



Numerical Simulation of Amplified Spontaneous Emission in Yb:YAG Thin-Disk Lasers

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Numerische Simulation von verstärkter Spontanemission in Yb:YAG Dünnscheibenlasern

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Chapter 1

Introduction

High-power lasers have a wide field of application, ranging from material processing [1, 2], to space debris removal [3–5] to the investigation of basic physics [6–8]. The thin-disk laser is a well established laser design for reaching a high laser output-power with high optical-optical efficiency and high brightness of the laser beam [9].

The efficient thermal management coming with the geometry of the thin-disk design allows a high pump intensity for an active medium with only moderate thermal properties [10]. A very good beam quality is achievable for typical thin-disk lasers, due to the good thermal management reducing the radial heat flux in the thin disk [11, 12]. The thin disk in combination with a multipass pumping scheme allows the efficient operation of a quasi-three-level medium like Yb:YAG [13].

The very small aspect ratio of the thin disk supports the buildup of parasitic oscillations in the thin disk [14]. Spontaneously emitted photons are amplified within the thin disk, which leads to an energy redistribution. Thus the population inversion is lowered in the pumped area, while being increased in unpumped regions of the thin disk [15]. Amplified spontaneous emission can be suppressed by an undoped cap, bonded to the front of the thin disk [16–18].

As amplified spontaneous emission is a strong nonlinear effect, many numerical models have been presented to estimate the influence by amplified spontaneous emission on the efficiency and power scaling ability of a thin-disk laser. Lowenthal *et al.* [19] presented a numerical model contributing for amplified spontaneous emission with a radially constant gain. Subsequent, Sasaki *et al.* [20] presented numerical simulations regarding a nonuniform spatial distribution of the gain for a high-power KrF-laser. Investigations on a slab-laser regarding the spectral power distribution and spatial distribution of amplified spontaneous emission were presented by Goren *et al.* [21] using Monte-Carlo-type simulations. Speiser [22] introduced an analytic model based on the laser rate equations regarding reflections at the top surface and the bottom surface by



introducing an effective volume for the calculation of amplified spontaneous emission. The transient numerical simulations by Speiser [23] consider reflections at the surfaces of the thin disk and scattering at the cylinder jacket in the calculations under the consideration of a radial gain distribution. Peterson *et al.* [17] accounts for the heating of the thin disk and a spectral power distribution by a Lorentzian spectral power distribution of amplified spontaneous emission. Scattering of amplified spontaneous emission at the cylinder jacket was investigated by Su *et al.* [24] with a bidirectional scattering model. [25, 26] focused on the non-uniform temperature distribution and Chen *et al.* [15] shows an analysis of the spectral power distribution of amplified spontaneous emission.

In the following, a numerical model for the transient evolution of the thermal and optical parameters of an Yb:YAG thin-disk laser is presented. A finite element method implemented in FEniCS [27] is used to calculate the non-uniform temperature distribution by local heating of the thin disk to regard the temperature dependent spectral and thermal properties of the quasi-three-level medium. As amplified spontaneous emission depends on the local gain along the amplification path, an angle-dependent reflectivity of the high-reflective coating and the anti-reflective coating is regarded in the numerical model. The spectral power distribution of amplified spontaneous emission is regarded by an adaptive frequency interval method.

An efficient implementation in *Python* provides a good platform for investigations under the variation of laser parameters.

The general structure of this thesis is outlined in the following. In Chapter 2 a brief introduction into the basic physics of a thin-disk laser and amplified spontaneous emission is given. Chapter 3 describes the implementation and consideration of the corresponding numerical model for amplified spontaneous emission in thin-disk lasers. The numerical model is then used to investigate the power scaling of the thin-disk laser with the radius of the thin disk ρ_{disk} and the intra-cavity laser intensity $I_{L;Intra}$ in Chapter 4. A conclusion of this parameter study and an outlook for further numerical simulations is drawn in Chapter 5.

Chapter 2

Theoretical Model of an Yb:YAG Thin-Disk Laser

Yb:YAG is a quasi-three-level laser medium frequently used for high-power thin-disk lasers [9]. In the following, the laser rate equations for a quasi-three-level laser are introduced. In addition, the partial differential equation governing the temperature distribution within the thin disk is briefly discussed.

2.1 Yb:YAG as Active Medium

The active medium under investigation in this work is an Yb:YAG thin-disk with a doping concentration of $c_{dot} = 9$ at %. Yb:YAG is a quasi-three level laser medium at room temperature with a small energy difference between lower laser level and lower pump level. Therefore, a thermal population of the lower laser level is present for Yb:YAG at room temperature. A diagram of the energy states and the corresponding energy levels, due to the energy split up of the energy states by the Stark-effect in the crystal lattice is shown in Figure 2.1. The ground energy state of Yb³⁺-ions in Yb:YAG is denoted as ${}^{2}F_{7/2}$ and the excited energy state is denoted as ${}^{2}F_{5/2}$ [28]. The population densities of the two energy states are related by the population density of the dopant of the active medium n_{dot} as

$$n_{dot} = n_0 + n_1, (2.1)$$

where n_1 is the population density of the excited state and n_0 is the population density of the ground state.

The lifetime of the energy levels in the excited state is assumed to be temperature indepen-





Figure 2.1: Energy level diagram of Yb³⁺-ions in YAG for transitions between ${}^{2}F_{5/2}$ and ${}^{2}F_{7/2}$.

dent and hence a combined lifetime of the excited state $\tau_{1\to0}$ is used for further calculations. $\tau_{1\to0}$ is in the order of one millisecond for Yb:YAG [29] and for this work assumed to be 951 µs [30].

The strength of an optical transition between energy levels is given by the spectroscopic cross section and the population density of the involved energy levels. This is summarized in the effective cross section σ_{eff} , which depends on the energies of the involved energy levels E_i and the temperature T by

$$\sigma_{eff}(E_0, E_1, T) = \sigma_{spec}(E_0, E_1) \cdot f(E_0, E_1, T),$$

where σ_{spec} is the spectroscopic cross section for transitions between energy levels E_1 and E_0 ($E_1 > E_0$) with an energy gap

$$\Delta E = E_1 - E_0 \tag{2.2}$$

$$=h(\nu_1 - \nu_0) = h\Delta\nu.$$
 (2.3)

For a light beam with photons of frequency ν , the effective cross section can be expressed by

$$\sigma_{eff}(\nu, T) = \sigma_{spec}(\nu) \cdot f(\nu, T),$$

whereas ν implicates the energy levels E_1 and E_0 of the active medium involved in the optical transition with an energy gap $\Delta E = h \Delta \nu$.

The spectroscopic cross section is independent of the population densities of each energy level, as the Boltzmann occupation factor f absorbs the temperature dependent population density of each energy level. The energy and temperature dependent effective emission cross section and effective absorption cross sections for six exemplary temperatures in the range between 300 K and 450 K are shown in Figure 2.2 [31]. The effective gain coefficient g and the effective absorption coefficient α for a light beam with photons of frequency ν are then expressed by the population density of the dopant n_{dot} , the population density of the excited state n_1 and the effective cross sections σ_{eff} by

$$\alpha(\nu) = \overbrace{\sigma_{eff;abs}(\nu, T) \cdot (n_{dot} - n_1)}^{\text{Effective absorption}} - \overbrace{\sigma_{eff;em}(\nu, T) \cdot n_1}^{\text{Effective emission}}$$
(2.4)

$$g(\nu) = \underbrace{\sigma_{eff;em}(\nu, T) \cdot n_1}_{\text{Effective emission}} - \underbrace{\sigma_{eff;abs}(\nu, T) \cdot (n_{dot} - n_1)}_{\text{Effective absorption}}.$$
(2.5)

The absorption coefficient $\alpha(\nu)$ and the gain coefficient $g(\nu)$ at transparency of the active medium for the frequency ν equals to zero. The population density of the excited state n_1 normalized to the population density of the dopant n_{dot} to reach transparency is defined as

$$\beta(\nu, T) = \frac{\sigma_{eff;abs}(\nu, T)}{\sigma_{eff;abs}(\nu, T) + \sigma_{eff;em}(\nu, T)}.$$
(2.6)

The absorption efficiency η_{abs} for a light beam with photons of frequency ν is defined by the effective absorption coefficient $\alpha(\nu)$ and the number of single-passes M of the light beam through the active medium with temperature T and thickness z_{disk} as

$$\eta_{abs}(\nu, T) = 1 - e^{-M \cdot \alpha(\nu, T) \cdot z_{disk}}.$$
(2.7)

For a low signal beam, $n_0 \gg n_1$ can be assumed and the single-pass (M = 1) absorption efficiency is simplified to the small signal absorption efficiency $\eta_{abs;0}$ defined as

$$\eta_{abs;0}(\nu,T) = \sigma_{eff;abs}(\nu,T)n_{dot}z_{disk}.$$
(2.8)



2.1. ACTIVE MEDIUM



(a) Effective absorption cross section of Yb:YAG



(b) Effective emission cross section of Yb:YAG

Figure 2.2: Effective cross section of absorption (a) and emission (b) for Yb:YAG in dependence of the wavelength at six exemplary temperatures between 300 K and 450 K. Data from Koerner *et al.*[31].

2.2 Laser Beam and Pump Beam

A diode-pumped Yb:YAG thin-disk laser is assumed with a pump beam of Super-Gaussian transversal intensity distribution of order ten with a peak wavelength of $\lambda_P = 940 \text{ nm}$. The laser beam is assumed to have a Gaussian transversal intensity distribution with a peak wavelength at $\lambda_L = 1030 \text{ nm}$ (see Figure 2.2). These are typical parameters for high brightness Yb:YAG thin-disk lasers and amplifiers.

Due to the low thickness of the thin disk z_{disk} , a multi-pass alignment is often used to achieve an efficient absorption of the pump beam [22]. This leads to a superposition of the corresponding single-passes of the pump beam in the thin disk, which can be regarded by an effective intensity I_{eff} . Assuming effective absorption of the investigated light beam in the thin disk, the intensity of the light beam decreases after each single-pass through the thin disk. In addition, the intracavity losses including the reflectivity of the high-reflective coating, the transmissivity of the anti-reflective coating and other losses, lower the intensity of the light beam during each roundtrip. The decrease of the intensity of the investigated light beam per pass (neglecting saturation effects) and the effective intensity of the investigated light beam is illustrated in Figure 2.3. If



Figure 2.3: Decrease of the intensity I for consecutive single-passes through the thin disk (black - solid) and the effective intensity I_{eff} (red - dotted).

no absorption within the active medium is considered and negligible losses can be assumed, the effective intensity I_{eff} is equal to the intensity I times the number of single-passes through the thin disk M. For a reasonable approximation of the effective intensity I_{eff} , absorption of

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the light beam within the active medium has to be considered [30]. The effective intensity is averaged along the propagation axis of the light beam. For negligible losses within the pump optics and a small single-pass absorption ($|\alpha(\nu) \cdot z_{disk}| \ll 1$), the effective intensity I_{eff} of a light beam with frequency ν passing M times through a thin disk of thickness z_{disk} can be approximated as

$$I_{eff}(\nu, T) = I(\nu) \cdot \frac{1 - e^{-M \cdot \overline{\alpha}(\nu, T) \cdot z_{disk}}}{\overline{\alpha}(\nu, T) \cdot z_{disk}},$$
(2.9)

where $\overline{\alpha}(\nu, T)$ is the absorption coefficient $\alpha(\nu, T)$ averaged along the propagation axis of the light beam. The effective intensity I_{eff} as shown in Equation 2.9 assumes a constant intensity of the forward beam and the reverse beam [32] and thus low round-trip losses.

The saturation intensity of a light beam with photons of frequency ν in an Yb:YAG laser medium with a temperature T is defined as the intensity reducing the small-signal absorption coefficient to one-half [33, p. 293] and is given by

$$I_{sat}(\nu, T) = \frac{E(\nu, T)}{\tau_{1 \to 0} \cdot (\sigma_{eff;abs}(\nu, T) + \sigma_{eff;em}(\nu, T))}.$$
(2.10)

2.3 Yb:YAG Thin-Disk Laser

The laser process for an Yb:YAG thin-disk laser is expressed in a good approximation by a system of two rate equations describing the population density of the excited state n_1 and the laser beam intensity I_L . This implicates a simplification of the atomic energy levels of the laser medium to the two multiplets of Yb:YAG (see Figure 2.1). Thermal relaxations within the two multiplets are assumed to be instant in time for the investigated time scales. This is reasonably fulfilled for Yb:YAG [29]. The population density of the excited state and the ground state is denoted as n_1 and n_0 , respectively. The population density of the excited state n_1 defines the absorption and the amplification by stimulated emission in the active medium. The laser beam intensity I_L is directly connected to the population density of the excited state n_1 by the process of stimulated emission for the amplification of the light beam within the active medium.

In general, the transient build up of the intra-cavity laser beam intensity of a thin-disk laser is treated in a separate ordinary differential equation. This ordinary differential equation is neglected for amplifiers, as the laser beam intensity of an amplifier (neglecting regenerative amplifiers) is assumed to be time independent.

2.3.1 Rate Equations of an Yb:YAG Thin-Disk Laser

The transient behavior of the population density of the excited state n_1 and the intra-cavity laser intensity I_L is described by

$$\frac{dn_{1}}{dt} = \underbrace{\overbrace{E_{P}}^{W_{P}} \cdot \alpha_{P}} - \underbrace{\overbrace{E_{L}}^{W_{L}} \cdot g_{L}}_{W_{L}} - \underbrace{\overbrace{n_{1}}^{W_{SE}}}_{\eta_{1} \to 0} \quad (2.11)$$

$$+ \int_{\nu} \alpha(\nu) \cdot \left[\int_{V_{em}} d\Phi_{ASE}(\nu, \vec{x}_{rec}, \vec{x}_{em}) \right] d\nu$$

$$\frac{dI_{L}}{dt} = \underbrace{I_{eff;L}}_{W_{L;I}} \cdot \underbrace{\frac{g_{L} \cdot z_{disk}}{T_{res}}}_{W_{Res}} - \underbrace{\frac{I_{L}}{\tau_{res}}}_{W_{Res}}. \quad (2.12)$$

The subscript P and L indicate the pump beam and the laser beam, respectively. E_P and E_L is the respective energy $E(\nu)$ of a photon with corresponding frequency ν . $\tau_{1\to 0}$ is the excited state lifetime of the active medium. $\Phi_{ASE;rec}$ is the received photon flux by amplified spontaneous emission within the active medium and treated more specific in the following section.

A rotational symmetric cylinder symmetry (ρ, z) is exploited, as the pump beam is of super-Gaussian transversal shape (rotational symmetric) with low divergence in the thin disk and no tilt of the light beam in respect to the thin disk is assumed.

The effective intensity $I_{eff;\nu}$ of the pump beam and the laser beam (see Equation 2.9) is defined as

$$\begin{split} I_{eff;L}(\rho) = & I_L(\rho) \cdot \frac{1 - exp\left(M_L \cdot \bar{g}_L(\rho) \cdot z_{disk}\right)}{-\bar{g}_L(\rho) \cdot d} \\ I_{eff;P}(\rho) = & I_P(\rho) \cdot \frac{1 - exp\left(-M_P \cdot \bar{\alpha}_P(\rho) \cdot z_{disk}\right)}{\bar{\alpha}_P(\rho) \cdot d}, \end{split}$$

where $\bar{g}(\rho)$ and $\bar{\alpha}(\rho)$ are the gain coefficient and the absorption coefficient averaged along the axis of propagation for the wavelength of the respective beam. A telecentric imaging of the multi-pass alignment of the pump beam and the laser beam with the focal plan at the back side of the thin disk is assumed. The local absorption coefficient of the pump beam α_P and the local gain coefficient of the laser beam g_L at radial position ρ and axial position z are defined as

$$\begin{aligned} \alpha_P(\rho, z) = &\sigma_{eff;abs;P}(T(\rho, z)) \cdot (n_{dot} - n_1(\rho, z)) - \sigma_{eff;em;P}(T(\rho, z)) \cdot n_1(\rho, z) \\ g_L(\rho, z) = &\sigma_{eff;em;L}(T(\rho, z)) \cdot n_1(\rho, z) - \sigma_{eff;abs;L}(T(\rho, z)) \cdot (n_{dot} - n_1(\rho, z)). \end{aligned}$$



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The laser threshold of a thin-disk laser is reached when the gain of the laser beam g_L per round-trip equals the resonator intern losses γ_{res} .

Amplified Spontaneous Emission

Spontaneously emitted photons propagate through the thin disk until they are emitted at boundaries of the thin disk or if they are absorbed by an Yb^{3+} -ion. The amplification of spontaneous emission is illustrated in Figure 2.4. Along the path through the active medium, these photons



Figure 2.4: Path of ASE between an emitting volume and an receiving element.

are amplified by stimulated emission of excited Yb^{3+} -ions. An observer point located within the thin disk receives an ASE-photon flux Φ_{ASE} from the volume of emitters V_{em} (here the volume of the active medium). The photon flux by these spontaneously emitted photons amplified along the path through the thin disk is an important factor for the power scaling of high-power thin-disk lasers. Hereof, the spatial emission profile of spontaneous emission is assumed to be isotropic.

This leads to the following expression for the ASE-photon flux Φ_{ASE} received at a point located at \vec{x}_{rec} from an infinitesimal volume dV_{em} located at \vec{x}_{em} [19, 20]

$$d\Phi_{ASE}(\nu, \vec{x}_{rec}, \vec{x}_{em}) = \underbrace{\exp\left(\int_{\vec{x}_{em}}^{\vec{x}_{rec}} g(\nu, \vec{x}') d\vec{x}'\right)}_{\cdot \underbrace{\frac{n_1(\vec{x}_{em}, \nu)}{4\pi \vec{\Lambda}_{amp}^2 \tau_{1 \to 0}} dV_{em}}.$$
(2.13)

Photon flux by spontaneous emission

 $n_1(\vec{x}_{em},\nu)$ is the population density of the excited state normalized to the emission spectrum of the active medium. The gain coefficient $g(\nu, \vec{x})$ at position \vec{x} for a frequency ν depends on the local excited state population density $n_1(\vec{x})$ and the local temperature $T(\vec{x})$ by $g(\nu, \vec{x}) =$ $\sigma_{eff;em;\nu}(T(\vec{x})) \cdot n_1(\vec{x},\nu) - \sigma_{eff;abs;\nu}(T(\vec{x})) \cdot n_0(\vec{x},\nu)$. $\vec{\Lambda}_{amp}$ is the distance from the infinitesimal emitter volume dV_{em} located at \vec{x}_{em} to the receiver located at \vec{x}_{rec} (length of the amplification path). The spectral power distribution of spontaneous emission is regarded in the spectral dependence of the population density of the excited state $n_1(\nu)$ and fulfills the relation $\int_0^\infty n_1(\nu)d\nu = n_1$, where n_1 is the total population density of the excited state.

2.3.2 Nondimensional Rate Equations

The magnitude of the different parameters of the rate equations vary by several orders in magnitude. Mathematical operations on numbers of very different magnitude can be numerically unstable. It is therefore numerically favorable to use normalized (nondimensionalized) parameters in the calculations.

The density of the excited state n_1 and the laser intensity I_L are hence normalized to

$$D_L = \frac{n_1}{n_{dot} \cdot \beta_L} \tag{2.14}$$

$$i_L = \frac{I_L}{I_{Sat;L}}.$$
(2.15)

 β_L is defined as the population density of the excited state n_1 normalized to the population density of the Yb^{3+} -ions n_{dot} necessary to reach transparency of the active medium for the laser wavelength λ_L . The saturation intensity of the active medium for the laser beam $I_{Sat;L}$ (see Equation 2.10) is defined by

$$I_{Sat;L} = \frac{E_L}{\tau_{1\to 0} \cdot (\sigma_{eff;abs;L} + \sigma_{eff;em;L})}$$

Transparency of the active medium for the laser beam wavelength λ_L is reached for $D_L = 1$. The normalized laser intensity i_L equals to one if saturation of the active medium by the laser beam with intensity I_L is reached. It has to be noted, that the transparency threshold at the laser wavelength β_L and the saturation intensity at the laser wavelength $I_{Sat;L}$ are not constant, as both are temperature dependent (see Appendix A).

