EUROPEAN SIGNAL PROCESSING CONFERENCE
30th EUSIPCO
BELGRADE/SERBIA
29 AUG - 2 SEP 2022

GROUP EQUIVARIANT NETWORKS FOR LEAKAGE DETECTION IN VACUUM BAGGING

Christoph Brauer, Dirk Lorenz and Lionel Tondji

Technische Universität Braunschweig

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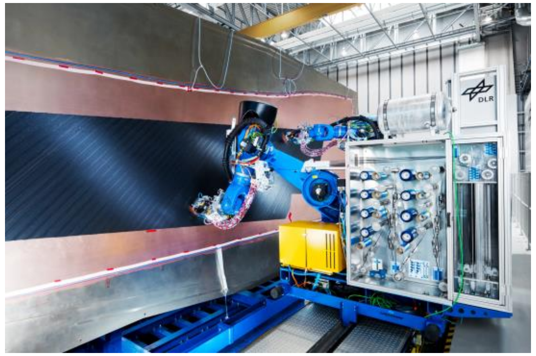
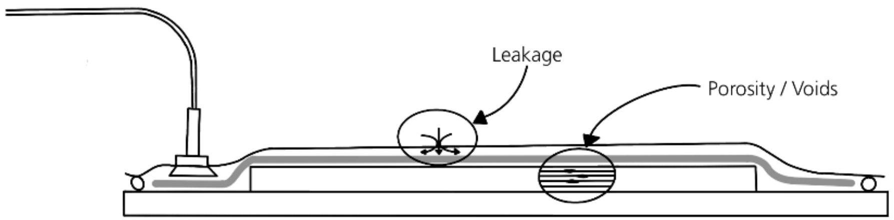
INTRODUCTION

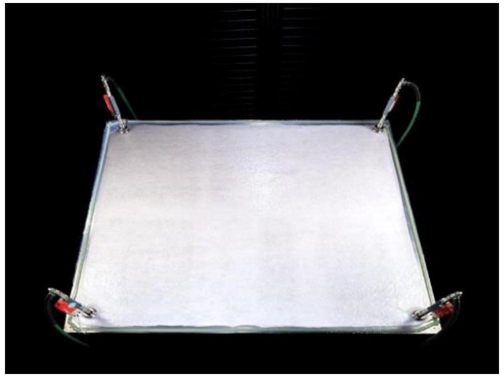
Leakage Detection

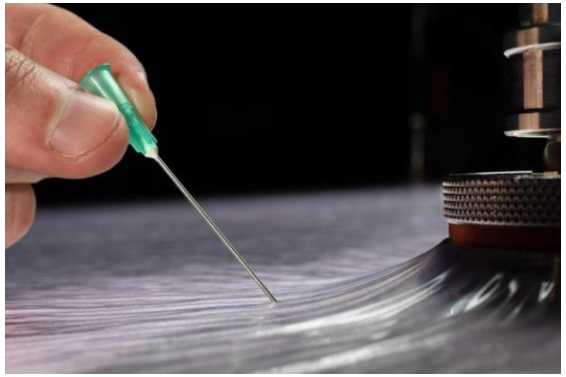


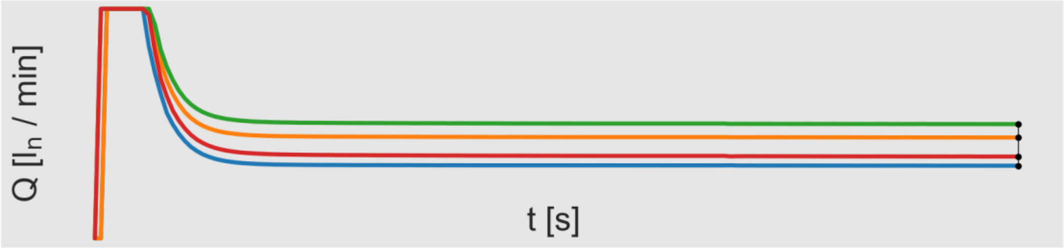


- Context: Production of fiber composite components
- Laminates are cured by means of heat and pressure
- Pressure is applied through vacuum setup
- Leakages in the vacuum bag lower the quality

