Experimental and Numerical Analysis of the Vibro-Acoustic Behavior of a Helmholtz Resonator with a Flexible Wall

Fleming Kohlenberg*, Vincent Radmann†, Julia Genßler‡, and Lars Enghardt§
Technische Universität Berlin, 10623 Berlin, Germany

Karsten Knobloch¶
German Aerospace Center (DLR), 10623 Berlin, Germany

Modern aircraft engines have large fans with a low-frequency noise signature, which conventional liners struggle to damp efficiently. New liner concepts are therefore needed to address low-frequency aircraft noise. In this paper, a Helmholtz resonator is combined with a flexible plate dividing the cavity into two. The flexible plate extends the conventional single-degree of freedom system to a multi-degree of freedom system, with resonances below the conventional Helmholtz resonance. The plate’s motion is investigated experimentally at the aero-acoustic wind tunnel DUCT-R with a vibrometer and compared to numerical simulations based on FEM, where good agreement is found. We showed that the face sheet and plate size is a crucial parameter of the concept which alters the number of resonance frequencies, their absorption amplitudes and their resonance frequencies. The flexible wall vibrates strongly and adds multiple damping peaks when the plate resonances lie in the vicinity of the Helmholtz resonance. Furthermore, we resolved the deflection shapes and the absorption of the Helmholtz resonator with a flexible wall of variable size numerically. The results help to fortify confidence in the proposed liner concept for future optimization and application.

I. Introduction

State-of-the-art aircraft are 75% quieter compared to the first civilian jets in operation 50 years ago [1]. The noise from first jet aircraft was completely dominated by roaring jet noise. Nowadays, not only jet noise but additional broadband and tonal noise radiated by fan, compressor, combustor, and airframe need to be taken into account. The relative contribution differs from the type of aircraft and engine in use as well as the specific flight stage such as takeoff or landing [2]. This evolution is largely due to the increase in fan size and thus bypass ratio, which is defined as the ratio between the mass flow rate of the bypass stream to the mass flow rate entering the core. Modern turbofan aircraft engines have a bypass ratio up to twelve and engine manufacturers are developing engines with even larger fans. The drawbacks of larger fans are increased size, weight and drag. Nevertheless, a larger fan is able to produce the same thrust with a reduced jet velocity and a slower rotation frequency, which reduces the environmental impact (fuel consumption and overall noise) of aviation. A slower rotation frequency, however, leads to a lower blade passing frequency and consequently to fan noise with tonal components at lower frequencies.

While liners are an efficient and established method to dampen aircraft noise, it is a challenge for them to address these new low-frequency components. Conventional liners consist of a face sheet backed with a honeycomb structure and a rigid end plate. They can be modelled similar to a Helmholtz resonator with a resonance below that of a $\lambda/4$-resonator. The resonance frequency of a $\lambda/4$-resonator only depends on its depth, while the Helmholtz resonance can be tuned by the cavity volume (cross section and depth) and the perforated face sheet parameters such as its thickness, hole number, hole pattern and hole size. The Helmholtz resonator can be tuned towards low frequencies by increasing the air mass in the face sheet’s holes. This, however, greatly decreases the resonance bandwidth. Alternatively, its resonance frequency can be lowered by enlarging the back cavity. Unfortunately, these cavity depths are limited due to installation space constraints, as a large cavity depth needs a large nacelle which would lead to more drag and consequently more fuel consumption. As a consequence, new liner concepts are needed to damp low-frequency noise.

*PhD candidate, Dept. of Turbomachinery Acoustics, AIAA Student Member, fleming.kohlenberg@dlr.de
†PhD candidate, Dept. of Engineering Acoustics
‡PhD candidate, Dept. of Turbomachinery Acoustics
§Professor, Dept. of Turbomachinery Acoustics, AIAA Senior Member
¶Scientist, Dept. of Engine Acoustics (DLR-AT-TRA), AIAA Member
One way to enable noise reduction at lower frequencies than conventional liners, is to combine hard walled Helmholtz resonators with additional mechanical elements such as bars, membranes or plates. These mechanical elements have the benefit, that their individual resonance frequencies can be tuned independently of the Helmholtz resonator. In this way low-frequency noise can be damped without increasing the overall resonator volume.

These mechanical elements differ in their elasticity. Membranes are elastic because of the applied tension, similar to a drum. The advantage of using a membrane is that it is tunable by a variable tension. The disadvantage is that this tension is challenging to determine and will most likely decrease over time. Successful implementations of resonators combined with membranes can be found for example in [3,7].

