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## A comment on different thermal shear-strain definitions in the literature and the consequences in composite design and application

### Wissenschaftlicher Bericht

Erik Kappel

Deutsches Zentrum
DLR für Luft- und Raumfahrt

**Institut für Faserverbundleichtbau und Adaptronik**

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Abteilungsleiter:
*Prof. Dr.-Ing. Christian Hühne*

Autor:
*Erik Kappel*

Deutsches Zentrum für Luft- und Raumfahrt

# A comment on different thermal shear-strain definitions in the literature and the consequences in composite design and application

Erik Kappel[a,*]

[a] *DLR, Institute of Composite Structures and Adaptive Systems, Lilienthalplatz 7, 38108 Braunschweig, Germany*

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## 1. Motivation

Shear strains in context of classical-laminate theory (CLT) are usually based on engineering strain formulation $\gamma_{xy}$, instead of the tensorial description using $\varepsilon_{xy}$. Some authors define thermal shear strains as $\gamma_{Txy} = \alpha_{Txy} \cdot \Delta T$. Others define $\varepsilon_{Txy} = \alpha_{Txy} \cdot \Delta T = \frac{1}{2} \cdot \gamma_{Txy}$, which causes a conflict. This investigation aims to highlight this confusing fact for uses. The individual formulations from literature are cross-checked and compared here with results of a simple ABAQUS FE model[1].

## 2. Formulations in the literature

Table 1 shows definitions found in the literature.

Table 1: Thermal shear strain definition

| Source | Definition |
|---|---|
| Reddy [2, p.99] | $\varepsilon_{xy} = \alpha_{xy} \cdot \Delta T$ |
| Nettles [2, p.44] | $\varepsilon_{xy} = \alpha_{xy} \cdot \Delta T$ |
| VDI [3] + HSB [4, p.4] | $\gamma_{xy} = \alpha_{xy} \cdot \Delta T$ |
| ESAComp [8] | $\gamma_{xy} = \alpha_{xy} \cdot \Delta T$ |
| Jones [7, p.242] | $\gamma_{xy} = \alpha_{xy} \cdot \Delta T$ |
| Schürmann [5, p.234] | $\gamma_{xy} = \alpha_{xy} \cdot \Delta T$ |
| Moser [10, p.252] | $\varepsilon_{xy} = \frac{1}{2} \cdot \alpha_{xy} \cdot \Delta T$ |

* Tel.:+49 531 295 2398

*Email address:* erik.kappel@dlr.de (Erik Kappel)

[1] Note, that the MSC NASTRAN documentation [11] indicates the use of the engineering strains as well, leading to the conclusion that the presented ABAQUS-related recommendations are valid for NASTRAN as well

It can be seen that $\alpha_{xy}$ is defined differently as $\gamma_{xy} = 2 \cdot \varepsilon_{xy}$. The VDI [3] notation, which is pursued in the industry-driven HSB as well, is used hereafter to show the relation between thermal expansion in the material coordinate system and the global system.

### *2.1. VDI background and notation*

According to VDI [3] and HSB [4, p.4] laminate CTEs are defined as followed.

$$\{\alpha_T^0\} = \left(\alpha_{Tx}^0, \alpha_{Ty}^0, \alpha_{Txy}^0\right)^T = [A]^{-1} \sum_{k=1}^{n} \left([Q']_k \cdot \{\alpha_T'\}_k \cdot t_k\right) \tag{1}$$

with the 'rotated' ply CTEs being linked to the ply material CTEs by

$$\{\alpha_T'\} = [T_\varepsilon] \cdot \left(\alpha_\parallel, \alpha_\perp, 0\right)^T \quad \text{with} \quad [T_\varepsilon] = \begin{bmatrix} m^2 & n^2 & -mn \\ n^2 & m^2 & mn \\ 2mn & -2mn & (m^2 - n^2) \end{bmatrix} \quad . \tag{2}$$

The strains in the UD-material COS and the laminate COS read

$$\{\varepsilon_T\} = \Delta T \cdot \left(\alpha_\parallel, \alpha_\perp, 0\right)^T \quad , \quad \{\varepsilon_T'\} = \Delta T \cdot \left(\alpha_{Tx}, \alpha_{Ty}, \alpha_{Txy}\right)^T \tag{3}$$

and can be transferred using

$$\{\varepsilon_T'\} = [T_\varepsilon] \cdot \{\varepsilon_T\} \quad . \tag{4}$$

Thus, $\alpha_{Txy}$ refers to engineering strain formulation, as $\gamma_{Txy} = \alpha_{Txy} \cdot \Delta T$.

## 3. Single-ply test case

Now, the expansion behavior of a single UD ply shall be examined. Lets assume a $\theta = 45°$ ply angle. For the single 45°-ply case, Equation 1 simplifies to:

$$\begin{bmatrix} \alpha_{Tx}^0 \\ \alpha_{Ty}^0 \\ \alpha_{Txy}^0 \end{bmatrix} = \begin{bmatrix} 0.5 & 0.5 & -0.5 \\ 0.5 & 0.5 & 0.5 \\ 1 & -1 & 0 \end{bmatrix} \cdot \begin{bmatrix} \alpha_\parallel \\ \alpha_\perp \\ 0 \end{bmatrix} = \begin{bmatrix} 0.5 \cdot (\alpha_\parallel + \alpha_\perp) \\ 0.5 \cdot (\alpha_\parallel + \alpha_\perp) \\ \alpha_\parallel - \alpha_\perp \end{bmatrix} \tag{5}$$

With the example UD-ply CTEs provided in Nettles [1]: $\alpha_\parallel = -0.04$ ppm/K and $\alpha_\perp = 18.0$ ppm/K the ply's expansion properties in the global coordinate system are

$$\begin{bmatrix} \alpha_{Tx}^0 \\ \alpha_{Ty}^0 \\ \alpha_{Txy}^0 \end{bmatrix} = \begin{bmatrix} 8.98 \\ 8.98 \\ -18.04 \end{bmatrix} \quad \text{ppm/K.} \tag{6}$$

Figure 1 shows the facts of the simple 100 mm x 100 mm ABAQUS model which considers an temperature step of $\Delta T = 100$ K.

