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INFLUENCE OF THE BLENDING PARAMETER σ ON THE HYBRID RECURSIVE REGULARIZED BGK COLLISION SCHEME FOR TURBULENT LBM SIMULATIONS

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Summary. This work studies the influence of the blending parameter σ on the Hybrid Recursive Regularized (HRR) BGK collision scheme. The comparison is carried over by modeling a turbulent channel flow at a friction Reynolds number, Re_τ of 1000. The wall is modeled via the implicit Musker profile, and the turbulent viscosity is taken into account via the Vreman model. The velocity space is discretized with both $D3Q19$ and $D3Q27$ stencil. The domain is discretized with different resolution leading to y^+ values of the first cell at the wall of $12.5, 25$ and 50. The σ parameter ranges in the interval $[0.900, 1.000]$.

1 INTRODUCTION

The Lattice Boltzmann Method (LBM) increased its visibility in the recent years both in industry and academia thanks to, among others, the latest development in the fields of high Reynolds number applications and aeroacoustic. Spinelli et al. [1, 2] conducted a systematic study of collision schemes, subgrid scale (SGS) models, and wall functions on accuracy and efficiency for turbulent channel flow and flow past a circular cylinder. The comparison has shown that the Cumulant [3] collision scheme delivers the best results in terms of both efficiency and accuracy.

Here we would like to contribute to this comparison by focusing our attention on the influence of the blending parameter σ on the HRR scheme [4]. This parameter influences the numerical dissipation of the collision scheme, and its influence might be problem dependent. Here we analyze the range of σ that delivers the best results in terms of accuracy. The HRR scheme is further enhanced by utilizing the correction term as proposed by Feng et al. [5]. The velocity space is discretized with both $D3Q19$ and $D3Q27$ stencils. The comparison is carried over by modeling the turbulent channel flow at a Re_τ of 1000. The domain is discretized with three different resolutions that lead to a y^+ value of the first element at the wall of $12.5, 25$, and 50.

2 METHOD

The Boltzmann equation is the fundamental pillar which the Lattice Boltzmann Method is based on [6]. This equation is discretized in time, space and velocity space, obtaining the so called lattice Boltzmann equation:

$$f_i(\boldsymbol{x} + \boldsymbol{c}_i \Delta t, t + \Delta t) = f_i(\boldsymbol{x}, t) + \Delta t \Omega(f_i), \tag{1}$$

where f represents the Probability Distribution Function (PDF), $\boldsymbol{x}$ is the cell center location, $\boldsymbol{c}$ is the streaming vector, t is the time, and Ω is the collision operator. The macroscopic variables, such as velocity $\boldsymbol{V}$ and density ρ, are computed as the velocity-moments of the PDFs [6].

The collision scheme used in this study is the Hybrid Recursive Regularized BGK scheme [4]. In this contest, the post-collision PDFs have the following form:

$$f_i^{\star}(\boldsymbol{x}, t) = f_i^{\mathrm{eq}}(\boldsymbol{x}, t) + (1 - \omega) f_i^{\mathrm{neq}}(\boldsymbol{x}, t) + 0.5 \Delta t \Psi_i, \tag{2}$$

where Ψ_i is the correction term, and

$$f_i^{\mathrm{eq}}(\boldsymbol{x}, t) = w_i \sum_{n=0}^{N} \frac{1}{n!} \boldsymbol{a}_0^{(n)} : \boldsymbol{\mathcal{H}}_i^{(n)}, \tag{3}$$

$$f_i^{\mathrm{neq}}(\boldsymbol{x}, t) = w_i \sum_{n=2}^{N} \frac{1}{n!} \boldsymbol{a}_1^{(n)} : \boldsymbol{\mathcal{H}}_i^{(n)}. \tag{4}$$

The explicit expressions of the Hermite polynomials $\boldsymbol{\mathcal{H}}_i^{(n)}$ and the coefficients $\boldsymbol{a}_0^{(n)}, \boldsymbol{a}_1^{(n)}$ are given in [5]. The same reference reports the formulation of the correction term for both $D3Q19$ and $D3Q27$ stencils. The Hermite polynomials that form the Hermite tensor basis for the above mentioned stencils are studied in [7]. Nevertheless, the authors prefer to use the basis formulation for $D3Q19$ as suggested in [8] due to the simplistic formulation of the correction term.

