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COMPARISON OF MODEL INTEGRATION APPROACHES FOR FLEXIBLE AIRCRAFT FLIGHT DYNAMICS MODELLING

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Abstract. Two approaches for the development of aircraft models integrating flight and airframe structural dynamics are compared. These models are intended for flight loads analysis and flight simulator application. The approaches combine available, agreed on flight mechanics and aeroelastic aircraft models from the respective engineering disciplines. The present work compares two methods, with different underlying assumptions, regarding accuracy, implementability, pre-processing and computing effort, as well as possibility to adapt the model components as a function of the flight condition.

Nomenclature

A	system matrix
$A_{0,1,2}$	approximation matrix
B	damping matrix, input matrix
D	output matrix of lag state space eq.
E	input matrix of lag state space eq.
F	force vector
g	gravity acceleration vector
J	total aircraft inertia tensor
M	moment vector, mass matrix
Q	aerodynamic influence coefficient
R	diagonal matrix with lag state poles
T_{be}	transformation matrix from inertial into body axes
V	(air-)speed
$\bar{c}$	mean aerodynamic chord length
m	total aircraft mass
n_z	vert. load factor in dir. of body z-axis
$\bar{q}$	dynamic pressure
q	decoupled modal states
s	Laplace variable
x	state vector
δ	variation
η	generalised coordinates
ϕ	Euler angle
θ	Euler angle
ψ	Euler angle
λ	eigenvalue
Ω	angular velocity vector
Φ	eigenvectors of system matrix, input matrix KS approach
Ψ	inverse of Φ

Abbreviations

AE	aeroelastic
Eq	equation
FM	Flight Dynamics
RFA	rational function approximation

Subscripts

E	elastic
R	rigid
L	lag state
x	control
0	trim, quasi-steady
b	in body-fixed frame
e	in inertial frame

Superscripts

T	transpose
-1	inverse

Other

$\dot{\chi}$	time derivative of χ
$\|\chi\|$	absolute value of χ
det	determinant

I. Introduction

Flight mechanic models (FM) are usually based on the nonlinear six degree of freedom Newton-Euler equations of motion, a detailed aerodynamics model, actuator models, etc. Aeroelastic models (AE) combine structural dynamics and unsteady aerodynamics in modal form. Aircraft tend to get larger with a lighter and more flexible airframe. Therefore the interaction between rigid and elastic motion becomes increasingly important and requires integrated models.

Various works describe approaches for integrated flight mechanics and aeroelastic modelling.[1–3,7,8,10–13] This paper is concerned with model integration approaches that combine available industrial agreed on flight mechanics and aeroelastic models.

The major task in model integration of available FM and AE models is the handling of overlaps. Flight mechanic models describe the rigid body motion but use aerdynamic databases that account for quasi-steady structural deformation. On the other hand aeroelastic models use a free-free modal analysis and include rigid body modes representing small amplitude rigid body motion. Two approaches are considered here. Both account for the model overlaps by adapting the AE model while leaving the FM model basically unchanged.

The first approach was developed by König and Schuler, *KS-method*,[11] and augments the FM and AE model differential equations. Hereby a modal transformation to the AE model is applied, so that rigid body state derivatives from the FM differential equations can be incorporated. The FM differential equations are left unchanged and describe the mean motion of the flexible aircraft center of gravity.

The second approach, the Residualised Model *RM-method*,[2,8] combines the FM Newton-Euler and AE modal equations of motion (flexible degrees of freedom), which is allowed under assumptions made in.[12] In combining the aerodynamic models, the quasi-steady contribution of the unsteady aerodynamics model is continuously subtracted from the flexible to rigid coupling, while leaving the FM quasi-flexible aerodynamic model unchanged. This approach basically removes the quasi-flexible rigid degrees of freedom from the AE data.

This present work reviews the integral model components followed by a review of the integration techniques, the KS- and RM-method. Then a comparison regarding assumption, pre-processing and implementation is presented. A numerical test case is studied next analyzing accuracy and computing effort.

II. Integrated Model Components

The integration approaches combine components of the flight mechanic (FM) models and aeroelastic (AE) models. The structure of the two basic elements is described this section.

Flight Mechanics Model

Flight mechanic models are based on the six degree of freedom Newton Euler equations of motion for a rigid body. Force and moment equations are given by:[4]

$$m\left[\dot{V}_b + \Omega_b \times V_b - T_{be}\, g_e\right] = F_{ext} \tag{1a}$$

$$J\dot{\Omega}_b + \Omega_b \times J\Omega_b = M_{ext} \tag{1b}$$

where V_b and Ω_b are the translational and angular velocity vectors resolved in aircraft body axes. The matrix T_{be} denotes the transformation from the geodetic reference frame into body axis. External force and moment vectors F_{ext}, M_{ext} contain the aerodynamic forces, thrust forces and other external forces. While the the Newton Euler equations of motion only contain six rigid degrees of freedom, the aerodynamic forces depend on the aircraft flight state and the quasi static deformation of the airframe[4] (Fig. 1). The aerodynamic coefficients are therefore corrected by "flex-factors" that depend on the vertical load factor n_z.