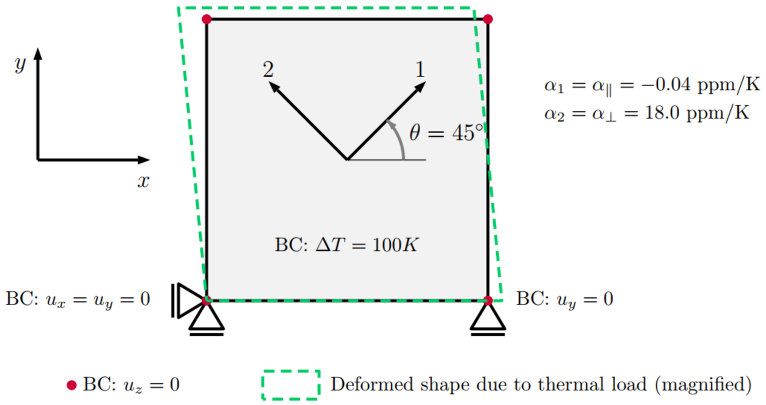
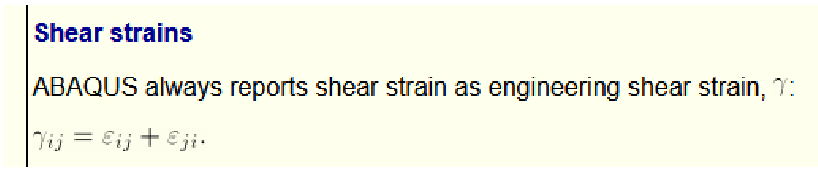



Figure 1: FE model description. 1 shows fiber direction, x,y present laminate coordinates

Figure 2 shows the important excerpt from the ABAQUS manual, saying that ABAQUS always provides shear strains in engineering notation.



**Shear strains**

ABAQUS always reports shear strain as engineering shear strain, $\gamma$:

$\gamma_{ij} = \varepsilon_{ij} + \varepsilon_{ji}$.

Figure 2: Engineering strain statement in ABAQUS manual [6]

Thus, following the VDI notation $\gamma^0_{Txy} = \Delta T \cdot \alpha^0_{Txy}$, the shear-strain in the global coordinate system are calculated to $\gamma^0_{Txy} = -18.04 \cdot 10^{-3}$.

Figure 3 shows the corresponding FE model result, with $E12$ in the legend referring to $\gamma_{xy}$.

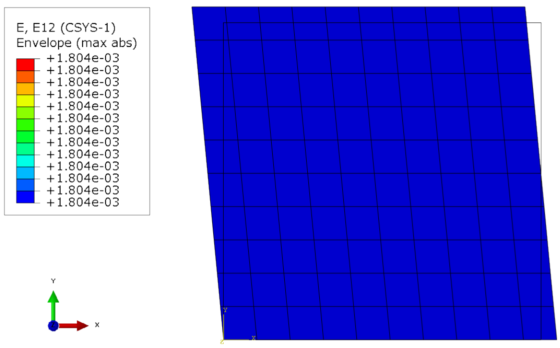
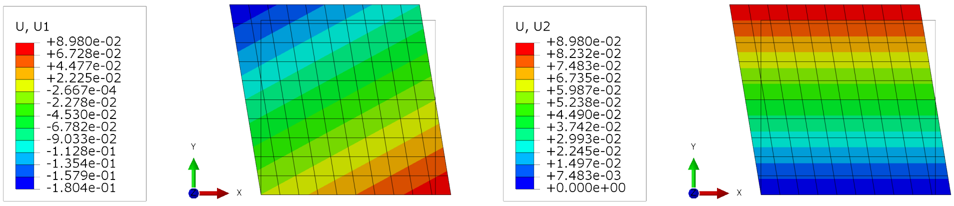



Figure 3: FE model shear strains in laminate coordinate system. Note that $E_{12}$ refers to $\gamma_{xy}$.

The corresponding deformation plots are provided hereafter for completeness.




(a) U1 (b) U2

Figure 4: In-plane displacements from FE model

## 4. Conclusion

The VDI notation and other authors defines $\gamma_{Txy} = \alpha^0_{Txy} \cdot \Delta T$. The FE model produced an identical result. According to the ABAQUS manual [6], the coefficient of thermal for in-plane shear strains can be defined as followed in the input file.

```
*Expansion, type=ANISO
```
$\alpha_{11}$, $\alpha_{22}$, $\alpha_{33}$, $\alpha_{12}^{Abaqus}$, $\alpha_{13}$, $\alpha_{23}$

When the definition of Nettles [1, p.44] and Reddy [2, p.99] ( $\varepsilon_{Txy} = \alpha_{xy} \cdot \Delta T$) is used $\alpha_{12}^{Abaqus} \overset{!}{=} 2 \cdot \alpha_{xy}$ shall be used to model correct thermal shear strains. When VDI notation is used, the determined coefficients can be directly used $\alpha_{12}^{Abaqus} \overset{!}{=} \alpha_{xy}$, leading to:

```
*Expansion, type=ANISO
8.98e-06, 8.98e-06, 0., -18.04e-06, 0., 0.
```

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