Plates on the other hand draw their elasticity from their flexural rigidity, similar to the top plate of a classical guitar[7]. In previous studies, flexible plates were often at the cavity end to improve attenuation [9–11]. Another approach is to use flexible walls to couple multiple resonator chambers [12–14]. Yet another possibility is to subdivide the resonator main cavity into two chambers with a flexible wall acting as a plate [15,16]. This represents the approach used in this work.

Both, the excitation as well as the measurement of the vibration of small flexible limb structures are challenging. Generally speaking, one can distinguish between contact and contactless methods. Contact methods to excite the structure such as shakers or impulse hammers offer a good signal-to-noise ratio. They, however, are not an option for flexible structures with a submillimeter thickness as the large excitation forces would damage the structure. A similar reasoning rules out accelerometers as a mean to measure the structure’s vibration, as the accelerometers’ mass would severely affect the structure’s response. A natural excitation choice is the non-contact plane wave acoustic excitation similar to [14]. This ensures a realistic, controllable amplitude with the drawback that the excitation is planar and not concentrated in one excitation point. For these reasons, we excited the resonator with an acoustic plane wave and measured the output, the plate vibration, with a (contactless) vibrometer without affecting the plate’s response.

Conventional liners are usually only investigated in the air domain and the acoustical effect of a possible structural response is not of interest. Thus, significantly less literature about the vibro-acoustic behavior of conventional liners can be found. The reason for this is the fact, that the underlying cavity structure prohibits vibrations of the face sheet or back plate in the relevant frequency range. When dealing with larger structures, however, the perforated face sheet’s motion does need to be taken into account [17,18]. In the proposed liner concept here, the acoustic-structure interaction between the flexible plate and the Helmholtz resonator cavities is a crucial effect which needs special experimental and numerical attention. Systems with acoustic-structure interaction are often modeled via the finite element method (FEM) [5,19,21], which is also pursued here.

In this work we investigate the acoustic properties of the resonator experimentally and numerically. The experimental setup allows us to analyze the system globally (by measuring the overall impedance and absorption) and locally (the vibration of the flexible plate). Furthermore, we make use of the FEM to investigate the plate behavior at characteristic frequencies and resolve the multi modal resonator behavior. Precisely, we want to investigate the following questions:

- How can one distinguish between Helmholtz associated resonances and flexible plate associated resonances?
- How does the flexible plate size affect the acoustic performance?
- What effect does a change in the face sheet have onto the flexible plate and the overall acoustic parameters?
- Is the FEM a suitable choice to resolve, analyze and extend the liner concept?

This paper is structured as follows: Details about the measurement setup can be found in Section II, while the numerical setup is presented in Section III. Section IV includes the presentation and discussion of the experimental results, the comparison to the results from the FE-Analysis and further numerical results. Finally, a conclusion is drawn in Section V.

II. Experimental setup

The vibro-acoustic experiments are conducted at the duct acoustic test rig (DUCT-R) facility of the German Aerospace Center (DLR) in Berlin. We investigate a Helmholtz resonator with a flexible wall which consists of a face sheet, a main cavity, a flexible plate and a second cavity. The resonator system is attached to the end of the upstream section of the test rig. A schematic view and a photograph of the measurement setup is depicted in Fig. 1.

The attachment of the resonator at the end of the duct effectively transforms it into a large impedance tube, which enables us to use the same resonator system as previously studied in [16]. We choose the direct incidence setup instead of the usual grazing setup, due to the accessibility of the vibrometer and the straightforward determination of the

---

*The body of a classical guitar can be viewed as a Helmholtz resonator with a flexible face sheet (top plate) with a single (sound) hole. In this case the resonator/body does not damp but, on the contrary, amplify sound [8].*
acoustic impedance of the resonator system. The duct has a rectangular cross section of 60 mm \times 80 mm and therefore a cut-on frequency of the first higher mode of 2142 Hz. The sound source is an upstream loudspeaker (BMS-4599-ND) attached to the side wall of the duct. The end opposite to the resonator system has an anechoic termination. We use a multi-microphone method with five microphones flush mounted in the measurement section to decompose the sound field in incoming and reflecting acoustic waves. The resulting equation system

\[
\begin{bmatrix}
    e^{-ikx_1} & e^{ikx_1} \\
    \vdots & \vdots \\
    e^{-ikx_5} & e^{-ikx_5}
\end{bmatrix}
\begin{bmatrix}
    p^+ \\
    \vdots \\
    p^-(x_5)
\end{bmatrix}
= \begin{bmatrix}
    p'(x_1) \\
    \vdots \\
    p'(x_5)
\end{bmatrix},
\]

is fitted in a least-square sense by calculating \( x = A^+ b \) with \( A^+ \) denoting the pseudo inverse of \( A \). In Eq. (1), \( i \) denotes the imaginary unit, \( k = \omega/c \) the (plane) wavenumber as the quotient of angular frequency \( \omega \) and speed of sound \( c \), \( p^\pm \) the incoming and reflected sound wave respectively and \( p'(x_n) \) the measured sound pressure at the distance \( x_n \) from the reference plane. The viscothermal losses inside the duct walls are taken into account as proposed by Dokumaci [22].