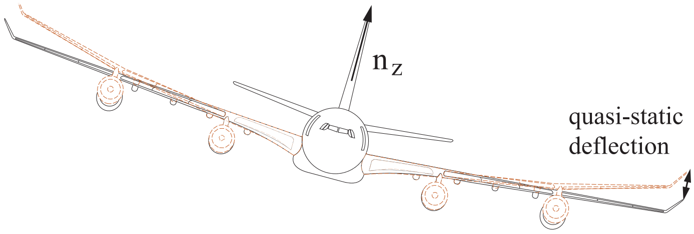

Figure 1. Aircraft in flight – quasi static deformation influences the total aerodynamic forces; this is taken into account via flex factors as a function of the vertical load factor

The correction of the quasi static deformation has to be kept in mind for subsequent coupling of the FM model with the AE model, because the quasi static deformation is also contained in the AE model. For later use in the coupling process the vector $\dot{x}_{FM}$ will be defined (note that this is not the state vector of the FM model):

$$\dot{x}_{FM}^T = [\dot{V}_b^T, \dot{\Omega}_b^T, V_b^T, \Omega_b^T] \tag{2}$$

Aeroelastic Model

The aeroelastic aircraft model contains of a structural finite element model and an unsteady aerodynamic model. The finite element model consists of condensed grid points for which a free-free modal analysis is performed. The structural degrees of freedom are transformed to modal degrees of freedom using six rigid body mode shapes (unit translations and rotations in the direction of the aircraft body axes) and a selection of elastic mode shapes. The aeroelastic equation of motion may then be written in the following generalized form:[10]

$$\begin{aligned} &\left\{ \begin{bmatrix} M_{RR} & 0 \\ 0 & M_{EE} \end{bmatrix} s^2 + \begin{bmatrix} B_{RR} & 0 \\ 0 & B_{EE} \end{bmatrix} s + \begin{bmatrix} K_{RR} & 0 \\ 0 & K_{EE} \end{bmatrix} \right\} \begin{bmatrix} \eta_R \\ \eta_E \end{bmatrix} \\ &\qquad = \bar{q} \left\{ \begin{bmatrix} Q_{RR}(s) & Q_{RE}(s) \\ Q_{ER}(s) & Q_{EE}(s) \end{bmatrix} \begin{bmatrix} \eta_R \\ \eta_E \end{bmatrix} + \begin{bmatrix} Q_{Rx}(s) \\ Q_{Ex}(s) \end{bmatrix} \eta_x \right\} \end{aligned} \tag{3}$$

where the rigid η_R and elastic η_E generalizes coordinates are:

$$\eta_R^T = [\delta x_b, \delta y_b, \delta z_b, \delta\varphi_{x_b}, \delta\varphi_{y_b}, \delta\varphi_{z_b}] \tag{4}$$

$$\eta_E^T = [\eta_{E_1}, \ldots, \eta_{E_n}] \tag{5}$$

and η_x is the vector of control inputs. The generalized stiffness, damping and mass matrices are denoted by K, B, M. The matrices of aerodynamic influence coefficients Q, Q_x are obtained from the Doublet Lattice theory[5] and initially depend on the mach number and the reduced frequency.

A rational function approximation[6,9] (RFA) is then applied to transform the unsteady aerodynamic forces and control forces from Laplace into time domain. For example:

$$Q_{EE}(s)\,\eta_E = \left\{ A_{0EE} + A_{1EE}\frac{\bar{c}}{V}\,s + A_{2EE}\frac{\bar{c}^2}{V^2}\,s^2 + D_{EE}\left(sI - \frac{V}{\bar{c}}R_E\right)^{-1} E_E\,s \right\} \eta_E \tag{6}$$

with the aerodynamics lag states x_{L_E}:

$$\dot{x}_{L_E} = \frac{V}{\bar{c}} R_E\, x_{L_E} + E_E\,\dot{\eta}_E \tag{7}$$

The aeroelastic equation of motion Eq. (3) can then be written in state space form with separated lag states (rigid, elastic and control lag states):[8]

$$\begin{bmatrix} \ddot{\eta}_R \\ \dot{\eta}_R \\ \ddot{\eta}_E \\ \dot{\eta}_E \\ \dot{x}_{L_R} \\ \dot{x}_{L_E} \\ \dot{x}_{L_x} \end{bmatrix} = \begin{bmatrix} A_{RR_1} & A_{RR_0} & A_{RE_1} & A_{RE_0} & A_{RL_R} & A_{RL_E} & A_{RL_x} \\ I & 0 & 0 & 0 & 0 & 0 & 0 \\ A_{ER_1} & 0 & A_{EE_1} & A_{EE_0} & A_{EL_R} & A_{EL_E} & A_{EL_x} \\ 0 & 0 & I & 0 & 0 & 0 & 0 \\ E_R & 0 & 0 & 0 & \frac{V}{\bar{c}}R_R & 0 & 0 \\ 0 & 0 & E_E & 0 & 0 & \frac{V}{\bar{c}}R_E & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \frac{V}{\bar{c}}R_x \end{bmatrix} \begin{bmatrix} \dot{\eta}_R \\ \eta_R \\ \dot{\eta}_E \\ \eta_E \\ x_{L_R} \\ x_{L_E} \\ x_{L_x} \end{bmatrix} + \begin{bmatrix} B_{R_1} & B_{R_0} \\ 0 & 0 \\ B_{E_1} & B_{E_0} \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ E_x & 0 \end{bmatrix} \begin{bmatrix} \dot{\eta}_x \\ \eta_x \end{bmatrix} \quad (8)$$

