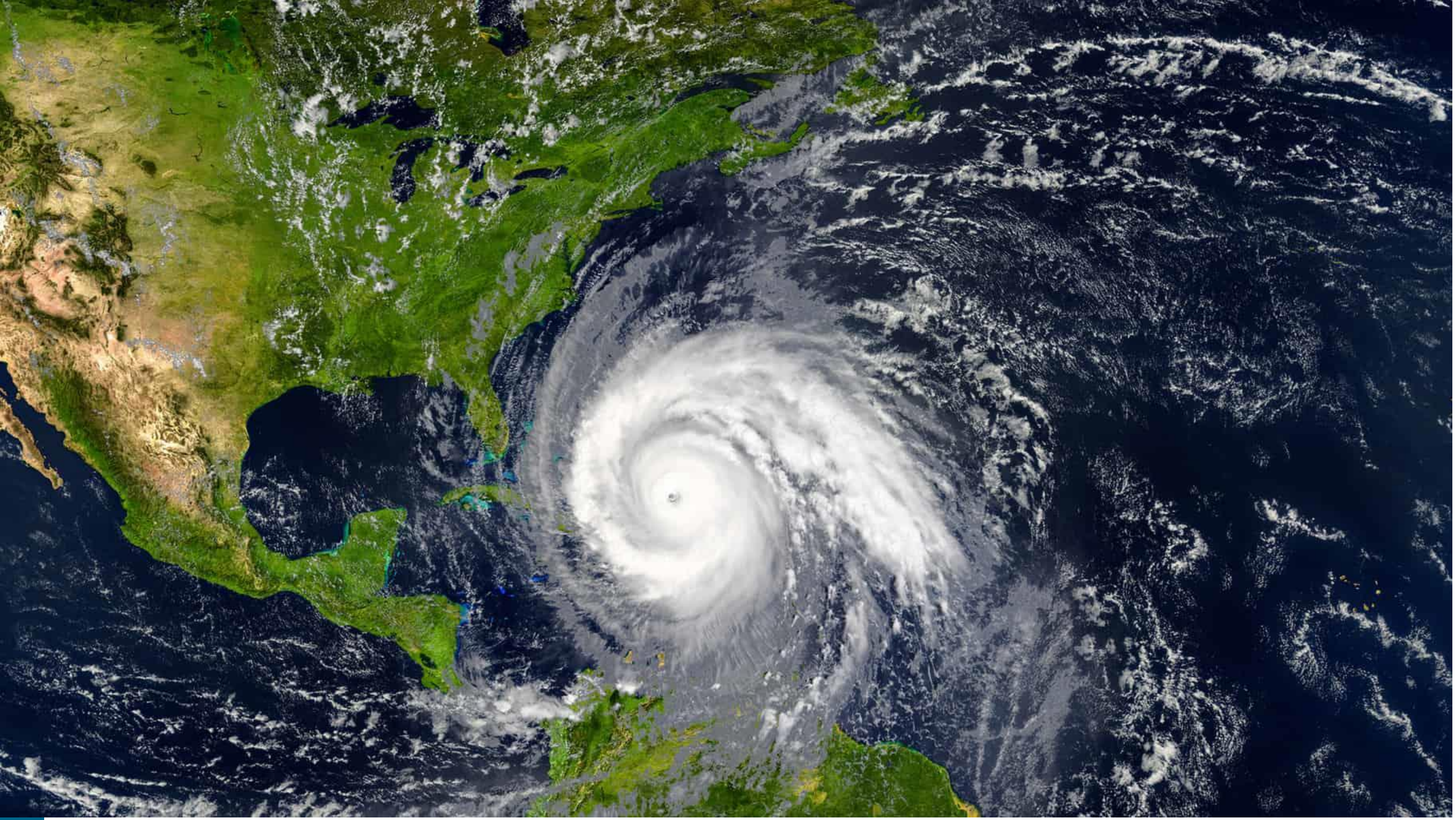
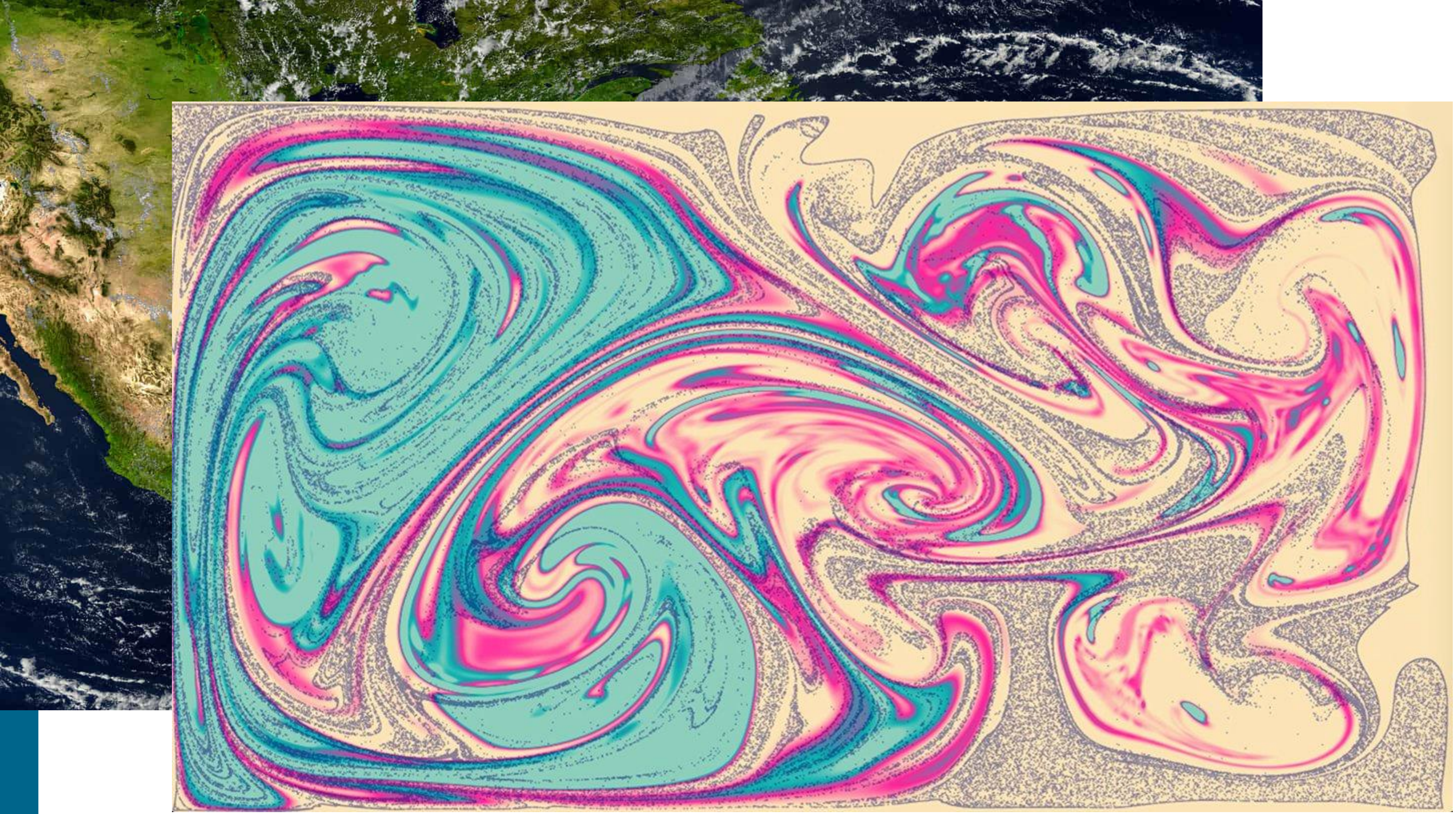
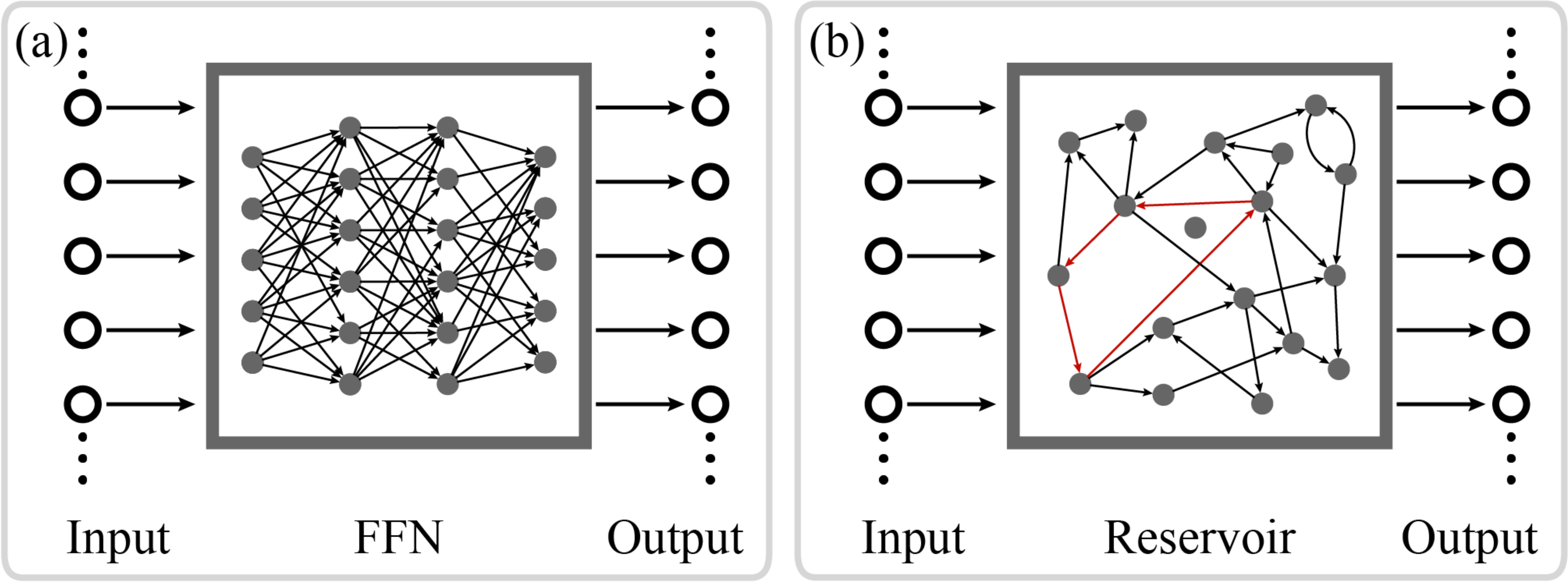
DLR






