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Towards the Prediction of Resilience: An Equation-based Resilience Representation

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The performance curve of a system during a disruption is widely used in the literature as an illustration of the system's resilience capabilities, especially in socio-technical works. To improve the resilience of a system, an important step is to develop methods and techniques to properly quantify relevant resilience metrics. Despite the importance, no final consensus has been reached regarding the mathematical definition of the concept of resilience. Against this backdrop, this works presents an analytic equation to fit the whole evolution of the system's performance curve during a disruption. This enables a decision maker to determine model parameters that are directly linked to the system's resilience capabilities. It can additionally serve as a base to predict resilience curves in future works. We propose to use two sigmoid functions to represent the resilience of a generic system.

Keywords: Resilience, Decision Analysis, Knowledge Management, Prediction, Critical Systems

1. Introduction

Within the context of protection of critical infrastructures, the concept of resilience has tremendously gained in importance over the past years. This work understands resilience as a sequence of different phases visualized by a resilience performance curve, which begins shortly before a disruption and ends after the system has recovered from the associated impacts (cf. Fig. 1). There are four phases associated with resilience: Prepare and plan, absorb, recover from and adapt to adverse events (National Academy of Sciences, 2012).

To improve the resilience of a system, an important step is to develop methods and techniques to properly quantify relevant resilience metrics (Häring, Ebenhöch and Stolz, 2016). This supports decision makers to evaluate and compare the different options of resilience enhancement and to determine the best measures. For this purpose, resilience needs to be quantified based on the development of the system's performance during a disruption. The equation presented in this work is the first to approximate general system performance over the whole resilience cycle. With this approach it is also possible to describe multiple resilience cycles, i.e. the evolution of system performance in case of multiple consecutive disruptions or in case of a stepwise progression of recovery of system performance.

An important aspect of the analysis of a system's resilience is the consideration of uncertainties regarding the examined scenarios. If the system performance drops due to unexpected disruptions, it is rational to always include the uncertainties about the further course of performance when considering countermeasures. Thus, during the disruption, a prediction about the resilience of the system in the further course would be desirable. Here, we demonstrate how stochastic system simulations can be used in conjunction with our new approach of quantifying resilience capabilities in order to estimate realistic ranges in which system performance might progress. These calculations can help to analyze the required intensity of potential countermeasures.

In order to demonstrate the application of our findings, we use simulation data of a traffic system to calculate the resilience under different circumstances and show how future developments of the system performance can be estimated while taking uncertainties into consideration.

2. Background

The following section presents the necessary knowledge and literature for a better understanding of the approach.

2.1. Performance-based Resilience Curve

It has become common practice to represent resilience, including its phases, using a curve of quality or performance as shown in Fig. 1. This was first proposed by Bruneau et al (2003) and widely extended by Cimellaro et al. (2009). These time-dependent curves describe, that after a sudden disruption resulting in a performance loss, the performance increases slowly and reaches its original level after some time. The more resilient a system behaves, the less deep is the dip and the faster is the recovery (Pimm, 1984). Some authors state that a system can learn from disruptions and perform even better afterwards so that the system performance after the disruption is corrected to above 100% (Francis and Bekera, 2014).

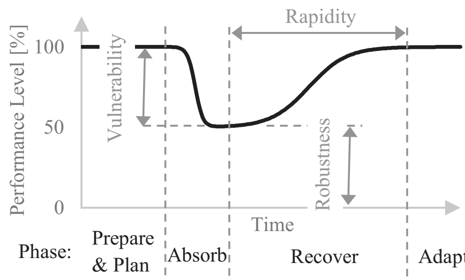

Fig. 1 Resilience performance curve during a disruption with related terms and phases.

Resilience is often approximated by the formulation first proposed by Bruneau et al. (2003):

$$R = \int_{t_b}^{t_e} [100 - Q(t)]dt$$

Here, R is the "loss of resilience", $Q(t)$ is the performance over time and t_b and t_e are beginning and end of one disruption cycle. To enhance the resilience the result of this equation needs to be minimized.

The performance curves $Q(t)$, which are used to estimate the system's resilience, are sometimes described in a phenomenological manner, mostly without a mathematical description. Thierney and Bruneau (2007) suggested the simplifying assumption that the performance drop can be approximated by a triangular-shaped curve (the "resilience triangle") that is defined by the performance minimum, the beginning of the disruption and the end of the performance loss. Other works that try to describe $Q(t)$ using mathematical expressions split the curve into a disruption and a recovery phase. For example, Sharma, Tabandeh and Gardoni (2017) discussed which function fits best for the description of the recovery phase.