The vector $\ddot{\eta}_x$ is not included in the formulation, since is usually not available from actuator models. The state space model can also be written in compact form with a single set of lag states:

$$\begin{bmatrix} \dot{x}_R \\ \dot{x}_E \\ \dot{x}_L \end{bmatrix} = \begin{bmatrix} A_{RR} & A_{RE} & A_{RL} \\ A_{ER} & A_{EE} & A_{EL} \\ A_{LR} & A_{LE} & A_{LL} \end{bmatrix} \begin{bmatrix} x_R \\ x_E \\ x_L \end{bmatrix} + \begin{bmatrix} B_R \\ B_E \\ B_L \end{bmatrix} \begin{bmatrix} \dot{\eta}_x \\ \eta_x \end{bmatrix} \quad (9)$$

with the state vectors $x_R^T = [\dot{\eta}_R^T, \eta_R^T]$, $x_E^T = [\dot{\eta}_E^T, \eta_E^T]$ and the vector x_L, containing all aerodynamic lag states. Note that Eq. (8) and Eq. (9) are alternative formulations of the same aeroelastic system.

III. Review of Integration Techniques

In this section the KS-approach and the RM-approach are reviewed and the connection with the flight mechanics model is shown.

KS-Approach

This approach was developed by König and Schuler.[11] It uses a modal analysis of the system matrix A (Eq. (9)) to decouple rigid and elastic states by modal transformations with the complete set of complex system eigenvectors. Fig. 2 shows a schematic distribution of the complex eigenvalues of the aeroelastic system matrix A. Rigid and elastic states as well as aerodynamic lag states can be recognized by their frequencies and damping factors. The

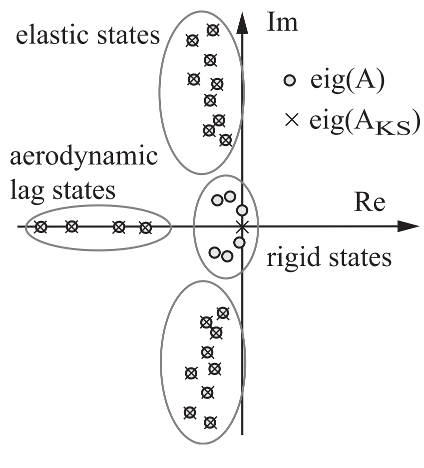

Figure 2. Eigenvalues of aeroelastic model A Eq. (9) and coupled model A_{KS} Eq. (10) system matrix

decoupled rigid states are replaced by the ones obtained from the flight mechanics model (see also the appendix for a derivation of the KS-approach). The coupled model can then be written in the following form:

$$\dot{x}_{KS} = A_{KS}x_{KS} + B_{KS}u + \Phi_{KS}(\dot{x}_{FM} - \dot{x}_{FM,0}) \tag{10}$$

where the index KS denotes data related to the coupled model and Φ_{KS} is the input matrix that incorporates the FM-model states. The new system matrix A_{KS} has the same elastic and aerodynamic lag states as the original AE model but does not contain the rigid body motion. All rigid body poles are zero after application of the coupling process; Fig. 2 also depicts the poles of the system matrix A_{KS}.

The connection of the nonlinear flight mechanics model Eq. (1) and the aeroelastic model with the KS-approach Eq. (10) is shown in Fig. 3. It starts with the computation of rigid states with the quasi flexible flight mechanics model. After removing the initial values at trim condition $\dot{x}_{FM,0}$ the data from the rigid model is introduced in the equation of the coupled model. The output of the coupled model $\dot{x}_{KS}$ contains dynamic increments for rigid and flexible states. It may subsequently be used for computation of loads and accelerations over the airframe as well as sensor signals.

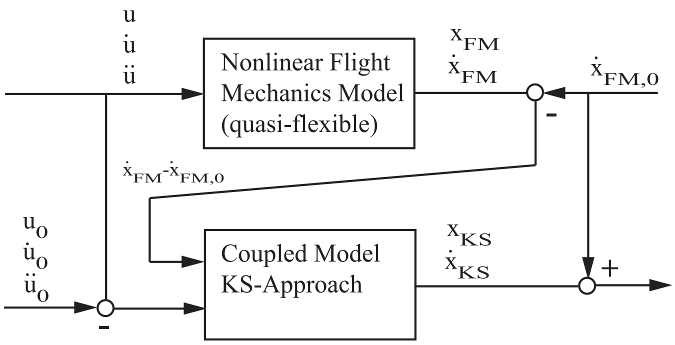

Figure 3. Connection of nonlinear flight mechanics model and aeroelastic model: KS-approach

RM-Approach

The residualised model approach (RM-approach) was developed by Winther[13] and extended by Looye[8] to be used with AE models that contain aerodynamic lag states. Various forms of the RM-approach are presented in.[8] In this work the form with variable dynamic pressure will be reviewed for completeness.

