

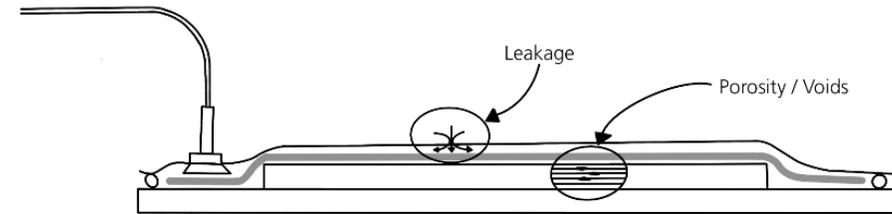
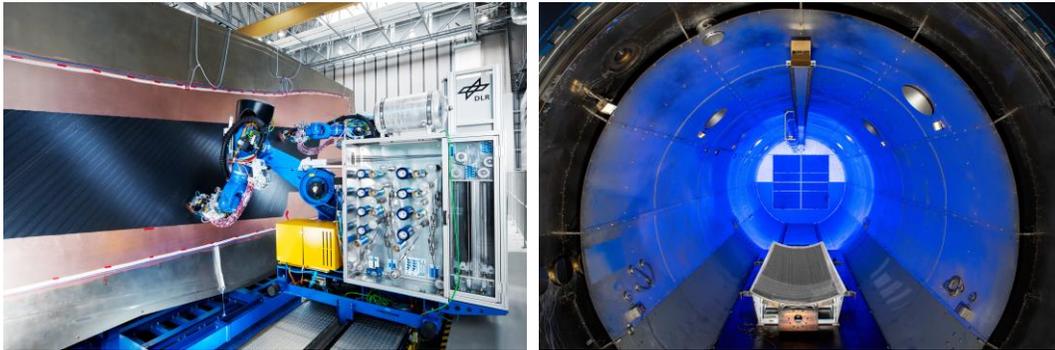
# GROUP EQUIVARIANT NETWORKS FOR LEAKAGE DETECTION IN VACUUM BAGGING

Christoph Brauer, Dirk Lorenz and Lionel Tondji

The background of the slide is a high-resolution photograph of a satellite in orbit above Earth. The satellite is a rectangular platform with two long, parallel solar panel arrays extending outwards. The panels are covered in a grid of small solar cells. The satellite's central body is complex, with various instruments and antennas visible. Below the satellite, the Earth's surface is visible, showing a mix of green landmasses and blue oceans, partially obscured by white clouds. The curvature of the Earth is visible at the bottom of the frame.

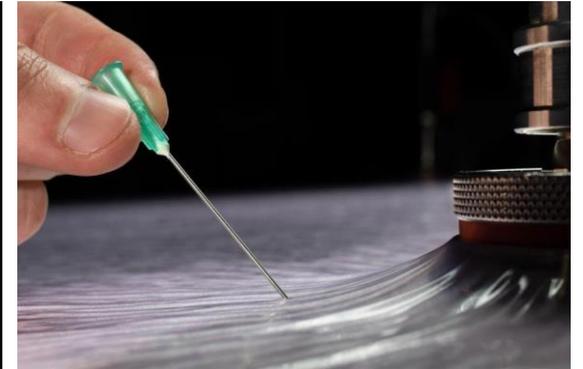
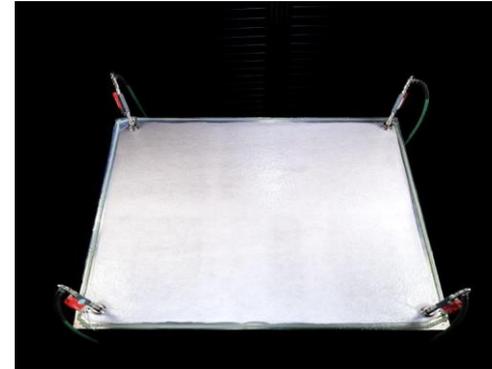
# INTRODUCTION

# Leakage Detection



- Context: Production of fiber composite components
- Laminates are cured by means of heat and pressure
- Pressure is applied through vacuum setup
- Leakages in the vacuum bag lower the quality