For, the pump beam the following nondimensionalized parameters are used to express the nondimensional rate equations

$$\begin{split} \beta_P = & \frac{\sigma_{eff;abs;P}}{\sigma_{eff;abs;P} + \sigma_{eff;em;P}} \\ \eta_{abs;P} = & 1 - e^{-M_P \cdot \alpha_P \cdot z_{disk}} \\ \eta_{abs;0;P} = & \sigma_{eff;abs;P} \cdot n_{dot} \cdot z_{disk}, \end{split}$$



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whereas the same relations with reference to the laser beam hold

$$\beta_L = \frac{\sigma_{eff;abs;L}}{\sigma_{eff;abs;L} + \sigma_{eff;em;L}}$$
$$\eta_{abs;L} = 1 - e^{M_L \cdot g_L \cdot z_{disk}}$$
$$\eta_{abs;0;L} = \sigma_{eff;abs;L} \cdot n_{dot} \cdot z_{disk}.$$

The subscripts L and P indicate optical properties for the laser beam with wavelength λ_L and the pump beam with wavelength λ_P , respectively. $\eta_{abs;\nu}$ is the absorption efficiency for a photon of frequency ν and $\eta_{abs;0;\nu}$ is the corresponding small signal absorption efficiency. The relation of the transparency threshold of the active medium for photons with the wavelength of the pump beam and the laser beam B_{PL} is defined as

$$B_{PL} = \frac{\beta_P}{\beta_L}.$$

The nondimensional coupled rate equations can then be rewritten as

$$\tau_{1\to0} \cdot \frac{dD_L}{dt} = \underbrace{\overbrace{i_P \cdot B_{PL} \cdot \frac{\eta_{abs;P}}{\eta_{abs;P;0}} + i_L \cdot \frac{\eta_{abs;L}}{\eta_{abs;L;0}}}_{W_{Ass;L;0}} - \underbrace{\int_{\nu} D(\nu) d\nu}_{V_{em}}$$
(2.16)
$$+ \underbrace{\int_{\nu} \eta_{abs;0}(\nu) \cdot (1 - D(\nu)) \int_{V_{em}} d\Phi_{ASE}(\nu, \vec{x}_{rec}, \vec{x}_{em}) d\nu}_{W_{ASE}}}_{W_{ASE}}$$

$$\tau_{1\to0} \cdot \frac{di_L}{dt} = \underbrace{i_L \cdot \frac{\tau_{1\to0}}{T_{res}} \cdot \eta_{abs;L;0} \cdot (D_L - 1)}_{W_{L;I}} - \underbrace{i_L \cdot \frac{\tau_{1\to0}}{T_{res}} \cdot \gamma_{res}}_{W_{Res}},$$
(2.17)

with the normalized pump intensity

$$i_P = \frac{I_P}{I_{Sat;P}},$$

and the corresponding saturation intensity

$$I_{Sat;P} = \frac{E_P}{\tau_{1\to 0} \cdot (\sigma_{eff;abs;P} + \sigma_{eff;em;P})}.$$

The photon flux by amplified spontaneous emission $d\Phi_{ASE}$ received at \vec{x}_{rec} from an infinitesi-

mal volume dV_{em} located at \vec{x}_{em} expressed in nondimensionalized parameters is

$$d\Phi_{ASE}(\nu, \vec{x}_{rec}, \vec{x}_{em}) = \underbrace{\exp\left(\int_{\vec{x}_{em}}^{\vec{x}_{rec}} \eta_{abs;0}(\nu, \vec{x'}) \cdot (D(\nu, \vec{s}) - 1)d\vec{s}\right)}_{\underbrace{D(\nu, \vec{x}_{em})dV'_{em}}_{\text{Photon flux by spontaneous emission}},$$
(2.18)

where dV'_{em} is the infinitesimal volume dV_{em} normalized to the surface of the sphere with radius Λ_{amp} and the thickness of the thin disk z_{disk} .

2.4 Heat Equation of an Yb:YAG Thin-Disk Laser

The quasi-three-level medium characteristics of Yb:YAG are reflected in a strong temperature dependence of the effective cross sections. Therefore, it is important to consider the non-uniform temperature distribution within the thin disk.

The temperature is obtained by solving the partial differential equation of the heat equation. The heat equation for a material with temperature T, mass density ρ_m , specific heat capacity c_P and thermal conductivity κ_T is given by

$$c_p \rho_m \frac{\partial T}{\partial t} - \nabla \left(\kappa_T \nabla T \right) = \dot{q}_V.$$
(2.19)

 \dot{q}_V is the volumetric heat flux and is defined by the heat sources and heat sinks of the system. The heat flux density \dot{q}_V can be separated into incoming heat flux density $\dot{q}_{V;in}$ and extracted heat flux density $\dot{q}_{V;ex}$. Both can be inferred from the rates of the laser rate equations (see Section 2.3.1) weighted by the corresponding energy. \dot{q}_V is then defined as

$$\dot{q}_V = \overbrace{W_P \cdot E_P}^{\dot{q}_{V;in}} - \overbrace{W_L \cdot E_L - W_{SE} \cdot E_{SE} - W_{ASE} \cdot E_{ASE}}^{\dot{q}_{V;ex}}, \tag{2.20}$$

where W_P and E_P is the rate by the pump beam and the energy of the photons of the pump beam, respectively. W_L , W_{SE} and W_{ASE} are the rates by the laser beam, spontaneous emission and amplified spontaneous emission, respectively. The corresponding energies are E_L , E_{SE} , and E_{ASE} . Identifications in Equation 2.20 by the introduced heat flux density $\dot{q}_{V;in}$ and the extracted heat flux density $\dot{q}_{V;ex}$ do not hold in general. The absorption of a photon introduces energy to the system, whereas stimulated emission and spontaneous emission extracts energy from the system. As the pump beam and the laser beam interact by both processes with the

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material, they both actually are heat sources and heat sinks of the system. The same holds true for amplified spontaneous emission, whereas spontaneous emission is the only real heat sink. However, under normal conditions for the operation of an efficient laser, the pump beam is introducing more energy to the system than it is extracting. Vice versa the laser beam usually extracts more energy from the system than it introduces.

The thermal relaxation time τ_T approximated by [34]

$$\tau_T = \frac{c_p \rho}{\kappa} \cdot \Delta z^2, \qquad (2.21)$$

provides a rough estimation of the transient temperature evolution of a material. Δz is the thickness of the thin disk.

Chapter 3

Implementation of the Numerical Model

The laser rate equations and the heat equation presented in Chapter 2 are used to implement a numerical model of an Yb:YAG thin-disk laser.

The programming language is *Python* in version 3.7.1. The program flow of the numerical model is shown in Figure 3.1. The transient calculation of the normalized population density of the excited state D_L , the laser intensity normalized to the saturation intensity i_L and the temperature T is calculated iteratively starting at time t_0 . If amplified spontaneous emission is considered, the rate by amplified spontaneous emission is calculated and regarded in Equation 2.16. Afterwards, an adaptive time step method based on the temporal temperature gradient decides whether to calculate the temperature in this iteration step or to skip the temperature calculation in this time step. The calculated values are assigned to the corresponding variables and if the time instance t_{n+1} has not yet reached the maximum time t_{end} , the next iteration $(t_n \to t_{n+1})$ is done. If the time instance t_{n+1} has reached the maximum time t_{end} , the iteration-loop is exited.

The *ODE*-package of the *SciPy*-library [35] is used to solve the initial value problem of the laser rate equations (see Equation 2.16). The *ODE*-package has an object-oriented interface to some older, but reliable explicit and implicit ordinary differential equation solvers. The laser rate equations considering the non-linear influence of amplified spontaneous emission are expected to be mathematically stiff. The *vode*-solver of the *ODE*-package wraps the *DVODE*-routine [36] written in Fortran 77 and offers a method for stiff initial value problems based on a backward differentiation formula.

The temperature calculation is done by solving the partial differential equation of the heat equation (see Equation 2.19) with FEniCS 2018.1.0 [27]. FEniCS [27] is an efficient open-source platform with a *Python*-interface for finite element methods.



3.1. GEOMETRICAL GRIDS



Figure 3.1: Flow chart of the implemented routine.

Finally, the combination with the multiprocessing-package (part of the *Python*-standard library) makes it possible to run several numerical simulations parallel. Due to the computationally costly operations coming with the consideration of amplified spontaneous emission, this is an important feature for running parameter studies within a reasonable time.

3.1 Considerations on the Geometrical Grids

Three-dimensional numerical calculations require a geometrical grid with discretized grid points at which functions are evaluated. A proper choice of the resolution of the geometrical grid can save computational operations and hence computation time. The computation time can be reduced by orders of magnitude in time if symmetries can be exploited to reduce the dimensionality of the geometrical grid. The total number of grid points is far less, but the resolution of the computation grid is not reduced.

A rotational symmetry of the parameters to describe the physical properties of the thin disk can be assumed. The local laser rate equations can then be solved on a two-dimensional grid spanned by the radial coordinates ρ and the axial coordinates z of the thin disk.

The heat equation is solved on a separate grid, as the partial differential equation solver implemented in FEniCS [27] expects a three-dimensional tetrahedron grid. The three-dimensional tetrahedron grid is created by Delaunay-triangulation with the free-to-use program *Gmsh* [37] in version 3.0.6. This geometrical grid is separated into different sub-grids¹ to account for the different thermal properties of the corresponding materials (see Appendix A). Sub-grids are defined for the thin disk, the high-reflective coating including the adhesive and the heat sink. These sub-grids share grid points at the geometrical boundaries, which is a numerical constraint for the heat exchange between the sub-grids.

The calculation of amplified spontaneous emission is separated from the problem of the local laser rate equations to achieve an efficient implementation. It is based on considerations to reduce the number of grid-points by [20]. Spontaneously emitted photons from a finite volume element ΔV_{em} are amplified along their geometrical path to a receiver element (see Figure 2.4). Each geometrical path is a function of the radial and axial coordinates. For geometrical paths with a finite distance to the center of the thin-disk, this leads to an azimuthal angle dependence φ of each geometrical path. Due to the radial symmetry of the parameters in the thin disk, no dependence on the azimuthal angle φ is observed for the start point and end point of each geometrical path. Therefore, a two-dimensional receiver grid in the ρ -z-plane in combination with a three-dimensional emitter grid spanned by the volume of the thin disk V_{disk} (see Figure 2.4) is sufficient to describe the process of amplified spontaneous emission in the thin disk. This method helps to keep the computation time for the integration of geometrical paths between emitting finite volumes ΔV_{em} and receiving elements within reasonable limits.

An overview of all four geometrical grids is given in Table 3.1. It has to be noted that the number of grid-points for the heat equation is not constant for different geometrical expansions of the thin disk, as the meshing algorithm searches for the optimum number and location of each grid point. The aspect ratio of the thin disk and the high-reflective coating is a critical parameter for the optimization of the tetrahedron-grid and has its limits in respect to the meshing algorithm for very low aspect ratios.

The different geometrical grids optimized for each problem entail the interpolation of parameter values between geometrical grids.

¹Due to the mesh-refinement algorithm inherited in *Gmsh*, the number of grid points depends on the aspect ratio z/ρ of the sub-grids.

3.2. RATE EQUATIONS

Coometrical Crid	Mesh	Resolution				
Geometrical Grid	Type (Geom Dim)	points				
Heat Equation						
Active Medium	Tetrahedron (3)	2235^{1}				
Coating	Tetrahedron (3)	2235^{1}				
Heat Sink	Tetrahedron (3)	4785^{1}				
Rate Equation						
Local Rate Equations	Plane (2)	$86\left(ho ight) imes15\left(z ight)$				
ASE-Receiver	Plane (2)	$15\left(ho ight) imes4\left(z ight)$				
ASE-Emitter	Cylinder (3)	$30\left(\rho\right) \times 24\left(\varphi\right) \times 5\left(z\right) \cdot \left(1 + N_{Refl}\right)$				

Table 3.1: Parameters of the used simulation grids.

3.2 Rate Equations

The nondimensional laser rate equations from Equation 2.16 are spatially discretized to be solved numerically on local grid points by the *ODE*-solver. Further considerations on the choice of the grid(s) are explained in Section 3.1. The local laser rate equations then read

$$\tau_{1\to0} \cdot \frac{\Delta D_L(\vec{x}_i)}{\Delta t} = i_P(\vec{x}_i) \cdot \frac{\eta_{abs;P}(\vec{x}_i)}{\eta_{abs;P;0}(\vec{x}_i)} \cdot B_{PL}(\vec{x}_i)$$

$$+ i_L(\vec{x}_i) \cdot \frac{\eta_{abs;L}(\vec{x}_i)}{\eta_{abs;L;0}(\vec{x}_i)}$$

$$- \sum_{l}^{L} D(\vec{x}_i, \nu_l) \Delta \nu_l$$

$$- \sum_{l}^{L} \eta_{abs;0}(\vec{x}_i, \nu_l) \cdot (D(\vec{x}_i, \nu_l) - 1)$$

$$\cdot \sum_{m}^{M} R_{m,i} \Delta \Phi_{ASE}(\vec{x}_i, \vec{x}_m, \nu_l) \Delta \nu_l$$

$$\tau_{1\to0} \cdot \frac{\Delta i_L(\vec{x}_i)}{\Delta t} = i_L(\vec{x}_i) \cdot \frac{\tau_{1\to0}}{T_{res}} \cdot \left[\eta_{abs;L;0}(\vec{x}_i) \cdot (D_L(\vec{x}_i) - 1) - \gamma_{res} \right],$$

$$(3.1)$$

where \vec{x}_i represents the coordinate position of point i ($i \in I$ for $I \subset \mathbb{N}$ the number of laser rate equation grid points). The temporal derivations dD_L/dt and di_L/dt are approximated by





Figure 3.2: Path of ASE between an emitting volume and an receiving element.

the discrete time difference $\Delta D_L/\Delta t$ and $\Delta i_L/\Delta t$ in the finite time interval Δt , respectively. This holds true for $\Delta D_L \ll D_L$ and $\Delta i_L \ll i_L$. The analytic integral $\int f(x)dx$ is expressed by the sum over the function values at discrete points x_n weighted by the interval spacing Δx and is written as $\sum_n f(x_n)\Delta x$. This is a reasonable approximation for sufficiently small intervals Δx . $R_{m,i} = R_{AR}(\Theta_{m,i})^{Num_{AR;m,i}}R_{HR}(\Theta_{m,i})^{Num_{HR;m,i}}$ is the accumulated reflectivity of amplification path between emitter volume m and receiver element i and defined by the number of reflections at the anti-reflective surface $Num_{AR;m,i}$ and the number of reflections at the highreflective surface $Num_{HR;m,i}$ at an angle $\Theta_{m,i}$ with reflectivity $R_{AR}(\Theta_{m,i})$ and reflectivity $R_{HR}(\Theta_{m,i})$, respectively.

The *ODE*-solver uses internal time steps Δt_{int} to optimize the time step according to the temporal gradient of the ordinary differential equation. This routine has the advantage of a dense and accurate calculation, where a steep gradient in the solution is present and a coarse and fast calculation for small temporal changes. The solution is then interpolated from the calculated solutions at internal time points t_{int} with irregular spacing to the regular spaced time points t_{out} .

To simplify the calculations, monochromatic beams with no divergence within the thin disk are assumed. For calculations of high-repetition pulsed lasers (MHz-repetition rate), the intensity can be temporally averaged, as the optical properties of the Yb:YAG crystal change with $\tau_{1\rightarrow0} = 951 \,\mu\text{s}$ (see Appendix A). If ultrashort pulsed low repetition rate lasers are investigated, the division into a slow time regime (inversion generation) and fast time regime (amplification of the laser pulse) could be a possible approach.

3.2.1 Amplified Spontaneous Emission

The photon flux by amplified spontaneous emission received at a grid point \vec{x}_i is calculated by transforming the analytic integrals of Equation 2.18 to the weighted sum over discrete parameters. Figure 3.2 illustrates the discretization into receiver elements located at \vec{x}_i on a twodimensional receiver grid in the thin disk and emitter volume elements located at \vec{x}_m on a threedimensional emitter grid [20].



3.2. RATE EQUATIONS



Figure 3.3: Illustration of the mirror-image technique used to regard reflections of amplified spontaneous emission at the top surface and the bottom surface.

Reflections at the top surface and at the bottom surface of the thin disk are regarded by using a mirror-image technique [24, 26], which is illustrated in Figure 3.3. Each finite emitter volume ΔV_{em} is mirrored at the top surface and at the bottom surface for a given number of reflections n_{refl} . Then the path between each finite emitter volume ΔV_{em} located at \vec{x}_m to the receiver element located at \vec{x}_i is drawn and then folded back into the original volume of the thin disk V_{disk} to regard the axial and radial dependence of the gain coefficient g_{ASE} of the corresponding path.

The spatial and spectral discretization of the analytic equation to calculate the ASE-photon flux at a receiver element is given by

$$\Delta \Phi_{ASE}(\vec{x}_i, \vec{x}_m, \nu_l) = \exp\left(\sum_{n=1}^N \eta_{abs;0}(\vec{x}_n, \nu_l) \cdot \left(D(\vec{x}_n, \nu_l) - 1\right) \Delta \Lambda_n\right)$$
$$\cdot D(\vec{x}_m, \nu_l) \Delta V_{em}'(\vec{x}_m)$$
(3.3)

with

$$D(\vec{x}_m, \nu_l) = D_L(\vec{x}_m) \cdot B_{L\nu_l}(\vec{x}_m)$$
$$B_{L\nu_l}(\vec{x}_m) = \frac{\beta_L(\vec{x}_m)}{\beta(\vec{x}_m, \nu_l)}$$
$$\beta(\vec{x}_m, \nu_l) = \frac{\sigma_{eff;abs}(\vec{x}_m, \nu_l)}{\sigma_{eff;abs}(\vec{x}_m, \nu_l) + \sigma_{eff;em}(\vec{x}_m, \nu_l)}$$
$$\Delta V_{em}'(\vec{x}_m) = \frac{\Delta V_{em}(\vec{x}_m)}{4\pi\Lambda_{amp}^2 \cdot z_{disk}}.$$

These values can be pre-calculated to improve the computation performance of the numerical model, as the geometric coordinates of the paths is not considered to be time dependent itself, in contrary to the gain coefficient g_{ASE} . $D(\vec{x}_m, \nu_l)$ is the density of the excited state normalized to population density of Yb^{3+} -ions n_{dot} and the transparency threshold $\beta(\vec{x}_m, \nu_l)$ at \vec{x}_m for a frequency ν_l . The nondimensionalized finite volume at \vec{x}_m denoted as $\Delta V_{em}'(\vec{x}_m)$ is given by the finite volume $\Delta V_{em}(\vec{x}_m)$ at \vec{x}_m normalized to the surface of a sphere with radius corresponding to the amplification path length Λ_{amp} and the thickness of the thin disk z_{disk} .

The spatial discretization of the emitter volume elements is indicated by the sum over the running index m. The sum with running index l denotes the discretization of the spectral power distribution of the ASE-photon flux. The path integral of each amplification path is given by the sum with running index n.

The frequencies ν_l are chosen according to the emission spectrum of spontaneous emission. The emission spectrum is divided into L intervals in the frequency domain. The Riemann-



Figure 3.4: Wavelength intervals (20) of the effective emission cross section of Yb:YAG (T = 300 K) and weighted mean wavelengths (20) of each interval. Calculations are carried out in the frequency domain.

3.2. RATE EQUATIONS

sum of each frequency interval is required to be equal. The weighted mean frequency ν_l of each frequency interval is then used in further calculations. The division of the spontaneous emission spectrum into intervals and the corresponding weighted mean of each interval is shown in Figure 3.4 (in the wavelength-domain).

As the emission spectrum of a quasi-three-level medium is temperature dependent, these frequency intervals are recalculated, if the temperature changes lie outside a given range. Figure 3.5 shows the weighted mean frequencies for a temperature of 300 K and 450 K.



Figure 3.5: Weighted mean wavelengths (20) of the effective emission cross section of Yb:YAG for T = 300 K (solid - blue) and T = 450 K (dashed - orange). Calculations are carried out in the frequency domain.