20.41
28.87
19.22
42.10
51.16
72.22
52.59
52.82
26.65
90.95

Reservoir Computing



Reservoir Computing

Update the reservoir state $\boldsymbol{r}(t)$:

$$\boldsymbol{r}(t+\Delta t) = \tanh(\boldsymbol{A}\,\boldsymbol{r}(t) + \boldsymbol{W}_{in}\boldsymbol{x}(t) + \mathbf{b})$$

Map $\boldsymbol{r}(t)$ onto the output $\boldsymbol{y}(t)$:

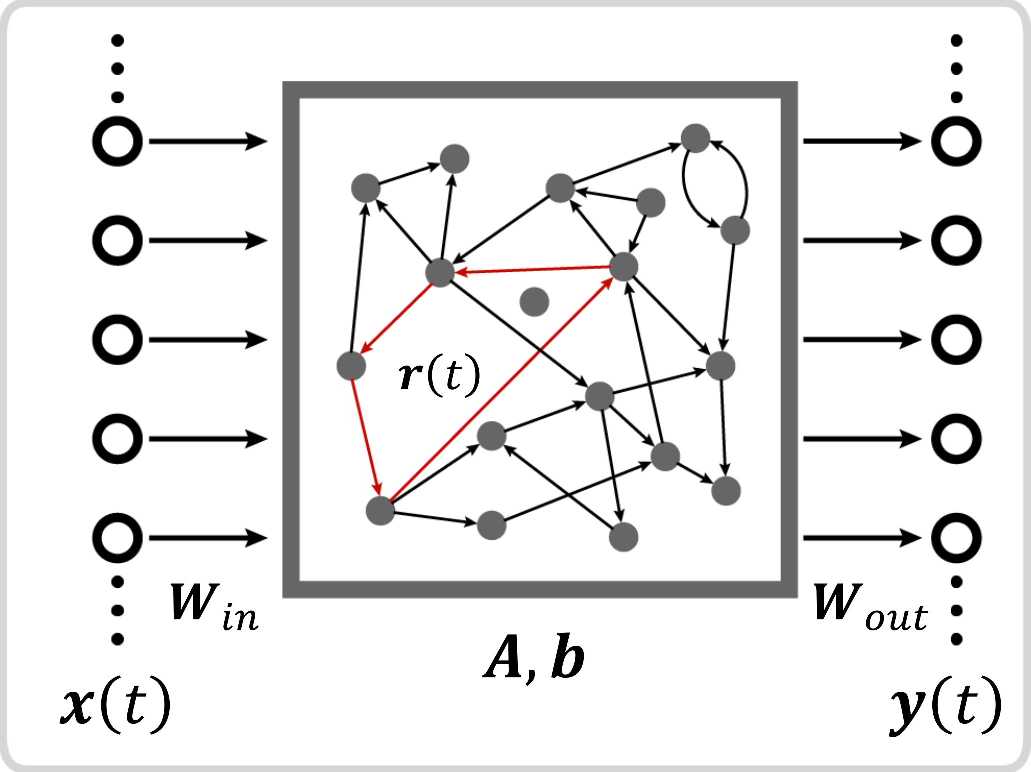
$$\boldsymbol{y}(t) = \boldsymbol{W}_{out}\boldsymbol{r}(t)$$

