Biomass Estimation and Uncertainty Quantification
From Tree Height

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Abstract—We propose a tree-level biomass estimation model approximating allometric equations by LiDAR data. Since tree crown diameter estimation is challenging from spaceborne LiDAR measurements, we develop a model to correlate tree height with biomass on the individual-tree levels employing a Gaussian process regressor. In order to validate the proposed model, a set of 8342 samples on tree height, trunk diameter, and biomass has been assembled. It covers seven biomes globally present. We reference our model to four other models based on both, the Jucker data and our own dataset. Although our approach deviates from standard biomass–height–diameter models, we demonstrate the Gaussian process regression model as a viable alternative. In addition, we decompose the uncertainty of tree biomass estimates into the model- and fitting-based contributions. We verify the Gaussian process regressor has the capacity to reduce the fitting uncertainty down to below 5%. Exploiting airborne LiDAR measurements and a field inventory survey on the ground, a stand-level (or plot-level) study confirms a low relative error of below 1% for our model. The data used in this study are available at https://github.com/zhubiomassUQ.

Index Terms—Above-ground biomass (AGB) estimation, allometric equation, Gaussian process regression, model uncertainty, tree height.

I. INTRODUCTION

According to the Food and Agriculture Organization (FAO), nearly 31% of the global land surface is covered by forests [1]. Woodland is a valuable resource on Earth. Among others, it regulates the circulation of air and water. According to EIP-AGRI1, forests in Europe provide around three million jobs, and forest biomass contributes to about half of the generated renewable energy [2]. Above-ground biomass (AGB) is a biophysical parameter for the total amount of accumulated organic material in an ecosystem. It has been applied widely as an index of forest volume. Moreover, it is key to monitoring ecosystems and modeling climate change [1], [3]. In addition, biomass provides a tool to evaluate carbon sequestration. Accurate biomass estimates help assess loss caused by wildfires [4].

Forest AGB evaluation is categorized into three levels: 1) fine, 2) middle, and 3) coarse-grained, cf. Fig. 1. In-situ measurements of biomass include tree harvest and desiccation to weigh the wood on a scale [5]. Despite errors in the harvesting process and scale uncertainties, this method most accurately evaluates biomass on the tree level. However, such a destructive approach is costly in labor and time. As an alternative, allometric equations provide estimates of biomass on individual-tree levels [6], [7], [8], [9]. Tree biomass is considered a function of tree height, trunk diameter at breast height (also known as trunk diameter or simply diameter), wood density, crown diameter, etc. Luo et al. [10] review state-of-the-art allometric equations applied to tree species in China. However, the method presented is not easily transferred to a global scale.

To generate large-scale (regional to global scale) biomass maps, spaceborne hyperspectral and synthetic aperture radar sensors with wide swath and global or near-to-global coverage have been applied in the literature [11], [12], [13], [14]. Based on a regression model trained with ground reference and remote sensing data pairs, the total amount of biomass for each pixel in the remote sensing imagery can get estimated. However, limited by the coarse-grained resolution of remote sensing imagery, ground truth reference data is costly to collect. According to the work in [18] approximately 234 trees per ha grow in the Black Forest, Germany. Also, since the values of pixels in remote sensing imagery are not related to trees’ biomass one-to-one, the relative errors (REs) might raise up to 37% [1].

A tradeoff compared with in-situ biomass measurements is the estimation of tree-level parameters (such as height, crown diameter, etc.) from high-resolution remote sensing data as input to allometric equations [19], [20], [21], [22]. Although highly correlated with biomass, parameters such as wood density and diameter cannot reliably get estimated by aerial imagery. Jucker et al. confirmed that the height and crown diameter of trees are sufficient to estimate the trunk diameter by a single equation. Crown diameter and height are easily derived from

1For example, a 25-m spatial resolution of spaceborne LiDAR sensor GEDI [15], [16], [17].

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Fig. 1. Illustration of biomass products with fine (individual), middle, and coarse-grained (stand) levels.

Fig. 2. (a) Sample of rasterized statistics of LiDAR return count (for details in methodology cf. [34]). (b) Corresponding aerial imagery over Prospect Park in Brooklyn, New York, USA, in 2017. A and B mark areas of an isolated tree versus a cluster of trees, respectively. LiDAR statistics and NAIP imagery got harmonized by the Big GeoData platform PAIRS [35]. (a) LiDAR return count statistics. (b) RGB-color-coded aerial imagery (NAIP, USDA Farm Service) with one-to-one correspondence of geospatial area in (a).

A few studies investigated stand-level height–biomass allometry [28], [29], [30]. Due to the lack of tree density information, these methods either focused on regional biomass estimation [28] or additional metrics such as percentile heights [31] and horizontal structure index [30] are integrated into the allometric relationships. Alternatively, tree-level height–biomass allometry may be beneficial as it directly includes tree density information. But as indicated in [32], the height–diameter relationship varies even within a small scale given the compositional diversity. Therefore, to build a general tree-level height–biomass allometric relationship for fast biomass estimation, it is necessary: 1) the relationship is derived based on a dataset collected over large areas; 2) the regression model should be less biased, so that aggregation of individual tree biomass within homogeneous forests would reduce biomass errors. In this article, we evaluate such an approach: We assemble a ground truth dataset including 8342 measurements of tree height, trunk diameter, and biomass drawn from global sampling. We proposed a Gaussian process regressor (GPR), which is a noise-aware model to capture the nonlinear relationship of biomass and tree height and reduce the estimation bias.