The rate by amplified spontaneous emission W_{ASE} of the nondimensional local laser rate equation from Section 2.3.2 then reads

$$W_{ASE}(\vec{x}_i) = \sum_{l=1}^{L} \eta_{abs;0}(\vec{x}_i,\nu_l) \cdot (D(\vec{x}_i,\nu_l) - 1) \sum_{m=1}^{M} R_{m,i} \Delta \Phi_{ASE}(\vec{x}_i,\vec{x}_m,\nu_l) \Delta \nu_l \qquad (3.4)$$

3.3 Heat Equation

The temperature T_{n+1} at a time instance t_{n+1} is calculated in dependence of the known temperature T_n and the heat flux density $\dot{q}_{V,n}$ at a previous time instance t_n with a corresponding time step $\Delta t = t_{n+1} - t_n$. The partial differential equation of the temperature T with respect to the time t (see Equation 2.19) is discretized by forward differentiation to

$$\left(\frac{\partial T}{\partial t}\right)_n = \frac{T_{n+1} - T_n}{\Delta t}.$$

The time discretized heat equation then reads

$$\frac{T_{n+1} - T_n}{\Delta t} = \frac{\kappa_T}{c_p \rho_m} \nabla^2 T_{n+1} + \frac{\dot{q}_{V;n}}{c_p \rho_m},\tag{3.5}$$

with the specific heat capacity c_p and the mass density of the material ρ_m . Rearranging Equation 3.5 for T_{n+1} on the left hand side and T_n on the right hand side yields the weak form of the heat equation given by

$$\overbrace{\int_{\Omega} \left(T_{n+1}v + \Delta t \frac{\kappa_T}{c_p \rho_m} \nabla T_{n+1} \cdot \nabla v \right) dV}^{a(T_{n+1},v)} = \overbrace{\int_{\Omega} \left(\Delta t \frac{\dot{q}_{V,n}}{c_p \rho_m} + T_n \right) v dV}^{L_n(v)}.$$

where v is a test function. The weak form of the heat equation is used by FEniCS [27] to numerically calculate the temperature distribution considering the different thermal properties of the thin disk, the high-reflective coating and the heat sink. The Dirichlet-boundary condition requires a temperature of 300 K at the back side of the heat sink.

The heat equation is solved at adaptive time steps Δt . These are defined by relative and absolute tolerances Δ_{rel} and Δ_{abs} (see Table 3.2) of the temperature change ΔT in one time step Δt . The time step Δt is checked after each calculation of the temperature T. If the temperature

Table 3.2: Lower and upper bounds for the relative and absolute tolerances in the adaptive time step routine of the temperature calculation.

Bound	Relative Tolerance (Δtol_{rel})	Absolute Tolerance (Δtol_{abs})
Lower	$\Delta tol_{rel;lb} = 0.001$	$\Delta tol_{abs;lb} = \Delta tol_{rel;lb} \cdot T_{min}$
Upper	$\Delta tol_{rel;ub} = 0.01$	$\Delta tol_{abs;ub} = \Delta tol_{rel;ub} \cdot T_{min}$

change ΔT is higher than an upper bound of the tolerances $\Delta tol_{ub} = \Delta tol_{rel;ub} \cdot T_{min} + \Delta tol_{abs;ub}$ provided to the solver, the time step Δt is bisected for the following temperature



3.3. HEAT EQUATION

calculation. For a temperature change ΔT lower than a provided lower bound for the tolerance $\Delta tol_{lb} = \Delta tol_{rel;lb} \cdot T_{min} + \Delta tol_{abs;lb}$, the time step Δt is doubled for the consecutive calculation of the temperature. If the temperature change ΔT is in between the provided tolerances for the lower bound and the upper bound, the time step Δt is kept constant.

This routine for the adaptive time step Δt for the temperature calculations speeds up the following calculations of the temperature for negligible temperature changes ΔT within the time interval Δt and provides a good time resolution for a strong gradient of the temperature ΔT with regard to the time interval Δt .

Chapter 4

Numerical Simulations of an Yb:YAG Thin-Disk Laser

This chapter presents transient numerical simulations based on the laser rate equation model described in Chapter 2. The implemented model (see Chapter 3) considers a nonuniform temperature distribution [26] and reflections of amplified spontaneous emission at the top surface and the bottom surface of the thin disk [22, 24], as well as a spectral power distribution of the spontaneous emission and of the amplification of spontaneous emission [17]. This numerical model gives insight to the strong nonlinear behavior of a thin-disk laser and the power scaling of high power thin-disk oscillators and thin-disk amplifiers.

Investigations on the power scaling limits of a thin-disk laser have been done by Speiser *et al.* [23] with an analytic model to estimate to upper limit of power scaling for thin-disk lasers. A good approach for power scaling of thin-disk lasers is to increase the pump spot size $w_{0;P}$ and the thin disk radius ρ_{disk} and while keeping the pump beam intensity $I_{P;In}$ constant [38].

The following numerical simulations have been carried out for an input peak intensity of the pump beam $I_{P;0;In} = 3 \text{ kW cm}^{-2}$ with $M_P = 48$ single-passes through the thin-disk by the pump beam. Hence, a high absorption of the pump beam is possible even for a moderate doping concentration of the active medium and a very low thickness of the thin disk [10]. The active medium is an Yb:YAG thin-disk with a doping concentration of $c_{dot} = 9 \text{ at }\%$ and a thickness of $z_{disk} = 130 \,\mu\text{m}$.

Numerical simulations have been made under the previously mentioned assumptions for a thin-disk oscillator and for a thin-disk amplifier. The parameter study investigating the influence of the thin disk radius ρ_{disk} and the intra-cavity laser intensity $I_{L;intra}$ is presented for a variation of five values for each parameter. Numerical simulations considering amplified spontaneous emission and numerical simulations neglecting amplified spontaneous emission are shown to

investigate the influence of amplified spontaneous emission on the transient behavior of a thindisk oscillator and a thin-disk amplifier and the spatial distribution of corresponding parameters.

4.1 Yb:YAG Thin-Disk Oscillator

Numerical simulations are done for a V-type resonator (see Appendix A for more details) resulting in a total number of laser beam single-passes through the disk per round trip of $M_L = 4$. No spatial shaping of the laser beam is assumed in these numerical simulations. The transverse profile of the laser beam is therefore only influenced by the transverse profile of the gain in the active medium. The variation of the intra-cavity laser intensity $I_{L;Intra}$ is tuned by the transmissivity of the output-coupler T_{OC} . The thin-disk radius ρ_{disk} and the transmissivity of the output coupler T_{OC} is varied to five different values each, resulting in a total number of 25 simulations. In addition, simulations of each parameter set (ρ_{disk} , T_{OC}) are done once considering amplified spontaneous emission and once neglecting amplified spontaneous emission.

The correlation of thin disk radius ρ_{disk} and transmissivity of the output coupler T_{OC} to the optical-optical efficiency $\eta_{opt.-opt.} = P_{L;Out}/P_{P;In}$ is shown in Figure 4.1. The optical-optical efficiency $\eta_{opt.-opt.}$ without considering amplified spontaneous emission shows a strong correlation to the transmissivity of the output-coupler T_{OC} , whereas the optical-optical efficiency $\eta_{opt.-opt.}$ can be considered independent of the thin disk radius ρ_{disk} up to a transmissivity of the output-coupler of $T_{OC} = 10\%$. For a transmissivity of the output-coupler of $T_{OC} = 10\%$, the optical-optical efficiency $\eta_{opt.-opt.}$ shows a strong correlation to the disk radius ρ_{disk} and for a thin disk radius of $\rho_{disk} = 5 \text{ mm}$ the highest result of optical-optical efficiency $\eta_{opt.-opt.}$. The numerical simulations for a transmissivity of the output-coupler of $T_{OC} = 10\%$ have to be treated with care, as the numerical model assumes low round-trip losses. This low-loss approximation is inherited in the effective intensity of the laser beam $I_{L;eff}$, assuming an equal intensity for the forward beam and the reverse beam (see Section 2.2) [32]. Up to a transmissivity of the output-coupler of $T_{OC} \lesssim 5.62 \,\%$, the optical-optical efficiency $\eta_{opt.-opt.}$ of the laser grows to an increasing transmissivity of the output-coupler T_{OC} . The maximum optical-optical efficiency $\eta_{opt.-opt;max} \approx 63.1\%$ is achieved for an output-coupler transmissivity of $T_{OC} = 5.62\%$ and a thin disk radius of $\rho_{disk} = 5 \text{ mm}.$

The optical optical efficiency $\eta_{opt.-opt.}$ for numerical simulations considering amplified spontaneous emission is shown in Figure 4.2. The results of the numerical simulations agree well with published data for high-power Yb:YAG thin-disk oscillators [39, 40]. Stewen *et al.* [40] showed an optical-optical efficiency of $\eta_{opt.-opt.} \approx 50\%$ for a beam waist of the pump beam of $w_{0;P} = 0.6$ mm and an optical-optical efficiency of $\eta_{opt.-opt.} \approx 45\%$ for a beam waist of the pump beam of $w_{0;P} = 3$ mm. The output-coupler transmissivity was $T_{OC} = 2.2\%$

CHAPTER 4. NUMERICAL SIMULATIONS



Figure 4.1: Optical-optical efficiency $\eta_{opt.-opt.}$ in dependence of the transmissivity of the output-coupler T_{OC} and the radius of the thin disk ρ_{disk} neglecting amplified spontaneous emission in the numerical simulations.

and the pump beam intensity for the respective data was $I_{P;In} = 3 \,\mathrm{kW \, cm^{-2}}$ in both cases. Experimental results by Stewen *et al.* [40] for a thin disk of thickness $z_{disk} = 230 \,\mu\mathrm{m}$ (numerical simulations in this thesis used $z_{disk} = 130 \,\mu\mathrm{m}$). The numerical simulations in this thesis (see Figure 4.2), a similar output-coupler transmissivity of $T_{OC} = 1.78 \,\%$ and thin disk radii of $\rho_{disk} = 1 \,\mathrm{mm}$ and $\rho_{disk} = 5 \,\mathrm{mm}$ with respective beam waist radii of the pump beam of $w_{0;P} = 0.66 \,\mathrm{mm}$ and $w_{0;P} = 3.33 \,\mathrm{mm}$ showed consistent results. For the small pump beam waist of $w_{0;P} = 0.66 \,\mathrm{mm}$ an optical-optical efficiency of $\eta_{opt.-opt.} = 47 \,\%$ was calculated and for the larger pump beam waist of $w_{0;P} = 3.33 \,\mathrm{mm}$ an optical-optical efficiency of $\eta_{opt.-opt.} = 48 \,\%$ was found. Ahmed *et al.* [39] showed experimental results for an output-transmissivity of $T_{OC} = 4 \,\%$ and pump beam waist of $w_{0;P} = 3.6 \,\mathrm{mm}$ for different pump beam powers. The thickness of the thin disk was about twice as thick with $z_{disk} = 215 \,\mu\mathrm{m}$ as the one used in the numerical simulations presented in this thesis ($T_{OC} = 5.62 \,\%$ and $\rho_{disk} = 5 \,\mathrm{mm} \rightarrow w_{0;P} = 3.33 \,\mathrm{mm}; \eta_{opt.-opt.} = 63 \,\%; P_{P;In} \approx 0.911 \,\mathrm{kW}$) is about twice as





Figure 4.2: Optical-optical efficiency $\eta_{opt.-opt.}$ in dependence of the transmissivity of the output-coupler T_{OC} and the radius of the thin disk ρ_{disk} considering amplified spontaneous emission in the numerical simulations.

high as the highest value of the pump beam power $P_{P;In}$ shown by Ahmed *et al.* [39], a direct comparison to the results is not possible. However, the optical-optical efficiency converged to a value of $\eta_{opt.-opt.} \approx 60\%$ for increasing pump powers $P_{P;In}$. Extrapolating this to the pump power used for the numerical simulation in this thesis will lead to consistent results.

A correlation between thin disk radius ρ_{disk} and the optical-optical efficiency $\eta_{opt.-opt.}$ is observed for a transmissivity of the output-coupler $T_{OC} \gtrsim 3.16$ %. Values of $T_{OC} = 10$ % are excluded for the aforementioned reasons of a laser rate equation model developed under the assumption of low losses. The optical-optical efficiency $\eta_{opt.-opt.}$ is almost equal for the case without considering amplified spontaneous emission (Figure 4.1) and considering amplified spontaneous emission (Figure 4.2) for a low output-coupler transmissivity ($T_{OC} \lesssim 1.78$ %). As amplified spontaneous emission is in competition with the intra-cavity laser intensity $I_{L;Intra}$ for the amplification by stimulated emission, a higher intra-cavity laser intensity $I_{L;Intra}$ can effectively suppress amplified spontaneous emission. For a higher transmissivity of the outputcoupler ($T_{OC} \gtrsim 3.16$ %), a negative correlation between thin disk radius ρ_{disk} and opticaloptical efficiency $\eta_{opt.-opt.}$ indicates an increasing influence of amplified spontaneous emission for increasing thin disk radii ρ_{disk} .

The correlation of the intra-cavity laser intensity $I_{L;Intra}$ to the transmissivity of the outputcoupler T_{OC} is shown in Figure 4.3. A strong negative correlation between the transmissivity



Figure 4.3: Intra-cavity laser intensity $I_{L;Intra}$ in dependence of the transmissivity of the output-coupler T_{OC} . These values are spatially averaged from the center of the thin disk to the radius of the pump beam waist $w_{0;P} = \rho_{disk} \cdot 66 \%$. Dash-dotted lines represent the corresponding numerical simulation without considering amplified spontaneous emission.

of the output-coupler T_{OC} and the intra-cavity laser intensity $I_{L;Intra}$ is present, regardless of considering amplified spontaneous emission or neglecting amplified spontaneous emission. No radial dependence of the intra-cavity laser intensity $I_{L;Intra}$ is observed, if amplified spontaneous emission is neglected ($T_{OC} = 10\%$ excluded). This is supported by the fact that the local laser rate equations do not show a direct dependence on the radius of the thin disk (indirect by the heat equation and the corresponding temperature distribution), if amplified spontaneous emission is neglected. If amplified spontaneous emission is considered in the numerical simulations, the local laser rate equations are no longer independent of the thin disk radius ρ_{disk} as the volume integral of amplified spontaneous emission depends on the thin disk radius ρ_{disk}

4.1. OSCILLATOR

(see Equation 2.18). For a low transmissivity of the output-coupler of $T_{OC} \leq 1.78$ % there is no effect on the intra-cavity laser intensity $I_{L;Intra}$, implicating that a high intra-cavity laser intensity $I_{L;Intra}$ effectively suppresses amplified spontaneous emission. If the transmissivity of the output-coupler T_{OC} is increased, a deviation from the radial independent case of neglecting amplified spontaneous emission is observed. This deviation increases with increasing transmissivity of the output-coupler T_{OC} .

The strong radial dependence is attributed to the rate of amplified spontaneous emission W_{ASE} . The rate of amplified spontaneous emission W_{ASE} normalized to the rate of the pump beam W_P (see Equation 2.16) is shown in Figure 4.4. The rates are spatially averaged up to the beam waist radius of the pump beam $w_{0:P}$. The normalized rate of amplified spontaneous emis-



Figure 4.4: Rate of amplified spontaneous emission W_{ASE} normalized to the rate of the pump beam W_P (see Equation 2.16) in dependence of radius of the thin disk ρ_{disk} . These values are spatially averaged from the center of the thin disk to the radius of the pump beam waist $w_{0;P} = \rho_{disk} \cdot 66\%$. Dash-dotted lines represent the corresponding numerical simulation without considering amplified spontaneous emission.

sion W_{ASE}/W_P is negligible for thin disk radii up to $\rho_{disk} = 5 \text{ mm}$. There is a massive increase of the rate of amplified spontaneous emission W_{ASE} for larger radii ($\rho_{disk} \gtrsim 5 \text{ cm}$) at a trans-


missivity of the output-coupler of $T_{OC} \gtrsim 3.16\%$. This indicates that amplified spontaneous emission is well suppressed until a certain threshold is reached, which depends on the intracavity laser intensity $I_{L;Intra}$ and the radius of the thin disk ρ_{disk} . A good suppression of amplified spontaneous emission is achieved for an output-coupler transmissivity of $T_{OC} = 3.16\%$ up to an radius of $\rho_{disk} = 5 \text{ cm}$. As ASE-photons are in concurrency with laser beam photons in regard to the amplification by stimulated emission in the thin disk, a high intra-cavity laser intensity $I_{L;Intra}$ is crucial to prevent amplified spontaneous emission to develop a significant photon population. For increasing radii of the thin disk ρ_{disk} , this plays a more important role, as the amplification path Λ_{ASE} of spontaneously emitted photons increases, whereas the amplification path of the laser beam photons is constant. The amplification path of the laser beam only depends on the number of passes through the thin disk ρ_{disk} .

Speiser et al. [22] formulated an analytic model for the influence of amplified spontaneous emission on an Yb:YAG thin-disk oscillator and introduced an effective lifetime of the excited state. This parameter is defined by the excited state lifetime $\tau_{1\rightarrow 0}$ reduced by amplified spontaneous emission. This can be summarized to $\tau_{1\to 0;eff}^{-1} = \tau_{1\to 0}^{-1} + \tau_{ASE}^{-1}$, where τ_{ASE} is defined by the corresponding rate in the laser rate equation W_{ASE} . As the rate of amplified spontaneous emission W_{ASE} is spatially dependent, the effective excited state lifetime is spatially dependent as well. Therefore, the effective excited state lifetime $\overline{\tau_{1\to 0:eff}}$ shown in Figure 4.5 is spatially averaged within the radius of the pump beam waist ($\rho < w_{0:P}$ and $w_{0:P} = \rho_{disk} \cdot 66\%$). A distinct radial dependence of the effective excited state lifetime $\overline{\tau_{1\to 0:eff}}$ is present, regardless of the output-coupler transmissivity T_{OC} . A drop of the effective excited state lifetime up to $\overline{\tau_{1\to 0;eff}} \approx 650 \,\mu s$ seems to have no distinct effect on the rate of amplified spontaneous emission W_{ASE} , whereas lower values of the excited state lifetime $\overline{\tau_{1\to 0;eff}}$ show distinct rates of amplified spontaneous emission (see Figure 4.4 and Figure 4.5). Values of the effective excited state lifetime $\overline{\tau_{1\to 0;eff}}$ lower than $\approx 200 \,\mu s$ raise the deposited energy in the active medium lost to amplified spontaneous emission to a level of $\approx 90\%$ preventing the laser from reaching the laser threshold (see Figure 4.2 and Figure 4.4).

A suppression of amplified spontaneous emission can be achieved (see Figure 4.4) by a high intra-cavity laser intensity $I_{L;Intra}$ or small radii of the thin disk ρ_{disk} (see Figure 4.3). The downside of an increasing intra-cavity laser intensity $I_{L;Intra}$ by a lower transmissivity of the output-coupler T_{OC} is a decreased optical-optical efficiency $\eta_{opt.-opt.}$ of the laser as the resonator intern losses are approximately constant. To achieve a high output power of the laser, it is advantageous to increase the thin disk radius ρ_{disk} in order to keep the power density of the pump beam and the power density of the laser beam at reasonable levels within the active medium [23]. However, increasing the radius of the thin disk ρ_{disk} leads to an increased rate of





Figure 4.5: Effective excited state lifetime $\overline{\tau_{ASE}}$, due to amplified spontaneous emission. These values are spatially averaged from the center of the thin disk to the radius of the pump beam waist $w_{0;P} = \rho_{disk} \cdot 66\%$.

amplified spontaneous emission, due to the increased amplification path Λ_{ASE} [22] of spontaneously emitted photons as pointed out above.

4.1.1 Transient Evolution in Dependence of the Output-Coupler Transmissivity

The parameter study discussed above indicates that amplified spontaneous emission can effectively be suppressed by a high intra-cavity laser intensity (see Figure 4.3 and Figure 4.4). In this section three selected numerical simulations, which are part of this parameter study, are discussed in further detail for their transient evolution. All three numerical simulations shown here are for a constant radius of the thin disk of $\rho_{disk} = 10$ cm. The transmissivity of the output-coupler of the three numerical simulations are $T_{OC} = 1.78 \%$, $T_{OC} = 3.16 \%$ and $T_{OC} = 5.62 \%$ showing a weak, a moderate and a strong influence of amplified spontaneous emission on the optical-optical efficiency $\eta_{opt.-opt.}$ (see Figure 4.1 and Figure 4.2).