Optimize $\boldsymbol{W}_{out}$ via Ridge regression. Minimize:

$$\sum_{t=-T}^{0} \parallel \boldsymbol{W}_{out}\boldsymbol{r}(t) - \boldsymbol{y}_{\boldsymbol{R}}(\mathrm{t}) \parallel^2 + \beta \parallel \boldsymbol{W}_{out} \parallel^2$$

$\boldsymbol{y}_{\boldsymbol{R}}(\mathrm{t})$: known real output

β: regularization parameter

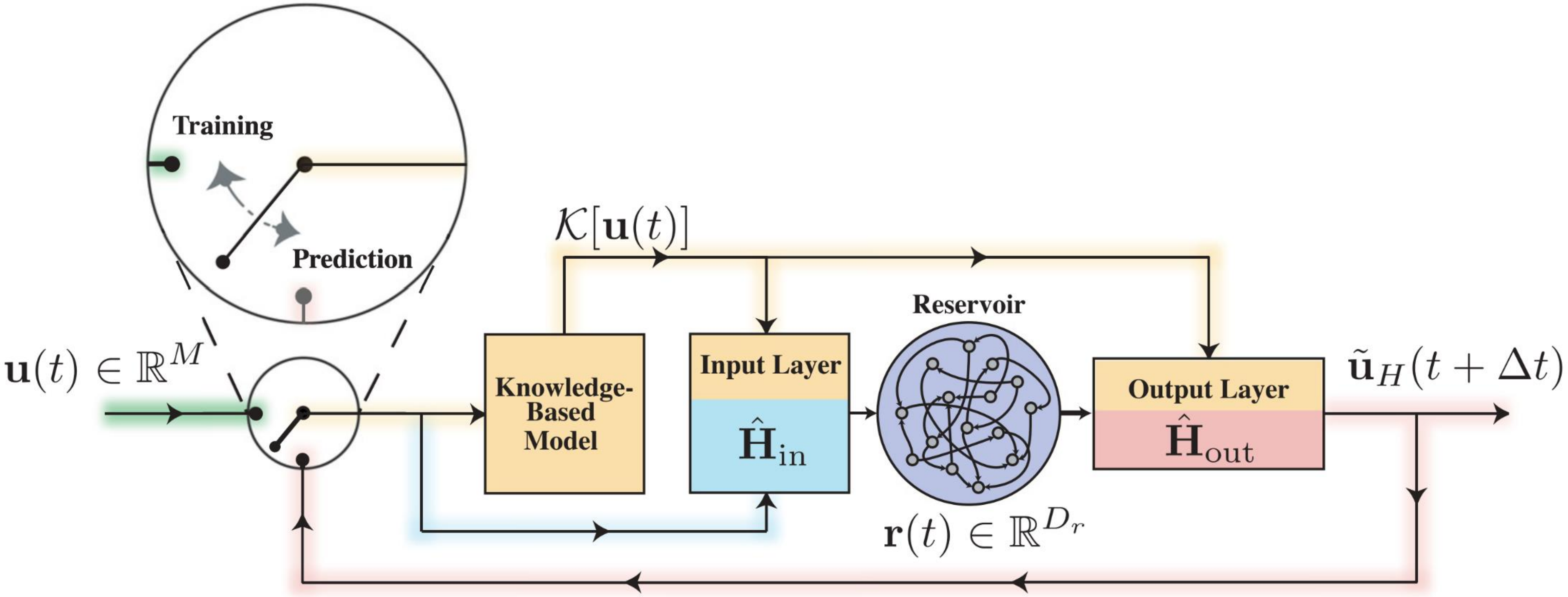


$\boldsymbol{W}_{out}$ is then obtained via:

$$\boldsymbol{W}_{out} = \boldsymbol{y}^{\intercal}\boldsymbol{r}(\boldsymbol{r}^{\intercal}\boldsymbol{r} + \beta\,\boldsymbol{I})^{-1}$$

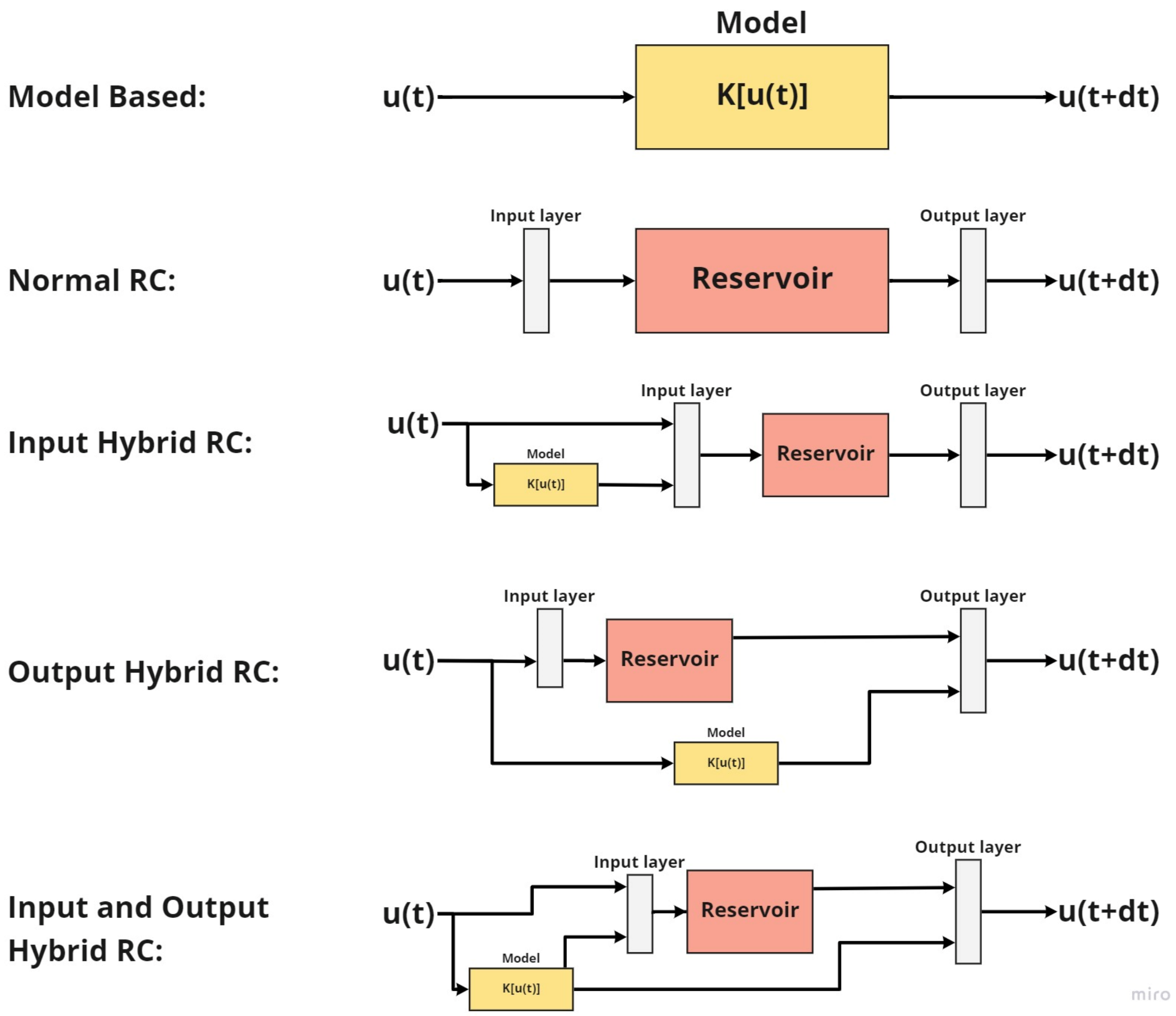
But what if you already have a decent model of your system?

