Master Thesis

Evaluation of Robot-assisted Spacecraft Alignment

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Declaration of academic honesty

I hereby declare that the master thesis "Evaluation of Robot-assisted Spacecraft Alignment" is my own work, that I have not presented it elsewhere for examination purposes, and that I have not used any sources or aids other than those stated. I have marked verbatim and indirect quotations as such. Further declarations which have to be signed within the context of the examination regulations of the University of Bremen can be obtained from the appendix A. Larger quantities of data and the software scripts developed are not appended due to unreasonable spatial requirements but made available on the institute server.

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Abstract

The Reusable Flight Experience (ReFEx) is a spacecraft developed by the German Aerospace Center (DLR), which aims to provide design and flight data as well as operational experience on aerodynamically controlled Reusable Launch Vehicles (RLV) stages. The constant knowledge of the position and orientation in 3D space is essential for a successful mission. The alignment and transformation matrices between the individual component's local coordinate systems within different subsystems of ReFEx must be determined before the flight. They are required as inputs for the Guidance, Navigation and Control (GNC) algorithms controlling the flight path during the spacecraft's mission.

A generalized alignment process was developed which guides the operator through the required measurements and concludes with the desired results. An industrial robot was implemented for an increased generalization with different Devices under Test (DUT). Additionally, the industrial robot was used to implement a feature that reduced the time effort for a performed alignment process by 42 min or up to 41.6% compared to a non robot-assisted alignment process.

A custom Python script was developed to support the operator with the numerous calculations performed in the background. The measured data and results of the alignment chain were visualized in a Graphical User Interface (GUI).

The alignment process was tested and verified using a custom designed and constructed test rig with a test alignment cube. The alignment cube had a measured angle between two autocollimated mirror faces of $90.0004297^{\circ} \pm 0.0003847^{\circ}$. That was a difference from the perfect 90° of $4.297 \cdot 10^{-4\circ} \pm 3.847 \cdot 10^{-4\circ}$ or $1.547 \operatorname{arcsec} \pm 1.385 \operatorname{arcsec}$.

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List of Acronyms

| 3D | Three Dimensional | 3 |
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| \mathbf{AC} | Autocollimation Lamp | 41 |
| \mathbf{AR} | Augmented Reality | 65 |
| ATR | Automated Target Recognition | 75 |
| CAD | Computer Aided Design | 23 |
| CCR | Corner Cube Reflector | 76 |
| COG | Center of Gravity | 39 |
| \mathbf{CS} | Coordinate System | 24 |
| DOF | Degrees of Freedom | 7 |
| \mathbf{DL} | Diode Laser | 41 |
| DLR | German Aerospace Center | 1 |
| DUT | Device under Test | 1 |
| \mathbf{EDM} | Electronic Distance Measurement | 15 |
| GNC | Guidance, Navigation and Control | 3 |
| GSE | Ground Support Equipment | 4 |
| GUI | Graphical User Interface | 7 |
| HNS | Hybrid Navigation System | 5 |
| Hz | Horizontal Angle | 27 |
| IMU | Inertial Measurement Unit | 3 |
| IT1CS | Initial Theodolite 1 Coordinate System | 33 |
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1. Introduction

An accurate position and orientation of a spacecraft must be known at any point in time to ensure a successful mission. The alignment of the local onboard measurement devices, as well as control devices relative to the internal local spacecraft's coordinate system is essential. This thesis describes the development and process of a generalized alignment process to determine transformation matrices of all measured devices to a common coordinate system. Furthermore, the developed alignment chain was improved and optimized by the implementation of an industrial robot. The device under test (DUT) considered in this thesis was the Reusable Flight Experiment (ReFEx) designed by the German Aerospace Center (DLR) and a designed test rig.

A more detailed system and problem description are explained in Chapter 2. Chapter 3 provides the required background knowledge for further descriptions. The developed general alignment chain is described in detail in Chapter 4. Practical measurements using the concept of the developed alignment chain are explained in Chapter 5. The results of the measurements are discussed in Chapter 6. A sensitivity analysis of the developed general alignment chain model with the measurements as inputs and calculated results as outputs is performed in Chapter 7. Chapter 8 focuses on the benefits of the integration of an industrial robot into the alignment chain. Chapter 9 summarizes the developed general alignment chain and further recommendations are discussed in Chapter 10.

2. System Description

2.1. Problem Description

ReFEx is a spacecraft developed by the DLR, which aims to provide design and flight data as well as operational experience on aerodynamically controlled reusable launch vehicle (RLV) stages. [1]

To ensure a successful space mission, the spacecraft must know its position and orientation at any point in time as well as be able to perform correctional manoeuvrers throughout the mission duration. Measurement devices, such as an Inertial Measurement Unit (IMU) or sun sensors, are responsible for collecting the required positional and orientational data of the spacecraft. The data is used within the Guidance, Navigation and Control (GNC) algorithms to align the actual spacecraft's flight path with the intended one. For the correctional manoeuvrers, ReFEx uses propulsive (thrusters) or aerodynamic (spacecraft's body, canards) control devices.

The IMU for example measures the acceleration and angular rates of the spacecraft in its local IMU three-dimensional (3D) coordinate system [2]. The IMU coordinate system however does not align with ReFEx's spacecraft coordinate system nor any other local coordinate system. Transformation matrices between the different local coordinate systems to a common reference coordinate system must be determined. Moreover, the exact position and orientation of each control device (e.g thrusters, canards) within the spacecraft is crucial for the GNC flight path correction algorithms. It is required to estimate the outcome of various correctional manoeuvrers before execution. The outer shape and therefore outer shell of ReFEx must be determined as well due to the influences on the aerodynamic behaviour within the atmosphere.

Additionally, due to an ongoing spacecraft's integration process, individual parts might be obstructed by other later integrated parts or ReFEx outer spacecraft's shell. Therefore, it must be possible to overlay multiple measurements performed at different points in times and integration stages.

2.2. Alignment Process

The alignment chain was defined as the measurement and calculation chain throughout the alignment process. It started with the required DUT as well as the required equipment and concluded with the aligned parts and resulting transformation calculations.

2.2.1. Alignment Chain Overview

Figure 2.1 gives a system level overview of the alignment chain. With the equipment, including measurement devices, the object and parts of the DUT could be measured. The resulting measured values were processed and the final coordinate system transformations between local coordinates were output as measurement results (Figure 2.1).



Figure 2.1.: Alignment chain overview

Figure 2.3 shows a more detailed overview of the used alignment chain. The orange smart ground support equipment (GSE) sub-chain within the equipment block can be ignored in the first place. This part will be discussed later in Chapters 4 and 8 as an addition to the alignment chain including an industrial robot and its benefits.

Device under Test

Within this thesis, the DUT was either ReFEx or the test rig (dashed line within DUT - Figure 2.3). The test rig was designed, constructed and used for measurements due to the unavailability of the primary ReFEx structure throughout the development of this thesis. Further details of the test rig can be found in Section 3.3.5.

The important components of ReFEx regarding the alignment chain have been discussed in Section 2.1. ReFEx's surface and its shape were of importance due to the aerodynamic influence. It was planned to measure multiple defined survey targets at ReFEx's surface creating a point cloud of the surface. More information about the used survey targets can be found in Section 3.3.1. The point cloud could provide an estimation of the outer ReFEx shape. The more points the higher the resolution and the more accurate the estimated surface. Additionally, the position and orientation of ReFEx's control devices (e.g. thrusters, canards) must be measured using survey targets. GNC is one of ReFEx subsystems. Within this subsystem was the Hybrid Navigation System (HNS) including an IMU (Figure 2.3). The axes' orientation of the local IMU axes must be accurately known for the GNC algorithms. Two alignment mirrors (Figure 2.2) at the HNS device could be autocollimated (Section 3.2) using theodolites (Section 3.3.3) determining the local IMU coordinate system.





The test rig (Section 3.3.5) was used to perform measurements mimicking the ReFEx measurements.

Equipment

All mentioned measurements could be performed using surveying instruments such as a total station (Section 3.3.2) and theodolites (Section 3.3.3). A total station could be used to measure the absolute 3D coordinates and calculate the coordinate system transformations of all targets including the surface and the control devices (green path -Figure 2.3). Theodolites could be used to autocollimate the IMU's alignment mirrors and determine the local IMU's coordinate system (blue path - Figure 2.3). The detailed calculations and performed measurements are described in more detail in Chapter 4 and Chapter 5.

The color schemes within the measurement results in Figure 2.3 are referred to later in Figure 4.3.



Figure 2.3.: Alignment chain detailed overview

2.2.2. Previous Alignment Chain

The previous alignment chain methodology had no defined process or structure to establish the fidelity or results required. Additionally, there was no comprehensive tool or application which took the measurements as inputs and conveniently determined the required outputs. The spatial movement of the DUT was based on a three degrees of freedom (DOF) mechanical ground support equipment (MGSE). A trolley table was used to move the DUT, in this case, ReFEx or the test rig, translationally in the horizontal plate (X-, Y-coordinates) and rotationally within the same plane (around the Z-axis).

2.2.3. Generalized Alignment Chain

The initial goal of this thesis was the development of an alignment chain, alongside a custom Python tool to complete the required calculations in the background. The alignment chain shall be applicable to multiple DUTs other than ReFEx or the designed test rig. The tool shall be user-friendly with a graphical user interface (GUI) and with visual representations of the measured data as well as the calculated results. The developed alignment chain is described in Chapter 4.

2.2.4. Industrial Robot Integration

An industrial robot was integrated to generalize the alignment chain even further and obtain three more DOFs to manipulate the DUT. A total of six DOFs were available with a highly accurate repeatability and positioning in 3D space. The used industrial robot is described in more detail in Section 3.3.4. Other benefits involving the industrial robot are described in Chapter 6.

3. State of the Art

This Chapter provides the required background knowledge to follow along further descriptions of the alignment chain, measurements and industrial robot integration.

3.1. Mathematical Background

3.1.1. Rotation Matrix

A rotation matrix is defined as a transformation matrix rotating a vector or matrix around a specific fixed coordinate system. In 3D space, the vector or matrix can be rotated around the X-, Y- or Z-axis or a combination of them. The rotations around the X-, Y- and Z-axis are often referred to as roll, pitch and yaw respectively. The rotations are clockwise in positive axis direction. Equations 3.1 - 3.3 give the rotation matrices around the X-, Y- and Z-axis. [3]

$$\mathbf{R}_{\mathbf{x}} = \mathbf{R}_{\mathbf{roll}} = \begin{pmatrix} 1 & 0 & 0\\ 0 & \cos(\theta) & -\sin(\theta)\\ 0 & \sin(\theta) & \cos(\theta) \end{pmatrix}$$
(3.1)

$$\mathbf{R}_{\mathbf{y}} = \mathbf{R}_{\mathbf{pitch}} = \begin{pmatrix} \cos(\theta) & 0 & \sin(\theta) \\ 0 & 1 & 0 \\ -\sin(\theta) & 0 & \cos(\theta) \end{pmatrix}$$
(3.2)

$$\mathbf{R}_{\mathbf{z}} = \mathbf{R}_{\mathbf{yaw}} = \begin{pmatrix} \cos(\theta) & -\sin(\theta) & 0\\ \sin(\theta) & \cos(\theta) & 0\\ 0 & 0 & 1 \end{pmatrix}$$
(3.3)

3.1.2. Euler Angles

Euler angles are the most common and intuitive way to perform a rotation of a coordinate frame. The Euler angles are three angles that define the rotation of a coordinate frame with respect to a fixed coordinate system. Each angle denotes a rotation around a specific axis. There are multiple conventions of the rotation axis order. Figure 3.1 displays the ZXZ-convention. In this convention, the first Euler angle α gives the rotation around the Z-axis followed by β and γ around the X-axis and again the Z-axis. By successively performing the individual rotations around each axis all possible rotations could be achieved. [4]



Figure 3.1.: Euler Angles

3.1.2.1. Gimbal Lock

Euler angles are intuitive but limited by gimbal lock. Gimbal lock is the loss of one or two DOF if two or three rotation axes align with each other in a 3D system. This effect can cause unexpected outcomes and singularities. Gimbal lock occurs if a gimbal is rotated $\pm 90^{\circ}$. Figure 3.2 shows a gimbal where no rotation is locked. Figure 3.3 shows a gimbal where two rotations are locked (blue and green) and one DOF is lost. A rotation around the blue or green gimbal results in the same rotation - they are locked. [5]

Therefore quaternions were used within this thesis, which is another method, to perform rotations of vectors or matrices around a predefined axis. Quaternions are further described in Section 3.1.3.