The sound waves are excited successively with a single tone and an incoming plane wave amplitude of 100 dB. In this region, the resonator system’s acoustic properties are independent of the incident amplitude. Due to the special setup of the DUCT-R as an impedance tube, measurements with grazing flow are not possible.

Based on the decomposed waves, the complex reflection factor of the sample \( r = p^-/p^+ \) can be calculated. This reflection factor is then used to calculate the absorption \( \alpha = 1 - |r|^2 \) and the complex normalized impedance

\[
\zeta = z/\rho c = \theta + i\chi = \frac{1 + r}{1 - r}.
\]

The laser Doppler vibrometer (Polytec OFV-5000 with a OFV-200 single point sensor head) is of a heterodyne-interferometer type. A Helium-Neon-laser emits a beam with a carrier signal which is focused by the sensor head to a small point on the plate. The reflected light of the vibrating object is subject to a Doppler shift proportional to the vibration velocity. This Doppler shift induces a frequency and phase modulation of the carrier signal which is registered by a light detector inside the vibrometer. This light detector then converts the fluctuation in light intensity into an electric signal. The vibrometer has two decoders. One decoder uses the frequency modulation to calculate the vibration velocity and the other uses the phase modulation to calculate the vibration deflection. We use the largest velocity sensitivity (50 mm/s/V) of the built-in velocity decoder to allow a maximum measurement range up to 500 mm/s. With the same idea in mind, we set the sensitivity of the deflection decoder to 5 mm/V. This enables deflection measurements up to 50 mm. Note, that the vibration velocity \( v_{vib} \) is the time derivative of the deflection \( x_{vib} \) and for harmonic signals it yields \( v_{vib} = \frac{\partial x_{vib}}{\partial t} = i\omega x_{vib} \). The acoustic excitation by the loudspeaker in the upstream section is held for five seconds prior to every measurement to ensure a stationary behavior of the flexible plate.
The experimental setup is similar to that introduced in [14], but here the modified channel is terminated with a resonator system instead of a flexible plate. This resonator system is made out of a face sheet, a main cavity, a flexible wall, a second cavity and a 5 mm thick transparent back wall made out of acrylic glass ( polymethyl methacrylat, PMMA). The main cavity with a quadratic cross sectional area of \( A_{\text{cav}} = 35 \text{ mm} \times 35 \text{ mm} \) and a depth of \( l_{\text{cav1}} = 60 \text{ mm} \) is attached to the duct end via an interchangeable perforated face sheet. Note, that the resonator cross section is smaller than the duct cross section and the overall impedance is the resonator impedance smeared with the hard wall area around the resonator. In the base configuration, the face sheet (FS1) is 2 mm thick, has 18 evenly spaced orifices with a diameter of \( d_{\text{FS1}} = 1.5 \text{ mm} \) and the open area ratio \( \sigma_{\text{FS1}} \) is 2.6 %. Additionally, we use a second face sheet (FS2) with a single hole with a diameter of \( d_{\text{FS2}} = 8.9 \text{ mm} \), yielding a porosity of \( \sigma_{\text{FS2}} = 5.1 \% \), which is roughly twice as much as the one in FS1. The regular cavity walls are made out of aluminum with a thickness of more than 5 mm. The flexible wall is a thermoplastic plate, which is clamped between two plate holders with a circular cut-out of variable diameter, followed by a second cavity and rigid back wall. We choose this plate material to make use of its suitable low flexural rigidity and high internal losses to ensure that the first eigenfrequency is within the investigated frequency range. The material values (\( E \): Young’s Modulus, \( \eta \): loss factor, \( \rho_p \): density, \( \nu_p \): Poisson’s ratio, \( h_{\text{plate}} \): plate thickness) are taken from [23].

In preliminary investigations, we measured the motion of the perforated face sheet, plate holder and the transparent back wall. We found that all of their vibration amplitudes are at least two orders of magnitude below that of the flexible plate and consequently they can be assumed rigid (not shown here). This is crucial, as the laser beam passes through the transparent back wall onto the flexible plate. Consequently, special care needs to be taken to ensure that the signal associated with the flexible plate’s motion is not corrupted by a possible back wall motion. We furthermore investigated whether the transparent back wall made out of PMMA is suited for laser vibrometer measurements. We measured the flexible plate vibration without the transparent back wall and with the transparent back wall at a sufficient distance to the resonator. Both vibration measurements agreed very well (not shown here) and the transparent back wall can therefore be assumed sufficiently transparent without adding any bias error.