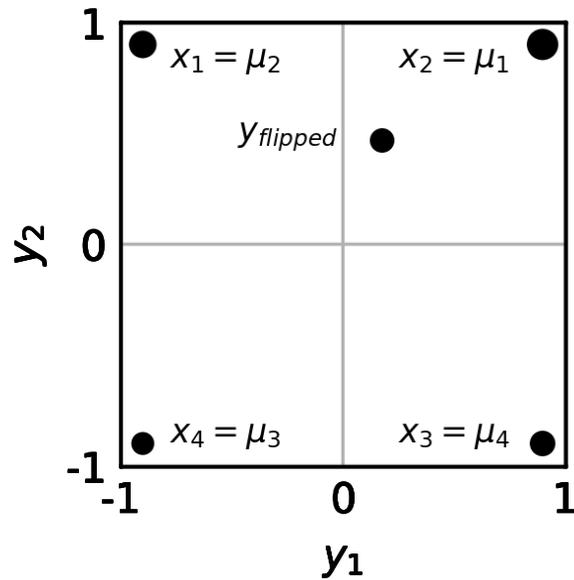
# Experimental Setup



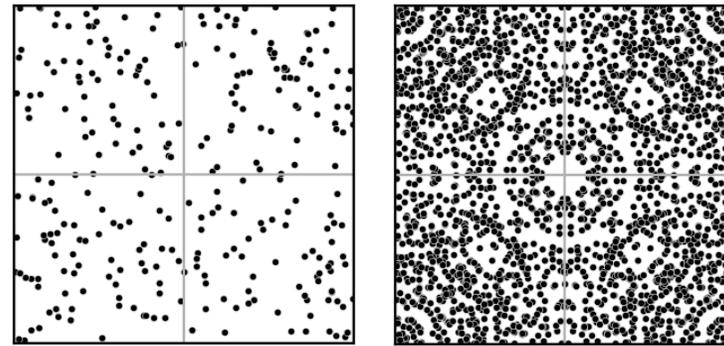
- Quadratic workpiece with edge length 1.5m
- Manually introduced leakage at random position
- One vacuum pump plus flow meter per corner
- Task: Predict leakage coordinates from residual flow

# Prior Knowledge

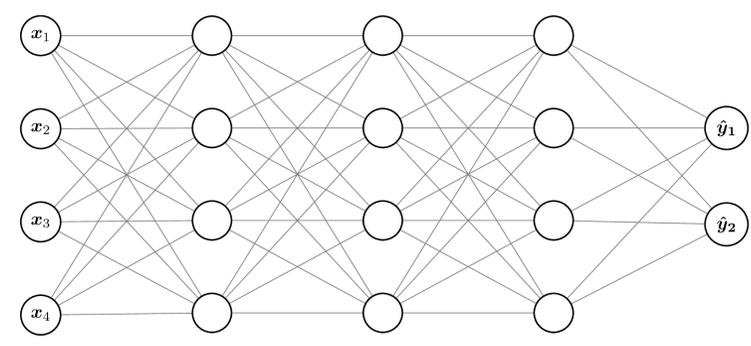
## Where is the knowledge integrated?



Training Data



Hypothesis Set



$$x = \frac{1}{\sqrt{2}} \begin{bmatrix} \cos(\frac{\pi}{4}) & \sin(\frac{\pi}{4}) \\ \sin(\frac{\pi}{4}) & \cos(\frac{\pi}{4}) \end{bmatrix} x$$

$$y = \frac{1}{\sqrt{2}} \begin{bmatrix} \cos(\frac{\pi}{4}) & \sin(\frac{\pi}{4}) \\ \sin(\frac{\pi}{4}) & \cos(\frac{\pi}{4}) \end{bmatrix} y$$

$$R = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \end{bmatrix} \quad r = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} \quad S = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix} \quad s = \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}$$