Experimental Setup

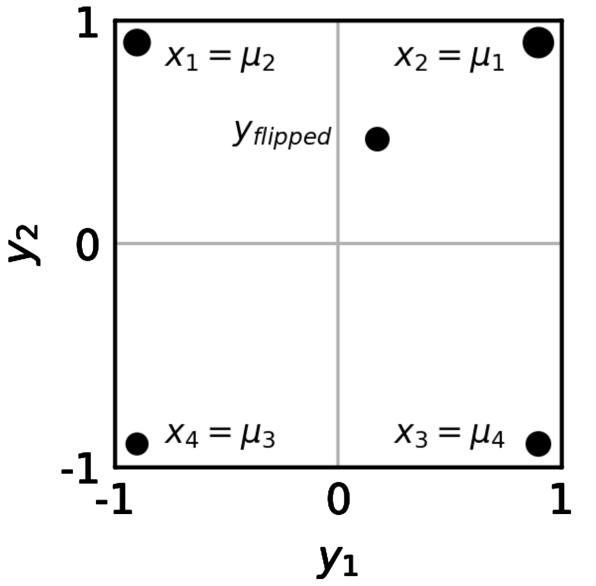






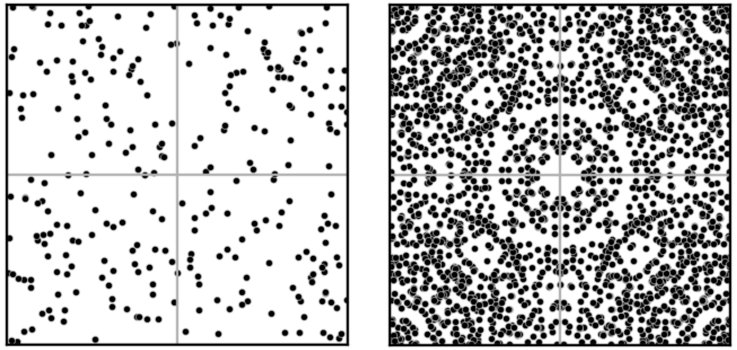
- Quadratic workpiece with edge length 1.5m
- Manually introduced leakage at random position
- One vacuum pump plus flow meter per corner
- Task: Predict leakage coordinates from residual flow

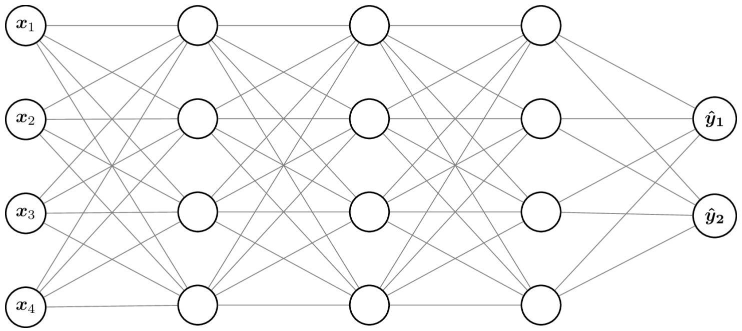
Prior Knowledge



Where is the knowledge integrated?

Training Data	Hypothesis Set





[illegible]