In addition, uncertainty quantification is crucial for evaluating the confidence level of derived models. Uncertainty related to plot-level biomass estimation based on tree allometry mainly comes from two sources [5]: independent tree variables airborne laser scanning (ALS) data [19], [20], [23], [24], [25]. However, the crown diameter estimation is a source of significant model error. Deciduous tree’s crown changes with seasons, and extraction of the crown profile for individual trees in dense forests is a major challenge. As demonstrated by Fig. 2, the crown profile of isolated trees may get reliably estimated, cf. label A. However, in densely populated areas such as labeled B, a reliable separation is close to impossible. The analysis of high-resolution aerial imagery in [26] revealed the accuracy of estimated crown diameter significantly varied with plots (0.63 and 0.85) when compared with height estimations. According to the work in [27], the RE of tree crown diameter derived from airborne/UAV-borne LiDAR data is significantly larger than that of derived tree height (19.22% and 20.7% for crown diameter, 11.70% and 10.97% for tree height estimation, respectively). In fact, the crowns of individual trees in e.g. rainforests may significantly intersect.
derivation uncertainty due to inaccurate and/or insufficient measurements and estimation uncertainty led by residual model noise and imperfect reference data. In this article, we focus on the estimation uncertainty. Most existing publications reported their model’s residual noise by metrics such as root mean square error (RMSE) [1], [21]. On the other hand, the Monte Carlo simulation approach was used to quantify the parameter uncertainty by selecting different samples as training data [33]. Usually, they assume that the reference data are noise-free. However, due to the imperfect sampling strategy, the impact of reference data on the uncertainty quantification is nonnegligible [5]. In this article, we propose an algorithm to decompose the estimation uncertainty into model uncertainty and fitting uncertainty. This way, we analyze the model uncertainties of four allometric equations in contrast to the fitting uncertainties of five candidate models. Stand-level uncertainties of the proposed GPR and two other models are evaluated based on LiDAR measurements and field surveys.

II. DATASETS

Our experiments employ a dataset collected from the open-source allometry databases [8], [36], [37]. The dataset provided in [8] consists of 4004 tree measurements at 58 sites in tropical forests over the globe. The biomass and allometry database (BAAD) [36] includes 258,526 measurements over the globe collected from 175 studies. Each measurement records the tree height, tree components’ biomass, etc. The biomass data collected in [37] include 6604 records of trees in Eurasian forests (mainly in Russia). Then, the part of the measurements in the datasets was removed so that the left ones obey the following characteristics.

1) The total height and trunk diameter of the tree are recorded.
2) The geographic location of the tree is recorded.
3) The tree is harvested to measure its biomass.
4) The tree’s diameter exceeds 5 cm.
5) The tree’s biomass passes the threshold of 2 kg.

Information on ecoregions defining geospatial boundaries of biome types (TEOW) has been downloaded from the world wide life fund [38]. According to geographic location, each measurement is allocated by one of the seven biome types: tropical and subtropical forests, temperate mixed forests, temperate coniferous forests, boreal forests, grasslands and shrublands, tundra, savannas, woodlands, and Mediterranean forests, or deserts and xeric shrublands. Based on the previous research, the parameters of the allometric equations depend on species, climate, and environmental conditions. Instead of training multiple models for each species and ecoregion, we explored the potential of using a single model to capture the variation caused by ecological factors.

Here, we utilize a total number of 8342 pairs of tree height–diameter–biomass measurements. In addition, we employ the Jucker data [21] as a reference for the proposed model. In addition to height, diameter, and biomass, the Jucker dataset [21] also records crown diameter. Although the Jucker data includes trunk diameter information, it contains 2395 samples, only. Fig. 3(a) visualizes the geographic distribution and the number of records for various sites. We set the diameter of the blue circles in proportion to the number of measurements. Distinct biome regions are colored differently. The dataset has global coverage containing all four forest types defined by the Food and Agriculture Organization of the United Nations (FAO) report. Fig. 3(b)–(d) presents violin and box plots including median values (blue circles) and outliers (gray circles) for each: height, diameter, and biomass of every biome type, respectively. Figures in plot Fig. 3(b) indicate the number of measurements in the biome regions. All three tree parameters cover a wide range of measured values: Biomass may get as little as 2 kg, and it may exceed 300 tons; tree height varies from 1.2 to 138 m; tree diameters span a range from 5 cm up to more than 2 m. Fig. 4 plots the distribution of tree diameter $D$ versus tree height $H$ in a double-logarithmic scale. Point colors indicate the log-scaled amount of biomass. Obviously, biomass increases with tree height and trunk diameter. We observe: Besides, a small number of outliers, tree height, and trunk diameter are highly correlated.

III. METHODOLOGY

A. Allometric Equation

Biomass refers to the total amount of dry weight of organic material in a unit area, i.e., the unit of biomass has a dimension of, e.g., kilograms per square meter ($m^2$) or tons (t) per hectare (ha). A tree’s biomass accumulates from the biomass of stump, trunk, branches, twigs, and leaves [39]. To accurately measure the biomass of trees, trees are felled, and dried at 105°C for scaling. Large trees are impossible to gauge. Instead, wood densities $\rho_i$ and volumes $V_i$ of all tree components labeled $i$ get recorded to estimate the biomass as $\sum_i \rho_i V_i$. In [40], [41], and [42], the authors elaborated on the process of dry biomass measurements.