Cimellaro et al. (2009) analyzed different mathematical approaches such as linear, trigonometric and exponential expressions and state that the recovery phase of resilient systems can best be described by an exponential curve.

2.2. Uncertainties

Only few works consider that the resilience performance curve is subject to many uncertainties. Decò, Bocchini and Frangopol (2013) state that every point of the performance curve during a disruption is associated with uncertainties regarding time of occurrence or performance level. Regardless of the method by which the data of the performance curve is gathered (via simulations or field-tests), there are always aleatoric and/or epistemic uncertainties that ought to be considered. While epistemic uncertainties might decrease with more data gathered, aleatoric uncertainties can never be reduced (Kiureghian and Ditlevsen, 2009).

2.3. Traffic System Performance Indicators

Traffic systems are a type of critical infrastructure that is well-suited for the study of general resilience properties. To evaluate a system's performance, the first step is to look for indicators to rate the system. The choice of indicator has a huge impact on the appearance of the performance curve. It might also be helpful to look at multiple indicators. An example of different traffic system indicators is given in (Kaparias et al., 2011). They describe an index for mobility that is essentially described by the average travel time to different destinations, normalized by the number of routes:

$$I_{MOB} = \frac{1}{|R|}\sum_{r\in R}^{|R|}\frac{ATT^{r}}{D_r}$$

Here, $|R|$ is the number of routes, r is a route, ATT^r is the average travel time of route r and D_r is the length of the route r.

3. Approach

The goal of this work is to identify a simple mathematical model that allows to fit the whole evolution of the system's performance curve during a disruption and to determine model parameters that are directly linked to the system's resilience capabilities. If both these requirements are fulfilled, the model is able to describe how the resilience capabilities affect the system's performance curve. The model thus allows to study how measures that change the resilience capabilities will affect the performance curve of the system.

3.1. Analytic Description of Resilience

This work proposes sigmoid functions (sometimes referred to as logistic functions) to describe the absorption phase and thereafter the recovery phase. It is possible to model the resilience cycle with one analytic equation, and it is thus not necessary to divide the cycle into different phases and use multiple, non-contiguous, equations:

$$Q(t) = P + \frac{L_1}{(1+e^{-k_1*(t-t_1)})} + \frac{L_2}{(1+e^{-k_2*(t-t_2)})}$$

Fig. 2 shows two different examples of curves that were generated using this expression. One advantage of using this equation is, that there are only six parameters needed to describe the process. Another advantage is, that the meaning of most of the model parameters has already been defined by the scientific community. P is the initial performance, mostly close to 100%. L_1 describes the depth of the performance drop and it is equal to what is sometimes called the vulnerability (Häring, Sansavini et al., 2017) or inversed robustness (Shinozuka et al., 2004). k_1 refers to the speed of loss of performance, sometimes called rapidity (Pimm, 1984) and t_1 is the time of the inflexion point. Because absorption and recovery phases are considered independently of each other but within the same mathematical expression, the same description is valid for the recovery phase. L_2 is the amount of recovered performance, k_2 characterizes the speed and t_2 again is the time of the recovery phase inflexion point. In spite of its simplicity, this expression can be used to approximate a variety of performance curves, as Fig. 2 shows. This can also be seen in other figures of this work.

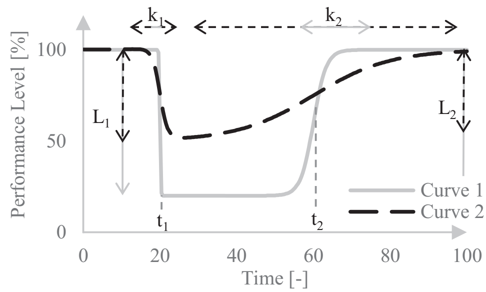

Fig. 2 Resilience performance curve and the equation parameters.