Hybrid Reservoir Computing



Pathak et al., Chaos, 2018

Combine Model-Based and RC-based Predictions



Input Hybrid RC:

$$f_{\text{inp}}(\tilde{\boldsymbol{u}}(t)) = \begin{bmatrix} \tilde{\boldsymbol{u}}(t) \\ K(\tilde{\boldsymbol{u}}(t)) \end{bmatrix}$$

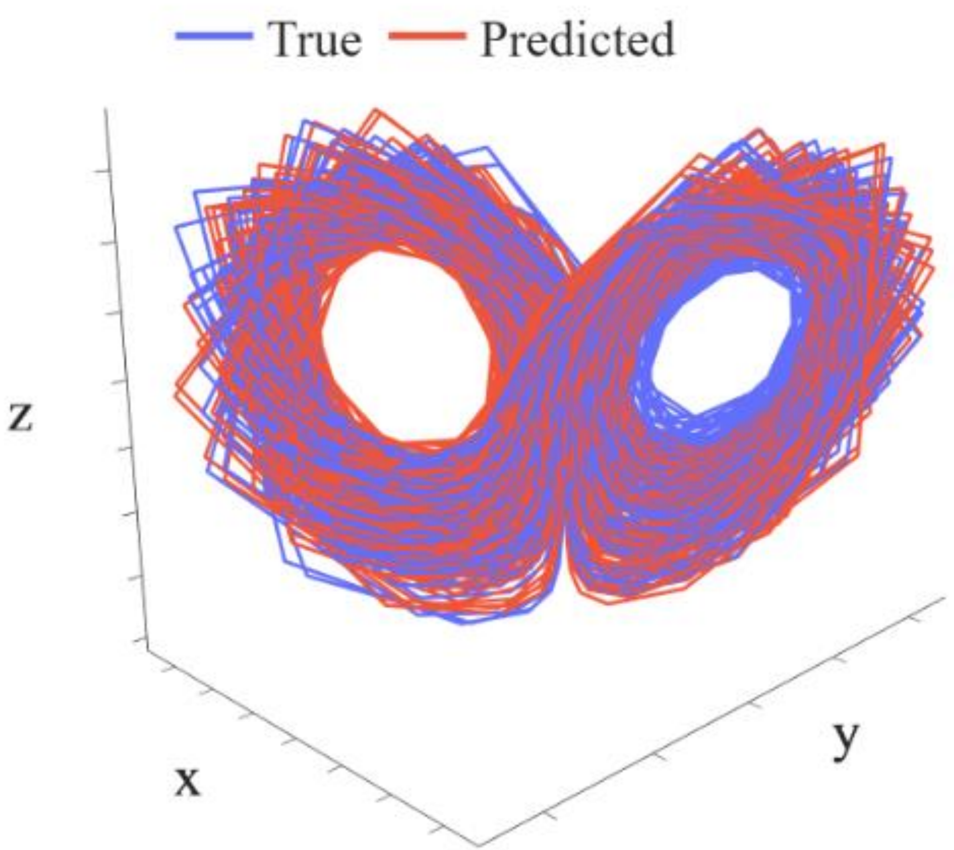
Output Hybrid RC:

$$f_{\text{out}}(\boldsymbol{r}(t), \tilde{\boldsymbol{u}}(t)) = \begin{bmatrix} \boldsymbol{r}(t) \\ K(\tilde{\boldsymbol{u}}(t)) \end{bmatrix}$$

Input and Output Hybrid RC:

$$f_{\text{inp}}(\tilde{\boldsymbol{u}}(t)) = \begin{bmatrix} \tilde{\boldsymbol{u}}(t) \\ K(\tilde{\boldsymbol{u}}(t)) \end{bmatrix} \qquad f_{\text{out}}(\boldsymbol{r}(t), \tilde{\boldsymbol{u}}(t)) = \begin{bmatrix} \boldsymbol{r}(t) \\ K(\tilde{\boldsymbol{u}}(t)) \end{bmatrix}$$

