# A COMPREHENSIVE APPROACH TO PULSE MODULATION FOR ATTITUDE CONTROL WITH THRUSTERS SUBJECT TO SWITCHING RESTRICTIONS

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## 1. INTRODUCTION

Launch and space vehicles may use on-off thrusters as actuators for attitude control. Examples are described in [1], [3], [6], [7], [8], [13] - [16]. Several types of on-off thrusters are available such as hydrazine, cold-gas and pulsed-plasma thrusters [1], [10], [11]. These thrusters produce discontinuous control actions and are subject to switching constraints. The most common approaches for thruster activation logic are direct bang-bang control and pulse modulation.

Bang-bang control is based on classical results from optimal control theory [4] and consists of the use of the non-linear actuator "as is". However, it is difficult to consider switching restrictions in the discontinuous optimal control setting when control laws in feedback form are sought for implementation in embedded controllers.

Pulse modulators convert continuous input commands in a sequence of switching signals suitable for the command of on-off thrusters. When subject to a constant modulator input, the average value of the switching signal at modulator output approximately attains the value prescribed by an arbitrary function of the constant input value. Such a function is chosen at system design stage based on considerations which are highly dependent on the application. It should be noted that the quality of the approximation might not be the same for all modulators.

Pulse modulators are mostly employed in a setup that entails averaged (output) control values which are proportional to modulator input, thus statically quasi-linearising the switching actuator. In control applications, quasi-linearisation is useful because physical plants show low-pass behaviour; the plant attenuates high frequencies introduced by switching actuators. Thus, quasi-linearisation through pulse modulation allows for the use of control methods originally designed for transient control of systems with proportional actuators. However, quasi-linearisation fails at low switching frequencies which typically occur close to dead-band regions during limit cycling.

Special pulse modulation schemes have been proposed and used for launching vehicles and spacecraft control (e.g. [2], [5] and [12]). In-depth discussion of pulse modulation schemes in general is available in the literature as well (e.g. [9]). However, most of these schemes do not consider actuator switching restrictions which were accommodated or accounted for with *ad-hoc* approaches.

Some pulse modulator implementations are composed of a Schmitt Trigger, a linear filter and a feedback loop (e.g. [2], [12] - [14] and [16]). Others make use of modulation curves implementable in software or firmware [5], [9]. Although recommendations on the choice of parameters for modulators based on a Schmitt Trigger are found in [12] and [16], such an implementation is complex [13] because the final values of the modulator parameters are often determined on the basis of a time-consuming trial-and-error approach, until a set of coefficients is found that meets the constraints on thruster activation. The design process is comparatively easier for modulators based on direct modulation curve implementations such as those discussed in [9].

Proposed in this paper is a pulse modulation scheme based on a set of modulation curves which explicitly considers switching restrictions and implements arbitrary bounded functions between constant modulator input and averaged modulator output. Such an approach is particularly well suited for application in attitude control systems. The use of this modulation scheme does not rely on time-consuming tuning strategies.

Given in section 2 is the problem statement which is solved by the pulse modulation scheme discussed in Section 3. Section 4 is devoted to its modification in cases where additional specifications that were not considered in the previous sections need to be taken into account. Finally, an example is found in section 5 while a conclusion follows in section 6.

## 2. PROBLEM STATEMENT

The problem statement refers to the block diagram reported in Figure 1, where a switching actuator with one output $u(t)$ that takes values in $\{-U; 0; U\}$ is commanded according to the switching command $w(t) \in \{-1; 0; 1\}$ delivered by a modulator, as a function of the modulator input $v(t)$. In the baseline problem, actuator switching is subject to the following restrictions:

- Minimum pulse duration (on-time): $t_{on}$
- Minimum rest between successive pulses: $t_{off}$

If all switching restrictions are satisfied, output will be $-U$ for negative actuator inputs, zero for zero input, and $U$ for positive inputs. Direct switching from $-U$ to $U$ (or vice-versa) is not allowed.

A pulse modulator shall be proposed that, from a given input signal $v(t)$, produces an actuator command signal, $w(t)$, with the following properties:

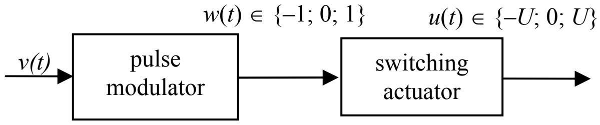



FIG 1. Pulse modulator and switching actuator.

- It guarantees proper actuator functioning, i.e. it will not command a change of actuator output in violation of the actuator's switching restrictions;
- It is such that, for constant modulator input $v(t) = v_c$, the average value of $u(t)$ is $u_m = f(v_c)$, where $f$: $\mathbb{R} \rightarrow [-U;\ U]$ is an arbitrary function chosen by the designer based on suitable engineering considerations or mission-dependent requirements. Let $f$ be a function without dead band, a restriction that will be relaxed later.

## 3. SOLUTION

Two quantities that allow the determination of a modulator output $w(t)$ which satisfies the constraints presented in the previous section are now defined according to the representation in Figure 2.

- $T_k$: time interval between start of pulse $k$ and start of pulse $k$+1 of $w(t)$;
- $\gamma_k$: duration of pulse $k$ of $w(t)$.