In Figure 4.6 the transient evolution of the laser beam output power $P_{L;Out}$ normalized







Figure 4.6: Transient evolution of the normalized output power of the laser beam $P_{L;Out}/P_{P;In}$ for a radius of the thin disk of $\rho_{disk} = 10$ cm. Three numerical simulations showing a weak ($T_{OC} = 1.78\%$; green - solid), a moderate ($T_{OC} = 3.16\%$; orange - dash-dotted) and a strong ($T_{OC} = 5.62\%$; blue - dash-dotted) influence by amplified spontaneous emission on the output laser power $P_{L;Out}$. Indicated by the vertical lines are the time instance the lasing threshold is reached τ_{th} and the consecutive time instance of relaxation τ_{relax} to a value of $P_{L;Out;relax} = (P_{L;Out;max} - P_{L;Out;steady})/e^2$.

power of the laser beam $P_{L;Out}$ stays intrinsically the same for all three presented numerical simulations and can be separated into three temporal phases, which are indicated by vertical lines of the corresponding numerical simulation. In the first phase no output by the laser beam is observed as the lasing threshold is not yet reached. The second phase is described by temporal spiking of the output power $P_{L;Out}$ after reaching the lasing threshold and a consecutive temporal relaxation until the steady state is reached in the third phase (the value of normalized output power $P_{L;Out}/P_{P;In}$ dropped to 1/e of the maximum).

The higher the transmissivity of the output-coupler T_{OC} is, the later the threshold of lasing is reached ($T_{OC} = 1.78 \%$: $\tau_{th} \approx 94 \,\mu$ s; $T_{OC} = 3.16 \%$: $\tau_{th} \approx 108 \,\mu$ s; $T_{OC} = 5.62 \%$: $\tau_{th} \approx 146 \,\mu$ s). This is in accordance to the expected behavior, as the laser threshold increases

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for an increasing resonator loss. The transmissivity of the output-coupler represents a (useful) resonator loss and thus increases the time instance to reach the laser threshold τ_{th} for an increasing transmissivity of the output-coupler T_{OC} . The following relaxation to a value of $P_{L;Out;relax} = (P_{L;Out;max} - P_{L;Out;steady})/e^2$ takes longer for an increasing transmissivity of the output-coupler ($\Delta \tau_{relax}$ ($T_{OC} = 1.78\%$) $\approx 60 \,\mu$ s, $\Delta \tau_{relax}$ ($T_{OC} = 3.16\%$) $\approx 83 \,\mu$ s and $\Delta \tau_{relax}$ ($T_{OC} = 5.62\%$) $\approx 190 \,\mu$ s).

The transient evolution of the spectral power distribution of the photon flux by amplified spontaneous emission $\overline{\Phi_{ASE}}$ for a transmissivity of the output-coupler of $T_{OC} = 1.78\%$, $T_{OC} = 3.16\%$ and $T_{OC} = 5.62\%$ is given in Figure 4.8, Figure 4.10 and Figure 4.12, respectively. The corresponding transient evolution for the spectral power distribution of the gain coefficient $\overline{g_{\lambda_{ASE}}}$ is given in Figure 4.7, Figure 4.9 and Figure 4.11. These values represent spatially averaged parameters within the pump beam waist $w_{0;P}$.

$\overline{g_{\lambda_{ASE}}}$

The spectral power distribution of the gain coefficient $\overline{g_{\lambda_{ASE}}}$ shows two temporal phases $(T_{OC} = 1.78 \%)$, see Figure 4.7). The first phase is described by a steep ascent in time, when lasing has not yet occurred and the ASE-photon flux $\overline{\Phi_{ASE}}$ has no significant amplitude. Then a consecutive slower ascent follows until the steady state is reached after $\tau_{steady} \approx 400 \,\mu\text{s}$. The spectral power distribution of the steady state shows a left-skewed spectral power profile with a peak at a wavelength of $\lambda_{ASE} \approx 1048 \,\text{nm}$ with an amplitude of $\overline{g_{\lambda_{ASE}}}_{max} \approx 220 \,\text{m}^{-1}$.

The photon-flux by amplified spontaneous emission $\overline{\Phi_{ASE}}$ ($T_{OC} = 1.78$ %, see Figure 4.8) shows a distinct peak at a wavelength of $\lambda_{ASE} \approx 1030$ nm and local minima at wavelengths of $\lambda_{ASE} \approx 969$ nm and $\lambda_{ASE} \approx 1036$ nm. The local minimum at $\lambda_{ASE} \approx 969$ nm is ascribed to the zero-phonon line of Yb:YAG, whereas the local minimum of the ASE-photon flux $\overline{\Phi_{ASE}}$ at $\lambda_{ASE} \approx 1036$ nm is not indicated by a sample point. Hence, this is ascribed to the interpolation of the spectral power distribution and therefore excluded from the physical interpretation. The same applies to the local maximum of the ASE-photon flux $\overline{\Phi_{ASE}}$ at $\lambda_{ASE} = 1042$ nm.

The ASE-photon flux $\overline{\Phi_{ASE}}$ shows a numerical artifact in the transient evolution (see transient evolution (right graph) in Figure 4.8 for a wavelength of $\lambda_{ASE} \approx 1030 \text{ nm}$ in the time range from 0.1 ms to 0.2 ms). This numerical artifact might be an indicator of too big time steps Δt of the *ODE*-solver resulting in an overshoot and a consecutive relaxation oscillation. As the numerical simulation itself is stable and the ASE-photon flux $\overline{\Phi_{ASE}}$ is not diverging, this is not an instability of the numerical model, but an inaccuracy in the transient solution of the ASE-photon flux $\overline{\Phi_{ASE}}$.

The spectral line-width of the peak at $\lambda_{ASE} \approx 1030 \,\mathrm{nm}$ shows a full-width half maximum of $\Delta \lambda_{ASE;FWHM} \approx 3.8 \,\mathrm{nm}$ and an amplitude of $\overline{\Phi_{ASE}}_{max} \approx 3.7 \times 10^{28} \,\mathrm{m}^{-2} \,\mathrm{s}$.

For a transmissivity of the output-coupler of $T_{OC} = 3.16$ %, no significant difference of the



Figure 4.7: Gain coefficient $\overline{g_{\lambda_{ASE}}}$ for a radius of the thin disk of $\rho_{disk} = 10 \text{ cm}$ and a transmissivity of the output-coupler of $T_{OC} = 1.78 \%$ (weak influence by amplified spontaneous emission on the optical-optical efficiency $\eta_{opt.-opt.}$). These values are spatially averaged from the center of the thin disk to the radius of the pump beam waist $w_{0;P} = \rho_{disk} \cdot 66 \%$. Dots in the spectral power distribution (top graph) represent sample points. The resolution of the spectral power distribution is 20 points in the interval from $\lambda_{ASE;ub} = 925 \text{ nm}$ to $\lambda_{ASE;ub} = 1075 \text{ nm}$.

spectral power distribution of the gain coefficient $\overline{g_{\lambda_{ASE}}}$ is observed (see Figure 4.9). The ascent following the steep rise of the gain coefficient $\overline{g_{\lambda_{ASE}}}$ is faster than for a lower output-coupler transmissivity T_{OC} (see Figure 4.7) and the steady state is already reached after $\tau_{steady} \approx 200 \,\mu s$.

The ASE-photon flux $\overline{\Phi_{ASE}}$ for a transmissivity of the output-coupler of $T_{OC} = 3.16\%$ shows a similar spectral power profile as for an output-coupler transmissivity of $T_{OC} = 1.78\%$ (see Figure 4.9). However, local minima are less pronounced, as the amplitude of the peak at $\lambda_{ASE} \approx 1030$ nm has increased more than one order of magnitude, whereas off-peak wavelengths do not show a significant difference to numerical simulation for an output-coupler transmissivity of $T_{OC} = 1.78\%$. The corresponding full-width half maximum of the peak of the spectral power distribution of the ASE-photon flux $\overline{\Phi_{ASE}}$ shows a significant increase after



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Figure 4.8: Photon flux by amplified spontaneous emission $\overline{\Phi_{ASE}}$ for a radius of the thin disk of $\rho_{disk} = 10 \text{ cm}$ and a transmissivity of the output-coupler of $T_{OC} = 1.78 \%$ (weak influence by amplified spontaneous emission on the optical-optical efficiency $\eta_{opt.-opt.}$). These values are spatially averaged from the center of the thin disk to the radius of the pump beam waist $w_{0;P} = \rho_{disk} \cdot 66 \%$. Dots in the spectral power distribution (top graph) represent sample points. The resolution of the spectral power distribution is 20 points in the interval from $\lambda_{ASE;lb} = 925 \text{ nm}$ to $\lambda_{ASE;ub} = 1075 \text{ nm}$.

the first steep ascent around $\tau_{th} \approx 100 \,\mu s$.

The spectral power distribution of the gain coefficient $\overline{g_{\lambda_{ASE}}}$ for a transmissivity of the output-coupler of $T_{OC} = 5.62 \%$ (see Figure 4.11) does not show a distinct difference to lower values for the output-coupler transmissivity ($T_{OC} \leq 3.16 \%$).

Figure 4.12 shows the spectral power profile of the ASE-photon flux $\overline{\Phi_{ASE}}$ for a transmissivity of the output-coupler of $T_{OC} = 5.62 \,\%$. The spectral power profile of the photon-flux by amplified spontaneous emission $\overline{\Phi_{ASE}}$ is similar for values of the transmissivity of the outputcoupler of $T_{OC} = 1.78 \,\%$, $T_{OC} = 3.16 \,\%$ and $T_{OC} = 5.62 \,\%$. The amplitude increases by a factor greater than 20 for a transmissivity of the output-coupler of $T_{OC} = 5.62 \,\%$ in comparison to a transmissivity of the output-coupler of $T_{OC} = 1.78 \,\%$. A further decrease in the full-width half maximum of the peak around $\lambda_{ASE} \approx 1030 \,\mathrm{nm}$ of the ASE-photon flux $\overline{\Phi_{ASE}}$ to a value



Figure 4.9: Gain coefficient $\overline{g_{\lambda_{ASE}}}$ for a radius of the thin disk of $\rho_{disk} = 10 \text{ cm}$ and a transmissivity of the output-coupler of $T_{OC} = 3.16\%$ (moderate influence by amplified spontaneous emission on the optical-optical efficiency $\eta_{opt.-opt.}$). These values are spatially averaged from the center of the thin disk to the radius of the pump beam waist $w_{0;P} = \rho_{disk} \cdot 66\%$. Dots in the spectral power distribution (top graph) represent sample points. The resolution of the spectral power distribution is 20 points in the interval from $\lambda_{ASE;lb} = 925 \text{ nm}$ to $\lambda_{ASE;ub} = 1075 \text{ nm}$.

of $\Delta \lambda_{ASE;FWHM} \approx 2.4 \,\mathrm{nm}$ is observed. The transient evolution of the ASE-photon flux $\overline{\Phi_{ASE}}$ for a transmissivity of the output-coupler of $T_{OC} = 5.62 \,\%$ reaches its maximum at the end of the first steep ascent of the ASE-photon flux $\overline{\Phi_{ASE}}$ and shows a successive fast relaxation to the steady state of $\tau_{relax} \approx 50 \,\mu\mathrm{s}$.

Unlike the output power of the laser beam $P_{L;Out}$ there is no significant temporal shift observed for the time instance the amplified spontaneous emission reaches the threshold at $\tau_{th} \approx 100 \,\mu\text{s}$ for the three different numerical simulations shown above. This is in accordance with the transient behavior of the gain coefficient $\overline{g_{\lambda_{ASE}}}$, which reaches the transparency threshold at a wavelength of $\lambda_{ASE} \approx 1030 \,\text{nm}$ after $\tau_{transparency} \approx 50 \,\mu\text{s}$, regardless of the transmissivity of the output-coupler. There is no significant ASE-photon flux $\overline{\Phi_{ASE}}$ before transparency of the active medium is reached, as photons are absorbed within the active medium. When



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Figure 4.10: Photon flux by amplified spontaneous emission $\overline{\Phi_{ASE}}$ for a radius of the thin disk of $\rho_{disk} = 10 \text{ cm}$ and a transmissivity of the output-coupler of $T_{OC} = 3.16\%$ (moderate influence by amplified spontaneous emission on the optical-optical efficiency $\eta_{opt.-opt.}$). These values are spatially averaged from the center of the thin disk to the radius of the pump beam waist $w_{0;P} = \rho_{disk} \cdot 66\%$. Dots in the spectral power distribution (top graph) represent sample points. The resolution of the spectral power distribution is 20 points in the interval from $\lambda_{ASE;lb} = 925 \text{ nm}$ to $\lambda_{ASE;ub} = 1075 \text{ nm}$.

transparency for the peak wavelength of amplified spontaneous emission $\lambda_{ASE} \approx 1030 \,\mathrm{nm}$ is reached the ASE-photon flux can develop and deplete the population inversion. As no amplification is given at transparency of the active medium, the ASE-photon flux $\overline{\Phi_{ASE}}$ is in the order of the photon flux by spontaneous emission $\overline{\Phi_{SE}}$. If the gain coefficient of the excited state increases further, the ASE-photon flux $\overline{\Phi_{ASE}}$ is effectively amplified and can reach values much higher than the photon flux by spontaneous emission $\overline{\Phi_{SE}}$.

The spectral linewidth for the ASE-photon flux $\Delta \lambda_{FWHM;ASE}$ is shown in Figure 4.13. Empty fields indicate a spectral power distribution of amplified spontaneous emission not showing a distinct peak at $\lambda_{ASE} \approx 1030 \text{ nm}$ and were excluded from the evaluation. For the other numerical simulations, a Lorentzian fit was done for the peak of the ASE-photon flux $\overline{\Phi_{ASE}}$ at $\lambda_{ASE} = 1030 \text{ nm}$. The extracted full-width half maximum $\Delta \lambda_{FWHM;ASE}$ representing the



Figure 4.11: Gain coefficient $\overline{g_{\lambda_{ASE}}}$ for a radius of the thin disk of $\rho_{disk} = 10 \text{ cm}$ and a transmissivity of the output-coupler of $T_{OC} = 5.62 \%$ (strong influence by amplified spontaneous emission on the optical-optical efficiency $\eta_{opt.-opt.}$). These values are spatially averaged from the center of the thin disk to the radius of the pump beam waist $w_{0;P} = \rho_{disk} \cdot 66 \%$. Dots in the spectral power distribution (top graph) represent sample points. The resolution of the spectral power distribution is 20 points in the interval from $\lambda_{ASE;lb} = 925 \text{ nm}$ to $\lambda_{ASE;ub} = 1075 \text{ nm}$.

spectral linewidth of the numerical simulation is obtained from these fits. The spectral linewidth $\Delta \lambda_{FWHM;ASE}$ shows a negative correlation to the transmissivity of the output-coupler T_{OC} and the radius of the thin disk ρ_{disk} . Values converge to a value of $\Delta \lambda_{FWHM;ASE} \approx 2.4$ nm.

Amplified spontaneous emission is a source of redistribution of the deposited energy by the pump beam in the active medium from regions with a normalized population density above the transparency threshold $D_L > 1$ to regions with a normalized population density lower than the transparency threshold $D_L < 1$. If energy is redistributed to regions outside the area of the laser beam waist $w_{0;L}$, this energy is potentially lost for the amplification of the laser beam. The ASEphoton flux $\overline{\Phi_{ASE}}$ received outside the pumped area $\rho_{disk} > w_{0;P}$ therefore defines a source of loss for the amplification process of the laser beam. Figure 4.14 shows the spatially averaged ASE-photon flux $\overline{\Phi_{ASE}}$ for radii greater than the beam waist of the pump beam $w_{0;P}$ for a



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Figure 4.12: Photon flux by amplified spontaneous emission $\overline{\Phi_{ASE}}$ for a radius of the thin disk of $\rho_{disk} = 10 \text{ cm}$ and a transmissivity of the output-coupler of $T_{OC} = 5.62 \%$ (strong influence by amplified spontaneous emission on the optical-optical efficiency $\eta_{opt.-opt.}$). These values are spatially averaged from the center of the thin disk to the radius of the pump beam waist $w_{0;P} = \rho_{disk} \cdot 66 \%$. Dots in the spectral power distribution (top graph) represent sample points. The resolution of the spectral power distribution is 20 points in the interval from $\lambda_{ASE;lb} = 925 \text{ nm}$ to $\lambda_{ASE;ub} = 1075 \text{ nm}$.

transmissivity of the output-coupler of $T_{OC} = 5.62 \,\%$ and a thin disk radius of $\rho_{disk} = 10 \,\text{cm}$. The spectral power distribution of the ASE-photon flux in the unpumped region $\overline{\Phi_{ASE}} (\rho > w_{0;P})$ does not differ from the ASE-photon flux within the pumped area $\overline{\Phi_{ASE}} (\rho < w_{0;P})$, showing a distinct peak at a wavelength of $\lambda_{ASE} \approx 1030 \,\text{nm}$. The same holds true for the spectral distribution of the gain coefficient $\overline{g_{\lambda_{ASE}}} (\rho < w_{0;P})$ with a peak of the spectral power distribution at $\lambda_{ASE} \approx 1048 \,\text{nm}$ (see Figure 4.15).

The transient evolution of the ASE-photon flux in the unpumped area of the thin disk $\overline{\Phi_{ASE}}$ ($\rho \gtrsim w_{0;P}$) shows a slower temporal evolution in comparison to the pumped area, starting at a time instance of $\tau_{th} \approx 100 \,\mu\text{s}$ and reaching the steady state at the maximum of the spectral power distribution at $\lambda_{ASE} \approx 1030 \,\text{nm}$ after about 350 µs. The transient evolution of the gain coefficient $\overline{g_{\lambda_{ASE}}}$ starts to develop in accordance to the ASE-photon flux $\overline{\Phi_{ASE}}$ after





Figure 4.13: Spectral linewidth $\Delta \lambda_{FWHM;ASE}$ of the peak of ASE-photon flux $\overline{\Phi_{ASE}}$ at $\lambda_{ASE} \approx 1030 \text{ nm}$ in dependence of the radius of the thin disk ρ_{disk} and the transmissivity of the output-coupler T_{OC} . Empty fields represent numerical simulations, which did not show a sufficient peak ($\overline{\Phi_{ASE}}_{max} > 2 \cdot \overline{\Phi_{ASE}}_{mean}$) to extract a full-width half maximum by fitting a Lorentzian curve to the data. Fits were made for a spatial average of the ASE-photon flux within the pump beam waist $\overline{\Phi_{ASE}}$ ($\rho < w_{0:P}$).

 $\tau_{th} \approx 100 \,\mu\text{s}$. The transparency threshold $\overline{g_{\lambda_{ASE}}} = 0$ at the wavelength of $\lambda_{ASE} \approx 1048 \,\text{nm}$ of the maximum of the spectral power distribution of the gain coefficient $\overline{g_{\lambda_{ASE}}}_{max}$ is reached approximately $\tau_{transparency} \approx 200 \,\mu\text{s}$ after the start of the development of a population density outside the pumped area at $\tau_{th} \approx 100 \,\mu\text{s}$. For the wavelength of the maximum of the spectral power distribution of the ASE-photon flux $\overline{\Phi_{ASE}}$ at $\lambda_{ASE} \approx 1030 \,\text{nm}$, the transparency threshold is not reached.