First the partitioned state space system Eq. (8) is reduced to elastic states and lag states:

$$\begin{bmatrix} \ddot{\eta}_E \\ \dot{\eta}_E \\ \dot{x}_{L_R} \\ \dot{x}_{L_E} \\ \dot{x}_{L_x} \end{bmatrix} = \begin{bmatrix} A_{EE_1} & A_{EE_0} & A_{EL_R} & A_{EL_E} & A_{EL_x} \\ I & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{V}{\bar{c}}R_R & 0 & 0 \\ E_E & 0 & 0 & \frac{V}{\bar{c}}R_E & 0 \\ 0 & 0 & 0 & 0 & \frac{V}{\bar{c}}R_x \end{bmatrix} \begin{bmatrix} \dot{\eta}_E \\ \eta_E \\ x_{L_R} \\ x_{L_E} \\ x_{L_x} \end{bmatrix} + \begin{bmatrix} B_{E_1} & B_{E_0} \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ E_x & 0 \end{bmatrix} \begin{bmatrix} \dot{\eta}_x \\ \eta_x \end{bmatrix} + \begin{bmatrix} 0 & A_{ER_1} \\ 0 & 0 \\ 0 & E_R \\ 0 & 0 \\ 0 & 0 \end{bmatrix} \delta\dot{x}_{FM} \tag{11}$$

where the aeroelastic rigid body states $\dot{\eta}_R$ are replaced by the corresponding states of the flight mechanics model $\delta\dot{x}_{FM} = \delta\dot{x}_{FM} - \dot{x}_{FM_0}$. The previous equation is used to compute elastic and lag states as a function of the control input and the flight dynamic states. Next the elastic modes are residualized:

$$\dot{\eta}_E = 0 \quad \ddot{\eta}_E = 0 \quad x_{L_E} = 0 \tag{12}$$

With Eq. (12) in Eq. (11) the quasi static elastic contribution then is:

$$\eta_{E_0} = -(A_{EE_0})^{-1} \left\{ \begin{bmatrix} A_{EL_R} & A_{EL_x} \end{bmatrix} \begin{bmatrix} x_{L_R} \\ x_{L_x} \end{bmatrix} + B_E \begin{bmatrix} \dot{\eta}_x \\ \eta_x \end{bmatrix} + A_{ER}\, \delta \dot{x}_{FM} \right\} \tag{13}$$

Dynamic load increments are computed in analogy to the unsteady aerodynamic forces (Eq. (6)) as follows:

$$\begin{bmatrix} \delta F \\ \delta M \end{bmatrix} = \bar{q} \left\{ A_{0RE} \left(\eta_E - \eta_{E_0}\right) + A_{1RE} \frac{\bar{c}}{V}\, \dot{\eta}_E + A_{2RE} \frac{\bar{c}^2}{V^2}\, \ddot{\eta}_E + D_{RE} x_{L_E} \right\} \tag{14}$$

where only elastic lag states are contained. Rigid lag states x_{L_R} would unintentionally couple into the rigid-elastic term. Therefore the separation of the lag states is important.

The incremental dynamic loads are then incorporated into the Newton Euler equations to complete the coupling process:

$$m \left[\dot{V}_b + \Omega_b \times V_b - T_{be}\, g_e \right] = F_{ext} + \delta F \tag{15a}$$

$$J \dot{\Omega}_b + \Omega_b \times J \Omega_b = M_{ext} + \delta M \tag{15b}$$

The RM coupling process is depicted in Fig. 4. The computation of the elastic states is governed by the flight mechanics model. The feedback of the dynamic load increments to the flight mechanics model augments the FM model.

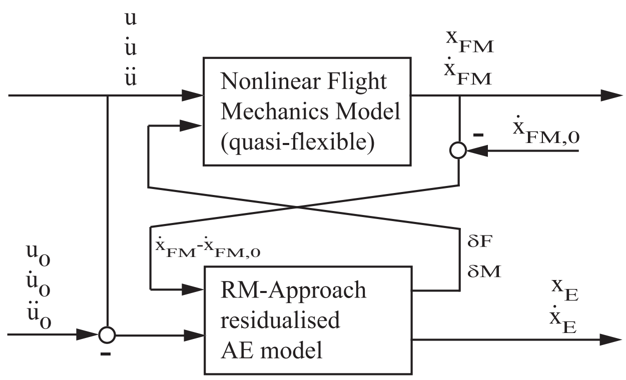

Figure 4. Connection of nonlinear flight mechanics model and aeroelastic model: RM-approach

IV. Comparison of the Integration Procedures

In this section the KS and RM-approach will be compared regarding underlying assumptions, model structure and accuracy.

Assumptions

The assumptions of the KS-approach (Table 1) is valid when rigid and elastic/lag states can be separated by modal decoupling. No minimum separation for the frequency of rigid and elastic effects is assumed. The RM-approach (Table 1) is based on static residualisation of the elastic mode shapes. Static residualisation implies a certain separation in frequency of the rigid and elastic motion. This is assured for most passenger aircraft but should be kept in mind and reviewed for extremely flexible airframe structures.

KS-Approach	RM-Approach
Flight mechanics states are *equivalent* to a linear combination of rigid body eigenvectors of the aeroelastic system matrix	The residualised linear aeroelastic model, containing rigid body dynamics only, is *equivalent* to the linearized flight mechanics model including quasi static corrections

Table 1. Coupling Approaches – Assumptions

Pre-Processing

The pre-processing of the KS-approach starts with the eigenvalue analysis of the system matrix that yields complex eigenvectors and complex eigenvalues (poles). Then the poles corresponding to the rigid states must be identified. This can be done by manual selection or by automated selection of the low frequency poles. An important check is to make sure, that no aerodynamic lag states have been mistaken for rigid body states. Then the coupling matrices are assembled. Numerical errors can be caused by the model transformation with complex matrices. Still existing small imaginary parts must be eliminated. The KS-approach can be applied even when the internal structure of the aeroelastic state space model is unknown.