Figure 3.2.: Gimbal Lock - not locked

Figure 3.3.: Gimbal Lock - two gimbals are locked (blue, green)

3.1.3. Quaternions

Quaternions are four-dimensional complex numbers that are used to perform rotations in 3D space. The numbers contain information about the rotation axis and rotation angle. They consist of a real component q_w and complex components $q_x \cdot i, q_y \cdot j, q_z \cdot k$. Equation 3.4 shows the Cartesian form of a quaternion. [6]

$$\mathbf{q} = q_w + q_x \cdot i + q_y \cdot j + q_z \cdot k \tag{3.4}$$

Where q_w is called the scalar part and $(q_x q_y q_z)$ is called the vector part. q_w, q_x, q_y , and q_z are real numbers and i, j, k are imaginary units that satisfy the relations of Equation 3.5. [6]

$$i^2 = j^2 = k^2 = i \cdot j \cdot k = -1 \tag{3.5}$$

The conjugate of a quaternion is defined with the identical scalar part and the negative of the vector part (Equation 3.6) [6]

$$\mathbf{q}^* = q_w - q_x \cdot i - q_y \cdot j - q_z \cdot k \tag{3.6}$$

An advantage of quaternions is the resistance to gimbal lock because rotation is carried out around the desired axis of rotation directly. A disadvantage of quaternions is their unintuitive way of angle representation. Generally, a quaternion is a useful tool in computer graphics and animation, robotics, and other fields that involve 3D orientation and rotation. [6]

3.1.3.1. Rotation Angle and Rotation Axis

Given a rotation angle θ and a normalized rotation axis $(\hat{x}, \hat{y}, \hat{z})^T$, the real values of the quaternion can be determined with Equation 3.7 to Equation 3.10. [6]

$$q_w = \cos\left(\frac{\theta}{2}\right) \tag{3.7}$$

$$q_x = \sin\left(\frac{\theta}{2}\right) \cdot \hat{x} \tag{3.8}$$

$$q_y = \sin\left(\frac{\theta}{2}\right) \cdot \hat{y} \tag{3.9}$$

$$q_z = \sin\left(\frac{\theta}{2}\right) \cdot \hat{z} \tag{3.10}$$

3.1.3.2. Multiplication

The multiplication of quaternions is used to perform a combination of two rotations in succession. It is not commutative but associative. Given two quaternions, \mathbf{a} and \mathbf{b} , the resulting quaternion \mathbf{c} can be calculated with Equation 3.11. [6]

$$\mathbf{c} = (c_w \ c_x \ c_y \ c_z) \tag{3.11}$$

With:

$$c_w = a_w \cdot b_w - a_x \cdot b_x - a_y \cdot b_y - a_z \cdot b_z$$

$$c_x = a_w \cdot b_x + a_x \cdot b_w + a_y \cdot b_z - a_z \cdot b_y$$

$$c_y = a_w \cdot b_y - a_x \cdot b_z + a_y \cdot b_w + a_z \cdot b_x$$

$$c_z = a_w \cdot b_z + a_x \cdot b_y - a_y \cdot b_x + a_z \cdot b_w$$

3.1.3.3. Vector Rotation

A 3D vector $\mathbf{v} = (v_x \ v_y \ v_z)^T$ can be rotated with a quaternion by transforming the 3D vector into a quaternion with q_w set to 0 (Equation 3.12). [6]

$$\mathbf{q}_{\mathbf{v}} = \begin{pmatrix} 0 \ v_x \ v_y \ v_z \end{pmatrix} \tag{3.12}$$

The resulting rotated vector quaternion $\mathbf{q_{rot}}$ can be determined by multiplying the vector quaternion $\mathbf{q_v}$ from the left with the normalized rotation quaternion $\hat{\mathbf{q}}$ and from the right with the normalized conjugate of the rotation quaternion $\hat{\mathbf{q}}^*$ (Equation 3.13). [6]

$$\mathbf{q_{rot}} = \mathbf{\hat{q}} \cdot \mathbf{q_v} \cdot \mathbf{\hat{q}}^* \tag{3.13}$$

The rotated vector $\mathbf{v_{rot}}$ is the vector part of the rotated vector quaternion $\mathbf{q_{rot}}$ (Equation 3.14). [6]

$$\mathbf{v_{rot}} = (q_{rot,x} \ q_{rot,y} \ q_{rot,z})^T \tag{3.14}$$

3.1.3.4. Rotation Matrix

Based on a quaternion's scalar q_w and vector part $(q_x q_y q_z)$, the corresponding rotation matrix **R** can be determined with Equation 3.15. [6]

$$\mathbf{R} = \begin{pmatrix} 2q_w^2 + 2q_x^2 - 1 & 2q_xq_y - 2q_wq_z & 2q_xq_z + 2q_wq_y \\ 2q_xq_y + 2q_wq_z & 2q_w^2 + 2q_y^2 - 1 & 2q_yq_z - 2q_wq_x \\ 2q_xq_z - 2q_wq_y & 2q_yq_z + 2q_wq_x & 2q_w^2 + 2q_z^2 - 1 \end{pmatrix}$$
(3.15)

3.1.4. Uncertainties

The uncertainty of a measurement is a quantitative measure of the doubt or degree of confidence associated with the result of the measurement. Every measurement in the real world has an uncertainty. Possible sources of uncertainties are the measurement device itself, the item being measured or the operator. [7]

There is the absolute uncertainty (u_{abs}) where an absolute value can be added or subtracted from the nominal value (a) written like 9.5 mm \pm 0.95 mm. If the absolute uncertainty is random and normally distributed among the range, it can also be referenced as standard deviation (SD) (σ). [7], [8]

$$(a \pm u_{rel}) = \left(a \pm \frac{u_{abs}}{a} \cdot 100\%\right) \tag{3.16}$$

The relative uncertainty is given in relation to the nominal value (a) and is usually given in a percentage (Equation 3.16). A written example would be $9.5 \text{ mm} \pm 10\%$ which is the same as $9.5 \text{ mm} \pm 0.95 \text{ mm}$. The relative uncertainty is used in error propagation calculations such as multiplication or division.

3.1.4.1. Error Propagation

All error propagation calculations used within this thesis are provided in this Section.

The error can propagate in the worst case if both SD add up and in the best case if they cancel each other out. Due to the uncertainty of the behaviour of the SD in each case, the SDs are added up using the Pythagorean theorem ($\sigma_a^2 + \sigma_b^2 = \sigma_c^2$). Graphically, the individual SDs can be seen as the legs of a right-angled triangle and the resulting total SD as the hypotenuse. Therefore, the total SD is in-between the largest individual SD and the sum of both SDs. Addition and subtraction of two nominal values associated with a SD is performed as in Equation 3.17 and Equation 3.18. The nominal values are added or subtracted regularly and the SDs are combined with the Pythagorean theorem in both cases. [9]

$$(a \pm \sigma_a) + (b \pm \sigma_b) = (a + b) \pm \sqrt{\sigma_a^2 + \sigma_b^2}$$
(3.17)

$$(a \pm \sigma_a) - (b \pm \sigma_b) = (a - b) \pm \sqrt{\sigma_a^2 + \sigma_b^2}$$
(3.18)

Multiplication and division are analogue to addition and subtraction with the difference that instead of the absolute SDs in the Pythagorean theorem the relative SDs are used (Equation 3.19, 3.20). [9]

$$(a \pm \sigma_a) \cdot (b \pm \sigma_b) = a \cdot b \pm \sqrt{\left(\frac{\sigma_a}{a}\right)^2 + \left(\frac{\sigma_b}{b}\right)^2}$$
(3.19)

$$\frac{a \pm \sigma_a}{b \pm \sigma_b} = \frac{a}{b} \pm \sqrt{\left(\frac{\sigma_a}{a}\right)^2 + \left(\frac{\sigma_b}{b}\right)^2} \tag{3.20}$$

Determining a power calculation with two nominal values associated with SDs is given in Equation 3.21. [9]

$$(a \pm \sigma_a)^{(b \pm \sigma_b)} = a^b \pm a^b \cdot \sqrt{\left(\frac{b}{a \cdot \sigma_a}\right)^2 + \left(ln(a) \cdot \sigma_b\right)^2}$$
(3.21)

The resulting SD of trigonometric functions is the derivative of the trigonometric function multiplied with the initial SD (Equation 3.22 - 3.24). [9]

$$\sin(a \pm \sigma_a) = \sin(a) \pm \cos(a) \cdot \sigma_a \tag{3.22}$$

$$\cos(a \pm \sigma_a) = \cos(a) \pm \sin(a) \cdot \sigma_a \tag{3.23}$$

$$\tan(a \pm \sigma_a) = \tan(a) \pm \left(\frac{1}{\cos(a)}\right)^2 \cdot \sigma_a \tag{3.24}$$

The inverses of trigonometric functions are analogue to the trigonometric functions. Again the resulting SD is the derivative of the inverse trigonometric function multiplied with the initial SD (Equation 3.25 - 3.27). [9]

$$\arcsin(a \pm \sigma_a) = \arcsin(a) \pm \frac{1}{\sqrt{1-a^2}} \cdot \sigma_a$$
 (3.25)

$$\arccos(a \pm \sigma_a) = \arccos(a) \pm \frac{1}{\sqrt{1 - a^2}} \cdot \sigma_a$$
 (3.26)

$$\arctan(a \pm \sigma_a) = \arctan(a) \pm \frac{1}{\sqrt{1+a^2}} \cdot \sigma_a$$
 (3.27)

3.2. Autocollimation

Autocollimation is the process to align a laseremitting device such as an optical surveying instrument with the normal vector of a reflecting surface. An optical surveying instrument such as a theodolite (Section 3.3.3) emits a laser beam through a semi-transparent mirror. The emitted laser beam reflects ideally at the reflecting surface such as an alignment cube. If the theodolite is perfectly orthogonal to the mirror face of the alignment cube, the laser beam is reflected directly back to the semi-transparent mirror in the theodolite (Figure 3.4). Therefore



Figure 3.4.: Autocollimation [10]

the theodolite is aligned with the normal of the alignment cube's mirror face. Slight horizontal or vertical misalignments result in not receiving the reflected laser beam back into the theodolite. This alignment process is called autocollimation. [11]

3.3. Equipment

For the measurements within this thesis certain equipment was required. This Section describes the used equipment and states their specific task within the alignment chain (top row - Figure 2.3).

3.3.1. Survey Targets

Survey targets or prisms are the genus of corner reflectors, which are designed to reflect signals back to its origin. Electronic Distance Measurement (EDM) devices such as total stations (Section 3.3.2) use this effect by emitting an infrared light beam and receiving the reflected beam. By determining the phase shift of the outgoing and incoming beam, the distance of the light travelled can be calculated. [13]



Figure 3.5.: Ball Prism [12]

Figure 3.5 shows a ball prism which is a subtype of survey prisms. These types of ball prisms with an outer diameter of 1.5 inch or 38.1 mm were used for the measurements within this thesis. They contain a glass triple prism with a diameter of 25 mm and a prism constant K = -16.9 mm or Leica = 17.5 mm [12]. There is an extra Leica prism constant given because the used Leica survey devices subtract a default constant of -34.4 mm prior to any other compensation. Therefore a constant, in this case of 17.5 mm, must be added to obtain the final prism constant K = -16.9 mm. [14]

Uncertainty

According to the manufacturer the used ball prisms have a prism constant variation of ± 0.1 mm for 90% of the tested prisms. The highest ever measured deviation was 0.3 mm. [15]

Additionally, there is an error produced if the surveying instrument is not perfectly aligned with the ball prism's Y-axis (Figure 3.6). The error has a maximum of 0.2 mm between the used -20° to 20° range (Figure 3.7). The expected error of the prism constant of ± 0.1 mm was combined with the error of the surveying instrument alignment of ± 0.2 mm using Equation 3.17. The total error for the ball prisms result in ± 0.224 mm.



Figure 3.6.: 1



Prism "zero-point" when rotating the prism around Z-axis (horizontal) in mm

Figure 3.7.: Prism constant error while rotating around Z-axis horizontally [16]

¹Ball Prism Coordinate System [16]

3.3.2. Total Station

A total station is a surveying instrument which precisely measures the distance, the horizontal and vertical angles between itself and a given target within its own local coordinate system.

This distance measurement however can only work if the target reflects the laser back to the device such as a ball prism target. The device emits an infrared light beam with a controlled phase. The shifted phase of the returning beam is determined and therefore the distance travelled by the light can be calculated. This measurement procedure is called EDM. [18]



Figure 3.8.: TDRA6000 Components [17]

The total station was placed with a

tribrach adapter on a tripod and levelled using the adjustable tripod legs and the tribrach foot screws. Within the telescope is a crosshair which can be used to align the total station with a target by looking through the eyepiece. The horizontal and vertical drive (i, k - Figure 3.8) could be used for fine adjustments while overlapping the device's crosshair with the target. After the establishment, the device could measure the horizontal and vertical and vertical and vertical and vertical and states to the target.

The measured vertical angle is the angle between the telescope and the internal vertical axis. The internal vertical axis can be referenced to the earth's gravitational vector with the provided total station's lateral and tangential tilt angles. The horizontal angle measurement is based on the internal horizontal axis which is not referenced to an absolute global vector such as the earth's gravitational vector. Therefore the horizontal angle must be treated as a relative angle.

Due to the distance measurement as well as the horizontal and vertical angle measurements from the total station to the target, the absolute 3D coordinates (X, Y, Z) of the target can be determined within the total station's local coordinate system by triangulation.

The device used for the measurements within this thesis was the Leica TDRA6000.

Uncertainty

The used TDRA6000 measured the horizontal and vertical angles as well as the distance to the target. The TDRA6000 had the same SD of $0.5 \operatorname{arcsec}$ or $1.389 \cdot 10^{-4\circ}$ for both the horizontal and the vertical angle. Inside the clean room with stable indoor conditions, the maximum permissible error (MPE) of the distance measurement EDM was $0.5 \operatorname{mm}$. Typically the current errors are within half of the MPE. [17]

3.3.3. Theodolites

A theodolite is also a surveying instrument which works similarly to a total station in the manner of measuring the horizontal and vertical angles between itself and a target. The theodolite however is not capable of directly measuring the distance between itself and the target.

The device was also placed with a tribrach adapter on a tripod with adjustable legs to align the device's vertical axis with the help of a circular level (12 - Figure 3.9) to the earth's gravitational vector. The tribrach foot screws (1 - Figure 3.9) could be used for fine adjustments. [20]





Figure 3.9.: TM5100A Components [19]

be aligned with the target using the eyepiece to be able to measure a target. Fine adjustment movements of the crosshair could be achieved with the drives for vertical and horizontal movement (10 - Figure 3.9). If the crosshair was aligned with the target, the horizontal and vertical angles can be read from the display.

Similar to the total station, the vertical angle measured was the angle between the panfocal telescope and the internal vertical axis. The internal vertical axis could be referenced to the earth's gravitational vector with the help of the lateral and tangential tilt angles provided by the spirit level. The measured horizontal angle must also be treated as a relative angle likewise the total station.
The device used for the measurements within this thesis was the Leica TM5100A. The A in the device's name stands for autocollimation which enables the device to perform the autocollimation alignment process, as described in Section 3.2, with a reflecting surface.

Uncertainty

The used TM5100A had a SD of $0.5 \operatorname{arcsec}$ or $1.389 \cdot 10^{-4\circ}$ for the measured horizontal and vertical angles as well as the device's tilt angles. [19]

3.3.4. Industrial Robot

An industrial robot is defined as an automatically controlled, reprogrammable multipurpose manipulator programmable in three or more axis [22]. The robot can be mobile or fixed in place. Often they are used for automated manufacturing processes such as car assembly.

The purpose of the 6-axis industrial robot within this thesis was the possibility to move the DUT freely in space without physical demand. Additionally, the fixed robot position increased repeatability due to the possibility of accurately saving its position and orientation in space. Further benefits are discussed in Chapter 8. [22]

The industrial robot used in this thesis was the Kuka KR500 R2830.

Figure 3.10.: Kuka KR500 R2830 [21]

Uncertainty

The used Kuka KR500 R2830 has a pose repeatability (ISO 9283) of ± 0.08 mm. [21]

3.3.5. Test Rig

The test rig (Figure 3.11) was designed and constructed to perform measurements within a real but known environment. The final to be measured ReFEx primary structure and internal HNS with the alignment mirrors were not available during the development of this thesis.