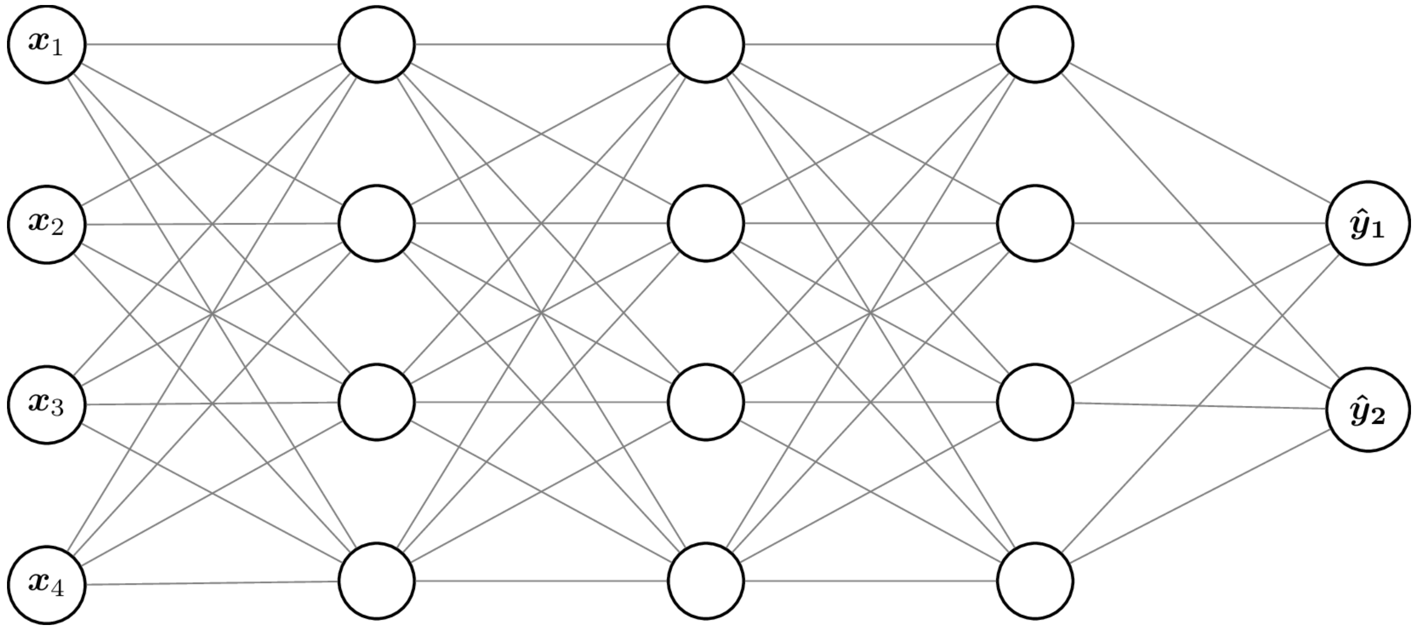
[illegible]

$$R = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \end{bmatrix} \quad r = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} \quad S = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix} \quad s = \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}$$

EQUIVARIANT ARCHITECTURE

From Standard to Equivariant

Standard Architecture



$a^0 = x$

$a^\ell = g\left(W^\ell a^{\ell-1} + b^\ell\right)$

for $\ell = 1, \dots, L-1$

$\hat{y} = f_\theta(x) = W^L a^{L-1} + b^L$

Equivariant Architecture

Requirement:

$f_\theta(Tx) = t f_\theta(x)$

for all $(T, t) = (R^k S^l, r^k s^l)$

Restrictions:

$W^\ell \in \mathbb{R}^{4\times4}$ for $\ell = 1, \dots, L-1$

$W^L \in \mathbb{R}^{2\times4}$

$b^\ell = 0$ for $\ell = 1, \dots, L$

Satisfy requirement layerwise by weight sharing in W^ℓ and W^L

Architecture Derivation

$$f_\theta(Tx) = tf_\theta(x)$$

$$g\left(W^\ell Ta\right) = Tg(W^\ell a) \quad \text{for } \ell = 1, \dots, L-1 \quad \text{and} \quad g(W^L Ta) = tg(W^L a)$$

$$g\left(W^\ell Ra\right) = Rg(W^\ell a) \quad \text{for } \ell = 1, \dots, L-1 \quad \text{and} \quad g(W^L Ra) = rg(W^L a)$$

$$g\left(W^\ell Sa\right) = Sg(W^\ell a) \quad \text{for } \ell = 1, \dots, L-1 \quad \text{and} \quad g(W^L Sa) = sg(W^L a)$$