Fig. 2, Curve 1 is expressed by:

$$Q(t) = 100 + \frac{-80}{(1 + e^{-10*(t-20)})} + \frac{80}{(1 + e^{-0.6*(t-60)})}$$

and Fig. 2, Curve 2 is described by:

$$Q(t) = 100 + \frac{-50}{(1 + e^{-1*(t-20)})} + \frac{50}{(1 + e^{-0.1*(t-60)})}$$

In contrast to former approaches, this formula also enables the user to take the idle time between a disruption and the recovery of performance into account. This is done by fitting t_1 and t_2 to the available performance data.

When simulating the implementation of potential resilience-enhancing measures, the new formula for *Q(t)* can be fitted to the resulting performance curve of each scenario (cf. example Fig. 7). Thereby, the system's resilience in each scenario can be characterized through the model parameters L_1, k_1, t_1, L_2, k_2, and t_2. This approach empowers decision makers to evaluate resilience-enhancing measures with a quantitative approach.

3.2. Resilience Assessment

Now that the equation Q(t) is defined, it is possible to insert this equation into the mathematical definition of the loss in resilience, first proposed by Bruneau (2003) and presented in Section 2.1. The indefinite integral results in:

$$R = (100 - P - L_1 - L_2) * t - \frac{L_1 * log\left(1 + e^{k_1*(t_1-t)}\right)}{k_1} - \frac{L_2 * log\left(1 + e^{k_2*(t_2-t)}\right)}{k_2}$$

With this information it is possible to calculate the loss of resilience instantly.

3.3. Function Extension

In case of performance curves that display a more complex pattern (e.g. due to multiple consecutive disruptions or in case of a stepwise progression of recovery of system performance), it is possible to supplement the formula for *Q(t)* by additional logistic expressions. This enables the user to look at multiple resilience cycles (Fig. 3, Curve 3) or implement sub-steps during the process (Fig. 3, Curve 4).

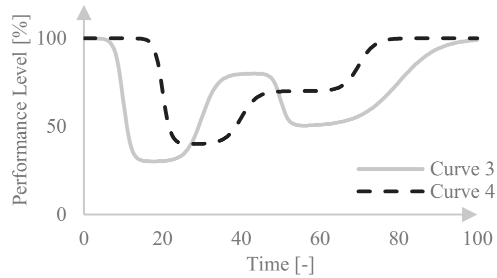

Fig. 3 More complex performance curves that can be fitted through additional logistic terms.

The amount of resilience cycles or sub-steps should be assumed by the operator, because until now it is not well-defined what is a complete cycle and when is it a sub-step.

When trying to fit the curve by an algorithm, it is important to set adequate boundaries for possible model parameter values, as algorithms will struggle to find the global optimum with more degrees of freedom through more parameters.

3.4. Performance Prediction considering Uncertainties

The prediction of a systems performance is always fraught with uncertainties as mentioned in Section 2.2. Using this approach, it is at least partially possible to account for these uncertainties because all model parameter values can be seen as probability distributions. This is especially important when comparing the effects of different resilience-enhancing measures. Applying the new formula for *Q(t)*, it is possible to determine the variability in model parameters that represent the system's resilience capabilities. It is thus possible to determine not only a single resilience value for each resilience-enhancing measure, but to assign a range of possible values of resilience capabilities to each measure.

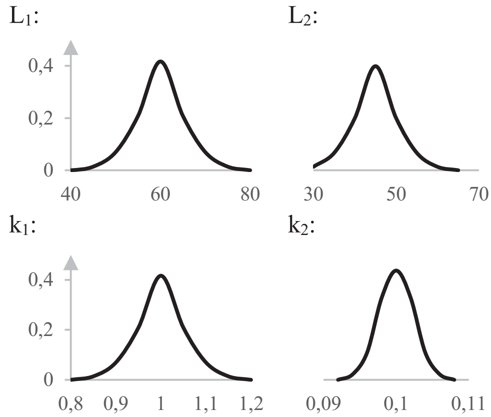

Fig. 4 Probability density functions of parameter values.

It is more accurate to use the probabilities given in Fig. 4, instead of using the expected values ($L_1 \approx 60$, $L_2 = 40$, $k_1 = 1$, & $k_2 = 0.1$).
Fig. 5 shows the variability in resulting performance curves when considering these probability density functions of parameter values shown in Fig. 4.