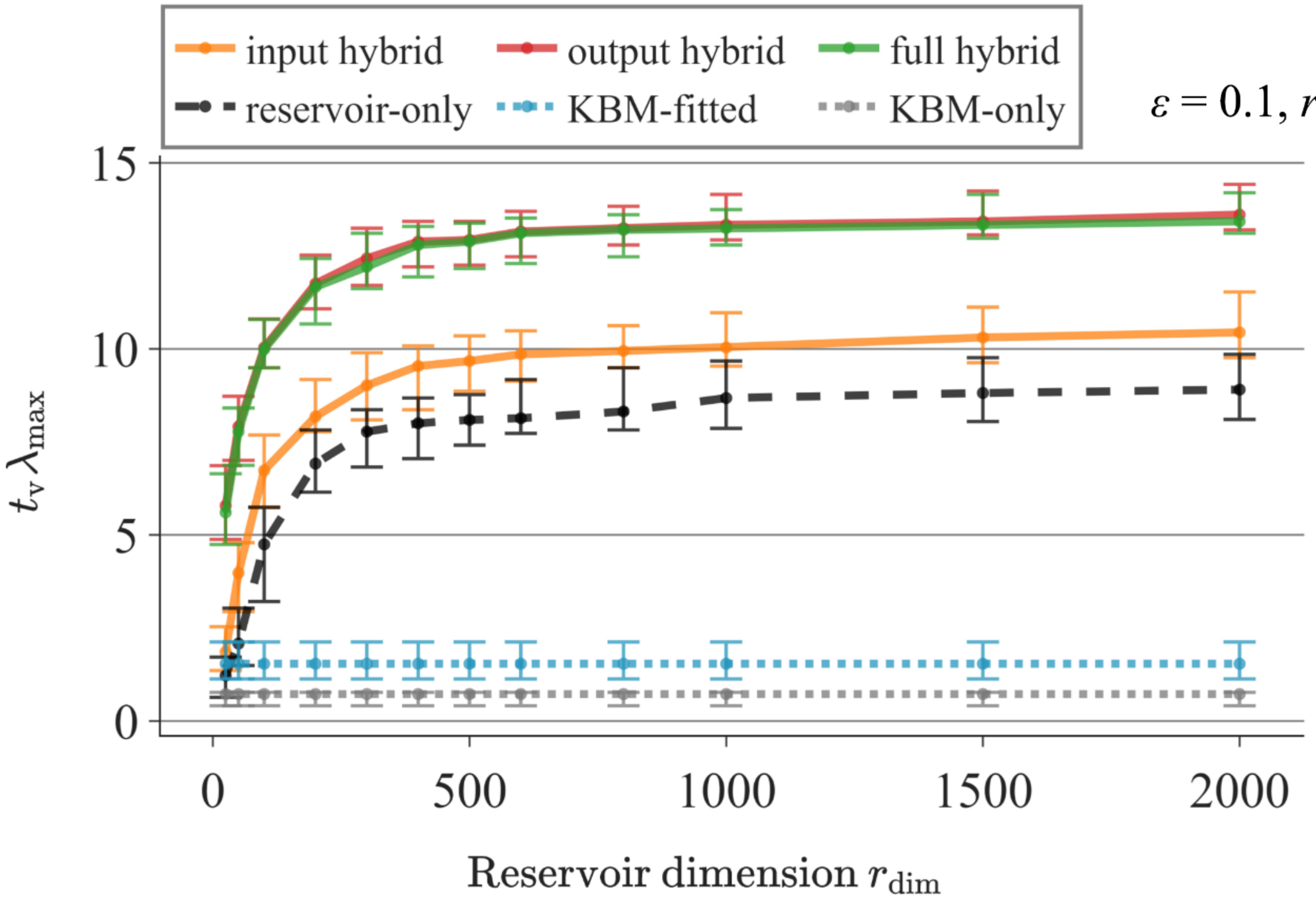
Adding Errors to our KBM



$$\dot{x}(t) = \sigma(y - x)$$
$$\dot{y}(t) = x(\rho - z) - y$$
$$\dot{z}(t) = xy - \beta z$$

ρ is modified to $\tilde{\rho} = 28(1 + \epsilon)$

Hybrid RC for the Lorenz System



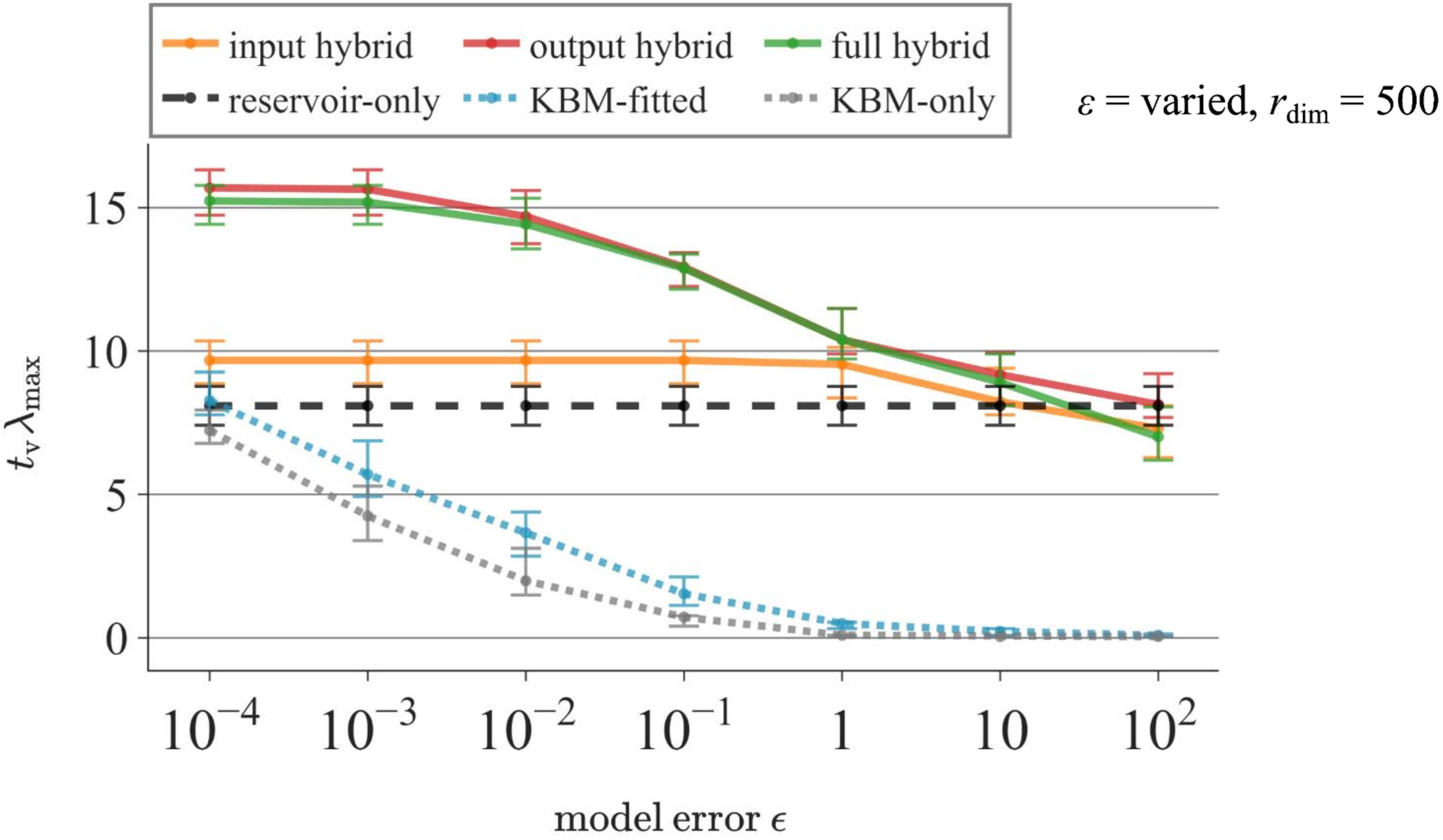
$\varepsilon = 0.1$, r_{dim} = varied

$$e(t) = \frac{\|\boldsymbol{y}(t) - \boldsymbol{y}_r(t)\|}{\langle \|\boldsymbol{y}(t)\|^2 \rangle^{1/2}}$$

