Mitteilung

Fachgruppe: Turbulenz und Transition

Study of Richardson Number in flows with mean-streamline curvature

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Several configurations such as flow over a curved surface, wake region behind behind an aircraft wing generate flow fields with high vorticity. In such highly vortical flows, mean-streamline curvature is known to make considerable changes in the turbulence structure of shear layers. [1] More insight into the phenomena of vortex regions is important in making modifications to the turbulence models for better accuracy.

Previously, Ströer and Knopp [2] explored the grid-point local and general formulation of the gradient Richardson number which is used to characterize and quantify rotation and mean-streamline curvature effects in a fluid flow. Originally, Richardson number is a dimensionless number that expresses the ratio of buoyancy to the flow shear. Bradshaw [1] suggested the gradient Ricardson number: $Ri_{Br} = 2S(S+1)$. Here S = $(U_\phi/r)/(\partial U_\phi/\partial r)$ is the vorticity ratio (Curvature/Shear). The classical formulation was converted into a generalised formulation using grid-point local quantities such as vorticity $|\Omega|$ and strainrate |S| and the directional information n_Ω which is related to the amplification or damping of the turbulence [2].

$$Ri_{Bradshaw} = \frac{2\left(\frac{U_\phi}{r}\right)\left(\frac{U_\phi}{r} + \frac{\partial U_\phi}{\partial r}\right)}{\left(\frac{\partial U_\phi}{\partial r}\right)^2} \rightarrow Ri_{local} = \frac{-n_\Omega|\Omega|\,(|S| - n_\Omega|\Omega|)}{\left(\frac{1}{2}|S| + \frac{1}{2}n_\Omega|\Omega|\right)^2}$$

Ströer and Knopp [2] used the above formulation to study a U-duct test case. Present work deals with an in-depth analysis of U-duct test cases with varying inner radius of the bend. This helps us understand the behaviour of Ri in convex and concave regions and give more accurate grid-point local formulation for Ri.

Two cases of U-ducts with inner radii at the bend(r) as a function of pipe radius(R) i.e. r = R (strong curvature) and 20R (moderate curvature) were simulated in DLR-TAU with negative Spalart-Almaras(SA-neg) turbulence model with ‘sarc’ rotational correction. Here, the inner wall of the bend acts as a convex curve $\left(U_\phi/r > 0, \partial U_\phi/\partial r > 0\right)$ where curvarture, shear have the same sign and the outer wall acts as a concave curve $\left(U_\phi/r > 0, \partial U_\phi/\partial r < 0\right)$ where curvature and shear have opposite sign. $\Omega_z(r) = U_\phi/r + \partial U_\phi/\partial r$ gives the distinction between convex and concave curvatures. A nondimensional parameter r_b introduced by Shur etal. [4] estimates convex and concave curvature regions accurately. This parameter is used in n_Ω to construct Ri_{local}.

Richardson Number as a function of Vorticity ratio (S) forms a parabolic curve where Ri < 0 only when $-1 \leq S \leq 0$ (see figure 2). Flow is destabilizing when Ri < 0 and stabilizing if Ri > 0. Hence, negative Ri amplifies turbulence and positive Ri supresses turbulence.

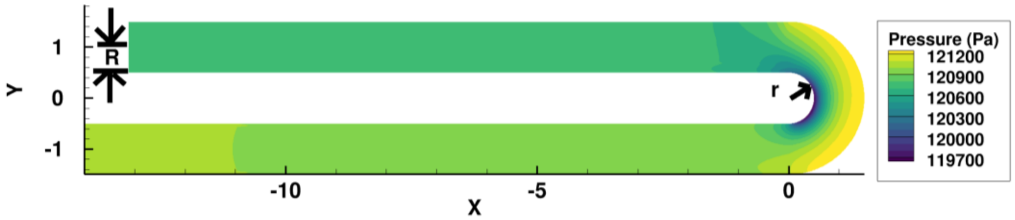

Figure 1: Pressure contours of U-turn test case with pipe radius R and inner bend radius r = R