4.1.2 Steady State Solution in Dependence of the Output-Coupler Transmissivity

The spatial profile of the laser beam defines the propagation properties and the spatial overlap to the pump beam. The spatial overlap to the pump beam is a crucial factor for the opticaloptical efficiency $\eta_{opt.opt.}$ of the laser. The power is related to the intensity over the area integral $P = \int_A I dA$. Thus the output power of the laser $P_{L;Out}$ has a quadratic dependence on the laser beam waist $w_{0;L}$. The laser output intensity for the steady state $I_{L;Out}$ is shown in Figure 4.16



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Figure 4.14: Photon flux by amplified spontaneous emission $\overline{\Phi_{ASE}}$ for a radius of the thin disk of $\rho_{disk} = 10 \text{ cm}$ and a transmissivity of the output-coupler of $T_{OC} = 5.62 \%$ (strong influence by amplified spontaneous emission on the optical-optical efficiency $\eta_{opt.-opt.}$). These values are spatially averaged from the radius of the pump beam waist $w_{0;P} = \rho_{disk} \cdot 66 \%$ to the cylinder jacket ρ_{disk} . Dots in the spectral power distribution (top graph) represent sample points. The resolution of the spectral power distribution is 20 points in the interval from $\lambda_{ASE;lb} = 925 \text{ nm}$ to $\lambda_{ASE;ub} = 1075 \text{ nm}$.

for the case of a weak, a moderate and a strong influence by amplified spontaneous emission on the laser output power $P_{L;Out}$ (represented by numerical simulations with a transmissivity of the output-coupler of $T_{OC} = 1.78$ %, $T_{OC} = 3.16$ % and $T_{OC} = 5.62$ %, respectively). A significant decrease in the amplitude of the laser output intensity $I_{L;Out}$ is observed for an increasing transmissivity of the output-coupler T_{OC} . A transmissivity of the output-coupler of $T_{OC} = 3.16$ % shows an amplitude of the laser output intensity $I_{L;Out}$ lowered only by a few percent in comparison to the case of a transmissivity of the output-coupler of $T_{OC} = 1.78$ %. The laser output intensity for a transmissivity of the output-coupler of $T_{OC} = 3.16$ % is mainly lowered by the radial compression of the transverse profile of the laser output intensity $I_{L;Out}$. The spatial overlap of the pump beam and the laser beam for a transmissivity of the outputcoupler of $T_{OC} = 1.78$ % is $A_L/A_P = (\pi \cdot w_{0;L}^2)/(\pi \cdot w_{0;P}^2) \approx 88$ %, whereas for a transmi-



Figure 4.15: Gain coefficient $\overline{g_{\lambda_{ASE}}}$ for a radius of the thin disk of $\rho_{disk} = 10$ cm and a transmissivity of the output-coupler of $T_{OC} = 5.62 \%$ (strong influence by amplified spontaneous emission on the optical-optical efficiency $\eta_{opt.-opt.}$). These values are spatially averaged from the radius of the pump beam waist $w_{0;P} = \rho_{disk} \cdot 66\%$ to the cylinder jacket ρ_{disk} . Dots in the spectral power distribution (top graph) represent sample points. The resolution of the spectral power distribution is 20 points in the interval from $\lambda_{ASE;lb} = 925$ nm to $\lambda_{ASE;ub} = 1075$ nm.

missivity of the output-coupler of $T_{OC} = 3.16\%$ the spatial overlap of the pump beam and the laser beam already decreased to $A_L/A_P \approx 49\%$. In addition, a low transmissivity of the output-coupler of $T_{OC} = 1.78\%$ shows in a good approximation a flat-top transverse profile, whereas a transmissivity of the output-coupler of $T_{OC} = 3.16\%$ transforms the transverse profile of the output laser intensity $I_{L;Out}$ to a lower order super-Gaussian transverse profile. For a transmissivity of the output-coupler of $T_{OC} = 5.62\%$ the beam waist of the laser beam $w_{0;P}$ is compressed even further to a value of less than one third of the pump beam waist $w_{0;P}$, leading to a spatial overlap of only $A_L/A_P \approx 4\%$.

The transverse profile of the laser output intensity is intrinsically defined by the transverse profile of the amplification within the active medium (see Equation 2.16) and hence by the radial profile of the normalized population density of the excited state D_L shown in Figure 4.17.



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Figure 4.16: Radial distribution of the laser output intensity $I_{L;Out}$ for a radius of the thin disk of $\rho_{disk} = 10 \text{ cm}$. Three numerical simulations showing a weak ($T_{OC} = 1.78\%$; green - solid), a moderate ($T_{OC} = 3.16\%$; orange - dash-dotted) and a strong ($T_{OC} = 5.62\%$; blue - dash-dotted) influence by amplified spontaneous emission on the output laser power $P_{L;Out}$.

 D_L shows increasing values at the center of the thin disk and at the cylinder jacket of the thin disk for an increasing influence by amplified spontaneous emission on the laser output power. A significant population density outside the pumped area $\rho \gtrsim w_{0;P}$ created by absorption of amplified spontaneous emission, almost reaching the transparency threshold ($D_L = 1$) is observed for a transmissivity of the output-coupler of $T_{OC} = 3.16$ %. Whereas the normalized population density of the excited state D_L up to the pump beam waist $w_{0;P}$ is constant for a transmissivity of the output-coupler of $T_{OC} = 1.78$ % and almost constant for a transmissivity of the outputcoupler of $T_{OC} = 3.16$ %, the numerical simulation for a transmissivity of the output-coupler of $T_{OC} = 5.62$ % shows a decrease with increasing radius rising again to a local maximum shortly before the pump beam waist $w_{0;P}$.

The rate of amplified spontaneous emission W_{ASE} shows a direct influence on the steady state of the normalized population density of the excited state D_L and vice versa (see Equation 2.16). The radial profile of the rate of amplified spontaneous emission W_{ASE} (see Fig-





Figure 4.17: Radial distribution of the normalized population density of the excited state D_L for a radius of the thin disk of $\rho_{disk} = 10$ cm. Three numerical simulations showing a weak ($T_{OC} = 1.78\%$; green - solid), a moderate ($T_{OC} = 3.16\%$; orange - dash-dotted) and a strong ($T_{OC} = 5.62\%$; blue - dash-dotted) influence by amplified spontaneous emission on the output laser power $P_{L;Out}$.

ure 4.18) differs in amplitude and radial distribution for the three cases of a transmissivity of the output-coupler of $T_{OC} = 1.78$ %, $T_{OC} = 3.16$ % and $T_{OC} = 5.62$ %. The rate of amplified spontaneous emission W_{ASE} for the case of a low transmissivity of the output-coupler $T_{OC} = 1.78$ % has a negligible amplitude for values within the pump beam waist $w_{0;P}$ in comparison to the rate of the pump beam W_P . There is a non-zero amplitude for values outside the pumped area ($\rho \gtrsim w_{0;P}$), but the amplitude is well below the amplitude of the pump beam rate W_P . The sign of the amplitude changes at $\approx w_{0;P}$. For a transmissivity of the output-coupler of $T_{OC} = 3.16$ % and a transmissivity of the output-coupler of $T_{OC} = 5.62$ % a non-negligible rate of amplified spontaneous emission is observed for the pumped area ($\rho \lesssim w_{0;P}$), as well as for the unpumped area ($\rho \gtrsim w_{0;P}$). The rate of amplified spontaneous emission in the unpumped area $W_{ASE}(\rho \gtrsim w_{0;P})$ increases towards the cylinder jacket of the thin disk, showing a higher rate for a transmissivity of the output-coupler of $T_{OC} = 3.16$ %. A transmissivity of the outputcoupler of $T_{OC} = 3.16$ % shows a minimum close to the pump beam waist at ≈ 55 mm. The



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Figure 4.18: Radial distribution of the rate of amplified spontaneous emission W_{ASE} (see Equation 2.11) for a radius of the thin disk of $\rho_{disk} = 10 \text{ cm}$. Three numerical simulations showing a weak ($T_{OC} = 1.78\%$; green - solid), a moderate ($T_{OC} = 3.16\%$; orange - dash-dotted) and a strong ($T_{OC} = 5.62\%$; blue - dash-dotted) influence by amplified spontaneous emission on the output laser power $P_{L;Out}$. The rate of the pump beam (red - solid) is shown for reference and does not differ significantly for the three different cases.

increase of the amplitude towards the center of the thin disk is ascribed to the laser beam, which is in concurrency to amplified spontaneous emission and higher towards the center of the thin disk (see Figure 4.16). For an increasing transmissivity of the output-coupler T_{OC} and hence a decreasing intra-cavity laser intensity $I_{L;Intra}$ (see Figure 4.3) a growing rate of amplified spontaneous emission is observed showing saturation effects by the development of a flat-top radial profile.

4.1.3 Summary of the Results for an Yb:YAG Thin-Disk Oscillator

A parameter study of an Yb:YAG thin-disk oscillator based on the numerical model introduced in Chapter 3 was presented in this section. All numerical simulations presented in this section are part of this parameters study. This parameter study involved the variation of the transmissivity of the output-coupler T_{OC} and the thin disk radius ρ_{disk} by five values each, making a total number of 25 numerical simulations. In addition, numerical simulations neglecting amplified spontaneous emission were done with the very same parameters.

No shift of the optimal transmissivity of the output-coupler $T_{OC} = 5.62\%$ is observed for numerical simulations neglecting amplified spontaneous emission and numerical simulations considering amplified spontaneous emission (see Figure 4.1 and Figure 4.2). A strong dependence on the thin disk radius ρ_{disk} has been observed for the optical-optical efficiency $\eta_{opt.-opt.}$ if amplified spontaneous emission is considered. For the increase of radii of the thin disk ρ_{disk} a decreased optical-optical efficiency $\eta_{opt.-opt.}$ is found, when considering amplified spontaneous emission compared to neglecting amplified spontaneous emission. This correlation is less concise for a lower transmissivity of the output-coupler T_{OC} . The output-coupler transmissivity is directly related to the intra-cavity laser intensity $I_{L:Intra}$ (see Figure 4.3). A higher intra-cavity laser intensity $I_{L;Intra}$ leads to a suppression of amplified spontaneous emission. Amplified spontaneous emission is increasing with the radius of the thin disk ρ_{disk} and the transmissivity of the output-coupler T_{OC} (see Figure 4.4). The correlation of the radius of the thin disk ρ_{disk} is a direct consequence of an increased amplification path Λ_{ASE} for increasing thin disk radii ρ_{disk} [22], whereas the correlation to the transmissivity of the output-coupler T_{OC} is related to a lower intra-cavity laser intensity $I_{L:Intra}$ for a higher transmissivity of the output-coupler T_{OC} . This would lead to a higher gain and thus support amplified spontaneous emission for large radii of the thin disk ρ_{disk} . Amplified spontaneous emission and the intra-cavity laser intensity are in competition.

The transmissivity of the output-coupler T_{OC} shifts the time instance to reach the laser threshold τ_{th} to later times, as the laser threshold depends on the losses in the laser resonator and the transmissivity of the output-coupler represents a (useful) loss for the laser resonator. The ASE-photon flux Φ_{ASE} does not depend on the laser threshold, but only on the transparency threshold. Therefore, the ASE-photon flux Φ_{ASE} starts earlier than the laser beam and can prevent the laser from lasing in the extreme case.

A constant central wavelength of the ASE-photon flux of $\lambda_{ASE} \approx 1030 \text{ nm}$ is observed for numerical simulations under the variation of the transmissivity of the output-coupler T_{OC} . The spectral linewidth of the peak of the spectral power distribution of the ASE-photon flux Φ_{ASE} at $\lambda_{ASE} \approx 1030 \text{ nm}$ decreases with an increasing transmissivity of the output-coupler T_{OC} converging to a value of $\Delta \lambda_{FWHM;ASE} \approx 2.4 \text{ nm}$.

The output intensity of the laser $I_{L;Out}$ is not only significantly lowered in its amplitude, but also in its beam waist $w_{0;P}$. Amplified spontaneous emission shows saturation effects by the transformation from a peak close to the pump beam waist $w_{0;P}$ to a flat-top profile reaching from the pump beam waist $w_{0;P}$ to the center of the thin disk.

4.2 Yb:YAG Thin-Disk Amplifier

Speiser *et al.* [9] pointed out, that power scaling by a master-oscillator power amplifier approach has several advantages over a single thin-disk oscillator. The scaling of the ASE-photon flux Φ_{ASE} with the thin disk radius ρ_{disk} showed a limitation in the achievable optical-optical efficiency $\eta_{opt.-opt.}$ for large radii of the thin disk $\rho_{disk} \gtrsim 5$ cm. Therefore, a parameter study investigating the influence of the thin disk radius ρ_{disk} and the intra-cavity laser intensity $I_{L;Intra}$ (represented by the normalized input intensity of the laser beam $i_{L;In}$) is presented in this section.

Numerical simulations are performed for a multi-pass amplifier (see Appendix A for more details) with a total number of laser beam single-passes through the thin disk of $M_L = 2$. No spatial shaping of the laser beam with a Gaussian transverse profile is assumed in these numerical simulations. The variation of the thin-disk radius ρ_{disk} and input laser peak intensity normalized to saturation intensity at the laser wavelength $i_{L;In}$ for five different values each make a total number of 25 simulations. In addition, simulations of each parameter set are done once considering amplified spontaneous emission and once neglecting amplified spontaneous emission. All numerical simulations discussed in this section are part of this parameter study.

The correlation of thin-disk radius ρ_{disk} and input laser beam intensity normalized to the saturation intensity of the laser $i_{L;In}$ to the optical-optical efficiency $\eta_{opt.-opt.} = \Delta P_{L;Out}/P_{P;In}$ is shown in Figure 4.19, if amplified spontaneous emission is neglected. There is no radial dependence for the relative amplification of the laser output power $\Delta P_{L;Out}/P_{P;In}$. This is in accordance with numerical simulations obtained for a thin-disk oscillator in Section 4.1 and shown in Figure 4.1. This would enable an unlimited power scaling with high optical-optical efficiency $\eta_{opt.-opt.}$ for the thin-disk laser by increasing the thin disk radius ρ_{disk} . The opticaloptical efficiency shows a maximum of $\eta_{opt.-opt.} \approx 53\%$ at a normalized laser input intensity of $i_{L;In} = 7.91$. The minimum (of the presented numerical simulations) is reached for a normalized input intensity of the laser of $i_{L;In} = 0.25$ at a value of $\eta_{opt.-opt.} \approx 12\%$. This is ascribed to the low number of single-passes of the laser beam through the thin disk. If the photon density of the laser beam is not sufficient, only a part of the energy deposited in the active medium by the pump beam can be retrieved by the laser beam. For a very high photon density of the laser beam, the energy deposited in the active medium by the pump beam is not sufficient and reabsorption of the laser beam lowers the optical-optical efficiency $\eta_{opt.-opt.}$.

If amplified spontaneous emission is considered, a strong correlation of the thin disk radius ρ_{disk} and the optical-optical efficiency $\eta_{opt.-opt.}$ is observed. Again, this is in accordance with the case for the thin-disk oscillator discussed in Section 4.1 and shown in Figure 4.2. No shift to

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Figure 4.19: Optical-optical efficiency $\eta_{opt.-opt.}$ in dependence of the normalized laser input intensity $i_{L;In}$ and the radius of the thin disk ρ_{disk} neglecting amplified spontaneous emission in the numerical simulations.

a lower or higher laser input intensities $i_{L;In}$ is observed for the maximum of the optical-optical efficiency $\eta_{opt.-opt.}$ located at $i_{L;In} = 7.91$.

The optical-optical efficiency $\eta_{opt.-opt.}$ is limited by the amplification of the laser beam and the normalized input intensity of the laser beam $i_{L;In}$. The amplification by stimulated emission in depends on the population density of the excited state n_1 . The normalized population density of the excited state D_L in dependence of the normalized input laser intensity $i_{L;In}$ is shown in Figure 4.21. For a large radius of the thin disk $\rho_{disk} \gtrsim 5$ cm, the normalized population density of the excited state D_L is approximately constant with the variation of the normalized input laser intensity $i_{L;In}$. For lower values of the thin disk radius $\rho_{disk} \lesssim 1$ cm, an exponential decrease of the normalized population density of the excited state D_L with increasing normalized input intensity of the laser beam $i_{L;In}$ is observed. For a high normalized input intensity of the laser beam $i_{L;In} \gtrsim 25$, the population density of the excited state D_L converges to a value of $D_L \approx 1.4$, regardless of the radius of the thin disk ρ_{disk} .

Fits were made according to the following simple considerations. Given the laser rate equa-





Figure 4.20: Optical-optical efficiency $\eta_{opt.-opt.}$ in dependence of the normalized laser input intensity $i_{L;In}$ and the radius of the thin disk ρ_{disk} considering amplified spontaneous emission in the numerical simulations.

tions (see Equation 2.16), a linear dependence of each rate to the normalized population density of the excited state D_L is assumed. No indirect relation of the laser input intensity $i_{L;In}$ is assumed for the normalized population density of the excited state D_L . Then the steady state solution of the laser rate equation can be expressed by

$$0 = \overbrace{A_1 + A_2 \cdot D_L}^{W_{Pump}} \overbrace{-B_1 \cdot i_{L;In} - B_2 \cdot i_{L;In} \cdot D_L}^{W_{Laser}} \overbrace{-D_L}^{W_{SE}} \overbrace{-C_1 - C_2 \cdot D_L}^{W_{ASE}}$$

$$= (\underbrace{A_1 - C_1}_{A} \underbrace{-B_1}_{B} \cdot i_{L;In}) - (\underbrace{-A_2 + 1 + C_2}_{C} + \underbrace{B_2}_{D} \cdot i_{L;In}) \cdot D_L.$$
(4.1)

Solving this equation for D_L on the left-hand side and $i_{L;In}$ on the right-hand side gives

$$D_L = \frac{A + B \cdot i_{L;In}}{C + D \cdot i_{L;In}}.$$
(4.2)

This function is used to fit the normalized population density of the excited state D_L in depen-

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Figure 4.21: Normalized population density of the excited state D_L in dependence of radius of the thin disk ρ_{disk} . These values are spatially averaged from the center of the thin disk to the radius of the pump beam waist $w_{0;P} = \rho_{disk} \cdot 66\%$. Dash-dotted lines represent the corresponding numerical simulation without considering amplified spontaneous emission (these overlay each other, as they are radially independent). A fit of the data with the function $(A + B \cdot i_{L;In})/(C + D \cdot i_{L;In})$ is drawn as solid line. Fitting parameters are given in Table 4.1.

dence of the normalized input intensity of the laser beam $i_{L;In}$ with the *curve-fit*-package from the *SciPy-library* [35]. Fits were made for each value of the radius of the thin disk ρ_{disk} and fitting parameters are shown in Table 4.1. Factors *B* and *D* define the linear relation of the normalized population density of the excited state D_L to the normalized input intensity of the laser beam $i_{L;In}$. For an increasing radius of the thin disk, these factors decrease significantly. The factor *C* is defined by the linear dependence of the normalized population density of the excited state D_L to the rate of the pump W_{Pump} (assumed to be constant), the rate of spontaneous emission and amplified spontaneous emission W_{SE} and W_{ASE} , respectively. Hence, this factor scales with the rate of amplified spontaneous emission W_{ASE} .

The suppression of amplified spontaneous emission, even for thin disks with very large radii, can be observed in Figure 4.22 showing the ratio of the rate of amplified spontaneous emission

$\rho_{disk}[\rm{mm}]$	Α	В	С	D
1	2.56	$138 imes 10^{-3}$	$326 imes 10^{-3}$	$139 imes 10^{-3}$
5	2.10	84×10^{-3}	$381 imes 10^{-3}$	88×10^{-3}
10	1.61	49×10^{-3}	405×10^{-3}	51×10^{-3}
50	0.99	12×10^{-3}	537×10^{-3}	12×10^{-3}
100	1.20	7×10^{-3}	816×10^{-3}	7×10^{-3}

Table 4.1: Fitting parameters for the population density of the excited state D_L in dependence of the laser input intensity normalized to the saturation intensity $i_{L;In}$ for five values of the thin disk radius ρ_{disk} .

 W_{ASE} normalized to the rate of the pump beam W_P . For an increasing laser intensity $i_{L;In}$



Figure 4.22: Rate of amplified spontaneous emission W_{ASE} normalized to the rate of the pump beam W_P (see Equation 2.16) in dependence of radius of the thin disk ρ_{disk} . These values are spatially averaged from the center of the thin disk to the radius of the pump beam waist $w_{0;P} = \rho_{disk} \cdot 66 \%$.

a decreasing rate of amplified spontaneous emission W_{ASE} is observed with a more distinct difference for larger radii of the thin disk ρ_{disk} . The rate of amplified spontaneous emission



 W_{ASE} grows to values well above 50 % of the rate of the pump beam W_P for a laser intensity below the saturation intensity $i_{L;In} \leq 0.79$ and radii of the thin disk $\rho_{disk} \geq 1$ cm. For a large radius of the thin disk of $\rho_{disk} = 10$ cm and a small normalized input intensity of the laser beam $i_{L;In} \leq 0.79$, the rate of amplified spontaneous emission converges to a value of $W_{ASE} \approx 92 \% \cdot W_P$.