The study [43] demonstrates trunk biomass constitutes about 83% of the total biomass of the tree. In addition, based on measurements with total AGB larger than 2 kg in BAAD [36], leaf mass accounts for about 10% of a tree’s biomass. Consequently, trunk biomass estimation needs the most attention. Assuming trunk biomass modeled by a cone, the biomass $B = \rho V = \frac{1}{12} \pi \rho D^2 H$, where $B$, $D$, and $H$ denote the tree’s biomass, diameter, and height, respectively. $\rho$ is the average wood density of dried trees that remote sensing is unable to capture. A central assumption of our work reads: Tree height is able to predict tree diameter such that biomass is predominantly determined by tree height. Our experiments in Section IV indicate complex relationships beyond a log–log linear model. Hence, we establish a nonlinear model $B \sim GP(H)$, where GP corresponds to a GPR detailed in Section III-B. The mapping of height $H$ to biomass $B$ is sensitive to average wood density and the biome-dependent relationship of tree diameter versus tree height.

B. Gaussian Process Regressor

In order to model noisy biomass $B = B(H)$ depending on tree height measurements $H$ based on a set of samples

3https://fra-data.fao.org/
Fig. 3. Summary of data collected [8], [36], [37]: Geospatial distribution of the measurements plotted on top of the biome classification map. (a) Circle diameters represent the number of records at each geolocation. (b) Violin plots of the distributions of tree heights in meters. (c) Tree diameters in centimeters. (d) AGB in kilograms for various biomes. The number of records for each biome is shown as a number to the right.

\((H_1, B_1), (H_2, B_2), \ldots, (H_n, B_n)\), we employ the Gaussian process regression.

Gaussian processes implement distributions over sequences of variables \(y' = (y'_1, y'_2, \ldots, y'_n, y'_n+1, \ldots)\) fully parameterized by mean values

\[ \mu'_i = \langle y'_i \rangle \quad \text{(1)} \]

and the (symmetric) two-point correlation function

\[ K'_{ij} = \langle y'_i y'_j \rangle - \mu'_i \mu'_j = K'_{ji} \neq 0 \quad \text{(2)} \]

where we introduced the statistical averaging operator

\[ \langle f \rangle = \int_y f(y') \mathcal{N}(y'|\mu', K') \quad \text{(3)} \]

to compute the mean and variance of \(y'\) over the Gaussian distribution \(\mathcal{N}\). Higher-order (centralized) moments

\[ \langle y'_i y'_j y'_k \rangle, \quad \langle y'_i y'_j y'_k y'_l \rangle, \ldots \quad \text{(4)} \]

can get expressed as products of two-point correlation functions [44]. Thus, for the following:

\[ y' \sim \mathcal{N}(\mu' = 0, K') \quad \text{(5)} \]

two samples \(y'_i\) and \(y'_j\) from a multivariate Gaussian distribution with zero mean and covariance matrix \(K'\) defined by matrix elements \(K_{ij}\).

It is observed that in most physical systems (spatial), correlations exponentially decay proportionally to the length scale \(l\), \(\propto e^{-l}\). In fact, algebraic decays, i.e., \(\propto l^{-\alpha}\), indicate strongly correlated systems close to phase transitions. It is, therefore, reasonable to model the kernel

\[ K'_{ij} \propto \exp\left(-\frac{(x'_i - x'_j)^2}{l^2}\right) \quad \text{(6)} \]

where \(x'_i\) and \(x'_j\) is associated with either height measurements \(H\) or model inputs \(\hat{H}\). Fig. 5 depicts the two-point correlation for two measurements \(y'_i = B_i\) and \(y'_j = B_j\) far apart \((x'_i = H_i \ll H_j = x'_j)\), close \((H_i \approx H_j)\), and identical \((H_i = H_j)\).
In order to predict $N - n$ values $\hat{y}$ given a set of $n$ observation pairs $(x_1, y_1), (x_2, y_2), \ldots, (x_n, y_n)$, we cast both into an $N$-variate Gaussian distribution over variables $y' = (y, \hat{y})$ and corresponding (given) parameters $x' = (x, \hat{x})$ to sample from the resulting conditional Gaussian probability distribution\(^4\)

$$y'|y \sim \mathcal{N}(\kappa K^{-1} y, \hat{K} - \kappa K^{-1} \kappa^T)$$

where we decomposed the covariance matrix $K'$ according to $y' = (y, \hat{y})$

$$K' = \begin{pmatrix} K & \kappa \\ \kappa^T & \hat{K} \end{pmatrix}$$

with $\kappa^T$ the transposed matrix of $\kappa$. Note that $K$ depends on $x$, only. Similarly, $\hat{K}$ takes prediction inputs $\hat{x}$, only. In contrast, $\kappa$ entails a mix of $x$ and $\hat{x}$.