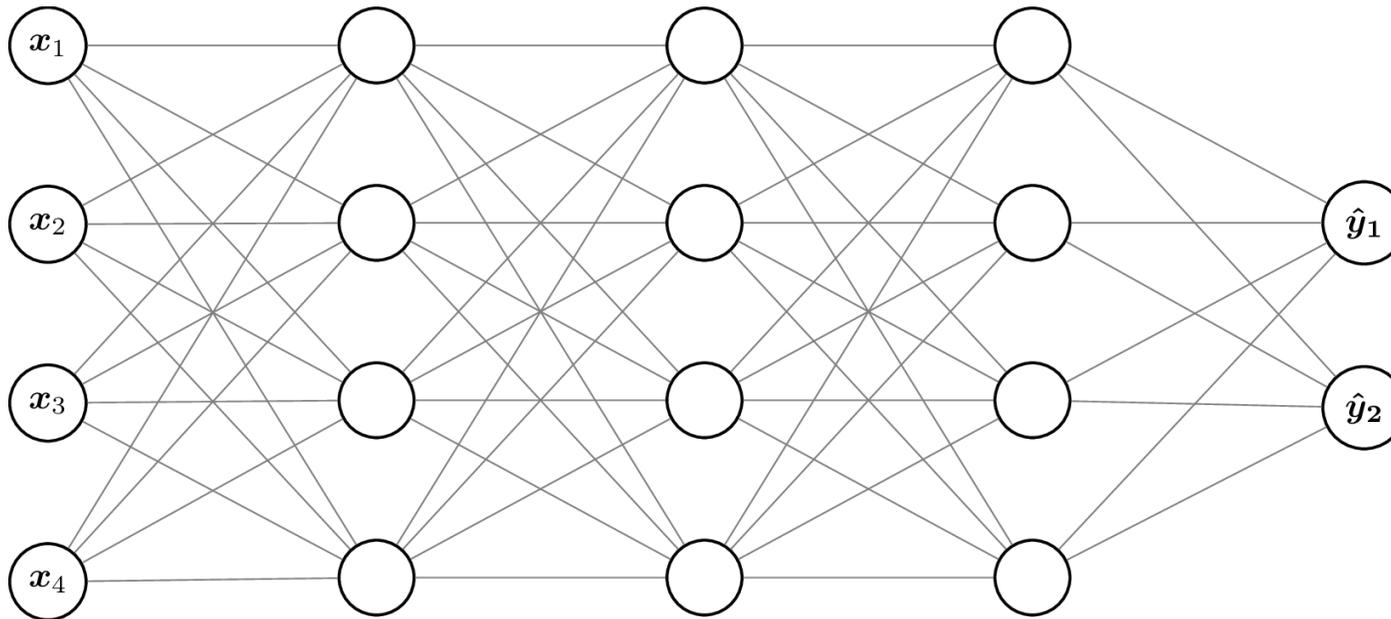
The background of the slide is a high-resolution photograph of a satellite in orbit above Earth. The satellite is the central focus, featuring a central body with various instruments and two long, rectangular solar panel arrays extending outwards. The Earth's surface below is a mix of green landmasses and blue oceans, partially obscured by white clouds. The curvature of the planet is visible at the top and bottom edges of the frame.

# EQUIVARIANT ARCHITECTURE

# From Standard to Equivariant



## Standard Architecture



## Equivariant Architecture

Requirement:

$$f_{\theta}(Tx) = tf_{\theta}(x)$$

for all  $(T, t) = (R^k S^l, r^k s^l)$

Restrictions:

$$W^{\ell} \in \mathbb{R}^{4 \times 4} \text{ for } \ell = 1, \dots, L-1$$

$$W^L \in \mathbb{R}^{2 \times 4}$$

$$b^{\ell} = 0 \text{ for } \ell = 1, \dots, L$$

$$a^0 = x \quad a^{\ell} = g(W^{\ell} a^{\ell-1} + b^{\ell}) \quad \hat{y} = f_{\theta}(x) = W^L a^{L-1} + b^L$$

for  $\ell = 1, \dots, L-1$

**Satisfy requirement layerwise  
by weight sharing in  $W^{\ell}$  and  $W^L$**

# Architecture Derivation



$$f_{\theta}(Tx) = tf_{\theta}(x)$$

$$g(W^{\ell}Ta) = Tg(W^{\ell}a) \text{ for } \ell = 1, \dots, L-1 \text{ and } g(W^L Ta) = tg(W^L a)$$

$$g(W^{\ell}Ra) = Rg(W^{\ell}a) \text{ for } \ell = 1, \dots, L-1 \text{ and } g(W^L Ra) = rg(W^L a)$$

$$g(W^{\ell}Sa) = Sg(W^{\ell}a) \text{ for } \ell = 1, \dots, L-1 \text{ and } g(W^L Sa) = sg(W^L a)$$

$$W^{\ell}R = RW^{\ell}$$

$$W^{\ell}S = SW^{\ell}$$

$$W^{\ell} = \begin{bmatrix} a & b & c & b \\ b & a & b & c \\ c & b & a & b \\ b & c & b & a \end{bmatrix}$$