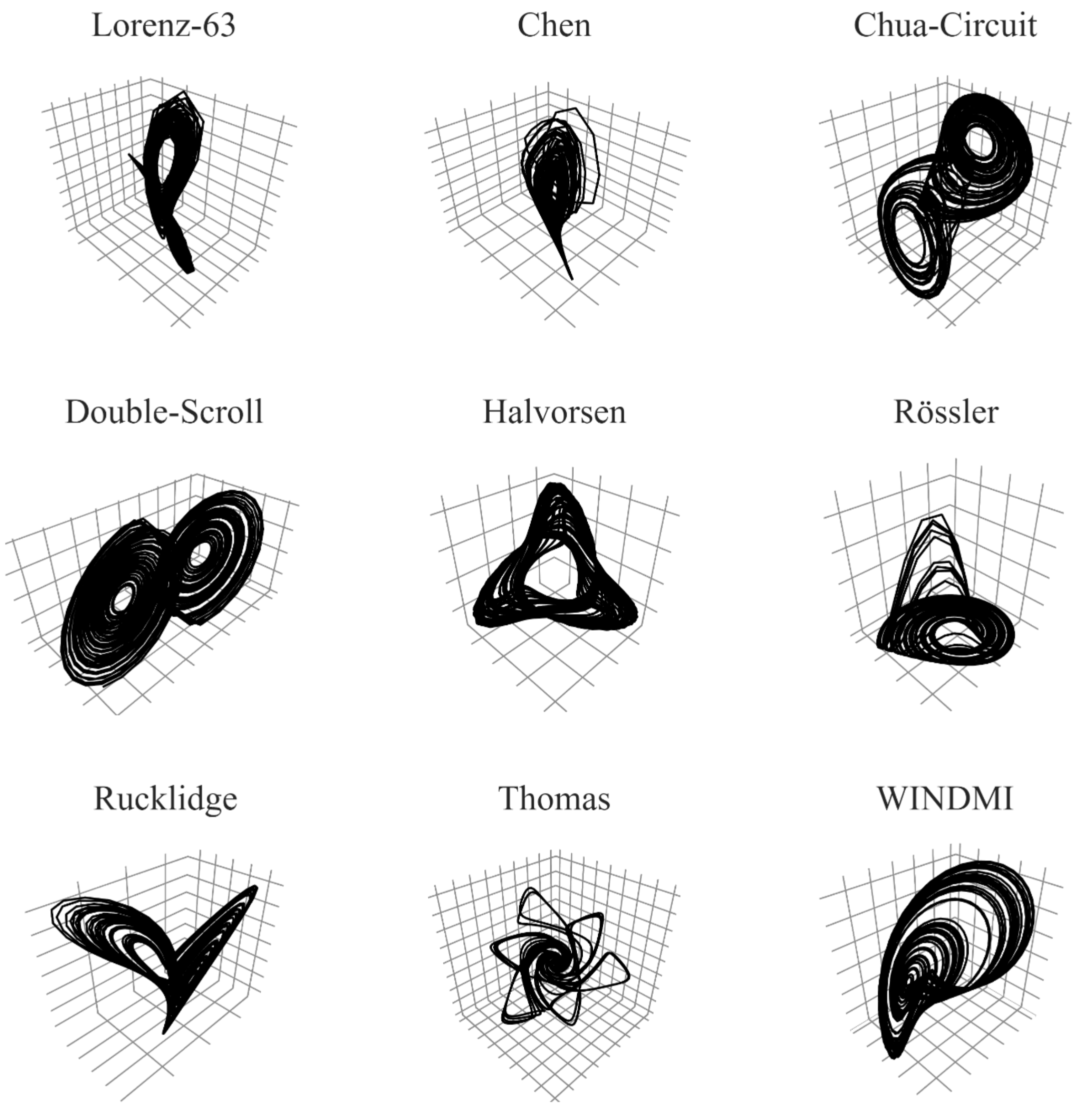
$$\boldsymbol{y}_{\text{kbm-fitted}}(t) = \mathbf{W}_{\text{kbm}} K(\tilde{\boldsymbol{u}}(t)) + \boldsymbol{w}_{\text{out}}$$

$$\boldsymbol{y}_{\text{kbm-only}}(t) = K_\varepsilon(\tilde{\boldsymbol{u}}(t))$$