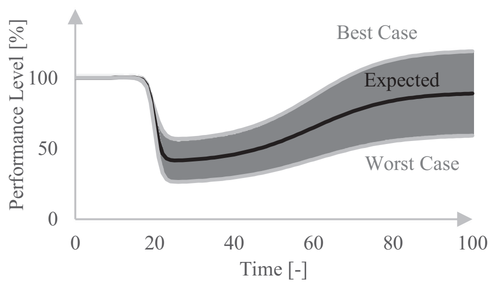

Fig. 5 Resilience performance curve with uncertainty range.

The expectation values of the model parameters yield the expected performance, which is the most probable one (Fig. 5, black line), but the figure shows how uncertainties regarding model parameter values relate to uncertainties associated with the progression of the performance curve. The best-case curve describes the case at minimum possible L_1 and maximum possible L_2. The parameter k_1 has a value of 0.8 and k_2 is equal to 0.11. For the worst-case curve all the parameters are inverted. Using the probabilities, it is possible to generate an overview over all the possible performance outcomes.
This can be transferred to the calculation of the loss of resilience presented in Section 2.1. The loss of resilience can also be described with another probability density function. This might serve as a valuable input for a subsequent decision analysis, in which a decision maker might need to choose between one option with the highest possible resilience but with a high uncertainty regarding its actual effect and another option in which the resulting resilience is somewhat lower but variability and thus uncertainty about outcomes is low as well.

4. Application

In the following, the proposed approach will be illustrated with the example of a traffic model. For this purpose, the private car traffic for the city of Cologne is simulated using “Sumo” (Alvarez Lopez, 2018) and the public “Tapas Cologne” dataset. The disruption is modeled as a sudden closure of the two major bridges, namely the Deutz Bridge and the Hohenzollern Bridge.

4.1. Traffic Performance Indicators

Traffic system performance can be described in various ways as described in Section 2.3. With Sumo it is easy to measure the performance by the time it takes vehicles to reach their destination. This work describes the system performance via the expected time without a disruption (d_{exp}) compared to the real time it takes cars to travel from start to destination (d_{real}). The difference is called time-loss. Expressed as a formula that means:

$$Q(t) = 100 \times \frac{d_{exp}}{d_{real}}$$

In a second step it might be necessary to translate the time into a comparable unit. Because this only serves as an example, all timescales in this work are converted to a dimensionless scale from 0 to 100. To focus only on data points that are affected by the disruption, a filter for the most severely affected vehicles is necessary, because depending on the system boundaries most of the cars are not affected even by heavy interferences.

An example result for the given scenario is shown in Fig. 6. The "Raw" points represent the performances of the individual vehicles based on their individual trip duration.

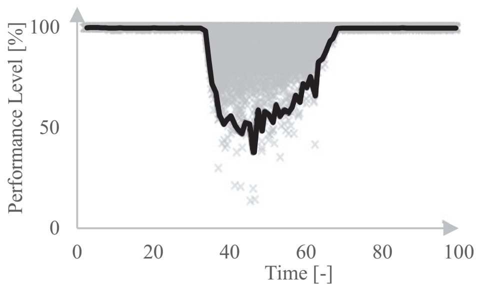

Fig. 6 Raw data and performance curve.

The resulting performance curve displays a very irregular pattern at a microscopic scale, but in general roughly follows the expected curve (cf. Fig. 1).

The first step is to look for the resilience triangle (Thierney and Bruneau, 2007) defined by the performance minimum, the beginning of the disruption and the end as shown in Fig. 7. A first evaluation can be done using this method. But it is more meaningful to fit the new formula for *Q(t)* to the raw data.

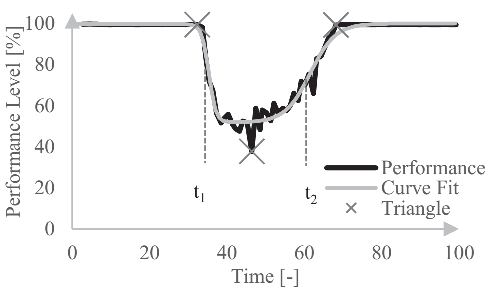

Fig. 7 Curve fit to raw data performance curve.

To fit a function to raw data, it is common practice to use least squares algorithms. The python package 'scipy.optimize' is a good choice for this because it brings a module for curve fitting using non-linear least squares. As the library sometimes lacks to find the global optimum, it is helpful to consider the resilience triangle and to define some boundaries for potential model parameter values. This can be done using an algorithm or by looking at the chart.