By design $K_{ii} = 0$ such that we may flexibly add uncorrelated Gaussian measurement noise to $x$ through

$$K_{ij} \rightarrow K_{ij} + \sigma^2 \delta_{ij}$$

where $\sigma \in \mathbb{R}$ quantifies the variance of the uncorrelated measurement noise. The Kronecker-delta $\delta_{ij}$ turns zero for all indices except for $i = j$ where it assumes the value 1. In addition, we may want to explicitly model a mean(expected) (biomass) function $m(x') = b(H')$ that translates into

$$\kappa K^{-1} y \rightarrow m(\hat{x}) + \kappa K^{-1} [y - m(x)]$$

\(^4\)The derivation follows standard statistics textbooks such as in \cite{46}, Sec. 2.3.1], where $K = \Sigma_{aa}, \hat{K} = \Sigma_{kb}$, and $\Sigma_{ab} = \kappa = \Sigma_{b0}^T$ with $\mu_a = \mu_b = 0$.

In summary: The following is given.

1) The one-dimensional radial basis function kernel

$$K'_{ij} = \exp(-(H'_i - H'_j)^2/2\lambda^2)$$

at length scale $\lambda$.

2) Measured height values $H = (H_1, H_2, \ldots, H_n)$ and model input values $\hat{H}$. We statistically model $B = B(\hat{H} | B, H)$ through a (mean-shifted) multivariate Gaussian-distributed $B \sim \mathcal{N}(\mu, \Sigma)$.

3) At mean value $\mu$

$$\mu - b_0 = \kappa (K + 1\sigma)^{-1} (B - b_0)$$

$$\log K_{ij} = -(H_i - H_j)^2/2\lambda^2$$

where $1$ denotes the unit matrix, $b_0$ is a constant hyperparameter mean biomass, i.e.,

$$m(x') = b(H') = b_0 = \text{const.}$$

getting optimized alongside with $\lambda$.

2) Associated covariance matrix $\Sigma$

$$\Sigma - \hat{K} = -\kappa (K + 1\sigma)^{-1} \kappa^T$$

Note that the various elements of $K'$ contain both, data tree heights $H_i$ of sample biomass terms $B_i$, and values $\hat{H}_i$ for biomass values $\hat{B}_i$ to predict. Also, the constant offset $b_0$ could get replaced by a more generic (known) functional dependence, e.g., a linear model $b(H') = b_1 H' + b_0$, etc.

The scalar hyperparameters $\lambda$ and $b_0$ get optimized by maximization of the predictor variables $\hat{B}$-marginalized likelihood

$$B|\hat{B} \sim p(B|H) = \int_B p(B, \hat{B}|H)$$

$$\propto \exp(-(B - b_0)^T (K + 1\sigma^{-1})^{-1} (B - b_0)/2 \sqrt{\det(K + 1\sigma^2)}$$

i.e., when taking the logarithm, it is minimized the scalar (loss) function

$$L(\lambda, b_0) = \log p(B|H)$$

$$\propto \sum_i \beta_i^2/(k_i + \sigma^2) + \log(k_i + \sigma^2)$$

where we exploited $\log \det = \text{Tr} \log [46]$, and defined the $l$-dependent eigenvalues $k_i$ of the symmetric matrix $K + 1\sigma$. $\beta_i$ denotes the $b_0$-dependent components of vector $B - b_0$ in the eigenbasis of $K$.

When using the Gaussian process for tree biomass estimation, the mean biomass offset can be predicted as a linear combination of observed biomass offsets weighted by the covariance matrix (closer samples have higher weights), as formulated in (12). In (16), the prediction uncertainty (covariance matrix) consists
of two parts, the inherent noise level, and the height distances between the observed data and the prediction inputs.

Notice: GPRs can get cast into the framework of nonparametric Bayesian models. An approach that has proven efficient in many nonlinear regression tasks [47], [48], [49].

### C. Evaluation Methods

We compare the Gaussian process biomass–height model with a random forest (RF) model and three allometric equations, specifically: biomass–height–crown diameter (LR), biomass–height (LR2), and biomass–height–diameter (LR3). Random forest is a data-driven nonlinear regressor, which has been widely applied to biomass estimation [1]. The form of the three allometric equations read

\[
\text{LR}: \ln B = a \ln (H \times CD) + b + \epsilon \quad (20)
\]

\[
\text{LR2}: \ln B = a \ln H + b + \epsilon \quad (21)
\]

\[
\text{LR3}: \ln B = a \ln H + b \ln D + c + \epsilon \quad (22)
\]

where \(a, b, c\) are the coefficients and bias terms determined by the training data; CD refers to the crown diameter; and \(\epsilon\) is model residuals. Since no crown diameter measurements in curated data in Section II, we utilize an alternative biomass–diameter model

\[
\text{LR}: \ln B = a \ln (D) + b + \epsilon. \quad (23)
\]

1) **Tree-Level Results Evaluation:** To evaluate model accuracy, three indices get derived: \(R^2\)-squared (\(R^2\)), RMSE, and model bias. \(R^2\)-squared refers to the coefficient of determination, and is defined according to

\[
R^2(y, \hat{y}) = 1 - \frac{\sum_{i=1}^{n}(y_i - \hat{y}_i)^2}{\sum_{i=1}^{n}(y_i - \bar{y})^2} = \frac{\text{ESS}}{\text{TSS}} = 1 - \frac{\text{RSS}}{\text{TSS}}, \quad (24)
\]

where \(y_i\) and \(\hat{y}_i\) are the \(i\)th ground truth and predicted values. \(\bar{y}\) amounts for the average mean of ground truth. ESS, TSS, and RSS abbreviate the definitions of the explained sum of squares, total sum of squares, and residual sum of squares in line with

\[
\text{ESS} = \sum_{i=1}^{n}(\hat{y}_i - \bar{y})^2 \quad (25)
\]

\[
\text{TSS} = \sum_{i=1}^{n}(y_i - \bar{y})^2 \quad (26)
\]

\[
\text{RSS} = \sum_{i=1}^{n}(y_i - \hat{y}_i)^2. \quad (27)
\]