Due to the plane wave excitation of the flexible plate, we assume that the flexible plate can vibrate only in radial modes. In this case the center point is always a maximum in the deflection and velocity amplitude for a clamped circular plate, hence we choose the center point as the (only) focal point of the vibrometer’s laser beam.

The in vacuo eigenfrequencies of the flexible plate decrease inversely with the square of the plate diameter \( d \):

\[
 f_{ij} = \frac{2 \lambda_{ij}^2}{\pi d^2} \sqrt{\frac{E h_{\text{plate}}^2}{12 \rho_p (1 - \nu_p^2)}} 
\]

with \( \lambda_{ij} \) denoting the radial and azimuthal eigenfrequency parameter [23]. The theoretical radial eigenfrequencies of the investigated flexible plates are shown in Table 1.

### Table 1 Properties of the experimental resonator

<table>
<thead>
<tr>
<th>Cavity dimensions</th>
<th>Plate properties</th>
</tr>
</thead>
<tbody>
<tr>
<td>( l_{\text{cav1}} ) = 60 mm</td>
<td>( E ) = 15.36 MPa</td>
</tr>
<tr>
<td>( l_{\text{cav2}} ) = 15 mm</td>
<td>( \eta ) = 0.1</td>
</tr>
<tr>
<td>( A_{\text{cav}} ) = 35 mm \times 35 mm</td>
<td>( \rho_p ) = 1080 kg/m³</td>
</tr>
<tr>
<td>\</td>
<td>( \nu_p ) = 0.48</td>
</tr>
<tr>
<td>\</td>
<td>( h_{\text{plate}} ) = 0.3 mm</td>
</tr>
<tr>
<td>\</td>
<td>( d_{[1;2;3]} ) = [15; 22.8; 30] mm</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Face sheet 1</th>
<th>Face sheet 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>( d_{\text{FS1}} ) = 1.5 mm</td>
<td>( d_{\text{FS2}} ) = 8.9 mm</td>
</tr>
<tr>
<td>( N_{\text{FS1}} ) = 18</td>
<td>( N_{\text{FS2}} ) = 1</td>
</tr>
<tr>
<td>( \sigma_{\text{FS1}} ) = 2.6 %</td>
<td>( \sigma_{\text{FS2}} ) = 5.1 %</td>
</tr>
</tbody>
</table>

This is the author’s version (post-print) of the work that was accepted for publication in the proceedings of the 2023 AIAA AVIATION Forum held in San Diego, CA (USA). June 2023. The final version was published in the proceedings of the conference as paper no. 2023-3348: https://arc.aiaa.org/doi/10.2514/6.2023-3348 © 2023. This manuscript version is made available under the CC-BY-NC-ND 4.0 license; http://creativecommons.org/licenses/by-nc-nd/4.0/
Table 2  Theoretical radial eigenfrequencies of the investigated clamped circular plates

<table>
<thead>
<tr>
<th>d (in mm)</th>
<th>$f_1$ (in Hz)</th>
<th>$f_2$ (in Hz)</th>
<th>$f_3$ (in Hz)</th>
<th>$f_4$ (in Hz)</th>
</tr>
</thead>
<tbody>
<tr>
<td>15</td>
<td>426</td>
<td>1657</td>
<td>3712</td>
<td>6591</td>
</tr>
<tr>
<td>22.8</td>
<td>184</td>
<td>717</td>
<td>1607</td>
<td>2853</td>
</tr>
<tr>
<td>30</td>
<td>106</td>
<td>414</td>
<td>928</td>
<td>1648</td>
</tr>
</tbody>
</table>

III. Numerical setup

The experimental investigations are accompanied with numerical simulations conducted with the commercial finite element software Actran 19.0. The main purpose of the numerical investigation is to examine the plate vibration in order to compare its absorption spectrum with the measured data. Therefore, we conduct a direct frequency response (DFR), which allows us to calculate the absorption spectrum of the resonator system and the plate vibration as well. This enables future parameter variations not feasible with purely experimental methods.