Contrary to the KS-approach the RM-approach requires an aeroelastic state space model, where aerodynamic lag states are separated in those affecting rigid body and those affecting elastic dynamics (due to Eq. (14)). If the rational function approximation was performed using the approximation by Rogers[9] the lag states can be separated. In case of an approximation with Karpel's method[6] the lag states have to be repeated for each matrix partition. Once the state space model is available with separated lag states the coupling matrices can be assembled.

The pre-processing effort of the KS-approach is mainly based on the numerical handling and checking of the coupling matrices. It is convenient that all states, other than the rigid ones, can be treated as a single set of states. The RM pre-processing is determined by the separate handling and the partitioning of the aerodynamic lag states. Contrary to the KS method a numerical check of the assembled coupling matrices is not necessary.

Implementation

The KS-approach is implemented in an existing flight dynamics environment in form of the differential equations Eq. (10). The original flight mechanics model i.e. the implementation of the Newton Euler equations is not effected by the coupling process. This is favorable when the original flight dynamics environment is not accessible or changes to the original implementation is overly time consuming (e.g. for flight simulator applications). Since there is is no feedback on the flight mechanics equations, the KS-approach may also be applied in the post-processing; an advantage for load analysis. Further more a different integrator algorithm can be used for the FM model and the AE-model.

The RM-approach requires the implementation of an additional state space model to compute the elastic states and lag states Eq. (11). An algebraic equation is needed to calculate the residualised elastic states Eq. (13). With this information the dynamic load increments can be computed Eq. (14). The original flight mechanics equation is then augmented by the dynamic load increments. The RM-approach can be applied when altering the original flight mechanics equation is possible and acceptable. It is assumed that the quasi flexible aerodynamic model (flex factors) is not negatively influenced by the additional dynamic load increments.

In case the original model must not be affected the flex-rigid coupling may be removed.

Dynamic increments are then obtained for the elastic states only.

The RM-approach can be implemented in the present form with variable dynamic or in a simple state space form with constant dynamic pressure. The variation of dynamic pressure requires online matrix inversion. See Table 2 for a summary of the implementation aspects.

	KS-Approach	RM-Approach
Form of implementation	transformed	physical
Transformation of AE model	eigenvalue analysis and modal transformation	separate elastic lag states in case of Karpel RFA
Integrator	different integrator possible	same as FM integrator
Applicability	rigid modes separable from flexible modes and lag states	FM model equivalent to residualized AE model
interpolation w.r.t	mach number, dyn. pressure	mach number

Table 2. Implementation

V. Numerical Example

The coupling techniques described above are now applied in a numerical example. The study vehicle is a large passenger aircraft for which an industrial nonlinear flight mechanics model and an aeroelastic model is available. The coupling methods are compared in time and frequency domain.

In time domain simulation, elevator and aileron stair inputs (Fig. 5) are used as a test case. The output of interest here is the longitudinal and lateral response of flight mechanics and coupled rigid states. The response of the rigid states to the respective input is shown in Fig.

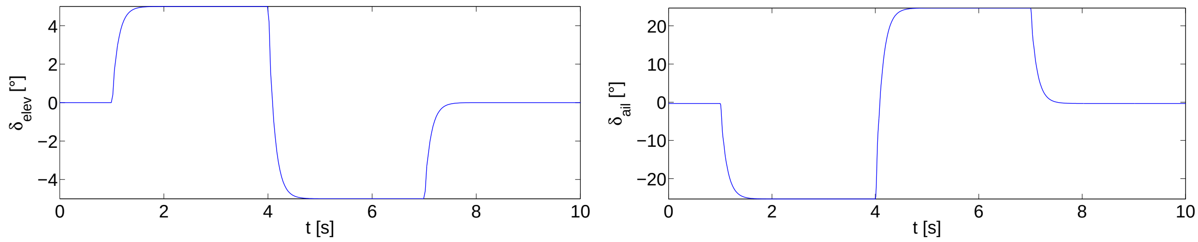

Figure 5. Elevator and aileron deflection due to input of stair command

6. Roll and pitch acceleration $\dot{p}, \dot{q}$ of the coupled models now contain dynamic increments that result from small rigid body motion of the AE model. The KS- and RM-approach lead to nearly identical rigid body accelerations that vary around the acceleration of the original FM model.

Differences between the two approaches can be identified for the Euler angles θ, ϕ and roll rates p, q. The mean value of the KS-approach more closely matches the FM-Model. The RM-approach response shows a difference in mean values due to feedback of incremental dynamic forces to the flight mechanics model.

The acceleration at cockpit (Fig. 7) is now analyzed, representing the airframe aeroelastic response. The KS and RM-approach again yields very similar results. In regions where the acceleration is dominated by its elastic contribution the results cannot be distinguished. With decreasing elastic acceleration slight differences can be noticed due to the differences in the rigid body response.

The transfer function of a rudder input to lateral cockpit acceleration is depicted in Fig.