$$W^\ell R = RW^\ell$$
$$W^\ell S = SW^\ell$$

$$W^\ell = \begin{bmatrix} a & b & c & b \\ b & a & b & c \\ c & b & a & b \\ b & c & b & a \end{bmatrix}$$

$$W^L R = rW^L$$
$$W^L S = sW^L$$

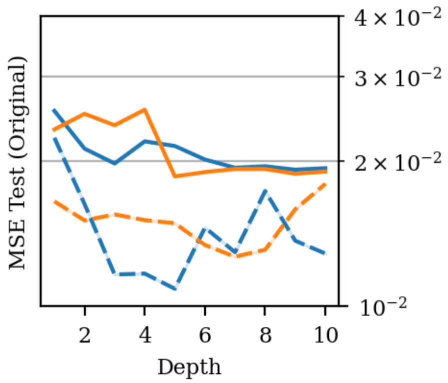
$$W^L = \begin{bmatrix} d & -d & -d & d \\ -d & -d & d & d \end{bmatrix}$$

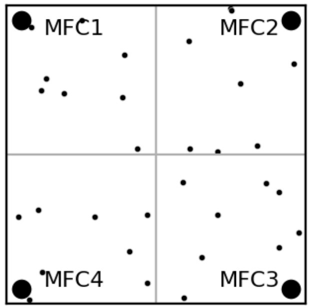
- Satisfy $f_\theta(Tx) = tf_\theta(x)$ layerwise by weight sharing in W^ℓ and W^L
- All representations of a group operation (T, t) can be generated in terms of (R, r) and (S, s)
- Permutation matrices and componentwise activation functions commute