Therefore, a part of the duct is modeled corresponding to the dimensions of the DUCT-R test rig. The model of the resonator and duct is built and meshed in the free 3D finite element mesh generator Gmsh and is imported in Actran afterwards. The perforated sheet, which separates the first cavity from the duct is modeled by an admittance boundary condition using the equivalent fluid model from Atalla and Sgard [25]. In order to increase the accuracy of the simulation results, the area around the perforated sheet is meshed significantly finer. This avoids numerical artifacts in the transition through the perforated sheet. The remaining air domain is meshed with six elements per smallest wavelength and a quadratic interpolation is used in Actran. The plate requires an even finer mesh, since different sound velocities have to be taken into account for airborne and structure-borne sound. In order to avoid unnecessary nodes, the meshes of the duct with the first cavity, the plate and the second cavity are modeled independently. This allows us to use a coarser grid for the fluid and a finer grid for the flexible plate without getting an unnecessary fine grid in the vicinity. We used coupling surfaces to couple the meshes. The Actran model is depicted in Fig. 2. The geometrical and material properties for the numerical model are listed in Table 1 as well.

![Overview of the mesh of the entire model](image1.png)  ![Detailed view of the mesh in the coupled resonator system](image2.png)

**Fig. 2**  FE-Mesh of the coupled resonator system at the end of the duct
IV. Results and Discussion

In this section the experimental results of the measurements with a vibrometer are presented first and afterwards compared to the finite element solution.

A. Experimental Results

Figure 3a displays the measured normalized impedance $\zeta = \theta + i\chi$ together with the absorption coefficient $\alpha$ for normal incidence of the resonator with face sheet one (FS1) and a flexible plate of diameter $d_1 = 15$ mm. The resonance associated with the flexible plate around 430 Hz as well as the Helmholtz resonance around 610 Hz are visible as maxima in the absorption (red) and minima in the absolute value of the reactance (blue, dashed). In between, the antiresonance is visible as a maximum in the normalized reactance (blue, solid) and a zero-crossing of the reactance with negative slope around 480 Hz. Consequently, in this configuration, the resonator acts as a 2DOF resonator with the air mass in the perforated plate as one degree of freedom and the first mode of vibration of the flexible plate as the other. The compressible air volumes in the main and back cavity can be seen as springs. The distinction between both resonances is clearly visible in Fig. 3b as the highest deflection and velocity in the center point of the flexible plate measured with the vibrometer is around 480 Hz, as well. Note that the deflection at an excitation of 100 dB already exceeds 10% of the plate thickness ($h_{\text{plate}} = 0.3$ mm). While these deflection amplitudes are still small compared to the plate thickness, excitation amplitudes of just 120 dB (ten times the acoustic pressure amplitude) suggest plate deflections in the same range as the plate thickness. This might pose problems when trying to model the flexible plate using linear plate theory. Additionally, the vibrometer measurements reveal that the plate is strongly excited around the Helmholtz resonance as well. This is plausible, as the flexible plate is driven by a pressure difference between the main and second cavity, which is increased near the Helmholtz resonance. This also means that very far from the Helmholtz resonance (excluding higher cavity modes) there is not enough pressure difference to excite the flexible plate. The antiresonance does not seem to affect the plate vibration. At frequencies higher than the Helmholtz resonance, no additional maxima are visible in the vibration spectra and one can conclude that no higher plate mode order is excited in this configuration. This is most likely due to the fact that the theoretical eigenfrequency of the second radial mode is too high for the investigated frequency range.

![Normalized Resistance and Reactance](image1)

![Absorption and Plate Deflection Velocity](image2)

Fig. 3  Measured normal incidence impedance, absorption, plate deflection and plate velocity for FS1, $d_1 = 15$ mm

Based on this reasoning, if multiple plate eigenfrequencies are near the Helmholtz resonance, then the resonator system should inhibit multiple absorption peaks, which is investigated in the following.

Figure 4a displays the absorption spectra of the resonator system with three different clamped circular flexible plates with a diameter of 15 mm, 22.8 mm and 30 mm respectively. One can see that plates with a larger diameter indeed show a multi modal behavior as the absorption spectrum of the plate with a diameter of $d_2 = 22.8$ mm (red) now consists of three local maxima around 330 Hz (first radial eigenmode), 590 Hz (Helmholtz resonance) and 740 Hz (second radial eigenmode). The absorption spectrum of an even larger plate with a diameter of $d_3 = 30$ mm (yellow) consists of even more characteristic points, which, however, are not as distinct. Two larger maxima around 430 Hz and 610 Hz are
accompanied by slope changes around 350 Hz and 870 Hz. Note, that these resonance frequencies are similar but do not match the in vacuo eigenfrequencies, listed in Table2. The a priori determined eigenfrequencies, therefore give a hint of the expected number of significant plate modes in the resonator system but are not sufficient to predict the complete resonance behavior. The first slope change may be a very weak excited first radial plate mode, even though one would expect its resonance frequency below that of $d_2$. A possible explanation is that the flexible plate with the largest diameter $d_3$ was slightly pre-stressed during application, as a tensed plate is stiffened and consequently its resonance frequency is shifted towards higher frequencies. The second slope change is most likely a weakly excited higher radial plate mode. Note, that the global absorption maximum has shifted towards lower frequencies by more than 150 Hz when comparing the resonator system with the largest and smallest flexible plate. The plate diameter is, consequently, a crucial parameter to tune both the broadband (multi modal) behavior as well as the low-frequency behavior.