The test rig consisted of multiple 40 I-Type Nut 8 profiles with different lengths, 40x40



mounting brackets with suiting M8 screws and M8 T-Nuts. The assembled test rig had a width of 280 mm, a height of 280 mm and a depth of 700 mm. It was mounted to the industrial robot's mounting plate with the same 40x40 mounting brackets and suitable M8 screws with nuts.

Additionally, a 3D printed alignment cube holder (Figure 3.12) was designed and 3D printed to securely attach the alignment cube to the test rig. The alignment cube was clammed in a designed hollow space of the holder by the elasticity of the used polylactic acid (PLA) for 3D printing. The holder had a height of 20 mm, a width of 60 mm, a total depth of 56.97 mm and was screwed with M8 screws onto the lower front part of the test rig. A picture of the entire assembly mounted at the industrial robot can be seen in the background of Figure 5.4.



Figure 3.11.: Test Rig

Figure 3.12.: ²

Figure 3.13.: Alignment Cube [23]

3.3.6. Alignment Cube

An alignment cube (Figure 3.13) is a small cube with mirrors on each face. It is used to qualify systems where critical optical alignment is crucial. The mirror faces are almost perfectly orthogonal to each other with small angular tolerances in the range of arcseconds. The normals of the mirror faces can be determined by autocollimation as described in Section 3.2. The used alignment cube for the measurements has a height, width and depth of 16 mm. [23]

 $^{^{2}}$ 3D printed alignment cube holder with alignment cube

3.4. Sensitivity Analysis

A sensitivity analysis is a technique to evaluate how changes in input variables affect the output of a model or system. The purpose of a sensitivity analysis is to identify which input variables are most influential in determining the outcome to increase the focus on these input variables. Within this thesis two analysis approaches were used - one local and one global approach. [24]

3.4.1. Local Sensitivity Analysis

Modifying only one of the multiple input variables and observing the model's output for each individual input variable is called local sensitivity analysis. This one-factor-at-atime (OFAT) approach provides a first overview of the influence to the model's output by the individual inputs. The higher the value, the higher the contribution to the change of the model's output.

3.4.2. Global Sensitivity Analysis

The disadvantage of the OFAT local sensitivity analysis is the ignored influence of interactions between multiple input variables. The change of one input variable might amplify or dampen the influence on the output of one or multiple other input variables. This could lead to cross interactions which change the individual influence of an input variable compared to the OFAT local sensitivity analysis. One global sensitivity analysis was performed within this thesis - namely the Sobol's Method

3.4.2.1. Sobol's Method

Sobol's method is a variance-based global sensitivity analysis determining the influence of individual input variables and their interactions on the total variance of the model's output. [25]

Two input matrices (\mathbf{A}, \mathbf{B}) were created and filled with random values within predefined boundaries. The number of input variables (i) determines the columns of the matrices and the number of samples (N) the rows. The model's output for each matrix row of the first matrix (\mathbf{A}) was determined and the model's output variance $(Var(\mathbf{Y}))$ was calculated with Equation 3.28. [26], [27]

$$\operatorname{Var}(\mathbf{Y}) = E[(f(\mathbf{A}) - E[f(\mathbf{A})])^2]$$
(3.28)

i further matrices were created where the i^{th} column of the first matrix (**A**) was replaced by the i^{th} column of the second matrix (**B**). These newly generated matrices were named $\mathbf{A}_{\mathbf{B}}^{(i)}$ matrices. Using Equation 3.29, the approximated variance for each input variable influencing the model's output variance could be determined. [26]

$$E_{X_{\sim i}}(\operatorname{Var}_{X_i}(Y|\mathbf{X}_{\sim i})) \approx \frac{1}{2N} \sum_{j=1}^N \left(f(\mathbf{A})_j - f\left(\mathbf{A}_{\mathbf{B}}^{(i)}\right)_j \right)^2$$
(3.29)

With both, the variance of the model's output determined with matrix \mathbf{A} , and the estimated variance for each input variable influencing the model's output variance, the Sobol's Total Effect Indices (S_{T_i}) for each input variable could be calculated using Equation 3.30. [26]

$$S_{T_i} = \frac{E_{X_{\sim i}}(\operatorname{Var}_{X_i}(Y|\mathbf{X}_{\sim i}))}{\operatorname{Var}(Y_i)}$$
(3.30)

The Sobol's Total Effect Index determines the contribution to the model's output variance caused by the input variable itself and by interactions with any other input variable. The Total Effect Index does not attribute the source of input variability. It solely denotes the impact and magnitude of the influence to the model's output. Thus, it is inadequate for determining the source of the variance with the Sobol's method.

n

$$n = N + i \cdot N \tag{3.31}$$

$$= N(i+1)$$

The Sobol's global sensitivity analysis has a higher computational effort compared to the OFAT local sensitivity analysis. The number of solved models n for the Sobol's Total Effect Indices can be determined with Equation 3.31. The first N left of the addition results from N calculated model outputs for matrix \mathbf{A} . The second summand $i \cdot N$ is for i different $\mathbf{A}_{\mathbf{B}}^{(i)}$ matrices with N model output calculations each. The computational effort increases linearly with both equation inputs (N, i). The sample size N however can be chosen freely and therefore the computational effort limited or increased. If the sample size N is too low, the Sobol's Total Effect Indices will be inaccurate. The higher the sample size N the more accurate the Sobol's Total Effect Indices but also the higher the computational effort. [25]

3.5. Sequential Least Squares Programming

Sequential Least Squares Programming (SLSQP) is an iterative optimization method for constrained non-linear equations. It is a sub-method of sequential quadratic programming (SQP) and minimizes a function of several variables with any combination of bounds, equality and inequality constraints. The SLSQP method is integrated into the SciPy Python library as a minimization optimization tool which was used in this thesis (Chapter 8). [28], [29], [30]

4. Alignment Chain



Figure 4.1.: Room overview in CAD with coordinate systems

This Chapter describes the developed alignment chain. The purpose of this alignment chain was to determine all coordinate systems and transformations shown in Figure 4.1 and measure selected targets as well as the alignment mirrors. In Figure 2.3 the alignment chain is represented by the green, blue and orange arrows starting from the total station, theodolite and industrial robot and finishing in the coordinate system transformation block. Figure 4.1 gives an overview of the final alignment setup modelled in computer aided design (CAD). As mentioned in Section 3.3.5, ReFEx was not available which resulted in the construction of the test rig. A setup of the real measurement process with the test rig can be seen in Figure 5.2 and Figure 5.4. A custom Python tool was developed to deal with the numerous calculations within the alignment chain. A flow chart overview of the Python script can be seen in Figure 4.3.

4.1. Reference Targets

Prior to any measurement or calculation five ball reflectors were mounted on the wall as reference targets (green spheres - Figure 4.1) with magnetic ball prism mounting systems (Figure 4.2). These reference targets were never moved which made it possible to overlay multiple measurements executed at different points in time.



Figure 4.2.: Magnetic Ball Prism Wall Mounting System [31]

4.2. Python Script Flow Chart Overview

A custom Python script was developed for the alignment chain. It provides a more convenient way to operate the alignment chain due to the numerous calculations performed in the background. An overview within a flow chart of each step in the alignment chain is visualized in Figure 4.3. Figure 4.3 could be inserted in the alignment chain overview in Figure 2.3 between the equipment and measurement results.

First, two custom Python libraries were developed and suited to the specific alignment tasks. As already mentioned in Section 3.1.4, each measurement deals with an uncertainty. Therefore, an uncertainty library was developed to perform the necessary error propagation calculations. The used equations are given in Section 3.1.4. Following with a custom quaternion library which was able to handle nominal values with SDs. The quaternion library was required to rotate the different coordinate systems (CS) and vectors.

The alignment chain was split into three main columns by the used measurement equipment: Theodolites, Total Station (TS) and Industrial Robot (Figure 4.3). After each measurement, multiple calculation operations were performed to be able to align the measurements with each other. The aligned data was the input of the main setup Python script which visualizes the data in a GUI. The inputs from the total station and industrial robot were only required once as an initial input and therefore marked as fixed (red). They were required for gathering the positions of the reference target (RT), the global room coordinate system and the industrial robot base position. They will be constant once defined and nothing changed inside the clean room. The theodolite inputs however were required for each alignment process and therefore marked with variable (green), due to the always changing DUT. The robot axis angles output from the setup script will be discussed later in Chapter 8.



Figure 4.3.: Flow chart overview of Python script alignment chain

4.3. Absolute 3D Coordinates

4.3.1. Total Station Measurements

For each measurement the horizontal angle (Hz) and vertical angle (V) between the total station and the target was measured. Additionally, the distance between the total station and the target was measured. Finally, the lateral (tilt_L) and tangential tilt angles (tilt_T) of the total station were noted for further compensation calculations.

4.3.2. Point Cloud Creation

3D Coordinates

An arbitrary initial coordinate system for a measurement series was created. The measured horizontal angle (Hz), vertical angle (V) and distance (d) were used to determine the 3D coordinates of each measured target (Equation 4.1 - 4.3).

$$X = \sin(\mathrm{Hz}) \cdot (\sin(\mathrm{V}) \cdot d) \tag{4.1}$$

$$Y = \cos(\text{Hz}) \cdot (\sin(\text{V}) \cdot d) \tag{4.2}$$

$$Z = \cos(\mathbf{V}) \cdot d \tag{4.3}$$

Tilt Compensation

A rotation axis (black dashed line - Figure 4.4) for the lateral tilt (up, down tilt) was created by calculating the cross product of the target vector (red dashed line - Figure 4.4) from the origin (black dot - Figure 4.4) and the projected target vector on the X,Y-plane (grey dashed line - Figure 4.4). A quaternion was used to rotate the target vector around the calculated rotation axis by the measured total station's lateral tilt angle (green dashed line - Figure 4.4). The projected target vector on the X,Y-plane was also used as rotation axis for the tangential tilt compensation (left, right tilt). Again a quaternion was used to rotate the already lateral tilted target vector (green dashed line - Figure 4.4) around the projected target vector axis by the measured total station's tangential tilt angle (orange dashed line - Figure 4.4).

Note that the lateral and tangential tilt angles in Figure 4.4 are exaggerated. Figure 4.5 shows the same as Figure 4.4 from a different viewing angle with the black dashed lateral rotation axis at the bottom and orthogonally the local Z-axis (solid blue line - Figure 4.4).

Finally, all 3D coordinates for each target were imported in a point cloud for further calculations which is described in more detail in Section 4.3.3. A custom point cloud Python class was developed to deal with these calculations.



Figure 4.4.: Total station tilt compensation

Figure 4.5.: 1

4.3.3. Global Point Cloud Alignment

At least a total of eight targets were measured within a total station measurement series - all five reference targets (green spheres - Figure 4.1) as well as three targets on the floor.

Figure 4.8 shows the final already aligned local point cloud of a total station measurement series with the global room coordinate system. The initial local point cloud however was not aligned with the global room coordinate system (Figure 4.6). The alignment process is described within this Section.



Figure 4.6.: Initial unaligned local point cloud of a total station measurement series

¹Total station tilt compensation with a different viewing angle

Figure 4.6, 4.7 and 4.8 show the same point cloud with different viewpoints and at different steps within the alignment process. The local point cloud from Figure 4.6 was based on the local total station's coordinate system which was unknown and therefore could be treated as arbitrary. To align the local point cloud from the total station measurement series with the global room coordinate frame the following steps were performed.

First, a floor plane was determined with points 5,6 and 7 (grey plane - Figure 4.7) and the normal to this floor plane. Additionally, two lines were created. The first line used points 0 and 1 (violet dashed line - Figure 4.7) and the second line used points 2, 3 and 4 (orange dashed line - Figure 4.7).



Figure 4.7.: Initial unaligned local point cloud of a total station measurement series with created floor plane and lines

The wall corner intersection point of both created lines was determined and denoted as point 8 in Figure 4.7. Given the wall corner intersection point, the normal of the floor plane and the floor plane itself, another intersection point could be determined. This new intersection point (9 - Figure 4.7) was located in the floor's plane and the floor's normal intercepts the wall corner intersection point. By subtracting the entire local point cloud from the coordinates of the new floor-wall corner interception point (9 - Figure 4.7), the defined local point cloud's origin aligned itself with the global room coordinate system's origin (point 9 of Figure 4.7 moved with the entire point cloud to the room coordinate system's origin).

Now a common point of the local total station point cloud and the global room was known. The axes of the local point cloud however did not align with the global room coordinate axes (coordinate systems - Figure 4.7). Therefore, the entire point cloud of the local total station was rotated around the axis generated by the cross product of the floor normal and the global Z-axis by the calculated angle between both. This operation compensated the tilt of the local point cloud and aligned the local total station point cloud's Z-axis with the global room's Z-axis. The point cloud was additionally rotated around the global Z-axis by the angle calculated between the first line (violet dashed line - Figure 4.7) and the global room's X-axis. This operation aligned the local total station point cloud's X-axis and the global room's X-axis. After these performed rotations the local total station's point cloud was aligned with the global room's coordinate system which can be seen in Figure 4.8.



Figure 4.8.: Local point cloud of a total station measurement series aligned with global room coordinate system

4.4. Industrial Robot Integration

4.4.1. Axes Measurements

The remote of the industrial robot was able to display the current axis angles for each of the six industrial robot axes. These robot axis angles were required for further calculations.

4.4.2. Modelling

The Kuka KR500 R2380 robot was modelled using a Python script. The axis dimensions from Table 4.1 were taken from Figure 4.9. The figure is located within the technical data sheet of the robot [21].

| Kuka KR500 R2830 | X-Offset in mm | Y-Offset in mm | Z-Offset in mm |
|------------------|----------------|----------------|----------------|
| Axis 1 | 500 | 1045 | 0 |
| Axis 2 | 0 | 1300 | 0 |
| Axis 3 | 1025 | -55 | 0 |
| Axis 4 | 0 | 0 | 0 |
| Axis 5 | 290 | 0 | 0 |
| Axis 6 | 0 | 0 | 0 |

Table 4.1.: Axis dimensions of Kuka KR500 R2830

With the robot axis angles and the robot axis dimensions, the robot could be modelled (Figure 4.10). An arbitrary coordinate system was created and the first axis dimensions were imported and rotated according to the provided robot axis 1 (A1) angle. The second robot axis dimension was placed at the end of the first rotated axis and rotated according to the A2 angle. This process was repeated with the remaining A3-A6. A4 and A6 do not have a dimension because they are rotational axis only (Table 4.1). A unique property of the Kuka KR500 R2830 robot was the negative Y-Offset of the third axis. This resulted in an offset of 55 mm between the rotation point of A2 and A3, and the A3 chord.