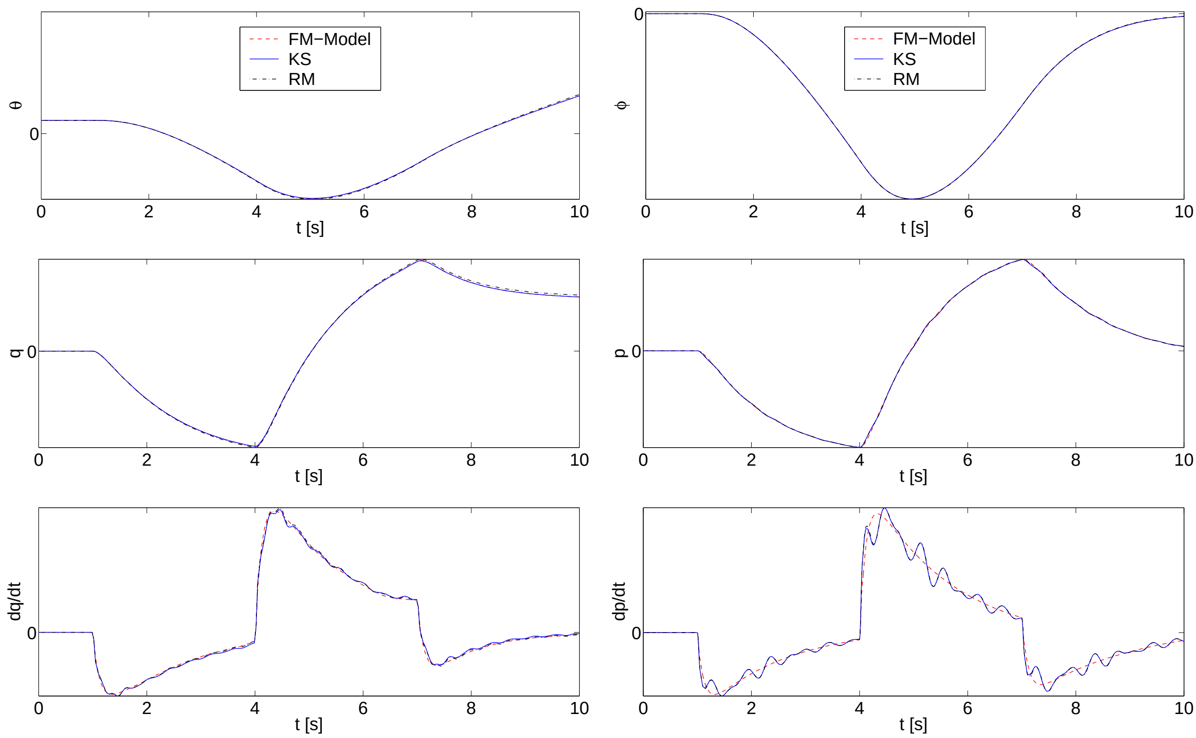

Figure 6. Rigid state response to elevator stair input (left column) and response to aileron stair input (right column)

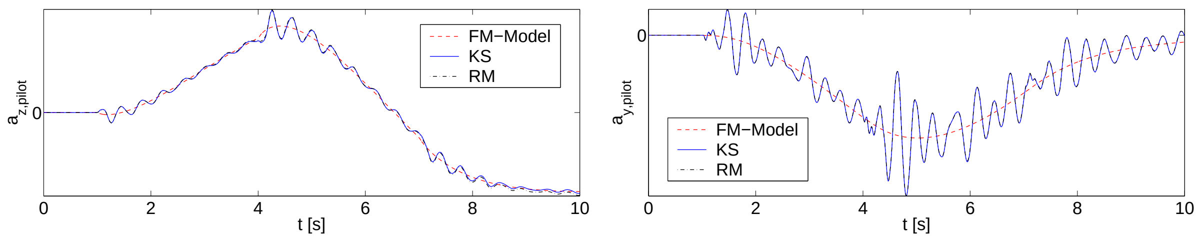

Figure 7. Acceleration response to elevator stair input (left column) and response to aileron stair input (right column)

8. The lower frequency range is dominated by the dutch roll mode. The higher frequency range by the elastic mode shapes. The aeroelastic model (AE) does not represent the rigid body motion very well compared to the quasi flexible flight mechanics model (FM). Both coupling techniques can accurately represent higher frequency range of the AE model. In the low frequency range the KS-approach exactly matches the dutch roll mode of the FM model. The RM-approach leads to a slightly increased amplitude.

Adaption to Flight Condition

Common flight mechanics models are valid for a wide range of flight conditions. Aeroelastic models are only valid for a linearisation point, specified by dynamic pressure and mach number. The KS-Method requires the transformation of the aeroelastic system matrix. Therefore coupling matrices are only valid at the specific working point. For coupled simulation in a wider range of working points a two dimensional interpolation of the coupling matrices over mach number and dynamic pressure is required. Any error that arises from interpolation can adversely affect the zero poles of the coupling matrix A_{KS} which may result in numerical problems during simulation. The RM-approach, if used in the form with variable dynamic pressure, requires the interpolation of the unsteady aerodynamic