The SGS turbulence is accounted by utilizing the Vremann model [9]. The coefficient of the model is set equal to 0.07. The wall is modeled via the Musker profile [10]. Since this profile is represented with an implicit function, we use the fixed-point iterative algorithm to solve for the friction velocity u_τ. In order to take into account the forcing term that drives the flow, we employed the force implementation as introduced in reference [11].

3 RESULTS AND DISCUSSION

A turbulent channel flow at a Re_τ of 1000 is used as testcase. The availability of DNS data for this testcase, makes it the perfect candidate for this study. The testcase description, boundary conditions and domain are described in [2]. As reference we use the DNS results of Lee and Moser [12]. The effects of a correction term [8] are studied for different y^+ of the first cell at the wall of 50, 25, and 12.5. Furthermore, the influence of the blending coefficient σ is studied.

Figure 1 shows the comparison of the normal Reynolds stress $\langle u'u' \rangle^+$ for different σ values of 0.90, 0.92, 0.94, 0.96, 0.98, and 1.00 at the considered y^+ values. The HRR scheme is discretized with the $D3Q19$. For all considered y^+ values, the peaks present in the DNS results are underestimated. In particular, the lower the σ, the more accurate are the peaks' values, but the location is shifted towards the boundary layer edge.

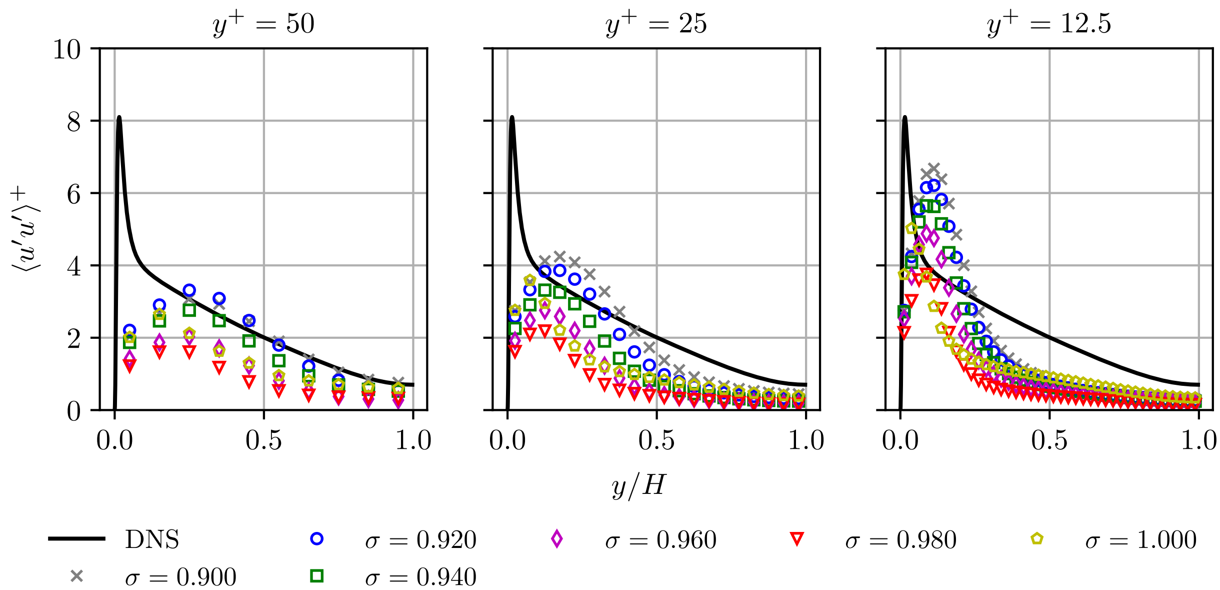

Figure 1: Comparison of the normal Reynolds stress $\langle u'u' \rangle^+$ for different σ values at the considered y^+ values. The corrected HRR scheme is discretized with the $D3Q19$ stencil.