According to these definitions, the \(R^2\)-squared score may receive impact by a single, strongly biased estimation. Thus, calculating \(R^2\), we exclude outliers when the corresponding absolute error exceeds the mean absolute error (MAE) by at least three times, cf. red circles in Fig. 7. RMSE is calculated as follows:

\[
\text{RMSE}(y, \hat{y}) = \sqrt{\frac{1}{n} \sum_{i=1}^{n}(y_i - \hat{y}_i)^2}. \quad (28)
\]

Bias relates to relative systematic error. It is defined as

\[
\text{Bias}(y, \hat{y}) = \frac{1}{n} \sum_{i=1}^{n} \frac{\hat{y}_i - y_i}{y_i}. \quad (29)
\]

The negative or positive value of the bias indicates biomass under- or overestimation.

In the following, we use a binning method to visualize prediction errors for input dimensions. That is, the residuals between observed and predicted biomass \(y_i - \hat{y}_i\) are calculated. According to the percentile, input values such as height get assigned on a logarithmic scale, and residuals are split into separate groups (bins). The interval spanning the mean plus–minus half a standard deviation for each bin is presented alongside the fitted curve. The result visually depicts the mean and standard deviation of the prediction errors. In addition, it illustrates the level of over- or underestimation.

2) **Plot-Level Results Evaluation:** We quantify uncertainty on stand level by RE and relative RMSE (denoted as \%RMSE). Since in-situ data for biomass are unavailable, the biomass obtained by the LR3 model trained on our curated dataset using the filed inventoried tree heights and diameters serves as ground truth.

By aggregating individual tree information, the RE denotes the ratio of the sum over residuals and the sum over predicted biomass values by LR3:

\[
\text{RE} = \frac{\sum_{i=1}^{N}[LR3(H_i, D_i) - f(x_i)]}{\sum_{i=1}^{N}LR3(H_i, D_i)}. \quad (30)
\]

Here, \(H_i\) and \(D_i\) represent tree height and diameter of the \(i\)th tree in the plot; \(f\) indicates one of the candidate models, and \(x_i\) signals model input parameter(s). The relative RMSE refers to the ratio of RMSE and the mean of biomass predicted by LR3

\[
\%\text{RMSE} = \sqrt{\frac{1}{N} \sum_{i=1}^{N} \frac{(LR3_i - f_i)^2}{\sum_{i=1}^{N}LR3_i}} \quad (31)
\]

where \(i\) indexes the \(i\)th plot.

3) **Uncertainty Evaluation:** In the following, we elaborate on the uncertainty evaluation algorithm in use. We concern with two sources of uncertainties, namely: 1) model uncertainty and 2) fitting uncertainty, cf. Fig. 6. Model uncertainty indicates variance rooted in model selection such as the choice of input parameters for allometric equations, etc. In practice, tree biomass depends on many factors such as annual rainfall, species, and average annual temperature. The model at hand might bear the limited capacity to capture such dependencies. As a result, the mapping from input to biomass remains noisy with the model unable to capture such residuals. We define model uncertainty by the variability of measured biomass. Concerning the wide range and heavy tail of the tree biomass distribution, we work with log-scaled quantities. Specifically, the model uncertainty index is calculated as follows: 1) measurements get sorted by the input parameter such as tree height, crown diameter, etc., and are grouped into \(n\) buckets; 2) for each of the \(n\) groups, the ratio of standard deviation to mean of the biomass is calculated on a logarithmic scale; 3) the overall model uncertainty is


Fig. 6. Illustration of model uncertainty versus fitting uncertainty. Dots and the solid lines refer to the sample measurements and the averaged biomass, respectively. Two sources of uncertainties we focus on are the model uncertainty that corresponds to the standard deviation of the sample measurements, and the fitting uncertainty—that is, the deviation of averaged biomass and the regressor-predicted biomass.

calculated as the averaged ratios. In some allometric models, biomass is correlated with multiple parameters, such as height and diameter. Subsequently, the measurements are sorted by one of the parameters.

In general, increasing the number of input parameters has the potential to decrease model uncertainty at the price of additional effort to collect data. An alternative provides training separate models for each biome, species, and age. On the downside, this approach requires vast amounts of in-situ measurements harvesting trees. Also, there exists an option to reduce model uncertainty for stand-level products where spatial aggregation of tree biomass may cancel over- and underestimation [1].

When employing various forms of regression models, the fitting precision of the regressors varies. We refer to the discrepancy in average biomass and predicted biomass as fitting uncertainty. Computation of the fitting uncertainty is similar to model uncertainty calculation: 1) The measurements are sorted by input parameter and assigned into \(n\) evenly spaced pockets; 2) the absolute error of predicted biomass subtracted by the mean observed biomass is determined for each measurement of every group; 3) for each of the \(n\) groups, the ratio of MAE to mean observed biomass is computed on a logarithmic scale; 4) the overall fitting uncertainty is computed as an averaged ratio.

We are going to demonstrate that GPRs reduce the fitting uncertainty with regard to the random forest and linear regression models.