NUMERICAL EXPERIMENTS

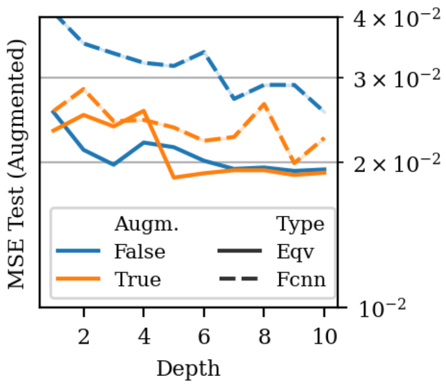
Test Performance

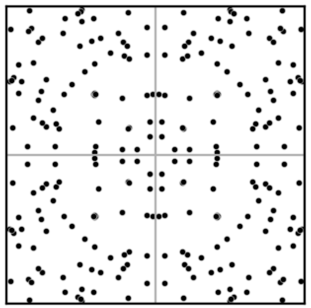
On Original Testset





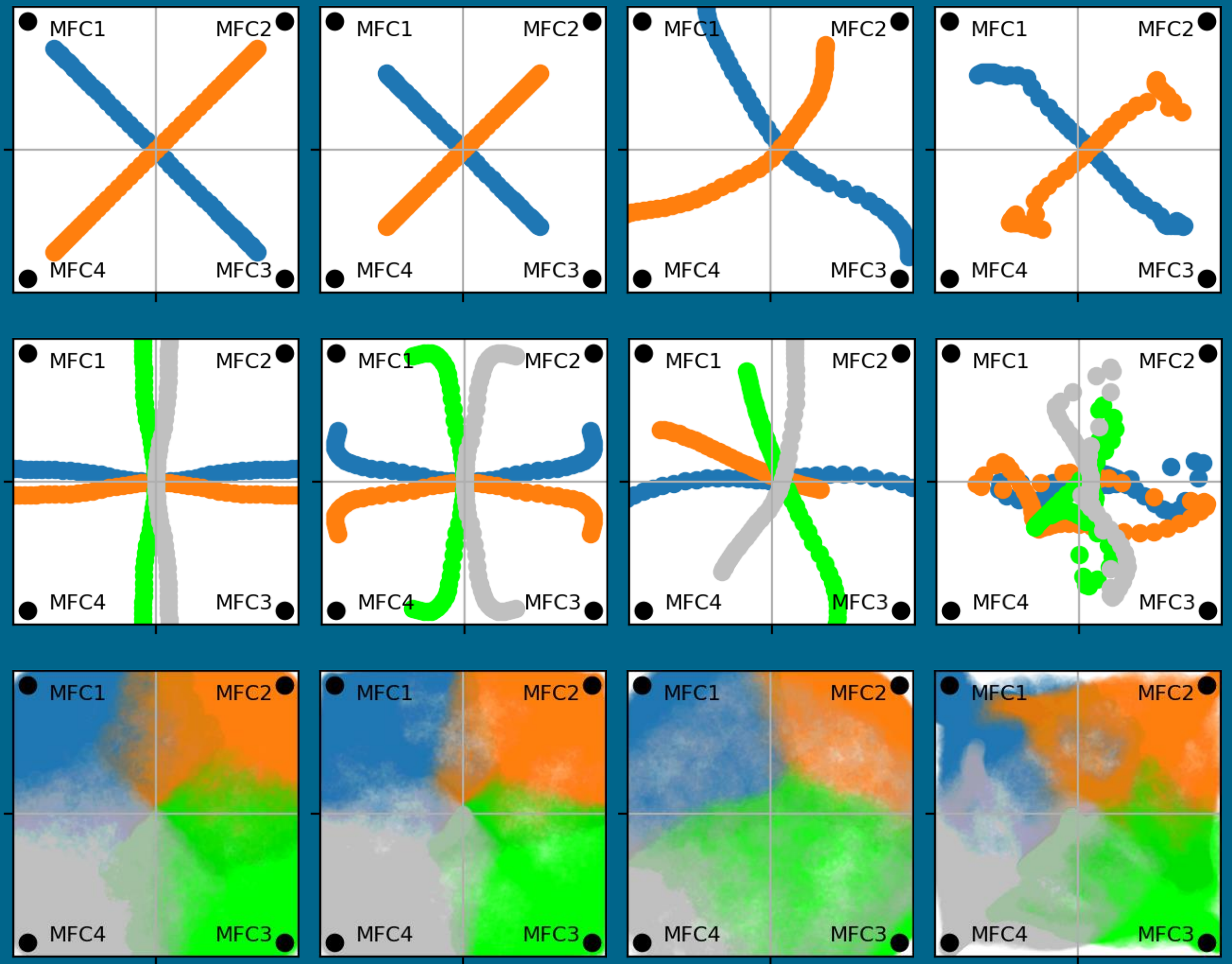
On Augmented Testset





- Standard networks perform best in terms of original test data
  → Arguably because real data does not reflect prior knowledge properly
- Equivariant networks outperform standard networks on augmented data
  → Also in case augmented training data is used for standard networks

Visual Inspection



(a) EQV (b) EQV DA (c) FCNN (d) FCNN DA

1. Trajectories under flow shift between diagonally opposed sensors

   Blue: $x_2 = x_4 = 0.25$
   Orange: $x_1 = x_3 = 0.25$

2. Trajectories under flow shift between adjacent sensors

   Blue: $x_3 = x_4 = 0.25$
   Orange: $x_1 = x_2 = 0.25$
   and so on…

3. Trajectories under flow shift between adjacent sensors

   Blue: $x_1 = \max(x_1, \dots, x_4)$
   Orange: $x_2 = \max(x_1, \dots, x_4)$
   and so on…

$$x_1 + x_2 + x_3 + x_4 = 1$$

SUMMARY

Summary

- Network architecture that is equivariant with respect to the dihedral group D_4
- Outperforms standard in terms of augmented data and visual inspection
- Construction can be transferred to symmetry groups of higher order (see paper)
- Equivariant blocks can be stacked to build wider equivariant networks (future)
- Transfer to more complex geometries and different tasks (future)

Authors: Christoph Brauer (German Aerospace Center, TU Braunschweig)
Dirk Lorenz (TU Braunschweig)
Lionel Tondji (TU Braunschweig)

Code: https://github.com/chrbraue/leakage_detection