The plate velocity for the same configurations is depicted in Fig.4b. Focusing first on the plate with a diameter of $d_2 = 22.8$ mm (red), one can see that the first mode around 330 Hz is excited the most and that both the Helmholtz resonance as well as the second radial mode show a similar velocity. The plate velocity amplitude of the largest plate $d_3 = 30$ mm (yellow) is the lowest of all plate configurations. The highest values are around 350 Hz near the first slope change, which gives further reason to expect the first plate mode at this frequency region. Additionally, a substantial vibration can be detected for higher frequencies around 900 Hz, near the second slope change. Nevertheless, the vibration velocity of the largest diameter is quantitatively only roughly one fourth of the values of the other plate diameters. Thus, no clear trend between vibration amplitude and plate size can be found.

The changes in the normalized impedance are depicted in Fig.5a. The most prominent difference is the peak values of the resistance in Fig.5a. These peaks do not stem from amplitude dependent behavior near the Helmholtz resonance, as they do not occur at the resonance frequencies. On the contrary, they can be attributed to the antiresonance. The resonator with $d_1$ only inhibits one resistance peak, while the resonator with $d_2$ inhibits two and $d_3$ has three. This gives further reason to suggest that the number of significant flexible plate modes increases with the larger plate diameters, as there is always an antiresonance between two resonances. Additionally, the antiresonance peak for $d_2$ with the highest vibration velocity is the strongest. This suggests that a higher plate velocity strongly affects the resonance as well as the antiresonance behavior.

The flexible plate is embedded into a resonator system consisting of a face sheet, a main cavity, the flexible plate itself and a second cavity. A crucial parameter for the acoustic performance is therefore the perforated face sheet. The absorption and plate velocity spectra under variation of the face sheet is depicted in Fig.6a. One can see, that the global maximum of the absorption (blue) is shifted towards lower frequencies in the resonator with FS2 (dashed, $\sigma_{FS2} = 5.1\%$), which has a higher porosity compared to FS1 (solid, $\sigma_{FS1} = 2.6\%$). In contrast, the flexible plate resonance around 430 Hz is not shifted. However, the absolute value is increased, as the Helmholtz resonance is closer to the plate resonance. These trends can also be found in the plate velocity spectrum, depicted in red in Fig.6a. The plate velocity is increased around the resonance and follows the downshift of the Helmholtz resonance. A higher porosity therefore seems to be beneficial to increase the plate vibration. A possible explanation is that a higher porosity...
Normalized impedance $\zeta = \theta + i\chi$ for different flexible plate diameters.

Effect of different face sheets on the resonator with the smallest flexible plate diameter $d = 15$ mm.

leads to higher pressure differences in the cavities. Fig. 6b shows a comparison of the impedance of the resonator with variable face sheets which reveals that the resistance is higher for FS1. On the contrary, the reactance is higher for FS2 while the difference increases with respect to the frequency.

Similar trends can be found in the absorption and plate velocity spectra under variation of the face sheet of the larger plates, which are depicted in Fig. 7. It is interesting to note that the change of the face sheet affects both local absorption maxima in Fig. 7a and Fig. 7b. It is therefore beneficial to tune the face sheet in such a way, that the Helmholtz resonance frequency lies in between multiple radial plate modes to obtain a broad absorption spectrum with multiple absorption peaks.
Fig. 7  Absorption $\alpha$ (blue) and center point plate vibration amplitude $|v|$ (red) for different face sheets FS1 (solid) and FS2 (dashed) of the resonator with a flexible plate of diameter $d_2 = 22.8$ mm (left) and $d_3 = 30$ mm (right)

B. Numerical results
The experimental results are compared to the numerical results obtained with the FEM Software Actran. The absorption spectra obtained experimentally and numerically for FS1 and the smallest diameter $d_1 = 15$ mm are presented in Fig. 8. Additionally, the absorption calculated with the analytical model presented in Kohlenberg et al. [16] is depicted.