Figure 4.9.: Kuka KR500 R2830 axis dimensions in mm [21]

Figure 4.10.: Modelled robot in the initial position (0, -90, 90, 0, 0, 0) and robot mounting plate

The robot base (RB) coordinate system on the left in Figure 4.10 is the local robot base coordinate system. The grey spheres in Figure 4.10 represent the axis joints and the grey dashed lines the axis dimension vectors. The black plate at the end is the robot mounting

plate with its own mounting plate coordinate system (lower right corner - Figure 4.10). The plate coordinate system was rotated to the RB coordinate system. An offset was added in positive Z-direction of the plate coordinate system to the plate center due to the offset of the mounting system of the ball prisms (black dot - Figure 4.10).

4.4.3. Base Location and Orientation

Additionally to the reference and floor targets, three targets at the robot plate were measured (11,12,13 - Figure 4.11). A plate plane could be determined with the three measured targets. The measured plate center was defined as the middle point of the left and right targets (11,13 - Figure 4.11). The measured plate center could be aligned with the modelled robot's plate center taking into account the offset of the ball prism's mounting system. With the cross product of the vector between points 11 and 13 and the vector between points 12 and 13 the normal or Z-axis for the measured plate coordinate system could be determined. Aligning both Z-axis of the measured points (Figure 4.11). The location of the local RB coordinate system defines the location and orientation of the RB in the global room coordinate system. This approach was used to determine the RB coordinate system due to the impossibility of a direct measurement because of physical obstructions (Figure 4.1).



Figure 4.11.: Robot in initial position aligned with measured point cloud

4.5. Relative 3D Coordinates

4.5.1. Theodolite Measurements

Two theodolites were required for the measurements. Analogue to the total station, the theodolites measured the horizontal angle (Hz) and vertical angle (V) between themself and each target. The theodolites cannot measure the distances like the total station. A unique required functionality of the theodolites was the possibility to autocollimate themself with a mirror or alignment cube (Section 3.2). Also, the Hz and V angles between both theodolites were required for further calculations. For each measurement, the current lateral (tilt_L) and tangential tilt (tilt_T) angles of the theodolites were recorded.

4.5.2. Relative Point Cloud





Figure 4.12.: Initial (grey), Measured (orange) and Tilted (colorful) Coordinate System of Theodolite 1 (T1CS) and 2 (T2CS). All angles are exaggerated with T1-T2-Hz,V (30°,15°), T1-T2-tilt_L,tilt_T (15°,15°), T2-T1-Hz,V (45°,-30°) and T2-T1-tilt_L,tilt_T (15°,15°)

Theodolite 1 - Initial, Measured and Tilted Coordinate System

First the location and orientation of both theodolite coordinate systems must be determined. An arbitrary local coordinate system was created which is later referred to as initial theodolite 1 coordinate system (IT1CS) (left grey coordinate system - Figure 4.12). The so-called measured theodolite 1 coordinate system (MT1CS) (left orange coordinate system - Figure 4.12) was created by rotating the IT1CS further by the measured Hz and V angles between the theodolites measured from theodolite 1 (T1-T2 Hz, V). A quaternion was used to rotate the MT1CS horizontally around the Z-axis of the IT1CS by the Hz angle starting at the X-axis towards the Y-axis of the IT1CS. Another quaternion was used to rotate the already horizontally rotated MT1CS around its new Y-axis by the V angle towards the Z-axis of the IT1CS.

The tilted theodolite 1 coordinate system (TT1CS) (left colorful coordinate system -Figure 4.12) was created by rotating the MT1CS further by the measured lateral and tangential tilt angles (tilt_L, tilt_T). A positive lateral tilt angle tilts the device forward resulting in a downward movement. A positive tangential tilt angle tilts the device to the left. A quaternion was used to rotate the TT1CS around the Y-axis of the MT1CS by the measured lateral tilt angle. The projected X-axis of the already laterally tilted TT1CS onto the X,Y-plane of the IT1CS was determined as a rotation axis for the tangential tilt. Another quaternion was used to rotate the laterally tilted TT1CS around the projected X-axis by the negative measured tangential tilt angle.

This concluded all rotations of the coordinate systems from theodolite 1 and produced the left part of Figure 4.12.

Theodolite 2 Origin

With the TT1CS, the origin of the second theodolite could be determined. The distance between the two theodolite coordinate system origins was set to 1. All following measurements were in relation to the distance between both theodolites and therefore referred to as relative distances. Following the X-axis of the TT1CS for a distance of 1, the relative origin of theodolite 2 was defined.

Theodolite 2 - Initial, Measured and Tilted Coordinate System

For the initial theodolite 2 coordinate system (IT2CS) (right grey coordinate system - Figure 4.12) the TT1CS was copied, placed at the relative theodolite 2 origin and rotated 180° around the Y-axis. The X-axis of IT2CS pointed towards theodolite 1 and the Y-axis was parallel to the Y-axis of TT1CS.

The measured theodolite 2 coordinate system (MT1CS) (right orange coordinate system -Figure 4.12) and tilted theodolite 2 coordinate system (TT2CS) (right colorful coordinate system - Figure 4.12) were determined with the same procedures as the theodolite 1 coordinate systems. Each angle however was negated due to the different point of view. For theodolite 1 an arbitrary coordinate system was created which was modified to the real theodolite 1 coordinate system. For theodolite 2 however, the ideal real coordinate system was already known with the IT2CS and the modifications to the "arbitrary" coordinate system must be determined. Therefore the operations were inverted by negating the individual measured angles.

Directional Vectors

With both so-called tilted coordinate systems, the initial relative 3D coordinates of the measured targets could be determined. A new arbitrary coordinate system was created and the direction vectors $\mathbf{v_1}$ and $\mathbf{v_2}$ were calculated with Equations 4.4 and 4.5 setting

the vector lengths to 1. Required are the horizontal and vertical angles between each theodolite and target $(T_1Hz, T_1V, T_2Hz, T_2V)$.

$$\mathbf{v_1} = \begin{pmatrix} \cos(\mathrm{T_1Hz}) & \sin(\mathrm{T_1Hz}) & \tan(\mathrm{T_1V}) \end{pmatrix}^T$$
(4.4)

$$\mathbf{v_2} = \begin{pmatrix} \cos(\mathrm{T_2Hz}) & \sin(\mathrm{T_2Hz}) & \tan(\mathrm{T_2V}) \end{pmatrix}^T$$
(4.5)

Tilt Compensation

These direction vectors were tilted by the measured lateral and tangential tilt angles from each theodolite with the same procedure as described in Section 4.3.2 for the total station. A rotation axis for the lateral tilt compensation (black dashed line - Figure 4.4) was created by the cross product of the direction vector (red dashed line - Figure 4.4) and the projection of the direction vector onto the X,Y-plane (grey dashed line - Figure 4.4). A quaternion was used to rotate the direction vector around this rotation axis by the corresponding lateral tilt angle (green dashed line - Figure 4.4). The projected direction vector (grey dashed line - Figure 4.4) was also used for another quaternion as a rotation axis to rotate the laterally rotated direction vector (green dashed line - Figure 4.4) further by the negative corresponding tangential tilt angle (orange dashed line - Figure 4.4). These tilted directional vectors were further rotated that the created arbitrary coordinate system aligned with the corresponding determined TT1CS and TT2CS.

Relative 3D Coordinates

With both modified direction vectors the relative 3D coordinates could be determined by calculating their intersection point of them. The intersection point was calculated using a second-order equation system with Equation 4.7. Required were the origins of both theodolites $(\mathbf{O_1}, \mathbf{O_2})$ and the modified direction vectors $(\mathbf{v_{1,m}}, \mathbf{v_{2,m}})$. There are three equations, one for each dimension (X, Y, Z), with only two unknowns (a, b). Therefore the system is over defined and only the first two equations (X, Y) were used to determine a and b.

$$\mathbf{O_1} + a \cdot \mathbf{v_{1,m}} = \mathbf{O_2} + b \cdot \mathbf{v_{2,m}} \tag{4.6}$$

$$a \cdot \mathbf{v_{1,m}} + b \cdot (-\mathbf{v_{2,m}}) = \mathbf{O_2} - \mathbf{O_1}$$

$$(4.7)$$

With a and b the relative 3D coordinates can be determined with Equation 4.6. The average of both sides of the equation was used as the interception point. The intersection point was imported into a local point cloud for the theodolite measurement series.

4.5.3. Alignment with Reference Targets

In each measurement series of the theodolites the reference targets at the walls were measured (green spheres - Figure 4.1). A scale factor could be calculated by determining the ratios of the distances between the reference targets measured by the total station (absolute distance) and measured by the theodolites (relative distance). This scale factor was used to scale the local point cloud created with the theodolite measurements. After the scaling, the distances between the reference targets of the theodolite point cloud match the distances of the total station point cloud but the point clouds were not aligned.



Figure 4.13.: Aligned theodolite point cloud with the point cloud of the total station. Green dashed lines are the extended modified direction vectors from theodolite 1 (left) to the intersection point. Analogue from theodolite 2 (right) the red dashed lines. Black dots and numbers are the measured points from the total station point cloud. Analogue the red dots and numbers from the theodolite point cloud.

Reference target pairs of the theodolite and total station point cloud were created. For example, point 0 from the theodolite point cloud was set to be the same as point 0 from the total station point cloud. The angles between the reference target pairs of each point cloud were calculated. The entire theodolite point cloud was rotated by these calculated angles aligning both point clouds with each other (Figure 4.13). With the scaling and alignment of both point clouds the theodolite measurements were transferred from the relative domain to absolute 3D coordinates.

5. Measurements

5.1. Requirements

The described alignment chain in Chapter 4 required certain input values to generate the desired outputs. The requirements for each measurement device are mentioned below.

• Surveying Targets

Prism Constant (K)Offsets (Wall, Floor, Robot)

• Total Station

Horizontal Angle (Hz) to each Target Vertical Angle (V) to each Target Distance (d) to each Target Lateral Tilt (tilt_L) for each Target Tangential Tilt (tilt_T) for each Target

• Industrial Robot

Axis Dimensions (A1-A6) Axis Angles (A1-A6)

• Theodolite 1, Theodolite 2

Horizontal Angle (Hz) to each other and each Target Vertical Angle (V) to each other and each Target Lateral Tilt (tilt_L) for each other and each Target Tangential Tilt (tilt_T) for each other and each Target

5.2. Measurement Process

5.2.1. 1.5" Ball Prisms

As surveying targets the 1.5 inch ball prisms (Figure 3.5) described in Section 3.3.1 were used. They have a prism constant K = -16.9 mm or Leica = 17.5 mm and a combined SD of $\pm 0.224 \text{ mm}$. [12], [14]

Five reference targets were mounted at the north and east wall (Figure 4.1). Four 40 I-Type Nut 8 profiles were screwed with mounting brackets to the walls. Each 40 I-Type Nut 8 profile holds one reference ball prism target except one which holds two. The magnetic ball prism wall mounting systems (Figure 4.2) with an M8 thread were screwed into M8 T-Nuts located in the 40 I-Type Nut 8 profiles (Figure 5.2).

The distance between the mounting face of the 40 I-Type Nut 8 profiles and the wall was measured using digital calipers. An average offset of 42.075 mm at the north wall (left wall - Figure 4.1) and 41.515 mm at the east wall (right wall - Figure 4.1) was measured. Additionally, the magnetic ball prism wall mounting system had an offset of 50 mm to the ball prism center [31]. The total offsets between the ball prism center and the wall results in 92.075 mm for the north wall and 91.515 mm for the east wall.



Figure 5.1.: Magnetic Ball Prism Floor/Robot Mounting System [31]

Three ball prism targets were placed on the floor as well as on the robot mounting plate. They were secured with a different mounting socket. The offset from the ball prism center to the bottom of the blue magnetic part in Figure 5.1 was 30.8 mm. The silver screwing part below had an offset of 5.49 mm. The total offset between the magnetic ball prism mounting system and the ball prism center, used on the floor and the robot was 36.29 mm. These offsets were considered within the custom Python script. [31]

5.2.2. TDRA6000

The TDRA6000 total station was used for all absolute 3D coordinate measurements to initially determine the global room coordinate system as well as the location of the industrial robot base (Section 4.3). The total station was secured on a tripod with adjustable legs and a tribrach adapter. The first goal was to level the total station and therefore align it with the earth's gravitational vector. The rough levelling of the total station can be performed with the adjustable legs of the tripod. Fine levelling was performed with the tribrach foot screws. Due to slight misalignments of the center of gravity (COG) of the coaxial optics (g - Figure 3.8) and the internal device's horizontal and vertical axis, the device tilted slightly when rotating the coaxial optics or the device itself. The unavoidable tilt was compensated as described in Section 4.3.2 with the measured lateral and tangential tilt angles for each individual target.

Further adjustments and settings performed prior to any measurement are described in more detail in the appendix (Section B.1).



Figure 5.2.: TDRA6000 Measurement Series Overview

After adjusting and setting up the total station, the measurement process could begin. An overview picture of the TDRA6000 measuring the ball prism targets mounted on the industrial robot can be seen in Figure 5.2. An individual measurement job was created for each measurement series to save the measured values orderly on the internal storage. The values could be additionally accessed later. The coaxial optics (g - Figure 3.8) of the TDRA6000 was used to home on a ball prism target. The crosshair within the coaxial

5. Measurements

optics was aligned with the center of the ball prism target using the horizontal and vertical drives (i,k - Figure 3.8). A picture of the aligned TDRA6000 with a ball prism target can be seen in Figure 5.3. The black crosshair within Figure 5.3 was amplified using an image modification software because the crosshair within the coaxial optics is hard to see due to its transparency. The original picture is within the appendix (Figure D.10).