$$W^L R = rW^L$$

$$W^L S = sW^L$$

$$W^L = \begin{bmatrix} d & -d & -d & d \\ -d & -d & d & d \end{bmatrix}$$

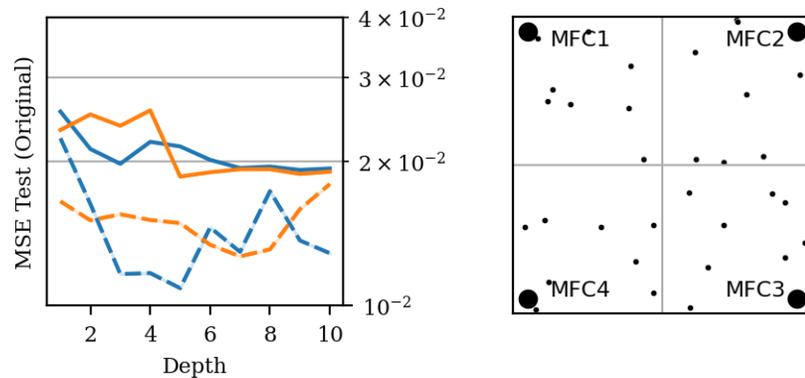
- Satisfy  $f_{\theta}(Tx) = tf_{\theta}(x)$  layerwise by weight sharing in  $W^{\ell}$  and  $W^L$
- All representations of a group operation  $(T, t)$  can be generated in terms of  $(R, r)$  and  $(S, s)$
- Permutation matrices and componentwise activation functions commute

The background of the slide is a high-resolution photograph of a satellite in orbit. The satellite is a rectangular platform with two long, thin solar panel arrays extending outwards. It is positioned in the center-right of the frame, with the Earth's surface below. The Earth shows a mix of green landmasses, blue oceans, and white cloud cover. The curvature of the planet is visible on the right side, where the atmosphere transitions into the blackness of space.