Fig. 8  Absorption $\alpha$ of the resonator with a flexible plate of diameter $d = 15$ mm and FS1 as determined experimentally (blue), numerically (red), and analytically (yellow)

All spectra show a good agreement regarding their resonance behavior. The analytical and FEM simulations predict both local maxima associated with the plate resonance (400 - 440 Hz) and Helmholtz resonance (610 Hz) respectively. The analytical and numerical simulations, however, predict a lower plate resonance and a stronger valley in around the antiresonance. An explanation for this might be that the plate used in the experiments was slightly pre-stressed.

A comparison between the measured and predicted absorption for the plate with an intermediate diameter of $d = 22.8$ mm is depicted in Fig. 9a. Again, a good agreement between the respective resonance frequencies is found. In
(a) Intermediate diameter \( d = 22.8 \text{ mm} \)

(b) Largest diameter \( d = 30 \text{ mm} \)

Fig. 9 Absorption \( \alpha \), as measured (blue) and numerically predicted (red) for different flexible plate diameters \( d = 22.8 \text{ mm} \) (left) and \( d = 30 \text{ mm} \) (right)

In this case a slight shift in the Helmholtz resonance (550 - 580 Hz) and in the second plate resonance (700 - 730 Hz) can be found, while the first resonance (340 Hz) matches the experiments very well. Nevertheless, the resonator system with two significant radial plate modes is predicted numerically reasonably well, too. The situation for the largest plate with the largest diameter of \( d_3 = 30 \text{ mm} \), shown in Fig. 9b, is more ambiguous. The highest absorption, namely at the second and third resonance peaks, are captured reasonably well in the simulation. However, the first peak in the simulation at 280 Hz is not as distinguishable in the experimental results and is most likely shifted towards 360 Hz, visible as a slope change in the blue curve. The highest absorption peak in the slope change is flattened in the experimental results but still visible as a slope change. Similar results can be found when using the other face sheet FS2 and are omitted here.

The experimental results with the impedance tube in combination with the single-point vibrometer hints that several absorption peaks are associated with different plate modes. Furthermore, we assumed that only radial plate mode shapes are excited; an assumption which needs proof. The numerically determined normalized plate deflection shape of the flexible plate inside the resonator system with the smallest diameter \( d_1 \) is presented in Fig. 10. The color map ranges from high deflection amplitude (deep red) to low deflection amplitude (deep blue). Note that the clamped boundary condition suppresses both edge rotation and displacement.

(a) First resonance (400 Hz)

(b) Second resonance (607 Hz)

Fig. 10 Numerically determined normalized plate deflection of the flexible plate inside the resonator evaluated at different resonance frequencies, smallest diameter \( d_1 = 15 \text{ mm} \)

In Fig. 10a we can see a very clear example of the first mode shape of a clamped circular plate with the highest deflection in the middle and the lowest at the boundary. The plate deflection shape at the Helmholtz resonance, depicted in Fig. 10b, is very similar. The Helmholtz resonance is in between the first and second radial plate resonance and we...
therefore expect the plate deflection to be a mélange of both. However, the resonance frequency of the second radial mode is too far away and the deflection shape at the Helmholtz resonance very much resembles the first mode shape.

(a) First resonance (340 Hz)  
(b) Second resonance (554 Hz)  
(c) Third resonance (702 Hz)

Fig. 11 Numerically determined normalized plate deflection of the flexible plate inside the resonator evaluated at different resonance frequencies, intermediate diameter $d_2 = 22.8$ mm

The situation is different for the plate with the moderate plate diameter $d_2$, depicted in Fig. 11. The first resonance (Fig. 11a) is still clearly associated with the first radial mode. The third resonance is the second plate resonance with one nodal circle, shown in Fig. 11c. At the second resonance (Fig. 11b), attributed to the Helmholtz resonance, the plate deflection is a mixture of both mode shapes as its frequency is almost in the middle between both plate associated resonances. Note that these mode shapes are purely radial without any circumferential dependency. Therefore, acoustic (circumferential) plane waves do not excite circumferential plate modes due to their symmetry.

The deflection shapes of the four distinct resonances in the numeric simulation are depicted in Fig. 12. The first resonance (Fig. 12a) has one nodal circle but seems to be a mixture between the first and second radial plate mode, as the amplitude of the outer ring is low in comparison to Fig. 11b. We usually expect the plate to be easily excited specifically at their resonance, however in this case the plate deflection shape at the first resonator resonance is a mixture between two resonances. We think that this is due to the influence of the face sheet, as the flexible plate is embedded into a resonator system and not in vacuo. The plate deflection shape at the second resonance (Fig. 12b), attributed to the Helmholtz resonance, is again a mixture of the adjacent radial mode shapes with two nodal circles. The plate deflection at the highest resonance peak (Fig. 12d) seems to be a mixture between second and third radial plate mode, as the outer ring is stronger excited compared to Fig. 12c but still with only two nodal circles. The overall absorption is a combination of all plate modes, and it becomes apparent that with more significant plate modes, their individual contributions are harder to distinguish.