IV. EXPERIMENTS AND RESULTS

A. Jucker Data

We adopt the Jucker data in order to benchmark the GPR in reference to the other models introduced. The dataset includes 2395 measurements including records on crown diameter. In order to compare with and validate trained models such as LR proposed by Jucker et al., we filter training data to exclude diameters smaller than 5 cm. We apply a random split into training and test sets in proportion to 9:1.

Table I summarizes the performance of the five regressors. LR3, the biomass–height–diameter in Section III-C performs best in terms of all three indicators picked. \(R^2\), RMSE, and Bias yield values 0.95, 424.68 kg, and 0.08, respectively. As detailed in Section III-A, a linear model on a log scale is insufficient to fit tree height–biomass data, and the LR2 model performance is reflected by an increased RMSE (1.47 Mg), the most prominent bias (0.29), and an \(R^2\) score equating to 0.53. Nonlinear models—such as random forest and GPR—reduce the RMSE to 1.15 Mg and 1.12 Mg, respectively.

Our experiment indicates a low RMSE for the LR, RF, and GPR models, namely: 1.11 Mg, 1.15 Mg, and 1.12 Mg. Compared with LR (20) (the most widely used model), RF and GPR yield higher R-square scores by margins of 21% and 27%, respectively. Consistently, the bias drops by 19% and 15%.

Based on the above findings, our tree height-only GPR provides a serious option for biomass modeling when compared with state-of-the-art biomass–height–crown diameter models.

The left column in Fig. 7 lists fitted curves (blue lines) and corresponding error distributions (blue areas) for the five models we did investigate. The background resembles density maps of biomass–input parameter pairs. We observe the LR3 model fits best with the data, it yields the lowest uncertainty, cf. it exhibits the most narrow range of green-dashed, vertical lines in the plots of the rightmost column of Fig. 7. In Fig. 7(a), although the fitted line does not align perfectly with the data, the actual biomass is linearly correlated with the product of tree height and crown diameter, which implies that a linear log–log model can describe the relationship between them.

In terms of single-parameter models, LR2 overestimates biomass predictions for medium-range tree height values, and it strongly underestimates the biomass for small and large heights—a linear model does not properly capture the nonlinear biomass–height relationship. Both, the random forest and GPR render well with the data. However, the fitted curve of the RF model is less regular compared with GPR bearing the risk of less robustness with respect to outliers.

The scatter plots of Fig. 7 (center column) contrast modeled biomass with observed ground truth. Red circles label outliers. All plots exhibit a strong correlation between predicted and observed biomass. Results in Fig. 7(h) is best aligned with the diagonal \(y = y(x) = x\) suggesting the biomass–height–diameter

<table>
<thead>
<tr>
<th>Model</th>
<th>(R^2)</th>
<th>RMSE (kg)</th>
<th>Bias</th>
</tr>
</thead>
<tbody>
<tr>
<td>LR</td>
<td>0.664463</td>
<td>1108.855</td>
<td>0.26314</td>
</tr>
<tr>
<td>LR2</td>
<td>0.53256</td>
<td>1466.553</td>
<td>0.2931</td>
</tr>
<tr>
<td>LR3</td>
<td>0.950643</td>
<td>424.6752</td>
<td>0.079669</td>
</tr>
<tr>
<td>RF</td>
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<td>1147.066</td>
<td>0.20911</td>
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<tr>
<td>GPR</td>
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<td>1117.9</td>
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</tbody>
</table>

We adopt the Jucker data in order to benchmark the GPR in reference to the other models introduced. The dataset includes 2395 measurements including records on crown diameter. In order to compare with and validate trained models such as LR proposed by Jucker et al., we filter training data to exclude diameters smaller than 5 cm. We apply a random split into training and test sets in proportion to 9:1.

Table I summarizes the performance of the five regressors. LR3, the biomass–height–diameter in Section III-C performs best in terms of all three indicators picked. \(R^2\), RMSE, and Bias yield values 0.95, 424.68 kg, and 0.08, respectively. As detailed in Section III-A, a linear model on a log scale is insufficient to fit tree height–biomass data, and the LR2 model performance is reflected by an increased RMSE (1.47 Mg), the most prominent bias (0.29), and an \(R^2\) score equating to 0.53. Nonlinear models—such as random forest and GPR—reduce the RMSE to 1.15 Mg and 1.12 Mg, respectively.

Our experiment indicates a low RMSE for the LR, RF, and GPR models, namely: 1.11 Mg, 1.15 Mg, and 1.12 Mg. Compared with LR (20) (the most widely used model), RF and GPR yield higher R-square scores by margins of 21% and 27%, respectively. Consistently, the bias drops by 19% and 15%.

Based on the above findings, our tree height-only GPR provides a serious option for biomass modeling when compared with state-of-the-art biomass–height–crown diameter models.

The left column in Fig. 7 lists fitted curves (blue lines) and corresponding error distributions (blue areas) for the five models we did investigate. The background resembles density maps of biomass–input parameter pairs. We observe the LR3 model fits best with the data, it yields the lowest uncertainty, cf. it exhibits the most narrow range of green-dashed, vertical lines in the plots of the rightmost column of Fig. 7. In Fig. 7(a), although the fitted line does not align perfectly with the data, the actual biomass is linearly correlated with the product of tree height and crown diameter, which implies that a linear log–log model can describe the relationship between them.