Amplified spontaneous emission can also be interpreted in the way of a reduced lifetime of the excited state $\tau_{1\to0;eff}$ [22]. This effective lifetime $\tau_{1\to0;eff}$ spatially averaged within the pump beam waist $w_{0:P}$ is shown in Figure 4.23. The effective lifetime of the excited state



Figure 4.23: Effective excited state lifetime $\overline{\tau_{ASE}}$, due to amplified spontaneous emission. These values are spatially averaged from the center of the thin disk to the radius of the pump beam waist $w_{0;P} = \rho_{disk} \cdot 66\%$. Dash-dotted lines represent the corresponding numerical simulation without considering amplified spontaneous emission.

 $\overline{\tau_{1\to 0;eff}}$ shows a significant dependence on the radius of the thin disk ρ_{disk} , as well as on the laser input intensity $i_{L;In}$. For a low normalized input intensity of the laser beam $i_{L;In} \lesssim 2.5$, an exponential decrease is observed for increasing radii of the thin disk ρ_{disk} . For a high normalized input intensity of the laser beam $i_{L;In} \gtrsim 7.91$, the decrease of the effective excited state lifetime $\overline{\tau_{1\to 0;eff}}$ with the thin disk radius ρ_{disk} transforms to a linear decrease with increasing radii of

the thin disk ρ_{disk} . The effective excited state lifetime converges to a value of $\overline{\tau_{1\to 0;eff}} \approx 100 \,\mu\text{s}$ for increasing radii of the thin disk ρ_{disk} .

Fits are done according to the following considerations. The effective lifetime of the excited state $\overline{\tau_{1\to 0;eff}}$ depends on the rate of spontaneous emission W_{SE} and the rate of amplified spontaneous emission W_{ASE} by

$$\begin{aligned} \frac{1}{\tau_{1 \to 0; eff}} = & \frac{W_{SE} + W_{ASE}}{D_L} \\ = & \frac{1 + \int_{V_{em}} \frac{B}{||\vec{s}||^2} \cdot \exp\left(b \cdot ||\vec{s}||\right) \cdot dV_{Em}}{\tau_{1 \to 0}}. \end{aligned}$$

A constant population density of the excited state is assumed as approximation. The integral over the volume of emitter elements V_{Em} is proportional to the radius of the thin disk squared ρ_{disk}^2 and the length of the amplification path $||\vec{s}||$ is proportional to the radius of the thin disk ρ_{disk} . The maximum amplification path is given by $||\vec{s}||_{max} = \sqrt{\rho_{disk}^2 + N \cdot z_{disk}^2}$, whereas the integral over the emitter volumes is approximated by an additional constant factor for regarding reflections of the ASE-photon flux along its path through the thin disk. A fit function of the effective excited state lifetime $\overline{\tau_{1\to0;eff}}$ based on these considerations gives

$$\tau_{1 \to 0; eff} = \frac{\tau_{1 \to 0}}{1 + B \cdot \frac{(\rho_{disk} + D)^2}{\rho_{disk}^2 + d^2} \cdot \exp\left(b \cdot \sqrt{\rho_{disk}^2 + d^2}\right)}$$

The factor *B* indicates the amplification of the ASE-photon flux $g_{\Phi_{ASE}}$, whereas the factor *b* indicates the amplification along the path of an ASE-photon $g_{\Lambda_{ASE}}$. These values are of the same order for each fit. Fitting parameters *D* and *d* represent the influence on the volume integral and the effective amplification path, respectively. The influence on the volume integral increases by a factor of ≈ 4 , whereas the influence on the effective amplification path within the thin disk increases by a factor of ≈ 18.3 . This indicates, that for larger radii of the thin disk ρ_{disk} the number of reflections within the thin disk plays a significant role and is the most important factor of an increasing rate of amplified spontaneous emission.

4.2.1 Transient Evolution in Dependence of the Laser Input Intensity

The transient evolution of the ASE-photon flux Φ_{ASE} is presented for three numerical simulations at a constant value of the normalized input intensity of the laser beam $i_{L;In} = 7.91$ under the variation of the thin disk radius to $\rho_{disk} = 1 \text{ cm}$, $\rho_{disk} = 5 \text{ cm}$ and $\rho_{disk} = 10 \text{ cm}$. These numerical simulations represent a weak ($\rho_{disk} = 1 \text{ cm}$), a moderate ($\rho_{disk} = 5 \text{ cm}$) and a strong ($\rho_{disk} = 10 \text{ cm}$) influence by amplified spontaneous emission on the optical-optical efficiency



$i_{L;In}$	Α	В	b	D	d
0.25	$986 imes 10^{-6}$	4.14	3.87	1.85×10^{-3}	$15.3 imes 10^{-3}$
0.79	984×10^{-6}	4.00	4.62	2.04×10^{-3}	17.6×10^{-3}
2.50	967×10^{-6}	4.25	3.30	3.97×10^{-3}	33.9×10^{-3}
7.91	962×10^{-6}	4.12	3.11	4.97×10^{-3}	64.0×10^{-3}
25.0	945×10^{-6}	4.60	3.18	$7.39 imes 10^{-3}$	280.5×10^{-3}

Table 4.2: Fitting parameters for the effective lifetime of the excited state $\overline{\tau_{1\to 0;eff}}$ in dependence of the thin disk radius ρ_{disk} for five values of the laser input intensity normalized to the saturation intensity $i_{L;In}$.

 $\eta_{opt.-opt.}$. All presented numerical simulations presented in this section are part of the parameter study presented in Section 4.2.

The ASE-photon flux $\overline{\Phi_{ASE}}$ and the corresponding gain coefficient $\overline{g_{\lambda_{ASE}}}$ for a thin disk radius of $\rho_{disk} = 1 \text{ cm}$ is shown in Figure 4.24 and Figure 4.25, respectively. Values are spatially averaged within the beam waist of the pump beam $w_{0;P}$. A Lorentzian curve-fit (see Section 4.1.1) of the peak of the spectral power distribution of the ASE-photon flux $\overline{\Phi_{ASE}}$ at $\lambda_{ASE} \approx 1030 \text{ nm}$ yields in a full-width half maximum of $\Delta \lambda_{FWHM;ASE} \approx 7.7 \text{ nm}$. The transient evolution shows a strong ascent followed by a slower rise of the ASE-photon flux $\overline{\Phi_{ASE}}$ until a steady state is reached. This is in general the very same transient behavior as already presented for the thin-disk oscillator (see Figure 4.8), but the buildup of the ASE-photon flux $\overline{\Phi_{ASE}}$ is much slower. This is ascribed to the fact, that the laser beam in the case of the amplifier actively reduces the population density of the excited state from the beginning, whereas the laser beam is not present until the lasing threshold of the oscillator is reached.

Similar findings are observed for the spectral power distribution of the gain coefficient $\overline{g_{\lambda_{ASE}}}$. A left-skewed Gaussian spectral power distribution is observed with a maximum of the gain coefficient $\overline{g_{\lambda_{ASE}}}$ at $\lambda_{ASE} \approx 1048$ nm. Whereas the spectral power distribution is not changed in general for the case of a thin-disk amplifier in comparison to a thin-disk oscillator, the transient evolution is slower. However, a significantly higher amplitude of the gain coefficient $\overline{g_{\lambda_{ASE}}}$ is observed for the case of a thin-disk amplifier ($\overline{g_{\lambda_{ASE}}}_{max} \approx 575 \text{ m}^{-1}$) being more than twice as high as for the case of a thin-disk oscillator ($\overline{g_{\lambda_{ASE}}}_{max} \approx 220 \text{ m}^{-1}$).

For a thin disk radius of $\rho_{disk} = 5 \text{ cm}$ the ASE-photon flux $\overline{\Phi_{ASE}}$ and the spectral power distribution of the gain coefficient $\overline{g_{\lambda_{ASE}}}$ are shown in Figure 4.26 and Figure 4.28. The transient behavior and the spectral power distribution of the ASE-photon flux $\overline{\Phi_{ASE}}$ is not changed in general in comparison to a smaller thin-disk radius of $\rho_{disk} = 1 \text{ cm}$. The amplitude of the





Figure 4.24: Photon flux by amplified spontaneous emission $\overline{\Phi_{ASE}}$ for a radius of the thin disk of $\rho_{disk} = 1 \text{ cm}$ and a normalized laser intensity of $i_{L;In} = 7.91$ (weak influence by amplified spontaneous emission on the optical-optical efficiency $\eta_{opt.-opt.}$). These values are spatially averaged from the center of the thin disk to the radius of the pump beam waist $w_{0;P} = \rho_{disk} \cdot 66\%$. Dots in the spectral power distribution (top graph) represent sample points. The resolution of the spectral power distribution is 20 points in the interval from $\lambda_{ASE;lb} = 925 \text{ nm}$ to $\lambda_{ASE;ub} = 1075 \text{ nm}$.

ASE-photon flux $\overline{\Phi_{ASE}}$ is by a factor of ≈ 25 greater than for a thin-disk radius of $\rho_{disk} = 1 \text{ cm}$. The full-width half maximum of the peak of ASE-photon flux at $\lambda_{ASE} \approx 1030 \text{ nm}$ is $\Delta \lambda_{ASE;FWHM} \approx 2.7 \text{ nm}$.

The transient evolution of the ASE-photon flux $\overline{\Phi_{ASE}}$ shows numerical artifacts for a wavelength of $\lambda_{ASE} = 1030 \text{ nm}$ (see transient evolution (right graph) in Figure 4.26). These already occurred for numerical simulations of the thin-disk oscillator (see Section 4.1.1) and are considered as inaccuracies not influencing the stability of the numerical simulation.

The full-width half maximum $\Delta \lambda_{FWHM;ASE}$ of the peak of ASE-photon flux $\overline{\Phi_{ASE}}$ at $\lambda_{ASE} \approx 1030 \text{ nm}$ is shown in Figure 4.27. The full-width half maximum $\Delta \lambda_{FWHM;ASE}$ is calculated by applying a Lorentzian curve-fit to the data within a small interval around the peak of ASE-photon flux $\overline{\Phi_{ASE}}$. Curve-fits were made in *Python* with the *curve-fit* package from the



Figure 4.25: Gain coefficient $\overline{g_{\lambda_{ASE}}}$ for a radius of the thin disk of $\rho_{disk} = 1 \text{ cm}$ and a normalized laser intensity of $i_{L;In} = 7.91$ (weak influence by amplified spontaneous emission on the optical-optical efficiency $\eta_{opt.-opt.}$). These values are spatially averaged from the center of the thin disk to the radius of the pump beam waist $w_{0;P} = \rho_{disk} \cdot 66 \%$. Dots in the spectral power distribution (top graph) represent sample points. The resolution of the spectral power distribution is 20 points in the interval from $\lambda_{ASE;lb} = 925 \text{ nm}$ to $\lambda_{ASE;ub} = 1075 \text{ nm}$.

SciPy-library [35]. There is a significant correlation between optical-optical efficiency $\eta_{opt.-opt}$ and the full-width half maximum $\Delta \lambda_{FWHM;ASE}$ of the ASE-photon flux $\overline{\Phi_{ASE}}$ observed. The full-width half maximum $\Delta \lambda_{FWHM;ASE}$ decreases for an increasing radius of the thin disk ρ_{disk} and a decreasing normalized input intensity of the laser beam $i_{L;In}$. The full-width half maximum $\Delta \lambda_{FWHM;ASE}$ converges to a value of $\Delta \lambda_{FWHM;ASE} \approx 2.4$ nm. The maximum value of full-width half maximum $\Delta \lambda_{FWHM;ASE} \approx 7.7$ nm also represents the numerical simulation (neglecting numerical simulations not showing a significant peak of ASE-photon flux $\overline{\Phi_{ASE}}$) showing the highest optical-optical efficiency $\eta_{opt.-opt.}$ (see Figure 4.20). This is in accordance with the findings from [15] that the spectral linewidth of amplified spontaneous emission $\Delta \lambda_{FWHM;ASE}$ is directly correlated to the amplitude of the ASE-photon flux $\overline{\Phi_{ASE}}$ and the effective lifetime of the excited state $\overline{\tau_{1\rightarrow 0:eff}}$.





Figure 4.26: Photon flux by amplified spontaneous emission $\overline{\Phi_{ASE}}$ for a radius of the thin disk of $\rho_{disk} = 5 \text{ cm}$ and a normalized laser intensity of $i_{L;In} = 7.91$ (moderate influence by amplified spontaneous emission on the optical-optical efficiency $\eta_{opt.-opt.}$). These values are spatially averaged from the center of the thin disk to the radius of the pump beam waist $w_{0;P} = \rho_{disk} \cdot 66\%$. Dots in the spectral power distribution (top graph) represent sample points. The resolution of the spectral power distribution is 20 points in the interval from $\lambda_{ASE;lb} = 925 \text{ nm}$ to $\lambda_{ASE;ub} = 1075 \text{ nm}$.

The spectral power distribution of the gain coefficient $\overline{g_{\lambda_{ASE}}}$ (see Figure 4.28) shows a faster ascent at the beginning as for a thin disk radius of $\rho_{disk} = 1$ cm. This transient behavior has already been observed for an increasing transmissivity of the output-coupler T_{OC} for a thin-disk oscillator (see Section 4.1.1). In contrary to the investigation of the thin-disk oscillator showing a constant amplitude for the spectral power distribution of the gain coefficient $\overline{g_{\lambda_{ASE}}}$ for an increasing transmissivity of the output-coupler T_{OC} (see Section 4.1.1), the numerical simulations for a thin-disk amplifier under the variation of the thin disk radius ρ_{disk} show a distinct dependence of the amplitude of the gain coefficient $\overline{g_{\lambda_{ASE}}}$ on the thin disk radius of ρ_{disk} . The gain coefficient $\overline{g_{\lambda_{ASE}}}$ for a thin disk radius of $\rho_{disk} = 5$ cm decreased by a factor of ≈ 1.4 in comparison to the gain coefficient $\overline{g_{\lambda_{ASE}}}$ for a thin disk radius of $\rho_{disk} = 1$ cm. For a thindisk oscillator the intra-cavity laser intensity $I_{L:Intra}$ is directly influenced by the population

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Figure 4.27: Full-width half maximum of the ASE-photon flux $\Delta \lambda_{FWHM;ASE}$ of the peak of the ASEphoton flux $\overline{\Phi_{ASE}}$ at $\lambda_{ASE} \approx 1030$ nm in dependence of the normalized input intensity of the laser beam $i_{L;In}$ and the radius of the thin disk ρ_{disk} . Empty fields represent numerical simulations, which did not show a sufficient peak ($\overline{\Phi_{ASE}}_{max} > 2 \cdot \overline{\Phi_{ASE;mean}}$) to extract a full-width half maximum by fitting a Lorentzian curve to the data. Fits were made for a spatial average of the ASE-photon flux within the pump beam waist $\overline{\Phi_{ASE}}$ ($\rho < w_{0;P}$).

density of the excited state n_1 , leading to a reduced intra-cavity laser intensity $I_{L;Intra}$, if the ASE-photon flux $\overline{\Phi_{ASE}}$ increases (see Equation 2.16). Hence, the population density of the excited state n_1 is constant for an increasing ASE-photon flux $\overline{\Phi_{ASE}}$, whereas the intra-cavity laser intensity $I_{L;Intra}$ decreases (see Figure 4.3). For a thin-disk amplifier, the intra-cavity laser intensity $I_{L;Intra}$ is assumed to be constant, entailing a reduced population density of the excited state n_1 to fulfill the laser rate equation (see Equation 2.16).

Hence, for a thin disk radius of $\rho_{disk} = 10 \text{ cm}$ a further decrease of the amplitude of the spectral power distribution of the gain coefficient $\overline{g_{\lambda_{ASE}}}$ by a factor of ≈ 2.3 to a value of $\overline{g_{\lambda_{ASE}}}_{max} \approx 255 \text{ m}^{-1}$ is observed in Figure 4.29 in comparison to a thin disk radius of $\rho_{disk} = 1 \text{ cm} (\overline{g_{\lambda_{ASE}}}_{max} \approx 575 \text{ m}^{-1})$. In addition, the first steep ascent of the transient evolution of the spectral power distribution of the gain coefficient $\overline{g_{\lambda_{ASE}}}$ is faster in comparison to the numerical





Figure 4.28: Gain coefficient $\overline{g_{\lambda_{ASE}}}$ for a radius of the thin disk of $\rho_{disk} = 5 \text{ cm}$ and a normalized laser intensity of $i_{L;In} = 7.91$ (moderate influence by amplified spontaneous emission on the optical-optical efficiency $\eta_{opt.-opt.}$). These values are spatially averaged from the center of the thin disk to the radius of the pump beam waist $w_{0;P} = \rho_{disk} \cdot 66\%$. Dots in the spectral power distribution (top graph) represent sample points. The resolution of the spectral power distribution is 20 points in the interval from $\lambda_{ASE;lb} = 925 \text{ nm}$ to $\lambda_{ASE;ub} = 1075 \text{ nm}$.

simulations for a smaller thin disk radius of $\rho_{disk} = 1 \text{ cm}$ and $\rho_{disk} = 5 \text{ cm}$ (see Figure 4.25 and Figure 4.28).

The spectral power distribution of the ASE-photon flux $\overline{\Phi_{ASE}}$ shown in Figure 4.30 only significantly changes in its amplitude of the peak at $\lambda_{ASE} \approx 1030$ nm, which is increased by a factor of ≈ 82 in comparison to a thin disk radius of $\rho_{disk} = 1$ cm. The full-width half maximum does not change significantly in comparison to a thin disk radius of $\rho_{disk} = 5$ cm and stays at a value of $\Delta \lambda_{ASE;FWHM} \approx 2.5$ nm.

The numerical artifacts observed in the transient evolution of the ASE-photon flux $\overline{\Phi}_{ASE}$ (see transient evolution (right plot) in Figure 4.30) occur for a longer time span in comparison to the numerical simulation for a thin disk radius of $\rho_{disk} = 5$ cm.

The transient evolution of the ASE-photon flux $\overline{\Phi_{ASE}}$ and the spectral power distribution



Figure 4.29: Gain coefficient $\overline{g_{\lambda_{ASE}}}$ for a radius of the thin disk of $\rho_{disk} = 10 \text{ cm}$ and a normalized laser intensity of $i_{L;In} = 7.91$ (strong influence by amplified spontaneous emission on the optical-optical efficiency $\eta_{opt.-opt.}$). These values are spatially averaged from the center of the thin disk to the radius of the pump beam waist $w_{0;P} = \rho_{disk} \cdot 66 \%$. Dots in the spectral power distribution (top graph) represent sample points. The resolution of the spectral power distribution is 20 points in the interval from $\lambda_{ASE;lb} = 925 \text{ nm}$ to $\lambda_{ASE;ub} = 1075 \text{ nm}$.

of the gain coefficient $\overline{g_{\lambda_{ASE}}}$ can be explained by the following considerations. The excited state population density increases until the laser threshold is reached and the following temporal evolution is then described by the increase of the temperature, due to the induced heat by the pump beam, which is by orders of magnitude slower than the excited state lifetime $\tau_{1\to 0}$.