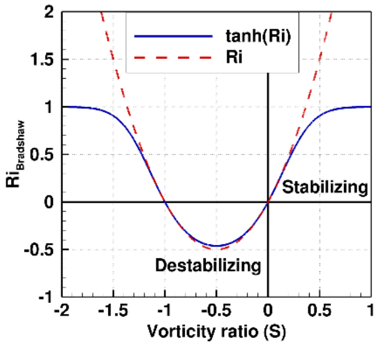

Figure 2: Behaviour of Richardson number showing Stable/Unstable zones

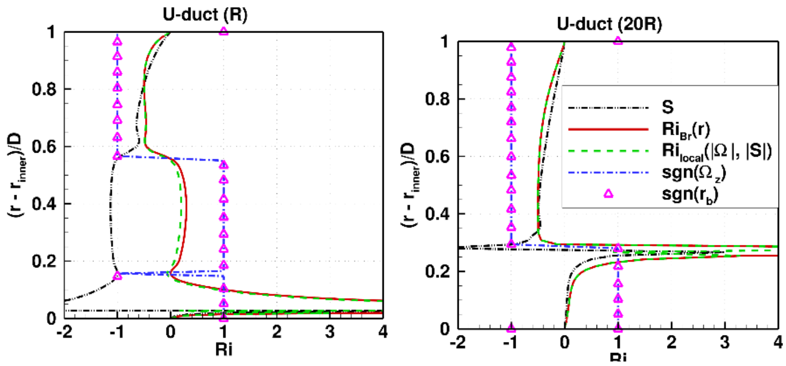

Figure 3: Spanwise line plots at the 90° bend for pipe with inner radii of the bend 3a (r = R), 3b (r = 20R)

Figure 3a, 3b show that non-dimensional parameter r_b captures the behaviour of $\Omega_z(r)$ very well. As a result, good agreement between Ri_{local} and $Ri_{Br}(r)$ can be observed. Here, Ri > 0 near the convex curve and Ri < 0 closer to concave region. Thus, convex curvature amplifies turbulence and concave curvature regions suppress turbulence.

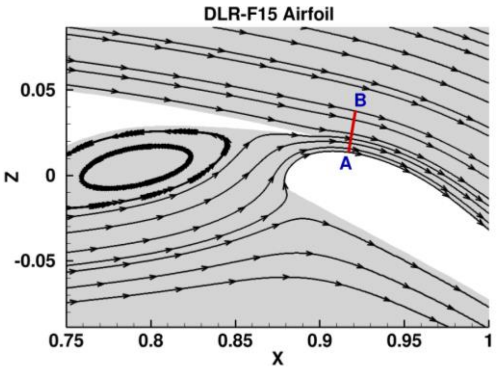

Figure 4: Streamlines around DLR F15 Airfoil and line segment AB

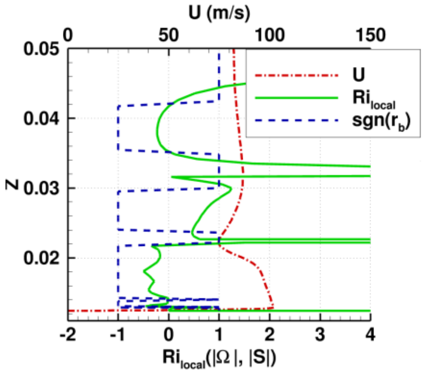

Figure 5: Horizontal velocity, local Ri on line segment AB

The line segment AB on the flap of the 3-element airfoil DLR-F15 covers the boundary layer of the flap, shear layer from the main wing and the bleeding jet emerging from the gap. Figure 5 shows Ri_{local} and sgn(r_b) predict the regions of destabilizing and stabilizing flow. But, Ri in the present form is heavily influenced by shear. The regions with low shear value or the point where shear switches in sign result in very high values of Ri which is observed in figures 3a, 3b and 5. It is necessary to modify Ri to make it a more reliable tool to be used in further modelling of components of turbulence budget.

Hyperbolic tangent(tanh) function could be a great alternative to deal with extreme values. Figure 2 shows how discontinuities can be avoided by clipping the values of Ri using tanh function. An interesting application of Ri is modifying the production term of the length scale equation. Rotational correction by Shur etal. [4] increases and decreases turbulence by a factor of 2 in concave and convex regions respectively. Similarly, the value of the production term can be modified between stabilizing and destabilizing zones to enhance and reduce turbulence based on mean-streamlined curvature.

References :

[1] Bradshaw, P., “The analogy between streamline curvature and buoyancy in turbulent shear flows,” Journal of Fluid Mechanics, Vol. 36, No. 177, 1969.

[2] Ströer, Philip, and Tobias Knopp. "General Formulation of the Gradient Richardson Number for RANS Modelling." AIAA SciTech 2023 Forum. 2023.

[3] Durbin, Paul A. "Some recent developments in turbulence closure modeling." Annual Review of Fluid Mechanics 50: 77-103, 2018.

[4] Shur, Michael L., Michael K. Strelets, Andrey K. Travin, and Philippe R. Spalart. "Turbulence modeling in rotating and curved channels: assessing the Spalart-Shur correction." AIAA journal 38, no. 5: 784-792, 2000.