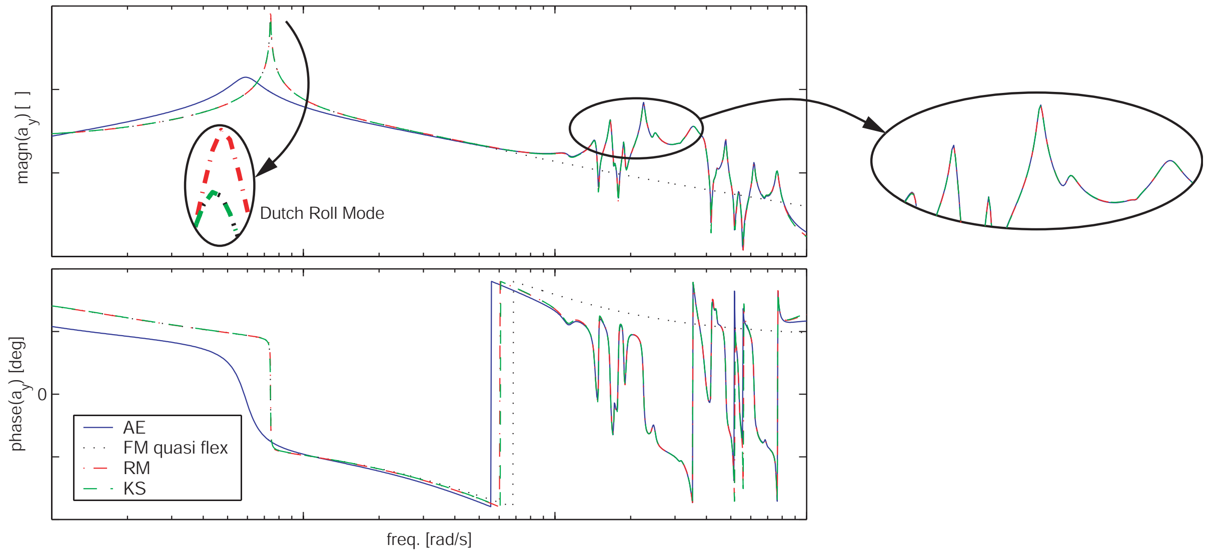

Figure 8. Bode plot of transfer function: rudder – y-acceleration at cockpit

forces $Q(Ma)$ only. Therefore interpolation is reduced to a one dimensional interpolation over the mach number.

Computing Effort

The RM-approach consists of two algebraic equations Eq. (13), Eq. (14) and the differential equation Eq. (11) containing elastic and lag states. The KS-approach consists of the differential equation Eq. (10) containing elastic and lag states as well as rigid body states. This leads to a similar computing effort for both methods (Table 3). The RM-approach with variable dynamic pressure requires the greatest computing effort since a matrix inversion has to be performed for every time step (or chance in dynamic pressure). However sice the variation of dynamic pressure is usually slow compared to the simulation rate, the matrix update can be done at slower rate without noticeable effect on the results.

Model	time for 10s simulation
FM-model	0.5 s
KS-approach	1.4 s
RM-approach ($\bar{q} = \text{const}$)	1.3 s
RM-approach ($\bar{q} = \bar{q}(t)$)	4.6 s

Table 3. Computing effort for test case; depends on FM aerodynamic model, number of elastic mode shapes in AE model and number of lag states

VI. Conclusion

In this paper a comparison of two model integration approaches has been presented. Both methods are based on the idea that the flight mechanics model should be left unchanged as far as possible.

The application of the KS-approach is convenient even if the structure of the aeroelastic model in unknown, e.g. for order reduced AE models. Only the rigid body modes have to be recognized. The preprocessing should always include numerical sanity checks of the coupling matrices. The original flight mechanics model remains unchanged which makes the KS-approach favorable for simple implementation in flight simulators. Different integrators may be applied to the FM and the AE model.

The RM-approach offers a variety of implementation forms. It can be used with or without feedback of dynamic force increments to the original flight mechanics equations. Its physical model structure takes the variation of dynamic pressure into account. Separate implementation of the equations of motion and aerodynamic forces is provided; accounting for equations of motion with inertial coupling. The RM-approach also extends to loads analysis, since the concept of quasi flexible deformation also applies for loads calculation.

It was shown that results of the KS-approach and the RM-approach yield close results, so the choice of a method is primarily based on the simulation environment.

Appendix – Derivation of the KS-Coupling

The derivation of the König-Schuler approach is reviewed from[11] for completeness. The aeroelastic equations of motion in state space form Eq. (9) can be written as follows:

$$\begin{bmatrix} \dot{x}_R \\ \dot{x}_{EL} \end{bmatrix} = \begin{bmatrix} A_{RR} & A_{R,EL} \\ A_{EL,R} & A_{EL,EL} \end{bmatrix} \begin{bmatrix} x_R \\ x_{EL} \end{bmatrix} + \begin{bmatrix} B_R \\ B_{EL} \end{bmatrix} u \tag{16}$$

where $x_{EL}^T = [x_E^T, x_L^T]$ denotes the vector of combined elastic and lag states. Then the eigenvalue problem:

$$A\Phi = \Phi\lambda \quad \Longrightarrow \quad \det\left|A - \lambda I\right| = 0 \tag{17}$$

is solved. In the above equation Φ denotes the matrix of (complex) eigenvectors and λ denotes the diagonal matrix of (complex) eigenvalues. The system matrix A can now be diagonalized using a particular case of a similarity transform of the matrix A :

$$\Phi^{-1} A \Phi = \lambda \tag{18}$$

The state vector x is transformed to modal states q as follows:

$$\begin{bmatrix} x_R \\ x_{EL} \end{bmatrix} = \begin{bmatrix} \Phi_{RR} & \Phi_{R,EL} \\ \Phi_{EL,R} & \Phi_{EL,EL} \end{bmatrix} \begin{bmatrix} q_R \\ q_{EL} \end{bmatrix} \tag{19}$$

For convenience the inverse of the matrix of eigenvectors $\Psi = \Phi^{-1}$ is introduced:

$$\begin{bmatrix} \Psi_{RR} & \Psi_{R,EL} \\ \Psi_{EL,R} & \Psi_{EL,EL} \end{bmatrix} = \begin{bmatrix} \Phi_{RR} & \Phi_{R,EL} \\ \Phi_{EL,R} & \Phi_{EL,EL} \end{bmatrix}^{-1} \tag{20}$$