The transient evolution of the temperature at a position located at $\rho = 0 \text{ mm}$ and z = 60 µmis shown in Figure 4.31. A steep ascent in time in the first few-hundred µs and a consecutive much slower rise of the temperature correlating with the transient evolution of the gain coefficient $\overline{g_{\lambda_{ASE}}}$ (see Figures 4.25, 4.28 and 4.29) is observed. As the laser beam is absorbed by the active medium for a gain coefficient $g_L < 0$, no energy is extracted by the laser beam. This makes spontaneous emission and amplified spontaneous emission the only dissipative sources left in the assumed model for $g_L < 0$. As the population density is low and the gain is neg-





Figure 4.30: Photon flux by amplified spontaneous emission $\overline{\Phi_{ASE}}$ for a radius of the thin disk of $\rho_{disk} = 10 \text{ cm}$ and a normalized laser intensity of $i_{L;In} = 7.91$ (strong influence by amplified spontaneous emission on the optical-optical efficiency $\eta_{opt.-opt.}$). These values are spatially averaged from the center of the thin disk to the radius of the pump beam waist $w_{0;P} = \rho_{disk} \cdot 66\%$. Dots in the spectral power distribution (top graph) represent sample points. The resolution of the spectral power distribution is 20 points in the interval from $\lambda_{ASE;lb} = 925 \text{ nm}$ to $\lambda_{ASE;ub} = 1075 \text{ nm}$.

ative (absorption), both factors are negligible. Therefore, a strong ascent induced by the heat deposited by the laser beam and the pump beam is observed at first. As the population density increases to values greater than the transparency threshold ($g_L = 0$), the laser beam is extracting energy from the active medium. In addition, amplified spontaneous emission can increase and redistributes deposited energy from the pumped area of the thin disk ($\rho < w_{0;P}$) to the unpumped area of the thin disk ($\rho > w_{0;P}$). This also explains that for a higher ASE-photon flux $\overline{\Phi_{ASE}}$ a lower temperature at the center of the thin disk is observed. In contrast to the center of the thin disk being cooled by amplified spontaneous emission, the unpumped area is heated by the redistribution of energy from the center of the thin disk to the cylinder jacket of the thin disk.

For a thin disk radius of $\rho_{disk} = 10 \text{ cm}$, there is a non-negligible gain coefficient $\overline{g_{\lambda_{ASE}}}$ observed for the unpumped area ($\rho \gtrsim w_{0;P}$) (see Figure 4.32). The spectral power distribution





Figure 4.31: Transient evolution of the temperature in the thin disk T at the radial center of the thin disk. The axial position is approximately at the center at $z = 60 \,\mu\text{m}$ of the thin disk of thickness $z_{disk} = 130 \,\mu\text{m}$. Three numerical simulations showing a weak ($\rho_{disk} = 1 \,\text{cm}$; green - solid), a moderate ($\rho_{disk} = 5 \,\text{cm}$; orange - dash-dotted) and a strong ($\rho_{disk} = 10 \,\text{cm}$; blue - dash-dotted) influence by amplified spontaneous emission on the optical-optical efficiency $\eta_{opt.-opt.}$.

of the gain coefficient $\overline{g_{\lambda_{ASE}}}$ does not differ from the spectral power distribution of the gain coefficient $\overline{g_{\lambda_{ASE}}}$ within the beam waist of the pump beam $w_{0;P}$, whereas the amplitude is lower, it still reaches transparency ($\overline{g_{\lambda_{ASE}}} > 0$) for $\lambda_{ASE} \gtrsim 1031$ nm. The peak of the ASE-photon flux $\overline{\Phi_{ASE}}$ ($\rho > w_{0;P}$) (see Figure 4.33) at $\lambda_{ASE} \approx 1030$ nm is by a factor of ≈ 1.5 lower than the ASE-photon flux within the pump beam waist $\overline{\Phi_{ASE}}$ ($\rho < w_{0;P}$). The first steep ascent is much slower in comparison to the very same ascent within the pump beam waist $w_{0;P}$. This is ascribed to the absorption of the ASE-photon flux $\overline{\Phi_{ASE}}$ in the unpumped area ($\rho \gtrsim w_{0;P}$), as the local absorption increases the local population density n_1 . This in turn decreases the local absorption and hence the ASE-photon flux $\overline{\Phi_{ASE}}$ is less absorbed in the unpumped area ($\rho \gtrsim w_{0;P}$), which yields in a higher ASE-photon flux $\overline{\Phi_{ASE}}$ in the unpumped area ($\rho \gtrsim w_{0;P}$). This process is reflected in the transient evolution of the gain coefficient $\overline{g_{\lambda_{ASE}}}$ of the unpumped area ($\rho \gtrsim w_{0;P}$).





Figure 4.32: Gain coefficient $\overline{g_{\lambda_{ASE}}}$ for a radius of the thin disk of $\rho_{disk} = 10 \text{ cm}$ and a normalized laser intensity of $i_{L;In} = 7.91$ (strong influence by amplified spontaneous emission on the optical-optical efficiency $\eta_{opt.-opt.}$. These values are spatially averaged from the radius of the pump beam waist $w_{0;P} = \rho_{disk} \cdot 66\%$ to the cylinder jacket ρ_{disk} . Dots in the spectral power distribution (top graph) represent sample points. The resolution of the spectral power distribution is 20 points in the interval from $\lambda_{ASE;lb} = 925 \text{ nm}$ to $\lambda_{ASE;ub} = 1075 \text{ nm}$.

4.2.2 Steady State Solution in Dependence of the Laser Input Intensity

The influence by amplified spontaneous emission on the optical-optical efficiency $\eta_{opt.-opt.}$ is not only induced by the modification of the amplitude of the gain coefficient g_L , but also strongly influenced by the modification of the transverse profile of the gain coefficient g_L (see Section 4.1.2). The radial distribution of the amplification of the laser beam $\Delta I_{L;out} = I_{L;out} - I_{L;In}$ is shown in Figure 4.34. The amplification of the laser beam $\Delta I_{L;Out}$ shows a significant decrease for an increasing thin disk radius ρ_{disk} . However, unlike for the case of the oscillator (see Section 4.1.2), there is no significant radial compression of the laser beam observed. The numerical simulation for a thin disk radius of $\rho_{disk} = 1$ cm shows a higher-order super-Gaussian transverse profile, induced by the transverse profile of the pump beam. For an increasing influence by amplified spontaneous emission the most significant loss of amplification $\Delta I_{L;Out}$ is



Figure 4.33: Photon flux by amplified spontaneous emission $\overline{\Phi_{ASE}}$ for a radius of the thin disk of $\rho_{disk} = 10 \text{ cm}$ and a normalized laser intensity of $i_{L;In} = 7.91$ (strong influence by amplified spontaneous emission on the optical-optical efficiency $\eta_{opt.-opt.}$. These values are spatially averaged from the radius of the pump beam waist $w_{0;P} = \rho_{disk} \cdot 66\%$ to the cylinder jacket ρ_{disk} . Dots in the spectral power distribution (top graph) represent sample points. The resolution of the spectral power distribution is 20 points in the interval from $\lambda_{ASE;lb} = 925 \text{ nm}$ to $\lambda_{ASE;ub} = 1075 \text{ nm}$.

observed for off-center radial positions $0.2 < \rho < w_{0;P}$. As a Gaussian transverse profile of the laser beam is assumed, the significant decrease of amplification at the slope of the radial intensity distribution of the laser beam indicates an insufficient laser beam intensity to suppress amplified spontaneous emission.

The ASE-photon flux Φ_{ASE} clears out the population density of the excited state n_1 for insufficiently saturated radial positions by the laser beam. The normalized population density of the excited state D_L is shown in Figure 4.35. For a thin disk radius of $\rho_{disk} = 1$ cm, a clear correlation of the transversal profile of the laser beam (Gaussian) and the normalized population density of the excited state D_L is observed. For a negligible rate of amplified spontaneous emission, spontaneous emission and the laser beam are the only dissipative terms left in the laser rate equation (see Equation 2.16). For a sufficiently strong laser beam, the rate of spontaneous

DLR

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Figure 4.34: Radial distribution of the amplification of the laser beam given by output intensity of the laser beam $I_{L;Out}$ less the input intensity of the laser beam $I_{L;In}$ for a normalized input laser intensity of $i_{L;In} = 7.91$. Three numerical simulations showing a weak ($\rho_{disk} = 1 \text{ cm}$; green - solid), a moderate ($\rho_{disk} = 5 \text{ cm}$; orange - dash-dotted) and a strong ($\rho_{disk} = 10 \text{ cm}$; blue - dash-dotted) influence by amplified spontaneous emission on the output laser power $P_{L;Out}$.

emission is negligible in comparison to the rate of the laser beam and therefore the strong radial correlation between the transverse profile of the laser beam and the radial distribution of the normalized population density of the excited state D_L is observed. A lower normalized population density of the excited state D_L is observed for off-center radial positions up to the beam waist radius of the pump beam $w_{0;P}$. For an increasing thin disk radius ρ_{disk} and thus a higher influence by amplified spontaneous emission (see Section 4.2.1), the curve within the pump beam waist $w_{0;P}$ transforms to a flat-top radial distribution, leading to a lower amplification at the slope of the laser beam.

For an increasing thin disk radius ρ_{disk} and thus a higher ASE-photon flux Φ_{ASE} , the population density of the excited state within the pump beam waist $D_L(\rho < w_{0;P})$ decreases, as the ASE-photon flux Φ_{ASE} redistributes the population density to unpumped areas ($\rho \gtrsim w_{0;P}$) (see Figure 4.35). For the center of the thin disk, the maximum length of the amplification path


Figure 4.35: Radial distribution of the normalized population density of the excited state D_L for a normalized laser intensity of $i_{L;In} = 7.91$. Three numerical simulations showing a weak ($\rho_{disk} = 1 \text{ cm}$; green - solid), a moderate ($\rho_{disk} = 5 \text{ cm}$; orange - dash-dotted) and a strong ($\rho_{disk} = 10 \text{ cm}$; blue - dash-dotted) influence by amplified spontaneous emission on the output laser power $P_{L;Out}$.

 Λ_{ASE} is approximately given by the pump beam waist $w_{0;P}$, if reflections at the surfaces are neglected. At a position ρ_{rec} the length of the amplification path Λ_{ASE} can grow up to a value of $w_{0;P} + \rho_{rec} > w_{0;P}$. Therefore, the ASE-photon flux Φ_{ASE} increases up to the radius, where the normalized population density of the excited state decreased below the transparency threshold $D_L < 1$.

The ASE-photon flux Φ_{ASE} redistributes the population density of the excited state, due to the absorption in the unpumped area ($\rho \gtrsim w_{0;P}$). This is observed in the increased normalized population density of the excited state D_L for $\rho \gtrsim w_{0;P}$ and the decreasing (negative) rate of amplified spontaneous emission W_{ASE} for off-center positions $0.2 < \rho < w_{0;P}$ shown in Figure 4.36. The minimum rate of amplified spontaneous emission W_{ASE} is observed close to the pump beam waist $w_{0;P}$ and is approximately constant for increasing radii of the thin disk ρ_{disk} (increasing ASE-photon flux Φ_{ASE}). Close to the center of the thin disk, the rate of amplified spontaneous emission W_{ASE} shows significantly higher values than the minimum rate of ampli-

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Figure 4.36: Radial distribution of the rate of amplified spontaneous emission W_{ASE} (see Equation 2.11) for a normalized laser intensity of $i_{L;In} = 7.91$. Three numerical simulations showing a weak ($\rho_{disk} = 1 \text{ cm}$; green - solid), a moderate ($\rho_{disk} = 5 \text{ cm}$; orange - dash-dotted) and a strong ($\rho_{disk} = 10 \text{ cm}$; blue - dash-dotted) influence by amplified spontaneous emission on the output laser power $P_{L;Out}$.

fied spontaneous emission W_{ASE} observed for each corresponding numerical simulation. The correlation and concurrency to the rate of the laser beam W_L is reflected in the radial distribution of the rate of amplified spontaneous emission W_{ASE} .

4.2.3 Summary of the Results for an Yb: YAG Thin-Disk Amplifier

A parameter study of an Yb:YAG thin-disk amplifier based on the numerical model introduced in Chapter 3 was presented in this section. All numerical simulations presented in this section are part of this parameter study. This parameter study involved the variation of the normalized input intensity of the laser beam $i_{L;In}$ and the thin disk radius ρ_{disk} by five values each, making a total number of 25 numerical simulations. In addition, numerical simulations neglecting amplified spontaneous emission were done with the very same parameters.

The comparison of the optical-optical efficiency $\eta_{opt.-opt.}$ for numerical simulations neglecting amplified spontaneous emission (see Figure 4.19) and considering amplified spontaneous



emission (see Figure 4.20) shows that power scaling by increasing the radius of the thin disk ρ_{disk} is limited in its optical-optical efficiency $\eta_{opt.-opt.}$. The rate by amplified spontaneous emission can grow up to $W_{ASE} = 92 \% \cdot W_P$. This limits the population density of the excited state and hence the achievable gain for a given number of laser beam passes through the thin disk. The effective lifetime of the excited state $\tau_{1\to0;eff}$ shows an exponential decrease with increasing radii of the thin disk ρ_{disk} for an normalized input intensity of the laser beam of $i_{L;In} \leq 7.91$. A linear decrease with increasing radius of the thin disk ρ_{disk} is observed for an input intensity of $i_{L;In} = 25$.

The transient evolution of the ASE-photon flux Φ_{ASE} is similar to the numerical simulations for a thin-disk oscillator (see Section 4.1). A peak of the ASE-photon flux Φ_{ASE} at $\lambda_{ASE} \approx 1030 \text{ nm}$ is observed for all three numerical simulations presented in this section. The numerical simulations under the variation of the thin disk radius ρ_{disk} show a negative correlation of the full-width half maximum of the ASE-photon flux $\Delta \lambda_{FWHM;ASE}$ to the amplitude of the ASE-photon flux Φ_{ASE} . This has also been observed by Chen *et al.* [15]. The full-width half maximum of the ASE-photon flux converges to a value of $\Delta \lambda_{FWHM;ASE} \approx 2.4 \text{ nm}$. The amplitude of the spectral power distribution of the gain coefficient $\overline{g_{\lambda_{ASE}}}$ shows a positive correlation to the radius of the thin disk ρ_{disk} .

The temporal evolution of the temperature T at the center of the thin disk shows a lower temperature for an increasing ASE-photon flux Φ_{ASE} . The ASE-photon flux Φ_{ASE} is redistributing heat from the center of the thin disk to the unpumped area of the thin disk ($\rho \gtrsim w_{0;P}$). The redistribution of the population density of the excited state n_1 from the center of the thin disk to the unpumped area ($\rho \gtrsim w_{0;P}$) decreases the achievable amplification at the slope of the laser beam $\Delta I_{L;Out}$. The amplification of the laser beam $\Delta I_{L;Out}$ at the center of the thin disk is only moderately lowered by a high ASE-photon flux Φ_{ASE} . This is caused by the Gaussian transverse profile of the input-laser beam and the super-Gaussian transverse profile of the pump beam.

Chapter 5

Conclusion

A numerical model of the laser rate equations considering amplified spontaneous emission implemented in *Python* was presented. Effort was made to implement a fast numerical model with a high temporal resolution and a high spatial resolution of the parameters. Calculations of the temperature were done with FEniCS, a highly efficient, open-source tool for finite element modeling with an interface to *Python*.

Local laser rate equations taking advantage of the rotational symmetry were used to compute the transient evolution and spatial distribution of the population density of the excited state, as well as the intra-cavity laser intensity for an oscillator. The efficient implementation allows performing numerical simulations with a high spatial and temporal resolution. The spectral power distribution of spontaneous emission and the amplification of spontaneous emission is regarded by an adaptive weighted frequency interval method.

Numerical simulations for an Yb:YAG thin-disk oscillator are presented in Section 4.1. A variation of the thin disk radius ρ_{disk} provides insight into the power-scaling behavior, whereas a variation of the intra-cavity laser intensity $I_{L;Intra}$ (tuned by the transmissivity of the output-coupler T_{OC}) gives the achievable optical-optical efficiency $\eta_{opt.-opt.}$.

The model reveals, that the ASE-photon flux Φ_{ASE} strongly depends on the intra-cavity laser intensity $I_{L;Intra}$ and the radius of the thin disk ρ_{disk} . Very large radii of the thin disk $\rho_{disk} \gtrsim 5 \text{ cm}$ increase the amplification path Λ_{ASE} of the ASE-photon flux Φ_{ASE} . The rate of amplified spontaneous emission can then grow up to values of 92% of the rate of the pump beam W_P , leading to a massive decrease of the optical-optical efficiency $\eta_{opt.-opt.}$ of the laser with increasing radius of the thin disk ρ_{disk} . A high intra-cavity laser intensity $I_{L;Intra}$ (by a small transmissivity of the output-coupler T_{OC}) can suppress amplified spontaneous emission, but comes with a decreased optical-optical efficiency $\eta_{opt.-opt.}$ as the population density of the excited state n_1 is kept low and hence the gain coefficient g_L is low.

The transient evolution of the spectral power distribution of the ASE-photon flux Φ_{ASE} shows a fast buildup with a peak of the spectral power distribution at $\lambda_{ASE} = 1030$ nm. The spectral linewidth $\Delta \lambda_{FWHM;ASE}$ of this signal decreases with an increasing amplitude of the ASE-photon flux Φ_{ASE} and converges to a value of $\Delta \lambda_{FWHM;ASE} \approx 2.4$ nm. The transient evolution of the spectral power distribution of the gain coefficient $g_{\lambda_{ASE}}$ shows the redistribution of the population inversion from the pumped area ($\rho < w_{0;P}$) to the unpumped area $\rho > w_{0;P}$. A maximum of the spectral power distribution of $g_{\lambda_{ASE};max} \approx 220 \text{ m}^{-1}$ at a wavelength of $\lambda_{ASE} = 1048 \text{ nm}$ is observed.

The transverse profile of the output-intensity of the laser beam $I_{L;Out}$ shows a significant radial compression of the transverse profile of the laser beam, caused by amplified spontaneous emission. This significantly lowers the achievable optical-optical efficiency $\eta_{opt.-opt.}$, as the output power of the laser $P_{L;Out}$ is quadratically dependent on the beam waist of the laser beam $w_{0;L}$.

Further numerical simulations for an Yb:YAG thin-disk amplifier with two single-passes through the thin disk of the laser beam are presented in Section 4.2. A parameter study under the variation of the thin disk radius ρ_{disk} and the variation of the input intensity of the laser beam $I_{L;In}$ was carried out.

The scaling of the normalized population density of the excited state D_L with the normalized laser input intensity $i_{L;In}$ showed a constant relation for very large radii of the thin disk of $\rho_{disk} \gtrsim 5 \text{ cm}$. The most important factor for this behavior was attributed to the increase of the amplification path of amplified spontaneous emission Λ_{ASE} . The effective lifetime of the excited state $\tau_{1\to0;eff}$ showed an exponential decrease with the radius of the thin disk ρ_{disk} for a low normalized input intensity of the laser beam $i_{L;In}$. This transformed to a linear decrease for a very high normalized input intensity of the laser beam $I_{L;In} = 25I_{Sat;L}$.

A significantly higher amplitude of the spectral power distribution of the gain coefficient $g_{\lambda_{ASE}} \approx 575 \,\mathrm{m}^{-1}$ for a low ASE-photon flux Φ_{ASE} was observed for the numerical simulation of an thin-disk amplifier in comparison to the numerical simulation for an thin-disk oscillator. This is induced by the constant laser input intensity $i_{L;In}$, and a decrease of the amplitude of the spectral power distribution of the gain coefficient $g_{\lambda_{ASE}}$ with increasing radius of the thin disk ρ_{disk} to values of $g_{\lambda_{ASE}} \approx 255 \,\mathrm{m}^{-1}$ (corresponding increase of the ASE-photon flux Φ_{ASE} by a factor of ≈ 82) is observed. The peak of the spectral power distribution of the gain coefficient $g_{\lambda_{ASE}} \approx 1048 \,\mathrm{nm}$. The spectral power distribution of the ASE-photon flux Φ_{ASE} shows a maximum at $\lambda_{ASE} \approx 1030 \,\mathrm{nm}$ with an decreasing full-width half maximum converging to a value of $\Delta\lambda_{FWHM;ASE} \approx 2.4 \,\mathrm{nm}$ for an increasing ASE-photon flux Φ_{ASE} . The redistribution of the population ($\rho < w_{0:P}$) and increases

the temperature for the unpumped area ($\rho > w_{0;P}$).

The amplification of the laser beam $\Delta I_{L;Out}$ shows the most significant decrease at the slope of the laser beam of Gaussian transverse profile. The insufficient saturation by the laser beam leads to an increase of amplified spontaneous emission in radial direction towards the pump beam waist $w_{0;P}$.