Figure 5.3.: TDRA6000 aligned with a ball prism target placed on the Floor

| | Hz in $^\circ$ | V in $^\circ$ | distance d in mm | tilt_L in $^\circ$ | tilt_T in $^\circ$ |
|----------------|----------------|---------------|--------------------|--------------------|--------------------|
| North Left RT | 321.0878 | 80.0004 | 7942.4 | -0.0006 | -0.0002 |
| North Right RT | 330.3198 | 80.4175 | 8292.6 | -0.0007 | 0.0001 |

7681.8

5732.1

5132.4

-0.0007

-0.0006

-0.0002

0.0004

0.0009

0.0016

79.6704

76.0709

74.4045

Table 5.1.: Example measurements of a total station measurement series (FG08)

| Table 5.1 shows an example of the measured reference targets of a total station mea- |
|--|
| surement series. The full table of this measurement series can be found in the appendix |
| (Table C.8). For each target the horizontal (Hz) and vertical (V) angles as well as the |
| distance were measured. Additionally, the lateral $(tilt_L)$ and tangential $(tilt_T)$ tilt angles |
| were noted. The five reference targets (RT), three floor targets and three robot base (RB) |
| targets were measured. Three targets mounted at the robot plate (RP) were measured |
| at three different robot locations each resulting in nine further measurements. |

East Left RT

East Center RT

East Right RT

350.5834

11.4121

29.7541

All measured values were inputs of the uncertainty Python script to add the corresponding SDs. The measured angles had a SD of $\pm 0.5 \operatorname{arcsec}$ or $\pm 1.389 \cdot 10^{-4\circ}$. The measured distances had a combined SD of $\pm 0.35 \operatorname{mm}$. Adding $\pm 0.25 \operatorname{mm}$ from the distance measurement of the total station itself (Section 3.3.2), $\pm 0.224 \operatorname{mm}$ from the ball prism constant (Section 3.3.1) and $\pm 0.1 \operatorname{mm}$ from the EDM averaging process (Section B.1) using Equation 3.17 with three quadratic terms under the square root.

5.2.3. Kuka KR500 R2830

The industrial robot was responsible for the free movement of the test rig using the robot's six DOF. After placing the test rig at the desired location and orientation, the robot axis angles could be accessed via the robot remote. The values can be found within the Settings \rightarrow Display \rightarrow Current Position.

5.2.4. TM5100A

The main unique task of the two TM5100A theodolites was the autocollimation process with the mirror faces of the alignment cube to obtain the internal IMU axis. Additionally, they were used to measure the reference targets to align the created theodolite point cloud with the total station point cloud.

Both theodolites were placed and secured using a tribrach adapter on tripods with adjustable legs like the total station (Figure 5.4). The theodolites must also be levelled using the adjustable legs of the tripod and the tribrach foot screws. Due to slight misalignments of the COG of the panfocal telescope (7 - Figure 3.9), the theodolites tilt when rotating the panfocal telescope or the devices themself. The resulting, unavoidable, tilt angle during operations was compensated as described in Section 4.5.2 with the recorded lateral and tangential tilt angles.

Adjustments and settings of each TM5000A device performed prior to any measurement series are described in more detail in the appendix (Section B.2).

After adjusting the instrument errors for both theodolites the autocollimation process with the mirror faces of the alignment cube could start. The test rig with the test alignment cube (Figure 5.4) was mounted to the industrial robot mounting plate. A plug lamp (Figure 5.4) must be attached to the panfocal telescope which creates the laser beam required for the autocollimation. The plug lamp must be switched on by enabling "Diode Laser (DL)/Autocollimation Lamp (AC)" within the theodolite's display settings [19].

5. Measurements



Figure 5.4.: TM5100A Measurement Series Overview

A laser beam was pointed in the same direction as the panfocal telescope, ideally hitting the alignment cube which reflected it back into the theodolite. Horizontal and vertical misalignments of the theodolite with the normal of the alignment cube's face result in not capturing the reflected laser beam.

Vertical misalignments were reduced by aligning the robot mounting plate's Y- and Z-axis parallel to the earth's gravitational vector using a spirit level. The theodolites were raised to the same height as the alignment cube using the tripods. From now on the robot plate was only moved translationally without rotation except the rotation around the robot mounting plate's X-axis (Figure 4.10). This approach reduced the vertical misalignment of the reflected laser beam.

For the first theodolite, the alignment cube was moved and yawed with the help of the industrial robot until the emitted laser beam was reflected back and captured by the theodolite. The focus ring of the theodolite must be rotated all the way to infinity which results in the appearance of another green laser crosshair. The green crosshair was emitted from the theodolite and reflected by the mirror face of the alignment cube (Figure 5.5). The original unedited picture of Figure 5.5 can be found in the appendix (Figure D.11).



Figure 5.5.: Autocollimated theodolite with a mirror face of the alignment cube. The black crosshair is within the panfocal telescope. The black crosshair is the theodolite's internal crosshair. The bright green crosshair is the reflected laser beam emitted from the theodolite and reflected at the mirror face of the alignment cube. The rest of the image is blurry due to setting the focus ring to infinity.

If the first theodolite was autocollimated with the alignment cube the autocollimation process with the second theodolite could begin. The industrial robot however cannot be moved or rotated anymore because otherwise, the autocollimation of the first theodolite would break. Therefore the entire second theodolite including the tripod must be shifted and rotated back and forth until the second theodolite also received the reflected laser beam emitted by itself. Manually finding the correct location of the second theodolite for autocollimation was a tedious and time-consuming task.

If the second theodolite was also autocollimated to the alignment cube, the robot axis angles were noted as well as the locations of the theodolites marked on the floor. This proved to be helpful which will be discussed later in Chapter 8.



Figure 5.6.: Internal target or crosshair within the panfocal telescope of a TM5100A [32]

| Hz in $^\circ$ | V in $^\circ$ | tilt _L in $^{\circ}$ | tilt _T in $^{\circ}$ |
|----------------|---|--|--|
| 359.99991 | -0.00700 | 0.001 | 0.001 |
| 359.99999 | 0.00583 | 0 | 0.001 |
| | | | |
| Hz in $^\circ$ | V in $^\circ$ | tilt _L in $^{\circ}$ | tilt _T in $^{\circ}$ |
| 277.93567 | 11.05991 | -0.001 | 0.001 |
| 287.67304 | 10.36065 | -0.001 | 0.001 |
| 309.60737 | 10.68535 | 0 | 0.001 |
| 333.25240 | 13.53721 | 0 | 0.001 |
| 351.40490 | 14.21779 | 0 | 0 |
| 310.93037 | -0.67818 | 0 | -0.001 |
| | | | |
| Hz in $^\circ$ | V in $^\circ$ | tilt _L in $^{\circ}$ | tilt _T in $^{\circ}$ |
| 68.00643 | 10.37368 | 0.001 | 0.001 |
| 78.20953 | 10.64142 | 0.001 | 0.001 |
| 98.69248 | 13.61532 | 0.001 | 0.001 |
| 117.17454 | 25.46229 | 0.001 | 0 |
| 152.89423 | 37.69892 | 0.001 | 0 |
| 40.92246 | -0.56583 | 0 | 0.001 |
| | Hz in $^{\circ}$ 359.99999 Hz in $^{\circ}$ 277.93567 287.67304 309.60737 333.25240 351.40490 310.93037 Hz in $^{\circ}$ 68.00643 78.20953 98.69248 117.17454 152.89423 40.92246 | Hz in °V in °359.99991-0.00700359.999990.00583Hz in °V in °277.9356711.05991287.6730410.36065309.6073710.68535333.2524013.53721351.4049014.21779310.93037-0.67818Hz in °V in °68.0064310.3736878.2095310.6414298.6924813.61532117.1745425.46229152.8942337.6989240.92246-0.56583 | Hz in °V in °tilt_ in ° 359.99991 -0.007000.001 359.99999 0.005830Hz in °V in °tilt_ in °277.9356711.05991-0.001287.6730410.36065-0.001309.6073710.685350333.2524013.537210351.4049014.217790310.93037-0.678180Hz in °V in °tilt_ in °68.0064310.373680.00178.2095310.641420.00198.6924813.615320.001117.1745425.462290.001152.8942337.698920.00140.92246-0.565830 |

Table 5.2.: Example measurements of a theodolite measurement series (03)

The theodolites were rotated to each other to measure their relative positions to each other. An internal target or crosshair (Figure 5.6) could be deployed by rotating a black screw at the end of the panfocal telescope. Both theodolites were homed on the internal crosshair of the other theodolite. The horizontal angles of both theodolites were set to 0° for more convenient calculations. The measured horizontal and vertical angles between the theodolites are referred later as T1-T2 Hz, V and T2-T1 Hz, V. Additionally the lateral (tilt_L) and tangential (tilt_T) tilt angles were recorded. Table 5.2 shows an example

of a theodolite measurement series measuring the five reference targets (two on the north wall and three on the east wall) (Figure 4.1) as well as the autocollimated alignment cube. The horizontal angles from the first theodolite must be inverted with 360 - Hz because the angles were measured from the other direction.

All measured angles were inputs to the uncertainty Python script and the SD of ± 0.5 arcsec or $\pm 1.389 \cdot 10^{-4\circ}$ was added.

6. Measurement Results

The alignment chain methodology described in Chapter 4 was used with the measured values described in Chapter 5. In Figure 2.3 the lower block represents the measurement results. Table C.8 shows the measured values from a total station measurement series to determine the reference targets as well as the industrial robot's base in the room coordinate system. Table 5.2 shows the measured values from a theodolite measurement series to obtain the normals of the alignment cube and align the theodolite point cloud with the total station point cloud. The measured industrial robot axis during the theodolite measurement series can be seen in Table 6.1. The test rig described in Section 3.3.5 with the alignment cube in the bracket, was mounted at the robot mounting plate for this measurement series.

Table 6.1.: Example measurements of industrial robot angles

| Robot Axis | Angle in $^{\circ}$ |
|------------|---------------------|
| Axis 1 | -14.67 |
| Axis 2 | -44.48 |
| Axis 3 | 99.1 |
| Axis 4 | 31.26 |
| Axis 5 | -58.86 |
| Axis 6 | -17.66 |

All values were imported in the custom Python script. The script visualized the imported data in a GUI and provided further functionalities to obtain important insights from the data. Figure 6.1 shows the different coordinate systems (Room, RB, T1CS, T2CS) and in black the measured total station point cloud. The green dashed lines represent the measured angles from the first (left) theodolite and the red dashed lines from the second (right) theodolite. The industrial robot is represented as the grey dashed lines with the black quadratic mounting plate at the end. A picture of the setup in the clean room can be seen in Figure 5.4.



Figure 6.1.: Final visualized imported data

6.1. Alignment Cube Distance

With the determined robot base coordinate system and the modelled industrial robot, the plate center of the robot's mounting plate was known. The distance between the robot mounting plate center and the alignment cube could be determined with the measured values. The distance was expected to be in the ranges shown in Table 6.2. The expected ranges were determined using calipers and folding rule measurements from the test rig and the mounting bracket. The ranges were large because the exact measured position of the alignment cube was not of interest. The focus was on the measurement of the alignment cube mirror face's normal. The measured normal could be determined at any point of the 16 mm alignment cube which results in these high expected ranges. The distances were given in the robot mounting plate coordinate system (Figure 4.10).

Table 6.2.: Distance ranges between the robot mounting plate and the alignment cube

| Axis | min. | Value in mm | max. | Value in mm |
|------|------|-------------|------|-------------|
| Х | | 82.000 | | 98.000 |
| Υ | | -11.314 | | 11.314 |
| Ζ | | 705.660 | | 728.287 |

The measured distance between the robot mounting plate and the alignment cube can be seen in Table 6.3. All measured values were within the expected range.

Table 6.3.: Measured distance between the robot mounting plate and the alignment cube

| X in mm | Y in mm | ${ m Z~in~mm}$ |
|---------|---------|----------------|
| 87.06 | 3.335 | 714.9 |

6.2. Alignment Cube Angle

With the absolute 3D coordinates of the alignment cube and the origin of both theodolites, the angle between the alignment cube mirror face's normals $(\mathbf{n_1}, \mathbf{n_2})$ could be determined (Equation 6.1).

$$\theta = \arccos\left(\frac{\mathbf{n_1} \cdot \mathbf{n_2}}{|\mathbf{n_1}| \cdot |\mathbf{n_2}|}\right) \tag{6.1}$$

Table 6.4.: Measured absolute 3D points

| Point | X in mm | Y in mm | Z in mm |
|----------------|----------------------|----------------------|----------------------|
| Theodolite 1 | 5576.063 ± 0.249 | 6832.919 ± 0.312 | 1629.964 ± 0.588 |
| Theodolite 2 | 1830.850 ± 0.237 | 7422.843 ± 0.250 | 1629.260 ± 0.630 |
| Alignment Cube | 3676.862 ± 0.185 | 5232.596 ± 0.251 | 1601.465 ± 0.842 |

The measured absolute 3D coordinates from theodolite 1, 2 and the alignment cube can be seen in Table 6.4. Vector $\mathbf{n_1}$ was defined as starting at the origin of theodolite 1 to the alignment cube. Vector $\mathbf{n_2}$ was defined as starting at the origin of theodolite 2 to the alignment cube. With these values and Equation 6.1 the angle between the alignment cube's mirror faces was calculated. The resulting angle was 90.000 429 7° ± 0.000 384 7°. That was a difference from the perfect 90° of $4.297 \cdot 10^{-4\circ} \pm 3.847 \cdot 10^{-4\circ}$ or 1.547 arcsec ± 1.385 arcsec.

7. Sensitivity Analysis

Multiple sensitivity analyses were performed with the concepts described in Section 3.4. The results of the sensitivity analyses are discussed in this Chapter.

All sensitivity analyses had the same single variable output being the angle between the autocollimated alignment cube's mirror faces. A total of 16 input variables were analysed within the sensitivity analyses. T1-T2 Hz, V were the horizontal and vertical angles measured from the first (left) theodolite to the second (right) theodolite. The other way around were the T2-T1 Hz, V measurements. Additionally the lateral and tangential tilt angles for each were considered. T1 Hz, V were the measured angles to one autocollimated alignment cube's mirror face from the first (left) theodolite. T2 Hz, V were the measured angles to another autocollimated alignment cube's mirror face from the second (right) theodolite. Again the measured lateral and tilt angles for every measurement were considered.

The measured values from Table 5.2 were used as the default values. All values were angle measurements with the same standard deviation of $0.5 \operatorname{arcsec}$ or $1.389 \cdot 10^{-4\circ}$. The standard deviation was used for the modification range during the sensitivity analyses. The same percentage of change for all angle values was decided as unsuitable due to the different absolute measured angles. If for example two angles of 1° and 100° were measured with a same applied sensitivity analysis percentage range of 1%, the ranges would differ significantly. This unproportional behaviour for different measured angles was eliminated by considering the same standard deviation range for all angles.