Hybrid RC for the Lorenz System



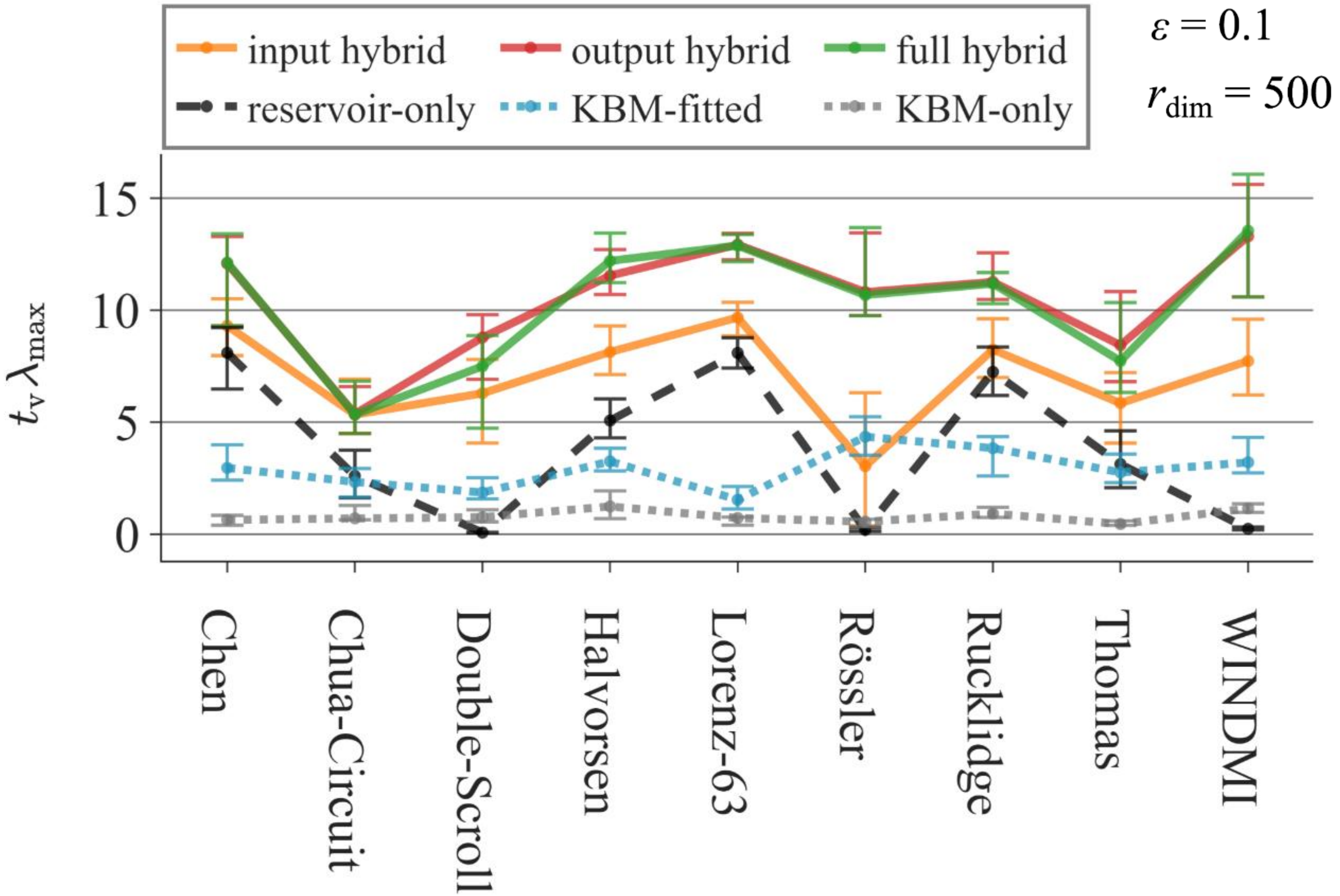
Multiple Chaotic Systems



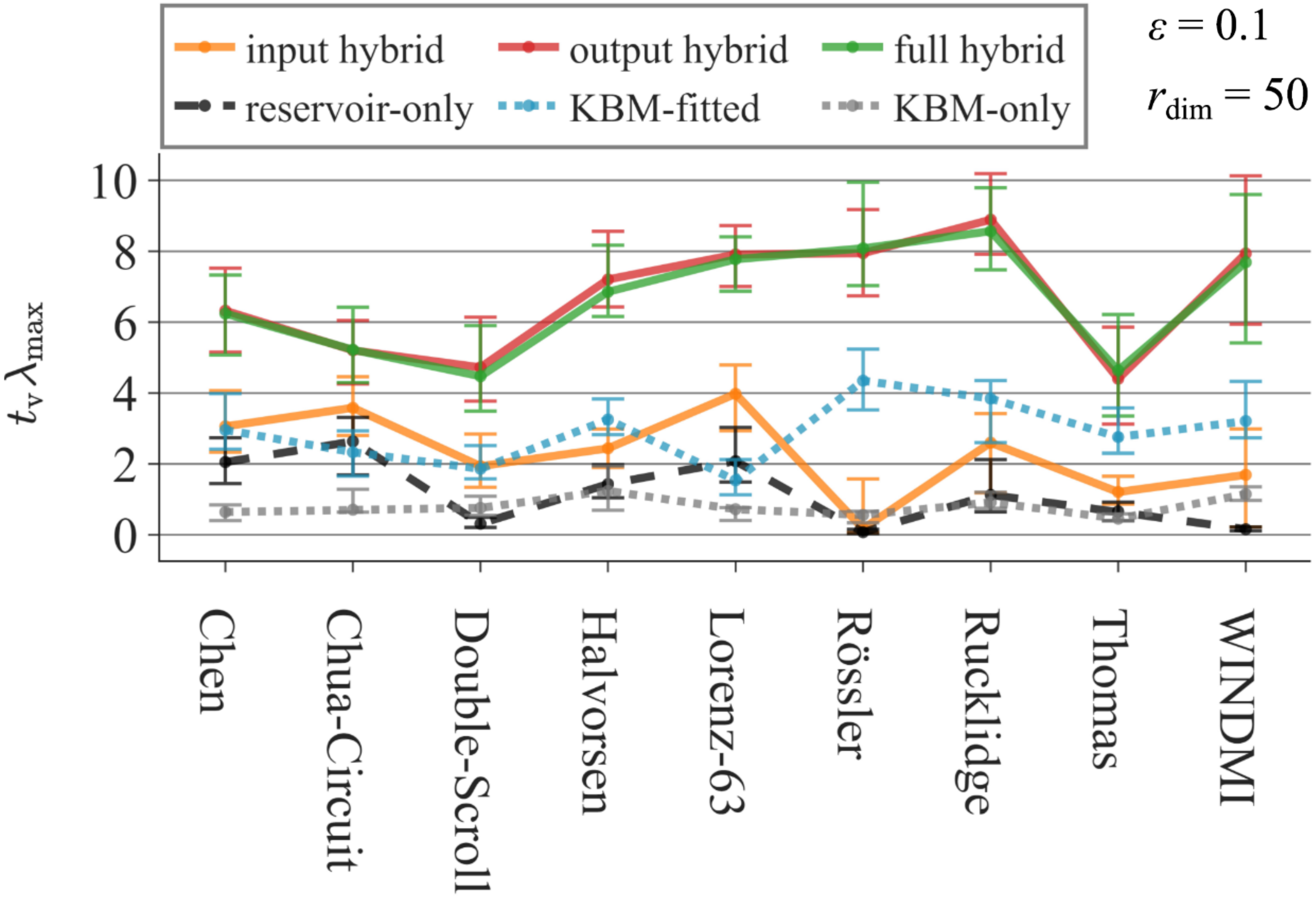
System	Eq.	ε-param
Lorenz-63	(A1)	ρ
Chen	(A2)	a
ChuaCircuit	(A3)	α
DoubleScroll	(A4)	a
Halvorsen	(A5)	a

System	Eq.	ε-param
Roessler	(A6)	c
Rucklidge	(A7)	κ
Thomas	(A8)	b
Windmi	(A9)	a
KS	(17)	a