In terms of single-parameter models, LR2 overestimates biomass predictions for medium-range tree height values, and it strongly underestimates the biomass for small and large heights—a linear model does not properly capture the nonlinear biomass–height relationship. Both, the random forest and GPR render well with the data. However, the fitted curve of the RF model is less regular compared with GPR bearing the risk of less robustness with respect to outliers.

The scatter plots of Fig. 7 (center column) contrast modeled biomass with observed ground truth. Red circles label outliers. All plots exhibit a strong correlation between predicted and observed biomass. Results in Fig. 7(h) is best aligned with the diagonal \(y = y(x) = x\) suggesting the biomass–height–diameter
Fig. 7. Plots of the fitted curves with corresponding prediction errors (left column), scatters of predicted and observed biomass shown in the middle column, and the distributions of errors depicted by the contents of the right column. The evaluation is based on the Jucker data. Each row corresponds to one of the five models—from top to down: (a)–(c) LR, (d)–(f) LR2, (g)–(i) LR3, (j)–(l) RF, (m)–(o) GPR.
model as preferred. Unfortunately, in many remote sensing scenarios, estimating tree diameter is out of reach. The data in Fig. 7(e) document the LR2 model tend to overestimate the biomass when the observed biomass is around 40 kg while underestimating above about 200 kg. In terms of height-only models, random forest and GPR, Fig. 7(k) and (n), are ruled by comparable performance with a lower bias for the full range of input data when referenced to their linear counterparts.

The right column in Fig. 7 evaluates the density distributions of residuals $B_i - \hat{B}_i$. Dashed lines are the 20th (left) and 90th (right) percentiles of the errors. Obviously, LR and LR2 models are significantly biased with positive errors dominating. Besides, RF and GPR distribute errors alike.

### B. Collected Data

The five candidate models are then trained and tested using the collected data in Section II. Since the dataset does not record crown diameter, as an alternative to LR in (20), we exploit the biomass–diameter model of (23). Table II and Fig. 8 present corresponding results. The plots in Fig. 8 are arranged in line with Fig. 7.

The LR and LR3 models yield significantly less bias—0.14 and 0.11, respectively—when compared with the other models exceeding 0.34. $R^2$ scores for LR and LR3 read 0.73 and 0.78, respectively. Our findings suggest the following:

1) tree diameter is relevant in biomass estimation;
2) tree height information improves model accuracy.

It seems a linear regressor is sufficient to render the biomass–tree diameter relationship. The dominant RMSE errors (8.2 Mg) stem from outliers (red circles) in Fig. 8(h).

A linear biomass–height model results in most poor performance, with $R^2$ as low as 0.25, and a bias of 0.50. The plot in Fig. 8(e) illustrates a significant underestimate of model predictions versus ground truth when the observed biomass exceeds 1 Mg. We conclude the log–log linear model misses to represent the AGB–tree height relationship. In fact, the nonlinear models outperform the linear model in terms of all three indicators. Moreover, the residual errors in Fig. 8(l) and (o) better center on zero compared with the results of Fig. 8(f); an indication of the nonlinear models more closely agree with the test data. Compared with the random forest model, the GPR is less biased. However, it ships with a larger RMSE of 5.0 Mg and a lower $R^2$ score equal to 0.66. The exceptionally high $R^2$ score roots in top generalization ability for $B > 2$ Mg. In Section V, we demonstrate that the GPR model outperforms RF.

### C. Uncertainty Evaluation

We consult the Jucker data [21] to quantify model uncertainties. Fig. 9 contrasts the model uncertainties of the four models: biomass–height, biomass–diameter, biomass–crown diameter, and biomass–height–crown diameter, respectively. It suggests that diameter is closely related to biomass. The biomass–diameter model exhibits the lowest model uncertainty of about 14%. The biomass–height model reaches medium performance at overall model uncertainty of 18.25%. The overall model uncertainty of the single-parameter biomass–crown diameter model is 30%. As a result, the biomass–height–crown diameter relationship—cf. the LR model in Section III-C—is plagued by major model uncertainty of about 20.6%. Here, we contrast single-parameter models, only. Multiple-parameter models, such as the biomass–height–diameter model of Section III-C, reduce uncertainty.

Fig. 10 aggregates fitting uncertainties of our five candidate biomass models learned from the Jucker data. In general, fitting uncertainties stay below model uncertainties. The overall fitting uncertainties of LR, LR2, LR3, random forest, and the proposed GPR read 8.80%, 11.45%, 6.13%, 6.90%, and 4.50% respectively. We conclude estimation errors are dominated by model uncertainty: All five models exhibit higher fitting uncertainty when the observed biomass is less than 2.5 log kg) with the GPR model (marked by a star) performing best. Because of the nonlinear biomass–height relationship, the LR2 model (marked by a diamond) scores highest with respect to fitting uncertainty, LR indicates medium performance, whereas LR3 and RF unveil performance scores on equal level. The GPR model demonstrates the lowest overall fitting uncertainty. Moreover, it constantly performs in all the groups suggesting the GPR over the other models in terms of low fitting uncertainty.

### V. VALIDATION BY LiDAR DATA

Finally, we study the uncertainty of trained models on stand level. We utilize a dataset that get assembled from forests in Baden-Württemberg, Germany in the years 2019 and 2020 [27]. It embraces 12 separate plots, each covering a spatial area of about one hectare. For each plot, point clouds of individual trees get segmented from terrestrial, UAV-borne, and airborne LiDAR devices. Field inventory measurements are available for a fraction of trees, too. We exclude from the validation process three out of the 12 plots with less than 20 trees available.