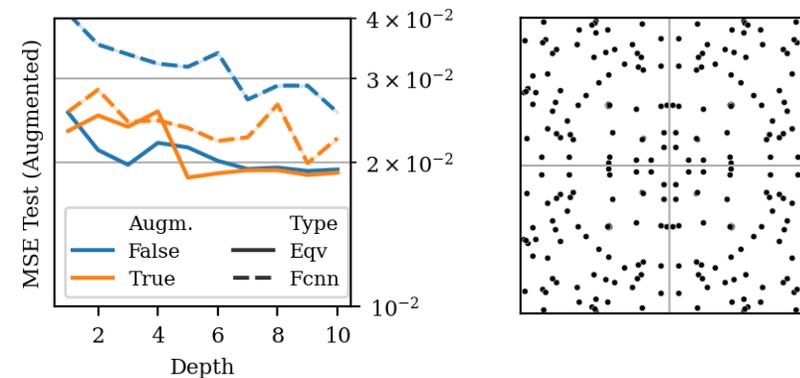
# NUMERICAL EXPERIMENTS

# Test Performance

## On Original Testset

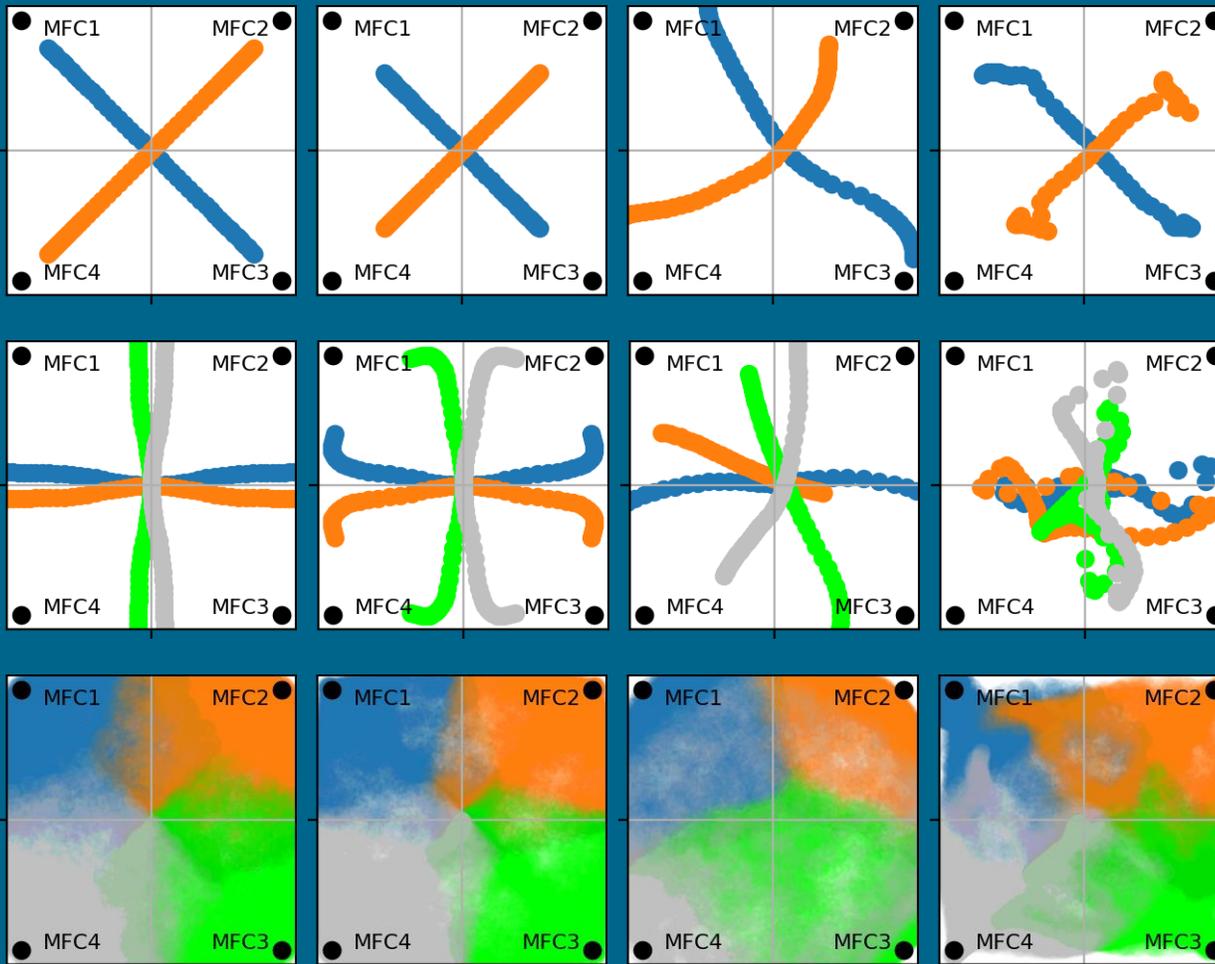


## On Augmented Testset



- Standard networks perform best in terms of original test data  
→ Arguably because real data does not reflect prior knowledge properly
- Equivariant networks outperform standard networks on augmented data  
→ Also in case augmented training data is used for standard networks

# Visual Inspection



(a) EQV

(b) EQV DA

(c) FCNN

(d) FCNN DA

1. Trajectories under flow shift between diagonally opposed sensors

Blue:  $x_2 = x_4 = 0.25$   
 Orange:  $x_1 = x_3 = 0.25$

2. Trajectories under flow shift between adjacent sensors

Blue:  $x_3 = x_4 = 0.25$   
 Orange:  $x_1 = x_2 = 0.25$   
 and so on...

3. Trajectories under flow shift between adjacent sensors

Blue:  $x_1 = \max(x_1, \dots, x_4)$   
 Orange:  $x_2 = \max(x_1, \dots, x_4)$   
 and so on...

$$x_1 + x_2 + x_3 + x_4 = 1$$

A satellite with two long solar panel arrays is shown in orbit above Earth. The satellite's body is gold-colored with various instruments and antennas. The solar panels are silver and extend far from the central body. The Earth below shows green landmasses, blue oceans, and white clouds. The curvature of the planet is visible at the bottom of the frame.

# SUMMARY

# Summary



- Network architecture that is equivariant with respect to the dihedral group  $D_4$
- Outperforms standard in terms of augmented data and visual inspection
- Construction can be transferred to symmetry groups of higher order (see paper)
- Equivariant blocks can be stacked to build wider equivariant networks (future)
- Transfer to more complex geometries and different tasks (future)

Authors: Christoph Brauer (German Aerospace Center, TU Braunschweig)  
Dirk Lorenz (TU Braunschweig)  
Lionel Tondji (TU Braunschweig)

Code: [https://github.com/chrbraue/leakage\\_detection](https://github.com/chrbraue/leakage_detection)