7.1. OFAT - Local Sensitivity Analysis

Starting with the local sensitivity analysis, analysing the impact of only one input variable on the output at a time (Section 3.4.1). Figure 7.1 shows the output change in percentage changing independently each input variable's default nominal value by the standard deviation between -100% and 100%. The maximum absolute output change is $1.543 \cdot 10^{-4}\%$. This maximum absolute output change was created by the measured horizontal angles except for T1-T2 Hz (Figure 7.3). T1-T2 Hz had almost no contribution to the output's change of $4.010 \cdot 10^{-11}\%$ due to the calculation approach. T1-T2 Hz and T1-T2 V only influence the origin location of the second (right) theodolite and therefore had no direct contribution to the output. The fourth highest absolute output's change

comes from T1 tilt_L with $3.179 \cdot 10^{-6}$ % which was two orders of magnitude smaller than the first three. Therefore, especially the three horizontal angles should be measured precisely.



Figure 7.1.: OFAT local sensitivity analysis adding -100% to 100% of the standard deviation to the measured default nominal values

All individual angle measurements seemed to have a linear contribution to the output's change. This was not the case if the added standard deviation was increased significantly. Figure 7.2 shows the same plot as Figure 7.1 just with a significantly increased added standard deviation of $1.5 \cdot 10^8$ %. Figure 7.2 shows that all input variables have a non-linear behaviour. The visualized pattern was periodically extendable in positive and negative Y-direction after about $1.3 \cdot 10^8$ %. The spiky jumps within Figure 7.2 were caused by the calculation process using absolute values of angles.

Figure 7.2 was however just used to prove the non-linear behaviour of the input variables on the output's change. Such high standard deviation errors were unrealistic. For the zoomed more realistic Figure 7.1, a linear behaviour of the input variables could be assumed.



Figure 7.2.: OFAT local sensitivity analysis adding $-1.5 \cdot 10^8 \%$ to $1.5 \cdot 10^8 \%$ of the standard deviation to the measured default nominal values

Figure 7.3 shows three main contributors to the output's change - namely T2-T1 Hz, T1 Hz and T2 Hz. T2-T1 Hz was proportional to the output's change due to an increase of T2-T1 Hz resulted in an increase of the output. T1 Hz and T2 Hz were anti-proportional to the output's change due to an increase in them resulted in a decrease of the output.

The influence of the vertical angles was much lower compared to the influence of the horizontal angles. One possible reason for this effect could be the orientation of the alignment cube measurement. The alignment cube was measured horizontally instead of vertically. A vertical measurement could theoretically be performed if one theodolite aims at the top or bottom alignment cube's mirror face. A theodolite below or above the alignment cube however was unrealistic.

Another possible reason for this effect could be the calculation method. The intersection of the two modified directional vectors originating in each local theodolite coordinate frame was determined using a 2D equation system as described in Section 4.5.2. The 2D equation system was overdetermined due to two unknowns with three equations. The Xand Y-axis equations proved to produce the most accurate results. Therefore the Z-axis equation was not considered which could reduce the influence of the vertical angle on the output.



Figure 7.3.: OFAT bar sensitivity analysis adding 100% of the standard deviation to the measured default nominal values

7.2. Sobol's Method - Global Sensitivity Analysis

The global sensitivity analysis took also into account the interactions between multiple input variables. Sobol's method was used to analyse the developed model as described in Section 3.4.2.1.

For the Sobol's method the sample number N must be defined. Therefore a study was conducted. The focus was on the three measured horizontal angles (T2-T1 Hz, T1 Hz, T2 Hz) as they were the main contributors to the output's change discovered in the
OFAT local sensitivity analysis (Section 7.1).

The study aimed to find a responsible sample size N for this problem. The sample size was doubled with each iteration starting with 25. Based on the current sample size, the Sobol's Total Effect Indices for the input variables were determined. The Sobol's method is based on random input values within a predefined boundary. Therefore it was assumed to generate more accurate Total Effect Indices with increasing sample size.

Figure 7.4 shows the Sobol's Total Effect Indices over the inverse of the sample size (1/N). It can be seen that a higher sample size or a lower inverse of the sample size results in a convergence. Based on the Figure 7.4, a sample size N of at least 1000 $(1 \cdot 10^{-3} \text{ inverse})$ should be used for this specific problem. Figure 7.5 shows the Sobol's Total Effect Indices for all 16 input variables with 10 000 samples.



Figure 7.4.: Sobol's global sensitivity analysis sample size study ranging from 25 to 10000



Figure 7.5.: Sobol's global sensitivity analysis with a sample size of 10 000 and 16 inputs

The Sobol's Total Effect Indices and therefore the contribution to the model's output variance for all 16 input variables except the three main contributors were almost 0 (Figure 7.5). T1 Hz seemed to have the highest contribution to the model's output variance followed by T2 Hz and T2-T1 Hz. An order change however might occur with another calculation using 10 000 samples or even more samples due to the randomly selected input values. This effect can be seen with lower samples in Figure 7.4. No further sample size increases were checked due to the high computational effort.

According to Equation 3.31 with 16 input variables (i) and 10000 samples (N), a total of 170000 models must be solved. Each model consists of the entire alignment chain process described in Chapter 4, 5 and 6. Each solved model took an average of 25.5 ms using the custom developed Python script. This resulted in a required time of about 72 min for all calculations with 10000 samples.



Figure 7.6.: Comparison of normalized OFAT and Sobol's Total Effect Indices

Figure 7.6 shows both sensitivity analyses in one bar chart. The absolute values of the normalized OFAT local sensitivity analysis and the normalized Sobol's Total Effect Indices with 10 000 samples were used and plotted in Figure 7.6. The main three contributors from the Sobol's method seemed to have a higher influence on the output compared to the OFAT local sensitivity analysis. This could be explained due to input variable interactions which are not considered in the OFAT sensitivity analysis.

Overall the difference between the local and global sensitivity analysis was minor for this specific model. This statement however cannot be generalized and applied to other models.

8. Robot-assisted Alignment

8.1. Robot Axis Angles Optimization

The alignment chain described in Chapter 4 used the industrial robot for the first (left) theodolite autocollimation process with a mirror face of the alignment cube mounted at the test rig. The second (right) theodolite was already autocollimated manually due to the interruption of the first autocollimation if the industrial robot would have been moved. The usage of an industrial robot for the initial alignment chain setup had therefore almost no beneficial impact because the first autocollimation process could also be performed on a table trolley. There was a small benefit if the same setup would be measured again at a different point in time and the locations of the theodolites were marked on the floor as well as the robot axis angles saved. With these conditions, the manual autocollimation process could be skipped due to the already known equipment locations. With an addition to the custom Python script, the industrial robot could be even more beneficial.

With the known room coordinate system, the origin and orientation of the robot base coordinate system as well as the robot axis angles, the robot can be modelled and the location and orientation of the robot mounting plate calculated (Section 4.4.2). This process could be reversed if the final location and rotation of the robot mounting plate were provided and an optimization algorithm used to search the suitable robot axis angles fulfilling this objective.

The SLSQP (Section 3.5) algorithm from the SciPy Python library was used. The defined objective function, which was minimized, determined the distance between the current mounting plate center with an additional predefined offset and the target coordinate. The offset was the distance between the robot mounting plate center and the alignment cube attached at the test rig given in the robot mounting plate's coordinate frame. A constrain function was implemented to align the mounting plate's X- and Y-axis parallel to the clean room's floor which could be modified if needed. This approach determined the robot axis angles for every possible target coordinate. Required was the offset, the target coordinate and that the target coordinate was within the working envelope of the industrial robot.

After the first initial theodolite measurement series, the locations of the theodolites

were marked at the clean room's floor. The initial absolute 3D coordinates of the theodolites and the alignment cube in the room coordinate frame were known (Chapter 6). With another location of the alignment cube at the test rig or another rig, the alignment cube's offset to the robot mounting plate will change. Knowing the new alignment cube's offset and using the described optimization, new robot axis angles were determined which place the new alignment cube to the same old absolute target coordinate in the room coordinate frame. Therefore the marked theodolite locations on the floor could be used to immediately autocollimate them with the new DUT's alignment cube. Figure 8.1 shows the initial robot position with the initial theodolite measurement series (Table 5.2). The measured alignment cube's offset to the robot mounting plate coordinate frame. Figure 8.2 shows the determined robot axis angles based on the new example offset of (0 mm 0 mm 0 mm)^T. All possible offsets fulfilling the mentioned requirements could be inserted. Usually, the offset of the alignment cube to the robot mounting plate can be extracted from the digital object in the used CAD tool.





Figure 8.1.: Industrial robot in initial theodolite measurement series position Offset: $(87.06\ 3.34\ 714.90)^T$ Robot Axis Angles: $(-14.67\ -44.48\ 99.10$ $31.26\ -58.86\ -17.66)$ Figure 8.2.: Determined industrial robot axis angles Offset: $(0 \ 0 \ 0)^T$ Robot Axis Angles: $(-22.41 \ -19.96 \ 45.14$ $27.28 \ -28.01 \ -24.41)$

This approach reduced the time needed for the autocollimation process drastically (Table 8.1) and increased convenience compared to a non robot-assisted alignment process. The time of the autocollimation process was reduced from 45 min for the initial autocollimation process to 3 min for the optimized autocollimation process predetermining the robot axis angles. This resulted in a reduction of 41.6 % of the total time measuring only one target. Measuring five targets still had a time reduction of 37.2 %. The more targets were measured, the lower the saved time in percentage due to an absolute reduction of -42 min.

| | Initial Setup | Optimization Setup |
|---------------------------|-----------------------|--------------------|
| Setup Theodolites | $2 \cdot 7.5 \min$ | $2 \cdot 7.5 \min$ |
| Adjust Theodolites | $2 \cdot 10 \min$ | $2 \cdot 10 \min$ |
| Autocollimation | $45 \min$ | $3 \min$ |
| Theodolites to each other | $3 \min$ | $3 \min$ |
| Measure Targets | $3 \min/target$ | $3 \min/target$ |
| Clean up | $2\cdot7.5~{\rm min}$ | $2 \cdot 7.5 \min$ |
| | | |
| Minimum Total Time | $101 \min$ | $59 \min$ |
| 5 Targets Total Time | $113 \min$ | $71 \min$ |

Table 8.1.: Estimated required time for a theodolite measurement series

8.2. Augmented Reality Initial Theodolite Locations



Figure 8.3.: Virtual ReFEx positioned on the robot mounting plate

The physical primary ReFEx structure as well as the HNS with the alignment mirrors were not available during the development of this paper. Therefore another method was used to determine the theodolite positions for the autocollimation process. The virtual ReFEx body with the alignment mirrors was exported from the CAD software and imported into a Microsoft HoloLens 2 (MSHL2). The ReFEx model was placed at the robot mounting plate using a quick response (QR) code which was read by the MSHL2 (Figure 8.3). The industrial robot was moved until the virtual alignment mirrors were in the desired location. A three-axis laser level tool was placed exactly at the virtual alignment mirrors beaming lasers orthogonally in all directions. The laser beams were orthogonally aligned with the virtual alignment mirrors. Following the laser beams on the floor, two points with enough operational space were selected and marked with labelled tape. This method allowed performing the positing of the theodolites and the finding of suitable robot axis angles without the actual physical object. Therefore the predetermined theodolites locations could be used if the physical ReFEx structure is available.

9. Summary

This thesis describes the developed alignment chain with the implementation of an industrial robot. Starting from the description of the required equipment and equipment's settings (Chapter 3, Chapter B) over the developed alignment chain (Chapter 4) towards the measurements (Chapter 5) and results (Chapter 6). Further improvements and optimizations can be found in Chapter 8.

An overview of the developed alignment chain can be seen in Figure 2.3. Figure 4.1 shows the CAD setup of the ReFEx alignment process. A real picture of the alignment with the test rig can be seen in Figure 5.4 and Figure 5.2. A custom Python tool was developed including a GUI and visual representations of the alignment process (Figure 6.1). A flow chart overview of the script's tasks can be seen in Figure 4.3. The Python tool took the measurements from the different devices as inputs and handled all calculations, rotations and alignments in the background.

Additionally, the industrial robot was modelled and visualized to be able to generalize the alignment chain by moving the alignment cube or mirrors accurately to the desired location. Therefore a feature was implemented to be able to determine the required industrial robot axis angles (Section 8.1). This generalization feature reduced the required time for all measurements of the alignment chain with a new DUT by 42 min or up to 41.6 % compared to a non robot-assisted alignment process.

The alignment process was tested and verified with a designed test rig and a test alignment cube. The alignment cube had a measured angle between two autocollimated mirror faces of 90.000 429 7° \pm 0.000 384 7°. That was a difference from the perfect 90° of $4.297 \cdot 10^{-4\circ} \pm 3.847 \cdot 10^{-4\circ}$ or 1.547 arcsec ± 1.385 arcsec (Chapter 6).

A sensitivity analysis was conducted of all 16 input variables to the output which was the angle between the two autocollimated alignment cube's mirror faces (Chapter 7). The horizontal angle measured from the second to the first theodolite and the measured horizontal angles towards the alignment cube from both theodolites had the most impact on the output.

Suggestions for further tests and improvements are discussed in the Outlook (Chapter 10).

10. Outlook

As mentioned in Section 3.3.5 the primary ReFEx structure and components were not available during the development of this thesis. Therefore a test rig was designed and constructed to imitate the expected ReFEx alignment measurements with the main focus on the IMU coordinate axes determination. If ReFEx will be available, the entire developed alignment chain could be tested with the actual DUT. Also, the initial augmented reality (AR) based theodolite location determination using the MSHL2s (Section 8.2) could be verified.

Measurements of real control devices including canards and thrusters were not performed within this thesis. Also, a physical determination of a sun sensor's position and orientation was not conducted. Both processes could be tested in the future with the available ReFEx structure and components.

The generalization of the alignment cube placement using the industrial robot described in Section 8.1 was only tested theoretically due to the lack of different DUT with different alignment cube positions. This alignment chain optimization could be tested and verified with a greater amount of different DUTs and differences in relative distances between the industrial robot's mounting plate and alignment cube resulting in different offsets. Additionally, a different orientation of the alignment cube to a horizontal measurement could be tested for example a 45° tilted alignment cube.