L_1 has to be located in between the first two points of the resilience triangle in vertical direction and t_1 has to be between these two points horizontally. The same applies for L_2 and t_2 and the two points of the triangle to the right. For the curve presented in Fig. 7, the following boundaries can be derived from the resilience triangle:

Table 1 Boundaries for curve fit.

Parameter	Upper Bound	Lower bound
L1	-40	-60
t1	50	30
L2	60	40
t2	70	50

This ensures that the parameters of the equation are inside of these ranges. The final double logistic function from this example is:

$$Q(t) = 100 + \frac{-48}{(1 + e^{-1.1*(t-35)})} + \frac{48}{(1 + e^{-0.3*(t-62)})}$$

If a sufficient number of stochastic simulations are performed in order to account for aleatoric uncertainties, probability density functions of parameter values (cf. Fig. 4) can be estimated, in order to calculate a range of potential performance curves as shown in Fig. 5.

The model parameters can also be used to evaluate resilience-enhancing measures. Fig. 8 shows the traffic system's performance as presented before (Fig. 7) and also displays the performance curve for a scenario in which a resilience-enhancing measure has been implemented (rerouting of traffic as a response to the disruption). Qualitatively, it is easy to evaluate the change in resilience based on these performance curves. However, a quantitative comparison of the system's resilience in those two scenarios is not possible without a descriptive equation. With the equation for *Q(t)* proposed in this paper, it becomes better possible to compare the different states of the system.

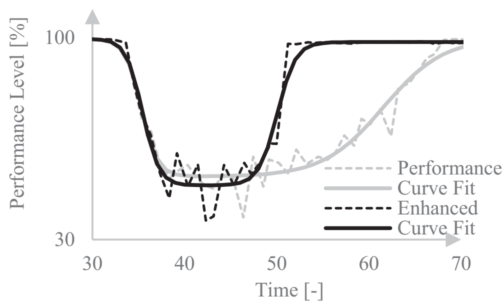

Fig. 8 Resilience enhancement.

The average performance in case of the implementation of the resilience-enhancing measure is described by:

$$Q(t) = 100 + \frac{-51}{(1 + e^{-1.1*(t-35)})} + \frac{50}{(1 + e^{-1.0*(t-50)})}$$

The following table shows the model parameters of the two scenarios that facilitate a quantitative comparison.

Table 2 Parameter changes for enhanced resilience.

Parameter	initial condition	resilience-enhancing measure
L_1	-48	-51
t_1	35	35
k_1	1.1	1.1
L_2	48	50
t_2	62	50
k_2	0.3	1.0

From Table 2 and Fig. 8 it can be seen that the disruption phase is nearly the same, resulting in similar parameter values L_1, t_1 and k_1. The enhanced resilience is reflected by a shorter time until half of the recovery is done, resulting in a smaller value of t_2, and by a higher speed of recovery, resulting in a higher value of k_2.

The overall loss of resilience for the initial state, calculated with the equation given by Bruneau, is:

$$R_1 = 1296$$

When implementing the resilience-enhancing measure, the value decreases to:

$$R_2 = 815$$

A decision maker is now able to evaluate that the resilience loss was reduced by:

$$R_{ges} = 1 - \frac{R_2}{R_1} = 0.37$$

However, as described in Section 3.4, this evaluation should take uncertainties into consideration and the values determined for R_1, R_2 and R_{ges} also depend on the choice of the indicator as described in Section 2.3.

5. Conclusion

This work proposes to express the performance of a system in the context of resilience using logistic functions:

$$Q(t) = P + \frac{L_1}{(1 + e^{-k_1*(t-t_1)})} + \frac{L_2}{(1 + e^{-k_2*(t-t_2)})}$$

This formulation enables users to describe the performance over time with six simple model parameters that are already defined by the scientific community involved in resilience research. P is the initial performance, mostly close to 100%. L describes the change in performance, k is the speed of the change and t sets the time of the performance change. Therefore, these model parameters characterize important features associated with the system's resilience capabilities. Due to the reduced complexity of the model, it might not be possible to adequately capture all subtle changes of performance during a disruption. But we think that it is this reduced model complexity that might help to compare a systems resilience in different situations (e.g. with or without implemented resilience-enhancing measures) in a simple and straightforward manner. This work shows how curve fitting with this equation and parameter boundaries can be executed. It is possible to extend this equation for multiple resilience cycles or performance sub-steps. Finally, it is also possible to take uncertainties regarding disruption intensities and effectiveness of counter-measures into account in order to predict potential changes in the system's performance and thus to identify the most suitable resilience-enhancing measures.