Hybrid RC for Multiple Chaotic Systems

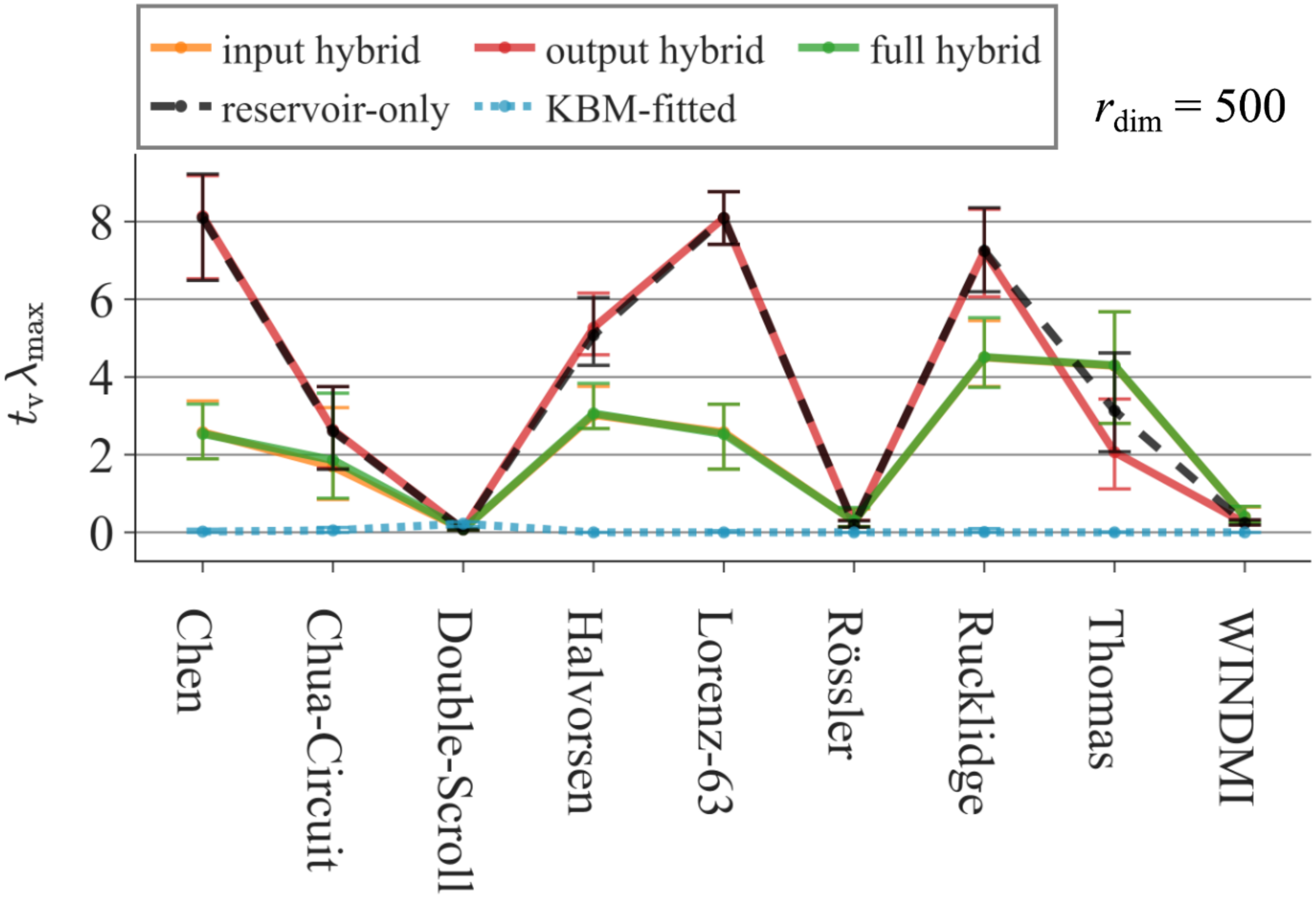


Hybrid RC for Multiple Chaotic Systems



But what if your knowledge based model is just really bad?

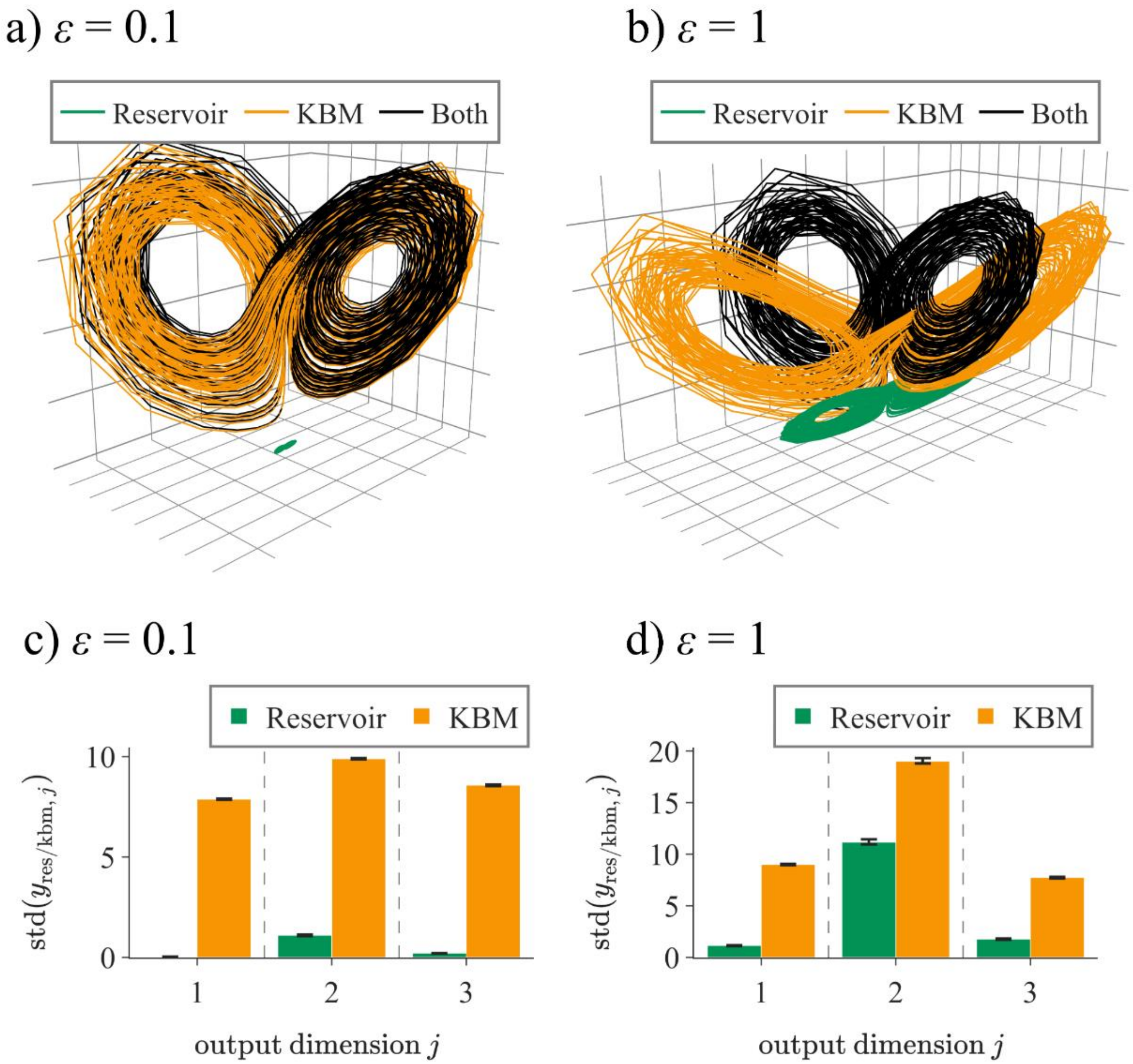
Using a Horrible KBM



$$K_{\sin}(\boldsymbol{u}(t)) = \sin(\boldsymbol{u}(t))$$

Interpretability?

Interpretability



Conclusion

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- Input hybrid (IH), output hybrid (OH), and full hybrid (FH), all outperform non-hybrid RC for sufficiently accurate models.
- For accurate models, OH and FH results are equivalent and significantly outperform the IH results, especially for smaller reservoir sizes.
- For totally inaccurate models the predictive capabilities of IH and FH may decrease drastically, while the OH architecture remains as accurate as the purely data-driven results.
- Output hybrid basically always better than input hybrid and about as good as full hybrid for good, and preferable for bad knowledge based models.
- OH allows for the separation of the reservoir and the model contributions to the output predictions.
- OH approach is the most favorable architecture for hybrid reservoir computing, when taking accuracy, interpretability, robustness to model error, and simplicity into account.

- Outlook: Hybrid-NG-RC, Hybrid-Minimal-RC, Real Data

Special Thanks:

- Christoph Räth
- Dennis Duncan: First author of the paper, but sadly couldn’t be here today.
- “Optimizing the combination of data-driven and model-based elements in hybrid reservoir computing”
  - Soon to be published in Chaos

THANK YOU FOR YOUR ATTENTION

DLR