To further reduce the uncertainties and therefore also the error propagation, certified survey targets could be used. The main contributor to the distance measurement of the total station TDRA6000 was the uncertainty of the ball prisms' prism constant (Section 5.2.2). This could result in more accurate absolute 3D coordinate measurement resulting in less uncertainties and better alignments of different point clouds.

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B. Equipment Settings

B.1. TDRA6000 Settings

Prior to any measurement, the device was adjusted to the current environmental conditions. All measurements were performed in an environmentally controlled clean room with stable conditions compared to the outside. Still, temperature, pressure and humidity changes occurred between the individual measurement series. The instrument errors in Table B.1 were adjusted prior to each measurement series. [17]

Table B.1.: Adjustable TDRA6000 instrument errors [17]

| Symbol | Description |
|-------------------------|---|
| l,t | Compensator longitudinal and transversal index errors |
| i | Vertical index error, related to the standing axis |
| с | Hz collimation error, also called line of sight error |
| a | Tilting axis error |
| ATR | ATR zero point error for Hz and V - option |

Certain key settings must be set within the total station to obtain accurate measurements. The Leica prism constant of 17.5 mm must be set within the device for the used 1.5 inch ball prisms (Section 5.2.1) to measure the distance between the total station and target accurately. An accurate distance measurement is of importance as all further calculations are based on it (Equation 4.1 - 4.3). The automated target recognition (ATR) was deactivated to manually home on the center of the ball prism target. This proved to be more accurate than using the ATR function. The EDM mode was set to average with 10 measurements. The device took 10 EDM distance measurements to the ball prism target and averaged them. The majority of the SDs due to the averaging process was less than ± 0.05 mm and therefore displayed as ± 0 mm on the total station due to an insufficient resolution. The highest ever recorded SD created by the averaging process was ± 0.1 mm.

B.2. TM5100A Settings

Prior to any measurements, the theodolite's instrument errors were adjusted to the current environmental conditions. All measurements were performed in an environmentally controlled clean room with stable conditions compared to the outside. Still, temperature, pressure and humidity changes occurred between the individual measurement series. According to the manual, adjusting the instrument errors was recommended within the following categories: [19]

• Daily:

Compensator (l, t) Vertical Index (i)

• Changing prism or Corner Cube Reflector (CCR) type:

ATR collimation

• Monthly, before the first use, after long distance transport, and after temperature changes >20 °C:

Compensator (l, t) Vertical index (i) Line-of-sight (c) Tilting axis (k) ATR collimation

It was decided to perform the monthly adjustment procedure prior to any measurement series even if the mentioned recommended conditions did not apply for each measurement series. It was believed that this approach generated the most accurate and comparable measurements with additionally a comparable time effort of each measurement series.

C. Measured Values

C.1. Total Station Measurement Series

Total Station TDRA6000 Measurement Series FG01

| | Hz in $^\circ$ | V in $^\circ$ | distance d in mm | tilt _L in $^{\circ}$ | tilt _T in $^{\circ}$ |
|-----------------------|----------------|---------------|--------------------|---------------------------------|---------------------------------|
| North Left RT | 178.9642 | 83.4013 | 11982.4 | 0.0004 | -0.0004 |
| North Right RT | 185.3221 | 83.4711 | 12123.7 | 0 | -0.0007 |
| East Left RT | 198.0515 | 82.7121 | 10841.7 | -0.0001 | -0.0005 |
| East Top RT | 206.2712 | 80.2036 | 8096.0 | -0.0004 | -0.0005 |
| East Right RT | 214.1381 | 78.0251 | 6637.2 | -0.0007 | -0.0004 |
| Floor Left | 176.3376 | 98.9064 | 10298.0 | 0.0015 | -0.0007 |
| Floor Center | 193.6348 | 98.7930 | 10438.9 | 0.0009 | -0.0009 |
| Floor Right | 215.6012 | 106.7309 | 5541.6 | 0.0002 | -0.0010 |
| RP INI Left | 176.6452 | 81.3485 | 8385.8 | 0.0013 | -0.0007 |
| RP INI Top | 177.6305 | 79.8499 | 8279.3 | 0.0013 | -0.0008 |
| RP INI Right | 178.6530 | 81.0569 | 8110.4 | 0.0013 | -0.0008 |
| RP ReFEx Left | 178.0682 | 90.5173 | 10049.2 | 0.0013 | -0.0007 |
| RP ReFEx Top | 179.2035 | 89.3862 | 10052.7 | 0.0013 | -0.0008 |
| RP ReFEx Right | 180.3431 | 90.5138 | 10053.3 | 0.0014 | -0.0007 |
| RP FG01 Left | 181.6959 | 92.2028 | 6481.1 | 0.0011 | -0.0009 |
| RP FG01 Top | 183.4285 | 90.5378 | 6546.7 | 0.0010 | -0.0009 |
| RP FG01 Right | 185.1581 | 92.2906 | 6547.4 | 0.0009 | -0.0008 |

Table C.1.: Total Station TDRA6000 Measurement Series FG01

| | Hz in $^\circ$ | V in $^\circ$ | distance d in mm | tilt _L in $^{\circ}$ | tilt_T in $^\circ$ |
|---------------------|----------------|---------------|--------------------|---------------------------------|--------------------|
| North Left RT | 175.7961 | 81.4698 | 9289.3 | 0.0001 | 0 |
| North Right RT | 184.1562 | 81.4076 | 9232.9 | 0 | -0.0007 |
| East Left RT | 200.5769 | 79.7163 | 7708.9 | -0.0006 | -0.0006 |
| East Center RT | 212.0522 | 73.8208 | 4947.6 | -0.0009 | -0.0001 |
| East Right RT | 227.3201 | 67.4053 | 3588.0 | -0.0008 | 0.0002 |
| Floor Left | 128.9261 | 109.7199 | 4721.8 | 0.0017 | -0.0008 |
| Floor Center | 193.3489 | 102.6202 | 7300.4 | 0.0004 | -0.0003 |
| Floor Right | 167.3602 | 101.6292 | 7900.9 | 0.0008 | -0.0009 |
| RP INI Left | 166.4642 | 77.6169 | 5886.3 | 0.0008 | -0.0001 |
| RP INI Top | 167.4293 | 75.3231 | 5762.1 | 0.0008 | -0.0003 |
| RP INI Right | 168.4615 | 76.8533 | 5546.2 | 0.0005 | -0.0007 |
| RP ReFEx Left | 172.2027 | 90.6959 | 7398.1 | 0.0008 | -0.0003 |
| RP ReFEx Top | 173.7259 | 89.1550 | 7359.9 | 0.0008 | -0.0009 |
| RP ReFEx Right | 175.2550 | 90.6987 | 7319.9 | 0.0008 | -0.0006 |
| RP FG02 Left | 175.7184 | 93.0187 | 3582.6 | 0.0006 | -0.0004 |
| RP FG02 Top | 177.6999 | 88.9821 | 3560.2 | 0.0006 | -0.0008 |
| RP FG02 Right | 181.2778 | 90.6152 | 3690.3 | 0.0003 | -0.0003 |

Table C.2.: Total Station TDRA6000 Measurement Series FG02

Total Station TDRA6000 Measurement Series FG03

Table C.3.: Total Station TDRA6000 Measurement Series FG03

| | Hz in $^\circ$ | V in $^\circ$ | distance d in mm | tilt _L in $^{\circ}$ | tilt _T in $^{\circ}$ |
|------------------|----------------|---------------|--------------------|---------------------------------|---------------------------------|
| North Left RT | 267.0071 | 80.1569 | 8059.4 | 0.0013 | -0.0009 |
| North Right RT | 276.6876 | 79.9153 | 7876.4 | 0.0003 | -0.0014 |
| East Left RT | 295.7514 | 77.2039 | 6212.6 | -0.0004 | -0.0015 |
| East Center RT | 310.7853 | 66.7289 | 3488.9 | -0.0009 | -0.0011 |
| East Right RT | 339.3258 | 54.0535 | 2348.5 | -0.0016 | -0.0004 |
| Floor Left | 230.7482 | 106.6574 | 5553.6 | 0.0035 | -0.0012 |
| Floor Center | 235.3505 | 127.3852 | 2625.8 | 0.0038 | -0.0014 |
| Floor Right | 286.3852 | 105.6136 | 5926.5 | 0.0008 | 0.0030 |
| RB Left | 263.6790 | 100.2680 | 5578.8 | 0.0015 | -0.0018 |
| RB Center | 270.6898 | 102.0646 | 4765.1 | 0.0010 | -0.0020 |
| RB Right | 285.8474 | 99.3682 | 6110.3 | 0.0009 | -0.0011 |

| | Hz in $^\circ$ | V in $^\circ$ | distance d in mm | tilt _L in $^{\circ}$ | tilt_T in $^\circ$ |
|-----------------------|----------------|---------------|--------------------|---------------------------------|--------------------|
| North Left RT | 278.9595 | 80.3462 | 8220.3 | 0.0007 | -0.0013 |
| North Right RT | 287.9688 | 80.6888 | 8529.1 | 0.0007 | -0.0012 |
| East Left RT | 307.5803 | 79.8426 | 7807.4 | 0.0001 | -0.0012 |
| East Center RT | 327.2036 | 76.0731 | 5730.6 | -0.0002 | -0.0010 |
| East Right RT | 345.2456 | 74.0503 | 5018.5 | -0.0004 | -0.0007 |
| RP INI Left | 279.5835 | 74.2199 | 4644.7 | 0.0007 | -0.0011 |
| RP INI Top | 281.7045 | 71.4542 | 4592.4 | 0.0006 | -0.0014 |
| RP INI Right | 283.9271 | 73.4423 | 4429.3 | 0.0007 | -0.0013 |
| A1 -30° Left | 291.7382 | 72.9513 | 4302.7 | 0.0003 | -0.0014 |
| A1 -30° Top | 294.4772 | 70.5762 | 4388.0 | 0.0006 | -0.0013 |
| A1 -30° Right | 297.1914 | 73.1857 | 4359.8 | 0.0003 | -0.0013 |
| A1 -45° Left | 298.3893 | 73.3048 | 4389.6 | 0.0004 | -0.0012 |
| A1 -45° Top | 300.7461 | 71.2559 | 4540.5 | 0.0004 | -0.0014 |
| A1 -45° Right | 303.0021 | 74.0165 | 4580.2 | 0.0001 | -0.0014 |
| A1 30° Left | 273.9031 | 76.6229 | 5464.8 | 0.0008 | -0.0012 |
| A1 30° Top | 274.5966 | 74.0855 | 5331.5 | 0.0006 | -0.0013 |
| A1 30° Right | 275.3478 | 75.6460 | 5097.6 | 0.0006 | -0.0014 |

Table C.4.: Total Station TDRA6000 Measurement Series FG04

Total Station TDRA6000 Measurement Series FG05

Table C.5.: Total Station TDRA6000 Measurement Series FG05

| | Hz in $^\circ$ | V in $^\circ$ | distance d in mm | tilt_L in $^\circ$ | tilt_T in $^\circ$ |
|----------------|----------------|---------------|--------------------|--------------------|--------------------|
| North Left RT | 278.9531 | 80.3470 | 8220.3 | 0.0008 | -0.0008 |
| North Right RT | 287.9773 | 80.6902 | 8529.0 | 0.0003 | -0.0008 |
| East Left RT | 307.5784 | 79.8435 | 7807.5 | 0.0002 | -0.0006 |
| East Center RT | 327.2029 | 76.0734 | 5730.6 | -0.0002 | -0.0008 |
| East Right RT | 345.2418 | 74.0495 | 5018.5 | -0.0003 | -0.0007 |

| | Hz in $^\circ$ | V in $^\circ$ | distance d in mm | tilt _L in $^{\circ}$ | tilt_T in $^\circ$ |
|------------------------|----------------|---------------|--------------------|---------------------------------|--------------------|
| North Left RT | 278.9501 | 80.3472 | 8220.3 | 0.0011 | -0.0008 |
| North Right RT | 287.9716 | 80.6903 | 8529.0 | 0.0008 | -0.0009 |
| East Left RT | 307.5744 | 79.8436 | 7807.5 | 0.0005 | -0.0010 |
| East Center RT | 327.2028 | 76.0737 | 5730.6 | 0 | -0.0009 |
| East Right RT | 345.2482 | 74.0492 | 5018.6 | -0.0005 | -0.0007 |
| RP INI Left | 279.5810 | 74.2211 | 4644.5 | 0.0009 | -0.0010 |
| RP INI Top | 281.6921 | 71.4548 | 4592.4 | 0.0010 | -0.0009 |
| RP INI Right | 283.9244 | 73.4422 | 4429.3 | 0.0009 | -0.0010 |
| A1 -30° Left | 291.7274 | 72.9508 | 4302.8 | 0.0009 | -0.0012 |
| A1 -30° Top | 294.4751 | 70.5717 | 4388.3 | 0.0008 | -0.0010 |
| A1 -30° Right | 297.1825 | 73.1827 | 4359.9 | 0.0008 | -0.0011 |
| A1 -45° Left | 298.3864 | 73.3019 | 4389.6 | 0.0008 | -0.0012 |
| A1 -45° Top | 300.7453 | 71.2523 | 4540.6 | 0.0006 | -0.0011 |
| A1 -45° Right | 302.9954 | 74.0165 | 4580.2 | 0.0006 | -0.0011 |
| A1 30° Left | 273.9034 | 76.6190 | 5464.6 | 0.0011 | -0.0011 |
| A1 30° Top | 274.5893 | 74.0806 | 5331.3 | 0.0011 | -0.0009 |
| A1 30° Right | 275.3393 | 75.6453 | 5097.8 | 0.0010 | -0.0010 |
| A2 -50° Left | 275.6034 | 88.7512 | 4096.0 | 0.0010 | -0.0012 |
| A2 -50° Top | 276.6309 | 86.4437 | 3877.1 | 0.0012 | -0.0011 |
| A2 -50° Right | 280.1337 | 88.6800 | 3850.6 | 0.0012 | -0.0010 |
| A2 -130° Left | 286.6282 | 71.0825 | 5722.3 | 0.0009 | -0.0014 |
| A2 -130° Top | 289.1160 | 69.7224 | 5790.6 | 0.0009 | -0.0012 |
| A2 -130° Right | 290.4413 | 70.4760 | 5549.1 | 0.0009 | -0.0012 |
| A3 130° Left | 282.1135 | 85.2635 | 4779.7 | 0.0011 | -0.0012 |
| A3 130° Top | 283.3065 | 83.1563 | 4580.2 | 0.0011 | -0.0012 |
| A3 130° Right | 286.2872 | 85.0552 | 4570.8 | 0.0011 | -0.0011 |
| A3 50° Left | 281.6427 | 65.3420 | 5172.6 | 0.0011 | -0.0010 |
| A3 50° Top | 284.5239 | 63.8281 | 5236.7 | 0.0011 | -0.0011 |
| A3 50° Right | 285.8559 | 64.3300 | 4980.5 | 0.0011 | -0.0012 |
| A5 40° Left | 280.1827 | 77.0865 | 4653.5 | 0.0011 | -0.0010 |
| A5 40° Top | 281.3441 | 74.5744 | 4477.9 | 0.0012 | -0.0012 |
| A5 40° Right | 284.4860 | 76.4575 | 4438.6 | 0.0011 | -0.0012 |
| A5 -40° Left | 280.1879 | 71.8576 | 4772.2 | 0.0012 | -0.0011 |
| A5 -40° Top | 283.1762 | 70.1168 | 4815.0 | 0.0011 | -0.0011 |
| A5 -40° Right | 284.4920 | 71.0045 | 4563.0 | 0.0012 | -0.0011 |