The eigenvalues λ of the system matrix A is partitioned in its rigid λ_R and elastic/lag state λ_{EL} eigenvalues. Inserting Eq. (19) in Eq. (16) and pre-multiplication by Ψ (Eq. (20)) we obtain the decoupled modal form of the original aeroelastic model:

$$\begin{aligned} \begin{bmatrix} \dot{q}_R \\ \dot{q}_{EL} \end{bmatrix} &= \begin{bmatrix} \Psi_{RR} & \Psi_{R,EL} \\ \Psi_{EL,R} & \Psi_{EL,EL} \end{bmatrix} \begin{bmatrix} A_{RR} & A_{R,EL} \\ A_{EL,R} & A_{EL,EL} \end{bmatrix} \begin{bmatrix} \Phi_{RR} & \Phi_{R,EL} \\ \Phi_{EL,R} & \Phi_{EL,EL} \end{bmatrix} \begin{bmatrix} q_R \\ q_{EL} \end{bmatrix} + \begin{bmatrix} \Psi_{RR} & \Psi_{R,EL} \\ \Psi_{EL,R} & \Psi_{EL,EL} \end{bmatrix} \begin{bmatrix} B_R \\ B_{EL} \end{bmatrix} u \\ &= \begin{bmatrix} \lambda_R & 0 \\ 0 & \lambda_{EL} \end{bmatrix} \begin{bmatrix} q_R \\ q_{EL} \end{bmatrix} + \begin{bmatrix} \Psi_{RR} & \Psi_{R,EL} \\ \Psi_{EL,R} & \Psi_{EL,EL} \end{bmatrix} \begin{bmatrix} B_R \\ B_{EL} \end{bmatrix} u \end{aligned} \tag{21}$$

The aeroelastic model will now be coupled to the flight mechanics model $\dot{x}_{FM} = G(x_{FM}, u, ...)$ Eq. (1). It is *assumed* that the derivative of the rigid states of the flight mechanic model $\dot{x}_{FM}$ can be approximated by:

$$\dot{x}_{FM} = \Phi_{RR}\, \dot{q}_R \tag{22}$$

Whith Eq. (22) in Eq. (19) the modal approach can be partitioned as follows:

$$\begin{bmatrix} \dot{x}_R \\ \dot{x}_{EL} \end{bmatrix} = \begin{bmatrix} \dot{x}_{FM} & \Phi_{R,EL}\dot{q}_{EL} \\ \Phi_{EL,R}\Phi_{RR}^{-1}\dot{x}_{FM} & \Phi_{EL,EL}\dot{q}_{EL} \end{bmatrix} = \begin{bmatrix} I \\ \Phi_{EL,R}\Phi_{RR}^{-1} \end{bmatrix} \dot{x}_{FM} + \begin{bmatrix} \Phi_{R,EL} \\ \Phi_{EL,EL} \end{bmatrix} \dot{q}_{EL} \tag{23}$$

The vector $\dot{q}_{EL}$ is obtained from the elastic part (second row) of Eq. (21). With

$$q_{EL} = \Psi_{EL,R}\, x_R + \Psi_{EL,EL}\, x_{EL}$$

from Eq. (19) the elastic part of Eq. (21) can be written as:

$$\begin{aligned} \dot{q}_{EL} =& \lambda_{EL}\, (\Psi_{EL,R}x_R + \Psi_{EL,EL}x_{EL})\; + (\Psi_{EL,R}B_R + \Psi_{EL,EL}B_{EL})\, u \\ =& \begin{bmatrix} \lambda_{EL}\Psi_{EL,R} & \lambda_{EL}\Psi_{EL,EL} \end{bmatrix} \begin{bmatrix} x_R^T & x_{EL}^T \end{bmatrix}^T + (\Psi_{EL,R}B_R + \Psi_{EL,EL}B_{EL})\, u \end{aligned} \tag{24}$$

The equation for the coupled model is now obtained from Eq. (23) and Eq. (24):

$$\begin{aligned} \begin{bmatrix} \dot{x}_R \\ \dot{x}_{EL} \end{bmatrix} = \begin{bmatrix} I \\ \Phi_{EL,R}\Phi_{RR}^{-1} \end{bmatrix} \dot{x}_{FM} + \begin{bmatrix} \Phi_{R,EL}\lambda_{EL}\Psi_{EL,R} & \Phi_{R,EL}\lambda_{EL}\Psi_{EL,EL} \\ \Phi_{EL,EL}\lambda_{EL}\Psi_{EL,R} & \Phi_{EL,EL}\lambda_{EL}\Psi_{EL,EL} \end{bmatrix} \begin{bmatrix} x_R \\ x_{EL} \end{bmatrix} \\ + \begin{bmatrix} \Phi_{R,EL}(\Psi_{EL,R}B_R + \Psi_{EL,EL}B_{EL}) \\ \Phi_{EL,EL}(\Psi_{EL,R}B_R + \Psi_{EL,EL}B_{EL}) \end{bmatrix} u \end{aligned} \tag{25}$$

or in compact form

$$\dot{x}_{KS} = \Phi_{KS}\, \dot{x}_{FM} + A_{KS}\, x_{KS} + B_{KS}\, u \tag{26}$$

where KS denotes the states and matrices related to the König-Schuler approach.

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