For each tree, its height, diameter, and crown diameter are derived either by field measurements and LiDAR point cloud data, and discard from the analysis trees without field measurements. For a single tree, there may exist multiple LiDAR measurements and the number of measurements fluctuates from plot to plot. Therefore, we average all measurements. Note that these measurements are incomplete, for example, tree heights were not inventoried (or measured by LiDAR data). In those cases, LiDAR-measured (or inventoried) variables were used instead. In Fig. 11(a), we present box plots of the three biomass derived either by field measurements and LiDAR point cloud data, and discard from the analysis trees without field measurements. For a single tree, there may exist multiple LiDAR measurements and the number of measurements fluctuates from plot to plot. Therefore, we average all measurements. Note that these measurements are incomplete, for example, tree heights were not inventoried (or measured by LiDAR data). In those cases, LiDAR-measured (or inventoried) variables were used instead. In Fig. 11(a), we present box plots of the tree biomass grouped by stands where it is indicated the following biomass statistics from bottom to top: minimum, first quartile, median, third quartile, and maximum. Outliers get represented by black,
Fig. 8. Plots of model fits including corresponding prediction errors (left column), scatters of predicted and observed biomass (middle column), and the distributions of errors (right column) based on curated data, cf. Section II. Each panel corresponds to one of the five models, i.e., LR: (a)–(c), LR2: (d)–(f), LR3: (g)–(i), RF: (j)–(l), GPR: (m)–(o).
empty circles. The tree biomass value dominantly varies from 0 to 2 Mg. Its distributions exhibit distinct characteristics for each plot. We note that the fluctuating number of outliers has the potential to impact model uncertainty.

The computation serves as a basis to compare three candidate models: LR from Section III-C trained on Jucker data, and models RF and GPR trained on the data curated in Section IV-B. In order to derive stand-level biomass estimates, we sum up the values of individual trees. This way, the over- and underestimation of biomass, in large parts, cancel. For each plot, Fig. 11(b) presents the resulting RE values of the three models RF, GPR, and LR. RF most poorly performs for all plots except KA09. However, LR underestimates the biomass in seven out of nine plots; thus, there is a significant bias for errors to accumulate. In Table III, we compute the RE of the three models given all reference data. We notice that the LR model rendering is more biased compared with the GPR. The scatter plot in

Fig. 9. Study of model uncertainties when working with tree height $H$, tree diameter $D$, crown diameter $CD$, and the product $H \times CD$ as an input parameter of the allometric equation. The overall model uncertainties read: 18.25%, 14.13%, 29.81%, and 20.57% respectively.

Fig. 10. Biomass-dependent fitting uncertainties for the five candidate models trained on the Jucker data. The overall fitting uncertainties of the five candidate models reduce to 8.80%, 11.45%, 6.13%, 6.90%, and 4.50%, respectively.

Fig. 11. (a) Box plot of predicted biomass by the LR3 model on individual level grouped by the plot. (b) RE of the three models in each plot where the LR3 model is assumed ground truth. (c) Scatter plot of model predicted biomass versus LR3 model reference.
TABLE III
COMPARISON OF MODEL PERFORMANCE IN TERMS OF RE AND RELATIVE RMSE FOR CANDIDATE MODELS: LR, RF, AND GPR

<table>
<thead>
<tr>
<th></th>
<th>LR</th>
<th>RF</th>
<th>GPR</th>
</tr>
</thead>
<tbody>
<tr>
<td>RE</td>
<td>0.0677</td>
<td>-0.1712</td>
<td>0.0021</td>
</tr>
<tr>
<td>%RMSE</td>
<td>0.1693</td>
<td>0.4608</td>
<td>0.2446</td>
</tr>
</tbody>
</table>

Fig. 11(c) supports that LR tends to over- and underestimate tree biomass when assuming values less and larger than 500 kg, respectively. GPR-predicted biomass estimates well correlate with the predicted biomass of the LR3 model reference.

As listed in Table III, the relative RMSE of models LR, RF, and GPR assume values of 16.93%, 46.08%, and 24.46%, respectively. Although GPR is less accurate compared with the LR model, the relative RMSE of GPR is acceptable when referenced to the state-of-the-art biomass estimation errors on a national and global scale, cf. [1] quoting %RMSE values in 37%–67%.

VI. CONCLUSION

We proposed a GPR model to estimate biomass on individual-tree levels taking tree height as input only. It enables rapid regional-to-national AGB evaluation from high-resolution LiDAR data. As a single-input parameter model, a series of existing allometry databases contribute to model training. We benchmarked GPR against four established biomass models training on Jucker data and a dataset curated by this work. Results confirm GPR performs best when compared with two biomass–height models, and it achieves reasonable results in reference to a biomass–height–crown diameter model. GPR generates a low fitting uncertainty of 4.50%. The stand-level uncertainty analysis of GPR yielded an averaged relative RMSE of 24.46%. Moreover, GPR renders less biased at a mean RE of 0.0021. Future work may explore a stratified approach where biome-specific models [21] have the potential to decrease model uncertainty.

REFERENCES

SONG et al.: BIOMASS ESTIMATION AND UNCERTAINTY QUANTIFICATION FROM TREE HEIGHT


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