Table C.6.: Total Station TDRA6000 Measurement Series FG06

| | Hz in $^{\circ}$ | V in $^{\circ}$ | distance d in mm | $\mathrm{tilt_L}$ in $^\circ$ | tilt _T in $^{\circ}$ |
|-----------------------|------------------|-----------------|--------------------|-------------------------------|---------------------------------|
| North Left RT | 316.8365 | 82.6551 | 10798.1 | 0.0014 | 0.0007 |
| North Right RT | 323.8521 | 82.7735 | 10984.6 | 0.0016 | 0.0006 |
| East Left RT | 338.2906 | 81.9483 | 9843.2 | 0.0017 | 0 |
| East Center RT | 349.1500 | 78.9941 | 7235.0 | 0.0016 | -0.0002 |
| East Right RT | 359.7577 | 76.5337 | 5930.5 | 0.0016 | -0.0004 |
| Floor Left | 295.5452 | 111.3151 | 4377.3 | 0.0021 | 0.0013 |
| Floor Corner | 317.8376 | 102.4938 | 7356.5 | 0.0024 | 0.0005 |
| Floor Right | 333.9826 | 99.8443 | 9312.2 | 0.0023 | -0.0003 |
| RB Left | 319.8305 | 96.8319 | 8337.1 | 0.0023 | 0.0004 |
| RB Center | 325.3667 | 97.2980 | 7819.0 | 0.0024 | 0.0003 |
| RB Right | 332.2203 | 95.9720 | 9535.3 | 0.0025 | 0 |
| RP INI Left | 314.9573 | 79.8768 | 7197.1 | 0.0022 | 0.0007 |
| RP INI Top | 316.1529 | 78.1151 | 7102.2 | 0.0021 | 0.0007 |
| RP INI Right | 317.4008 | 79.4949 | 6934.7 | 0.0021 | 0.0007 |
| RP ReFEx Left | 316.1579 | 90.5658 | 8850.9 | 0.0022 | 0.0008 |
| RP ReFEx Top | 317.4421 | 89.2814 | 8861.1 | 0.0022 | 0.0007 |
| RP ReFEx Right | 318.7339 | 90.5603 | 8869.0 | 0.0022 | 0.0006 |
| RP FG07 Left | 315.8591 | 90.7408 | 5582.1 | 0.0021 | 0.0010 |
| RP FG07 Top | 318.4281 | 90.3449 | 5455.2 | 0.0021 | 0.0008 |
| RP FG07 Right | 318.2921 | 93.1025 | 5349.0 | 0.0021 | 0.0008 |

Table C.7.: Total Station TDRA6000 Measurement Series FG07

| | Hz in $^\circ$ | V in $^\circ$ | distance d in mm | tilt _L in $^{\circ}$ | tilt_T in $^\circ$ |
|------------------|----------------|---------------|--------------------|---------------------------------|--------------------|
| North Left RT | 321.0878 | 80.0004 | 7942.4 | -0.0006 | -0.0002 |
| North Right RT | 330.3198 | 80.4175 | 8292.6 | -0.0007 | 0.0001 |
| East Left RT | 350.5834 | 79.6704 | 7681.8 | -0.0007 | 0.0004 |
| East Center RT | 11.4121 | 76.0709 | 5732.1 | -0.0006 | 0.0009 |
| East Right RT | 29.7541 | 74.4045 | 5132.4 | -0.0002 | 0.0016 |
| Floor Left | 307.5779 | 109.9185 | 4668.8 | -0.0006 | -0.0003 |
| Floor Center | 13.8336 | 118.6338 | 3326.0 | -0.0007 | 0.0010 |
| Floor Right | 345.2288 | 102.6315 | 7285.9 | -0.0009 | 0.0004 |
| RB Left | 328.3440 | 100.2668 | 5571.1 | -0.0010 | 0.0001 |
| RB Center | 337.7665 | 101.0121 | 5207.3 | -0.0009 | 0.0002 |
| RB Right | 343.2403 | 97.9420 | 7189.3 | -0.0009 | 0.0004 |
| RP INI Left | 323.2801 | 73.2310 | 4379.4 | -0.0009 | -0.0001 |
| RP INI Top | 325.6240 | 70.3291 | 4341.1 | -0.0009 | 0 |
| RP INI Right | 328.0903 | 72.4391 | 4185.5 | -0.0009 | 0.0001 |
| RP ReFEx Left | 322.0130 | 90.8520 | 5963.3 | -0.0009 | -0.0001 |
| RP ReFEx Top | 323.8824 | 88.9535 | 6001.4 | -0.0008 | -0.0001 |
| RP ReFEx Right | 325.7498 | 90.8365 | 6039.3 | -0.0009 | -0.0001 |
| RP FG08 Left | 322.1499 | 90.9984 | 2924.1 | -0.0008 | -0.0002 |
| RP FG08 Top | 326.2063 | 87.3139 | 2942.4 | -0.0008 | -0.0001 |
| RP FG08 Right | 329.8661 | 90.8597 | 2826.8 | -0.0008 | 0.0001 |

Table C.8.: Total Station TDRA6000 Measurement Series FG08

C.2. Industrial Robot Axis Angle Measurements

| | A1 in $^\circ$ | A2 in $^\circ$ | A3 in $^\circ$ | A4 in $^\circ$ | A5 in $^\circ$ | A6 in $^\circ$ |
|-------|----------------|----------------|----------------|----------------|----------------|----------------|
| INI | 0 | -90.00 | 90.00 | 0 | 0 | 0 |
| ReFEx | 86.92 | -44.23 | 134.93 | 130.23 | -90.57 | 0.44 |
| FG01 | -30.20 | -0.62 | 17.65 | 33.23 | -33.99 | -30.49 |
| FG02 | -39.67 | -8.44 | 18.62 | 99.66 | -32.97 | -115.06 |
| FG03 | 0 | -90.00 | 90.00 | 0 | 0 | 0 |
| FG04 | 0 | -90.00 | 90.00 | 0 | 0 | 0 |
| FG05 | 0 | -90.00 | 90.00 | 0 | 0 | 0 |
| FG06 | 0 | -90.00 | 90.00 | 0 | 0 | 0 |
| FG07 | -21.06 | 4.23 | 5.89 | -17.39 | -34.25 | 54.64 |
| FG08 | -11.91 | -13.67 | 39.51 | 4.76 | -47.36 | -5.96 |

Table C.9.: Industrial Robot Axis Angle Measurements

C.3. Theodolite Measurement Series

Theodolite TM5100A Measurement Series 01

| | Hz in $^\circ$ | V in $^\circ$ | tilt _L in $^{\circ}$ | tilt _T in $^{\circ}$ |
|----------------|----------------|---------------|---------------------------------|---------------------------------|
| T1-T2 | 359.99990 | 0.00074 | N/A | N/A |
| T2-T1 | 0 | -0.00361 | N/A | N/A |
| | | | | |
| Theodolite 1 | Hz in $^\circ$ | V in $^\circ$ | tilt_L in $^\circ$ | tilt_T in $^\circ$ |
| Alignment Cube | 291.16760 | -0.82380 | N/A | N/A |
| | | | | |
| Theodolite 2 | Hz in $^\circ$ | V in $^\circ$ | tilt_L in $^\circ$ | tilt_T in $^\circ$ |
| Alignment Cube | 21.16419 | -0.28054 | N/A | N/A |

Table C.10.: Theodolite TM5100A Measurement Series 01

| | Hz in $^\circ$ | V in $^\circ$ | tilt _L in $^{\circ}$ | tilt_T in $^\circ$ |
|----------------------|----------------|---------------|---------------------------------|--------------------|
| T1-T2 | 0 | 0.00459 | N/A | N/A |
| T2-T1 | 0 | -0.00345 | N/A | N/A |
| | | | | |
| Theodolite 1 | Hz in $^\circ$ | V in $^\circ$ | tilt _L in $^{\circ}$ | tilt_T in $^\circ$ |
| North Left RT | 249.94757 | 13.69997 | N/A | N/A |
| North Right RT | 261.16454 | 12.33463 | N/A | N/A |
| East Left RT | 286.57747 | 11.85479 | N/A | N/A |
| East Center RT | 314.61250 | 13.64860 | N/A | N/A |
| East Right RT | 332.27416 | 13.17170 | N/A | N/A |
| Alignment Cube | 290.95223 | -0.85512 | N/A | N/A |
| Test Rig Back Left | 279.02929 | 5.07268 | N/A | N/A |
| Test Rig Front Left | 288.09155 | 6.65549 | N/A | N/A |
| Test Rig Front Right | 292.49400 | 6.16474 | N/A | N/A |
| RB Left | 269.54847 | -15.03380 | N/A | N/A |
| RB Center | 283.10158 | -14.31792 | N/A | N/A |
| RB Right | 282.52666 | -12.18267 | N/A | N/A |
| Floor Left | 320.91705 | -41.06400 | N/A | N/A |
| Floor Center | 329.44720 | -31.40352 | N/A | N/A |
| Floor Right | 308.79462 | -24.54955 | N/A | N/A |
| | | | | |
| Theodolite 2 | Hz in $^\circ$ | V in $^\circ$ | tilt_L in $^\circ$ | tilt_T in $^\circ$ |
| North Left RT | 33.34777 | 8.11643 | N/A | N/A |
| North Right RT | 41.25188 | 8.30519 | N/A | N/A |
| East Left RT | 55.86803 | 10.27193 | N/A | N/A |
| East Center RT | 62.06672 | 16.76501 | N/A | N/A |
| East Right RT | 71.47391 | 25.49359 | N/A | N/A |
| Alignment Cube | 20.95452 | -0.33646 | N/A | N/A |
| Test Rig Back Left | 24.31587 | 2.11728 | N/A | N/A |
| Test Rig Front Left | 20.25023 | 2.42734 | N/A | N/A |
| Test Rig Front Right | 21.96020 | 2.49885 | N/A | N/A |
| RB Left | 30.95253 | -7.87418 | N/A | N/A |
| RB Center | 35.86015 | -8.73487 | N/A | N/A |
| RB Right | 40.21630 | -8.37915 | N/A | N/A |
| Floor Left | 14.13253 | -18.68178 | N/A | N/A |
| Floor Center | 19.52254 | -21.89303 | N/A | N/A |
| Floor Right | 35.54836 | -18.83258 | N/A | N/A |

Theodolite TM5100A Measurement Series 02

Table C.11.: Theodolite TM5100A Measurement Series 02

Theodolite TM5100A Measurement Series 03

| | Hz in $^\circ$ | V in $^\circ$ | tilt _L in $^\circ$ | tilt_T in $^\circ$ |
|----------------|----------------|---------------|---------------------------------|--------------------|
| T1-T2 | 359.99991 | -0.00700 | 0.001 | 0.001 |
| T2-T1 | 359.99999 | 0.00583 | 0 | 0.001 |
| | | | | |
| Theodolite 1 | Hz in $^\circ$ | V in $^\circ$ | tilt_L in $^\circ$ | tilt_T in $^\circ$ |
| North Left RT | 277.93567 | 11.05991 | -0.001 | 0.001 |
| North Right RT | 287.67304 | 10.36065 | -0.001 | 0.001 |
| East Left RT | 309.60737 | 10.68535 | 0 | 0.001 |
| East Center RT | 333.25240 | 13.53721 | 0 | 0.001 |
| East Right RT | 351.40490 | 14.21779 | 0 | 0 |
| Alignment Cube | 310.93037 | -0.67818 | 0 | -0.001 |
| | | | | |
| Theodolite 2 | Hz in $^\circ$ | V in $^\circ$ | tilt _L in $^{\circ}$ | tilt_T in ° |
| North Left RT | 68.00643 | 10.37368 | 0.001 | 0.001 |
| North Right RT | 78.20953 | 10.64142 | 0.001 | 0.001 |
| East Left RT | 98.69248 | 13.61532 | 0.001 | 0.001 |
| East Center RT | 117.17454 | 25.46229 | 0.001 | 0 |
| East Right RT | 152.89423 | 37.69892 | 0.001 | 0 |
| Alignment Cube | 40.92246 | -0.56583 | 0 | 0.001 |

Table C.12.: Theodolite TM5100A Measurement Series 03

D. Figures

D.1. Point Cloud Alignment



Figure D.1.: Aligned total station TDRA6000 measurement series FG01



Figure D.2.: Aligned total station TDRA6000 measurement series FG02



Figure D.3.: Aligned total station TDRA6000 measurement series FG03

Total Station TDRA6000 Measurement Series FG04



Figure D.4.: Aligned total station TDRA6000 measurement series FG04



Figure D.5.: Aligned total station TDRA6000 measurement series FG05 $\,$



Figure D.6.: Aligned total station TDRA6000 measurement series FG06


Total Station TDRA6000 Measurement Series FG07

Figure D.7.: Aligned total station TDRA6000 measurement series FG07



Total Station TDRA6000 Measurement Series FG08

Figure D.8.: Aligned total station TDRA6000 measurement series FG08



All Total Station TDRA6000 Measurement Series Aligned

Figure D.9.: All total station TDRA6000 measurement series aligned

D.2. Original Pictures



Figure D.10.: TDRA6000 aligned with a ball prism target placed on the floor - original



Figure D.11.: Autocollimated theodolite with a mirror face of the alignment cube - original