Further investigations based on the implemented numerical model for amplified spontaneous emission in thin-disk lasers will help to gain a better understanding of the strong nonlinear behavior of a thin-disk laser. A recent development to suppress amplified spontaneous emission is the so-called anti-ASE cap, which introduces an undoped medium at the front of the thin disk and thus reduces the effect of back-reflected ASE-photons at the top surface [16–18]. Numerical simulations regarding an anti-ASE cap are assumed to expand the power scaling of the thin disk laser limited by amplified spontaneous emission. For thin-disk amplifier, it was found, that the transverse profile of the laser beam with an intensity I_L radially decreasing towards the cylinder jacket, leads to the development of a high ASE-photon flux Φ_{ASE} . An increased number of passes through the thin disk could help suppressing amplified spontaneous emission. In addition, numerical simulations investigating the influence of the spatial overlap of the pump beam and the laser beam might clarify the power scaling of high-power thin-disk oscillators and amplifiers.

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Appendix A

Yb:YAG Thin-Disk Laser

The basic assumptions used for the simulations are described in this chapter as a reference.

A.1 Geometrical and Optical Properties of the Cavity

The thin disk is often used as an active mirror of the resonator. The heat sink is mounted to the back base of the thin disk to cool it axially. The very low aspect ratio z_{disk}/ρ_{disk} for the assumed thin disk geometry makes it feasible to effectively discharge the induced heat flux density in the thin disk to the heat sink.

To use the active medium as mirror of the resonator, a high-reflective coating (*HR*) with reflectivity R_{HR} has to be added between the thin disk and the heat sink. To minimize losses of incident beams an anti-reflective (*AR*) coating with transmission T_{AR} at the front side of the thin disk should be added as well.

The single-pass amplification of the laser beam or the single-pass absorption of the pump beam is usually below 10% of the incident power, due to the short interaction length ($z_{disk} = 130 \,\mu\text{m}$) and the moderate doping concentration of $c_{dot} = 9 \,\text{at}$ %. To achieve a high absorption of the pump beam and the laser beam, multiple passes through the active medium have to be made to increase the gain-length product.

The resonator losses per round trip γ_{res} can be divided into useful and dissipative losses denoted as γ_{use} and γ_{diss} , respectively. Dissipative losses are defined by the combined reflectivity of the mirrors for the laser beam inside the resonator R_{res} and the disk R_{disk} and other losses summarized by L_{res} . The reflectivity of the thin disk is defined as

$$R_{disk} = R_{HR} \cdot (1 - T_{AR})^2.$$
 (A.1)

A.1. CAVITY

Parameters of the resonator are given in Table A.1, unless otherwise stated.

Table A.1: Parameters	s of a t	ypical	laser	resonator.
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Parameter	Value	Unit		
Optical Parameters				
Disk reflectivity (R_{disk})	99.985			
Combined mirror reflectivity (R_{res})	99.900	۲ <u>%</u> ٦		
Resonator losses (L_{res})	0.250			
Output-coupler transmissivity (T_{OC})	1.0, 1.78, 3.16, 5.62 and 10.0			
Geometrical Parameters				
Resonator length (l_{res})	1.0	[m]		
Pump beam single-passes (M_P)	48	-		
Laser beam single-passes (M_L)	4	-		

A.1.1 Yb:YAG Thin-Disk Oscillator

The simplest approach for a resonator with a thin disk as an active mirror is an I-resonator (see Figure A.1a). The energy is extracted through an output coupler, which has a transmissivity of the output-coupler T_{OC} optimized for the favorable working point.

The combined reflectivity of N mirrors with reflectivity R_i ($i \in I$ and $I \subset \mathbb{N}$ for I the number of mirrors) in the resonator R_{res} is defined analogously as

$$R_{res} = R_1 \cdot R_2 \cdots R_N. \tag{A.2}$$

The round-trip time T_{res} of a resonator of length l_{res} is approximated by

$$T_{res} = 2 \cdot l_{res}/c_0,\tag{A.3}$$

with the speed of light in vacuum c_0 . The mean resonator dwell time of a photon τ_{res} is then defined by [30]

$$\tau_{res} = \frac{T_{res}}{\underbrace{\gamma_{use} + \gamma_{diss}}_{\gamma_{res}}}.$$
(A.4)

APPENDIX A. THIN-DISK LASER

There is one double pass through the thin-disk in each round trip of the I-resonator. Another typical resonator for thin-disk lasers is the V-resonator, which is shown in Figure A.1b. Two double passes through the thin disk are made during each round trip in the V-resonator. The



Figure A.1: I-resonator design (a) and V-resonator design (b) of a thin-disk laser.

mean resonator dwell time of a photon au_{res} is then defined by the losses per round trip as

$$\tau_{res} = \frac{T_{res}}{-\ln(1 - T_{OC}) - \ln(R_{res} \cdot R_{disk}^{M_L/2} \cdot (1 - L_{res})^2)}.$$
 (A.5)

 M_L indicates the number of single-passes of the laser beam through the thin disk and is $M_L = 2$ for an I-resonator and $M_L = 4$ for a V-resonator. R_{res} is given by the combined reflectivity of the high-reflective mirrors, which is one for the I-resonator as there are no additional mirrors to the active mirror and the output coupler. R_{disk} is the reflectivity of the thin disk and defined by Equation A.1.

A.1. CAVITY

A.1.2 Yb:YAG Thin-Disk Multi-Pass Amplifier

An illustration of a thin-disk multi-pass amplifier $(M_L = 2)$ is shown in Figure A.2.



Figure A.2: Amplifier cavity with a thin disk as active mirror.

A.1.3 Transversal Modes of a Laser Beam

For a perfectly aligned cylinder symmetric cavity with infinite mirror radius, the transverse mode structure in paraxial approximation can be expressed by the generalized Laguerre polynomials $L_p^l(\rho)$ as [41]

$$I_{pl}(\rho, \Phi) = I_0 \cdot \rho^l \cdot \left(L_p^l(\rho) \cdot \cos(l\Phi) \right)^2 \cdot e^{-\rho}.$$

Typically, the cylinder symmetry of the cavity is not given as tilted or distorted optical elements entail a rectangular symmetry to the system [33, p. 648]. Then the transverse modes are better expressed by Hermite-polynomials $H_i(x)$ as [42]

$$I_{nm}(x,y,z) = I_0 \cdot \left[H_m\left(\frac{\sqrt{2}x}{w(z)}\right) \cdot e^{-\left(\frac{x}{w(z)}\right)^2} \right]^2 \times \left[H_n\left(\frac{\sqrt{2}y}{w(z)}\right) \cdot e^{-\left(\frac{y}{w(z)}\right)^2} \right]^2.$$

These transversal modes are called "cold cavity"-modes, as they suppose the propagation through free space [33, p. 798] and hence there is no amplification of the light beam within the resonator. The transverse modes of a laser are called "hot cavity"-modes and can be described by a linear combination of complex Hermite- or Laguerre-Gaussian modes.

The parameters to describe the pump beam and the laser beam are given in Table A.2.



Parameter	Value (Pump Beam)	Value (Laser Beam)	Unit
Wavelength (λ)	940	1030	[nm]
M^2	-	1	-
Beam waist radius (w_0)	$66\%\cdot ho_{disk}$	$66\%\cdot ho_{disk}$	[nm]
Gaussian Order (\mathcal{O}_{Gauss})	10	1	-

Table A.2: Parameters of the pump beam and the laser beam.

A.2 Optical and Thermal Properties of the Thin-Disk

A.2.1 Yb:YAG as Active Medium

Optical and thermal properties of an Yb:YAG thin disk of thickness $z_{disk} = 130 \,\mu\text{m}$ used in the numerical simulations (see Chapter 4) are summarized in Table A.3. The temperature dependent

Parameter		Value	Unit		
	Opti	cal Properties [3	31]		
Eff. cross section	Emission	$\lambda = 940\mathrm{nm}$	1.43×10^{-21}		
EII. cross section	Linission	$\lambda = 1030\mathrm{nm}$	2.14×10^{-20}	$[1/cm^2]$	
	Absorption	$\lambda=940\mathrm{nm}$	7.23×10^{-21}		
$(\sigma_{eff;T=300\mathrm{K}})$		$\lambda = 1030\mathrm{nm}$	1.23×10^{-21}		
Radiative excited state lifetime $(\tau_{1\to 0})$		951	[µs]		
	Thermal Properties				
Heat Conductivity $(\kappa_{T=300 \text{ K}})$		6.6	[W/(mK)]		
Specific Heat Capacity (c_p)		590	[J/(kg K)]		
Mass Density (ρ_m)		4560	$[kg/m^3]$		
Doping concentration (c_{dot})		Doping concentration (c_{dot})		9	[at %]

Table A.3: Optical and thermal parameters of Yb: YAG at 300 K.



A.2. OPTICAL AND THERMAL PROPERTIES

thermal conductivity κ_T of Yb:YAG is given by

$$\kappa_T = \kappa_{T=300 \,\mathrm{K}} \left(\frac{204 \,\mathrm{K}}{T - 96 \,\mathrm{K}} \right)$$
$$\kappa_{T=300 \,\mathrm{K}} = (7.28 - 7.3 \cdot c_{dot}) \mathrm{W} \,\mathrm{K}^{-1} \,\mathrm{m}^{-1},$$

where c_{dot} is the doping concentration and T is the temperature. Figure A.3 shows the thermal conductivity of Yb:YAG with a doping concentration of $c_{dot} = 9 \text{ at } \%$ in dependence of the temperature.



Figure A.3: Thermal conductivity κ_T of Yb:YAG ($c_{dot} = 9 \text{ at } \%$) in the temperature range from 300 K to 450 K.

A.2.2 CVD-Diamond as Heat Sink

The thin disk is mounted on a heat sink. The heat sink is a material with high thermal conductivity to effectively transport the heat out of the thin disk. Synthetic diamond is therefore a favorable material. The assumed thermal properties for a synthetic diamond made by chemical vapor deposition (CVD) of thickness $z_{sink} = 2 \text{ mm}$ are listed in Table A.4. The thermal conduc-

Parameter	Value	Unit
Heat Conductivity $(\kappa_{T=300 \text{ K}})$	2041^{1}	[W/(mK)]
Specific Heat Capacity $(c_{p;T=300 \text{ K}})$	504^{1}	[J/(kgK)]
Mass Density (ρ_m)	3515	$[kg/m^3]$
Extrinsic Parameters		

 Table A.4: Thermal properties of synthetic (CVD) diamond at 300 K.

tivity κ_T and the specific heat capacity c_p of CVD-diamond in dependence of the temperature T is shown in Figure A.4¹.

A.2.3 Anti-Reflective Coating

An anti-reflective coating is bonded to the front side of the thin disk. The transmissivity of the anti-reflective coating T_{AR} in dependence of the incidence angle is shown in Figure A.5². The anti-reflective coating is assumed as a single-layer of MgF_2 with a thickness of $z_{AR}(\lambda/4) = \lambda_{vac}/(4 \cdot n) \approx 187 \,\mathrm{nm}$. Calculation are made for a wavelength of $\lambda = 1040 \,\mathrm{nm}$. $n(\lambda = 1030 \,\mathrm{nm}) \approx 1.37$ [43] is the refractive index of MgF_2 .

A.2.4 High-Reflective Coating

The back side of the thin disk is coated with a high-reflective coating. The high-reflective coating usually consists of alternating layers of thickness $\lambda/4$ of a high refractive material and a low refractive material. λ is the wavelength in the material ($\lambda = \lambda_{vac}/n$ with the wavelength in vacuum λ_{vac} and the refractive index of the material n). The high refractive material and the low refractive material is assumed to be Ta_2O_5 with a refractive index of 2.08 at $\lambda = 1040$ nm [44] and SiO_2 with a refractive index of 1.45 at $\lambda = 1040$ nm [45]. The thermal properties of Ta_2O_5 and SiO_2 are listed in Table A.5³ and A.6, respectively. The values are for bulk material and it is assumed that the values do not differ for thin layers. The numerical model regards for

¹COMSOL Multiphysics[®], www.comsol.com, COMSOL AB, Stockholm, Sweden

²Anti-reflective coating: SPEKTRUM, 17.11.2007, www.lzh.de/de/dienstleistungen/details/software-tools-fuerdie-optische-duennschichttechnologie, Laser Zentrum Hannover e.V., Hannover, Germany

³Gas flow rates for sputtering: $5\% O_2$ and 95% Ar

 $^{{}^{3}}Ta_{2}O_{5}$: WebElements, www.webelements.com/compounds/tantalum/ditantalum_pentaoxide.html, accessed May 2019

³SiO₂: Crystran Ltd., www.crystran.co.uk/optical-materials/silica-glass-sio2, accessed May 2019



A.2. OPTICAL AND THERMAL PROPERTIES

Parameter	Value	Unit
Thermal Conductivity (κ_T)	2 [46] ³	[W/(mK)]
Specific Heat Capacity (c_p)	297 [47]	[J/(kg K)]
Mass Density (ρ_m)	8200 ³	$[kg/m^3]$

Table A.5: Thermal properties of Ta_2O_5 at 300 K.

Table A.6: Thermal properties of SiO_2 at 300 K.

Parameter	Value	Unit
Thermal Conductivity (κ_T)	1.38^{3}	[W/(mK)]
Specific Heat Capacity (c_p)	703.0^{3}	[J/(kgK)]
Mass Density (ρ_m)	2203^{3}	$[kg/m^3]$

an adhesive between the high-reflective coating and the heat sink. Values for a typical adhesive⁴ are listed in Table A.7. The thermal properties of the high-reflective coating (see Table A.8) are

Table A.7: Thermal properties of the adhesive at 300 K.

Parameter	Value	Unit
Thermal Conductivity (κ_T)	0.17^4	[W/(mK)]
Specific Heat Capacity (c_p)	1100	[J/(kg K)]
Mass Density (ρ_m)	1150	$[kg/m^3]$

approximated by the mean value of both layers with weights according to the thickness of the layers. The reflectivity of the high-reflective coating R_{HR} in dependence of the incidence angle Θ is shown in Figure A.6⁵. The composition of the Ta_2O_5 and SiO_2 multi-layer structure of the high-reflective coating is assumed as 15 layers of SiO_2 , Ta_2O_5 and SiO_2 with thickness of $z_{SiO_2}(\lambda/4) = \lambda_{vac}/(4 \cdot n) \approx 177 \text{ nm}$, $z_{Ta_2O_5}(\lambda/4) \approx 126 \text{ nm}$, $z_{SiO_2}(\lambda/2) \approx 354 \text{ nm}$, respectively. Calculation are made for a wavelength of $\lambda = 1040 \text{ nm}$.

⁴NOA81: Norland Products Inc., www.norlandprod.com/adhesives/NOA%2081.html, Cranbury, United States of America

⁵High-reflective coating: SPEKTRUM, 17.11.2007, www.lzh.de/de/dienstleistungen/details/software-toolsfuer-die-optische-duennschichttechnologie, Laser Zentrum Hannover e.V., Hannover, Germany



Parameter	Value	Unit	
Intrinsic Parameters			
Thermal Conductivity (κ_T)	1.32	[W/(mK)]	
Specific Heat Capacity (c_p)	658.9	[J/(kg K)]	
Mass Density (ρ_m)	3898	$[kg/m^3]$	
Extrinsic Parameters			
Thickness (z_{HR})	4.6	[µm]	

Table A.8: Thermal properties of the high-reflective coating at 300 K.



A.2. OPTICAL AND THERMAL PROPERTIES



(a) Thermal Conductivity of CVD-diamond



(b) Specific Heat Capacity of CVD-Diamond

Figure A.4: (a) Thermal conductivity κ_T^1 and (b) specific heat capacity c_p^1 of CVD-diamond in the temperature range from 300 K to 450 K.



APPENDIX A. THIN-DISK LASER



Figure A.5: Transmissivity of the anti-reflective coating T_{AR} ($\lambda = 1030 \text{ nm}$)² in dependence of the incidence angle of light Θ for s-polarized light, p-polarized light and unpolarized light.



Figure A.6: Reflectivity of the high-reflective coating R_{HR} ($\lambda = 1040 \text{ nm}$)⁵ in dependence of the incidence angle of light Θ for s-polarized light, p-polarized light and unpolarized light.

Appendix B

Parameters of the Numerical Simulations

The simulation parameters are summarized in Table B.1. For more information on the used sources for the parameters, please refer to Appendix A.

Parameter	Value	Unit	
Thin Disk (Yb:YAG)			
Intrinsic Parameters			
Em. cross section $(\sigma_{em;L;T=300 K})$	2.13×10^{-20}		
Abs. cross section $(\sigma_{abs;L;T=300 K})$	1.23×10^{-21}	$[1/cm^2]$	
Em. cross section $(\sigma_{em;P;T=300 K})$	1.43×10^{-21}		
Abs. cross section $(\sigma_{abs;P;T=300 K})$	7.23×10^{-20}		
Radiative excited state lifetime $(\tau_{1\rightarrow 0})$	951	[µs]	
Heat Conductivity $(\kappa_{T=300 K})$	6.6	[W/(mK)]	
Specif. Heat Capacity $(c_{p;T=300 K})$	590	[W/(kgK)]	
Doping concentration (c_{dot})	9	[at percent]	

Table B.1: Simulation parameters (please refer to Appendix A for information on the used sources).



Parameter	Value	Unit	
Mass Density (ρ_m)	4560	[kg/m ³]	
Extrinsic Parameters			
Radius (ρ_{disk})	1, 5, 10, 50 and 100	[mm]	
Thickness (z_{disk})	130	[µm]	
High-Reflective (Coating $(Ta_2O_5 \& SiO_2)$		
Intrinsic Parameters			
Heat Conductivity $(\kappa_{T=300 K})$	1.32	[W/(mK)]	
Specif. Heat Capacity $(c_{p;T=300 K})$	658.9	[W/(kgK)]	
Mass Density (ρ_m)	3898	[kg/m ³]	
Extrinsic Parameters			
Radius $(\rho_{coating})$	$ ho_{disk}$	[mm]	
Thickness $(z_{coating})$	3	[µm]	
Heat Sink (Diamond)			
Intrinsic Parameters			
Heat Conductivity $(\kappa_{T=300 K})$	2041	[W/(mK)]	
Specif. Heat Capacity $(c_{p;T=300 K})$	504	[W/(kgK)]	
Mass Density (ρ_m)	3515	[kg/m ³]	
Extrinsic Parameters			
Radius (ρ_{sink})	$1.47 \cdot \rho_{disk}$	[mm]	
Thickness (z_{sink})	2	[mm]	



APPENDIX B. SIMULATION PARAMETERS

Parameter	Value	Unit	
Resona	ator (Oscillator)		
Optical Parameters			
Disk reflectivity $(R_{disk}(\Theta = 0^{\circ}))$	99.985		
Mirror reflectivity $(R_{res}(\Theta = 0^{\circ}))$	99.900	<u>ا</u> %۱	
Output-coupler transm. (T_{OC})	1.00, 1.78, 3.16, 5.62 and 10.00	[70]	
Resonator losses (L_{res})	0.250		
Geometrical Parameters			
Pump beam single-passes (M_P)	48	-	
Laser beam single-passes (M_L)	4 (Oscillator); 2 (Amplifier)	-	
Light Beams			
Pump Beam: Super Gaussian (Transver	rsal); Continuous Wave (Temporal)		
Power $(I_{P;In})$	3	$[\mathrm{kWcm^{-2}}]$	
Wavelength (λ_P)	940	[nm]	
Beam waist radius $(w_{0;P})$	$66~\%\cdot ho_{disk}$	[mm]	
Gaussian Order ($\mathcal{O}_{Gauss;P}$)	10	-	
Laser Beam (Amplifier): Gaussian (Transversal); Continuous Wave (Temporal)			
Power $(i_{L;In})$	0.25, 0.79, 2.50, 7.91 and 25.00	-	
Wavelength (λ_L)	1030	[nm]	
Beam quality factor $(M^2{}_L)$	1	-	
Beam waist radius $(w_{0;L})$	$66 \% \cdot ho_{disk}$	[mm]	
Gaussian Order $(\mathcal{O}_{Gauss;L})$	1	-	

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Eigenständigkeitserklärung

Hiermit erkläre ich, die vorliegende Arbeit selbständig verfasst zu haben und keine anderen als die in der Arbeit angegebenen Quellen und Hilfsmittel benutzt zu haben.

Stuttgart, 15. Mai 2019:

Unterschrift