6. References:

Alvarez Lopez, Pablo, Michael Behrisch, Laura Bieker-Walz Jakob Erdmann and Yun-Pang Flötteröd, Robert Hilbrich, Leonhard Lücken, Johannes Rummel, Peter Wagner and Evamarie Wießner. Microscopic Traffic Simulation using SUMO. 2018.

Bruneau, Michel, Stephanie E. Chang, Ronald T. Eguchi, George C. Lee, Thomas D. O'Rourke, Andrei M. Reinhorn, Masanobu Shinozuka, Kathleen Tierney, William A. Wallace, Detlof von Winterfeldt „A Framework to Quantitatively Assess and Enhance the Seismic Resilience of Communities.“ Earthquake Spectra 19 (2003): 733–752.

Cimellaro, Gian Paolo, Cristina Fumo, Andrei M. Reinhorn, and Michel Bruneau. "Quantification of Disaster Resilience of Health Care Facilities." Quantification of Disaster Resilience of Health Care Facilities. September 2009.

Decò, Alberto, Paolo Bocchini, and Dan M. Frangopol. „A Probabilistic Approach for the Prediction of Seismic Resilience of Bridges.“ Earthquake Engineering & Structural Dynamics (Wiley) 42 (January 2013): 1469–1487.

Francis, Royce, and Behailu Bekera. „A metric and frameworks for resilience analysis of engineered and infrastructure systems.“ Reliability Engineering & System Safety 121 (2014): 90–103.

Häring, Ivo, Giovanni Sansavini, Emanuele Bellini, Nick Martyn, Tatyana Kovalenko, Maksim Kitsak, Georg Vogelbacher, Katharina Ross, Ulrich Bergerhausen, Kash Barker and Igor Linkov. „Towards a Generic Resilience Management, Quantification and Development Process: General Definitions, Requirements, Methods, Techniques and Measures, and Case Studies.“ In NATO Science for Peace and Security Series C: Environmental Security, 21–80. Springer Netherlands, 2017.

Häring, Ivo, Stefan Ebenhöch and Alexander Stolz. "Quantifying Resilience for Resilience Engineering of Socio Technical Systems." European Journal for Security Research 1 (April 2016): 21–58.

Kaparias, Ioannis, Michael G. H. Bell, Niv Eden, Ayelet Gal-Tzur, Oded Komar, Carlo Giacomo Prato, Leonid Tartakovsky, Boris Aronov, Yoram Zvirin, Marcus Gerstenberger, Antonios Tsakarestos, Silvio Nocera and Fritz Busch „Key Performance Indicators for traffic management and Intelligent Transport Systems Deliverable.“ 2011.

Kiureghian, Armen Der and Ove Ditlevsen. „Aleatory or epistemic? Does it matter?“ Structural Safety 31 (2009): 105-112.

National Academy of Sciences. Disaster Resilience: A National Imperative. Washington, DC: The National Academies Press, 2012.

Pimm, Stuart L. "The complexity and stability of ecosystems." Nature (Nature Publishing Group) 307 (January 1984): 321–326.

Sharma, Neetesh, Armin Tabandeh and Paolo Gardoni. „Resilience analysis: a mathematical formulation to model resilience of engineering systems.“ Sustainable and Resilient Infrastructure (Informa UK Limited) 3 (October 2017): 49–67.

Shen, Lijuan, Beatrice Cassottana, Hans Rudolf Heinimann and Loon Ching Tang. „Large-scale systems resilience: A survey and unifying framework.“ Quality and Reliability Engineering International (Wiley) 36 (June 2020): 1386–1401.

Shinozuka, Masanobu, Stepahanie E. Chang , Tsen-chung Cheng, Maria Feng, Thomas D. O'rourke, M. Ala Saadeghvaziri, Xuejiang Dong, Xianhe Jin, Yu Wang, Peixin Shi „Resilience of Integrated Power and Water Systems.“ 2004.

Thierney, Kathleen and Michel Bruneau. „Conceptualizing and Measuring Resilience: A Key to Disaster Loss Reduction.“ TR News 250 (2007): 14–17.