Figure 2 shows the same comparison described above, where the only difference is that we utilized the $D3Q27$ stencil. For all considered y^+ values, the peaks present in the DNS results are underestimated again. As previously observed, the locations of the peaks are shifted towards the boundary layer edge when σ decreases. However, for the case of $y^+ = 50$, the best result is given by $\sigma = 1.00$.

We analyzed the relative error between the target value of the friction velocity at the wall of $0.0509\,\mathrm{m\,s^{-1}}$, and the numerical value obtained from the simulations. In general, our results show that the $D3Q19$ stencil is more accurate when the $y^+ \geq 25$ regardless the value of σ. While the $D3Q27$ stencil is more accurate for simulations with $y^+ \leq 12.5$, namely when the wall profile is used to model the flow close to the viscous sublayer region. As a quantitative comparison , we computed the $L2$-norms of all Reynolds stresses with respect to the DNS results. The correction term improves the accuracy of the results on average by 0.1%. For few cases the accuracy worsened. As conclusion, the $D3Q19$ stencil delivered the most accurate results with a σ value of 0.998, while the $D3Q27$ stencil does so with $\sigma = 1.000$.

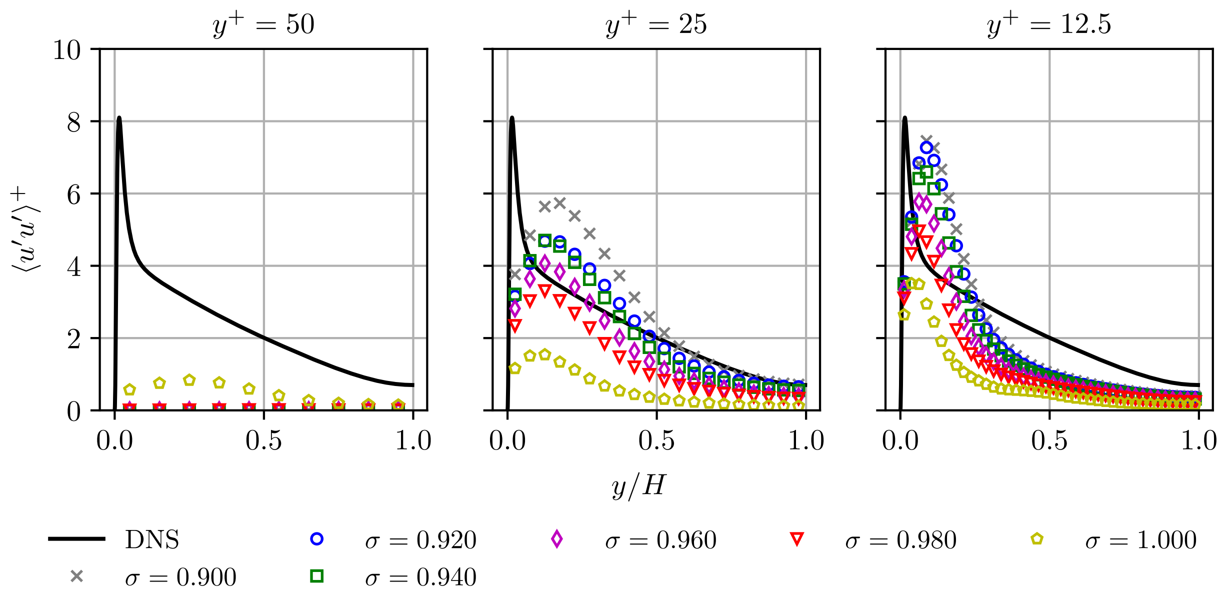

Figure 2: Comparison of the normal Reynolds stress $\langle u'u' \rangle^+$ for different σ values at the considered y^+ values. The corrected HRR scheme is discretized with the $D3Q27$ stencil.

4 CONCLUSIONS

We studied the influence of the blending parameter σ on the HRR collision scheme. The comparison is conducted by performing a wall modeled LES of a turbulent channel flow at a Re_τ of 1000. The wall profile utilized is the Musker, while subgrid scale turbulent is accounted with the Vreman model. Our results show that the $D3Q19$ stencil delivers the most accurate results for $y^+ \geq 25$ and with $\sigma = 0.998$. While, the $D3Q27$ stencil is suitable for simulations with $y^+ \leq 12.5$ with a $\sigma = 1.000$.

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