Finally, the numerical simulations can be used to resolve the resonator system with varying flexible plate diameter. The simulations are performed varying the diameters between 10 mm and 30 mm with 1 mm steps. The data are then interpolated in between in order to obtain a finer representation of the absorption map, which is depicted in Fig. 13.

At small diameters the plate resonance and the Helmholtz resonance coincide and only one broad absorption peak around 580 Hz is visible. Up to a diameter of 18 mm, the plate resonance is highly sensitive regarding a change in the plate diameter while the Helmholtz resonance frequency is constant. With plate diameters greater than 18 mm, the first resonance is nearly constant while a third resonance is appearing and decreasing rapidly in frequency. The third resonance becomes significant when it approaches the second resonance for diameters greater than 20 mm. However, both peaks never meet, as the second peak is shifted towards lower frequencies. For plate diameters higher than 26 mm, the first resonance is shifted again towards lower frequencies with rapidly decreasing absorption. Additionally, a fourth absorption peak is visible around 800 Hz. The occurrence and origin for the different regions with respect to the plate diameter are still under investigation and are most likely dependent on the other resonator system’s properties such as dimensions of the main cavity and second cavity.
Fig. 12 Numerically determined normalized plate deflection of the flexible plate inside the resonator evaluated at different resonance frequencies, largest diameter $d_3 = 30$ mm

Fig. 13 Influence of the flexible plate diameter $d$ on the numerically determined normal incident absorption $\alpha$ with FS1
V. Conclusion

In this study we investigated Helmholtz resonators with a flexible wall with a special focus on the plate’s motion using an impedance tube in combination with a vibrometer. In the second part of the paper we compared our experimental results with numerical simulations with the FEM software Actran to investigate the flexible plate deflection shape and to better resolve resonator system’s dependency on the plate size.

Our experimental setup enabled us to successfully combine acoustic resonator properties (impedance, absorption), obtained via impedance tube measurements with the plate vibration (deflection, velocity), obtained by simultaneous vibrometer measurements. We showed that for this concept, the plate size is a crucial parameter. In fact, this alters the number of resonance frequencies, their absorption amplitudes and their resonance frequencies. The face sheet mainly alters the Helmholtz resonance and only to a minor degree the plate resonance at higher frequencies. We showed with the vibrometer that the different resonances can be clearly separated between Helmholtz and plate resonances. Additionally, we found that the flexible plate vibrates stronger, when plate resonances and the Helmholtz resonance are closer together. We found the analytical eigenfrequencies of the clamped circular plate to be a reasonable starting point to guess the system’s resonant behavior. If the in vacuo eigenfrequencies are near the Helmholtz resonance, additional absorption due to the plate vibration is to be expected. However, analytical or numerical simulations are needed to predict the overall resonant behavior of the proposed liner concept. When larger plates with a sufficiently low flexural rigidity are used, it is necessary to take higher radial plate modes into account. Additionally, we showed that the concept’s resonance behavior can be successfully modelled via FEM, although for larger flexible plates with multiple resonances, the agreement was less accurate. At the Helmholtz resonance, the plate vibrates in a mixed mode shape of the neighboring radial plate modes. We showed that the plane wave excitation only excites radial plate modes and that circumferential modes can be omitted. We found different regions where the plate resonance has a varying degree of sensitivity towards the plate diameter. Consequently, the FEM was found to be a suitable choice to resolve, analyze and extend the proposed concept.

Measurements with further instrumentation, such as microphone inside the main and back cavities should reveal more insights into the plate vibration near the Helmholtz resonance and antiresonances. Additionally, different rectangular plate shapes, which might fit better into a conventional liner system, should be investigated for future optimization of the concept. Furthermore, future research on Helmholtz resonators with flexible walls should focus on the dependencies of the cavity dimensions, as well as nonlinear influences such as high excitation amplitudes and a grazing flow onto the flexible wall.

Acknowledgements

The authors want to thank Yiping Li for her assistance with the numerical simulations. This work is part of the project "Akustische Wirkmechanismen eines Helmholtz-Resonator-Liners mit flexiblen Strukturelementen" funded by the Deutsche Forschungsgemeinschaft (DFG, German Research Foundation) – 416728553. Their financial support is gratefully acknowledged.

References