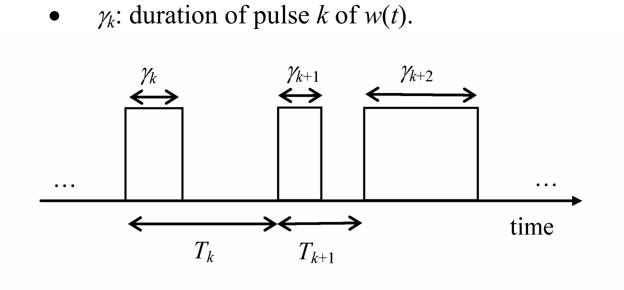



FIG 2. Generic pulse modulator output $w(t)$.

The proposed pulse modulator scheme is based on the implementation of two modulation curves which specify $T_k$ and $\gamma_k$ as functions of the modulator input $v$. Such modulation curves, illustrated in Figure 3, can be defined as functions of $|f(v)|$. The sign of modulator output $w(t)$ depends directly on the sign of $f(v)$, i.e. for $f(v) > 0$ pulses of $w(t)$ will be positive, while $w(t)$ will be negative for $f(v) < 0$. Modulator output is set to zero if $f(v) = 0$. The meaning of the symbol $f_{min}$ used in Figure 3 will be clarified later in this section.

The following restrictions for $T_k$ and $\gamma_k$ result from the problem statement:

(1) $$T_k \geq \gamma_k + t_{off}$$

(2) $$\gamma_k \geq t_{on}\,.$$

Inequality (1) guarantees that the specification of minimum rest between successive pulses is satisfied. Inequality (2) is required in order to enforce the specification of minimum on-time. If (1) and (2) hold, switching restrictions will be satisfied and actuator output will be $u(t) = w(t)U$.

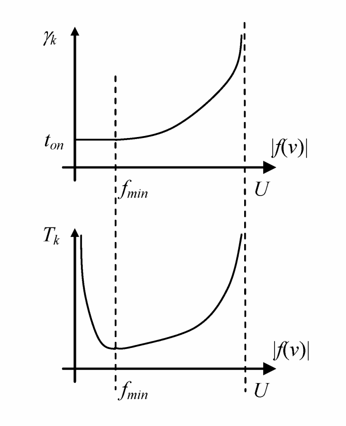



FIG 3. Modulation curves for $T_k$ and $\gamma_k$ ($f$ without dead band).

Since on the one hand for constant modulator input $v = v_c$, $u_m = f(v_c)$ is specified, and on the other hand according to Figure 2 it is $u_m$ = sign$[f(v_c)]U\gamma_k/T_k$, then modulation curves must be chosen such that

(3) $$\frac{U\gamma_k}{T_k} = |\,f(v)\,|\,.$$

Fastest possible switching is desirable in order to attain the average output prescribed by equation (3) also in the low frequency range. This choice implies that equation (1) shall hold with equality whenever possible. From equations (2) and (3) it is seen that equality in equation (1) is not possible if $\gamma_k = t_{on}$ and additionally

$$|\,f(v)\,| < \frac{Ut_{on}}{t_{on} + t_{off}}\,.$$

At this point, it is useful to define the quantity $f_{min}$ as follows:

$$f_{min} = \frac{Ut_{on}}{t_{on} + t_{off}}\,.$$

With these considerations in mind, an expression for the $\gamma_k$ modulation curve is proposed which combines equations (1), (2) and (3):

(4) $$\gamma_k = \begin{cases} \dfrac{t_{off}\,|\,f(v)\,|}{U - |\,f(v)\,|} & \text{if } v \in \{v \mid |\,f(v)\,| \geq f_{\min}\} \\ t_{on} & \text{otherwise.} \end{cases}$$

The modulation curve for $T_k$ follows from equation (3) as a function of $|f(v)|$ and $\gamma_k$,

(5) $$T_k = \frac{U\gamma_k}{|\,f(v)\,|}$$

or, using equation (4)

$$T_k = \begin{cases} \dfrac{Ut_{off}}{U - |\,f(v)\,|} & \text{if } v \in \{v \mid |\,f(v)\,| \geq f_{\min}\} \\ \dfrac{Ut_{on}}{|\,f(v)\,|} & \text{otherwise.} \end{cases}$$

Equations (4), (5) and the sign convention for $w(t)$ lead to the following calculation rules for modulator output calculation:

***Algorithm***

1) Modulator initialisation:

- At $t = 0$ assume $w(t \leq 0) = 0$, evaluate $f[v(0)]$ and calculate $\gamma_k$ from (4) and $T_k$ from (5).
- Initialise two auxiliary variables $t_{soff}$ (time of last switch off) and $t_{son}$ (time of last switch on) as follows: $t_{son} = -T_k$ and $t_{soff} = -T_k + \gamma_k$.

2) After initialisation, repeat the following calculation rule (quasi-) continuously to determine $w(t)$ for all positive $t$:

If $t \geq t_{son} > t_{soff}$ then
- Calculate $\gamma_k$ from (4).
- If $t - t_{son} \leq \gamma_k$ then $w(t) = \text{sign}[f(v)]$, else $w(t) = 0$ and $t_{soff} = t$.

Otherwise
- Calculate $T_k$ from (5).
- If $t - t_{soff} \leq T_k - \gamma_k$ then $w(t) = 0$, else $w(t) = sign[f(v)]$ and $t_{son} = t$.

## 4. SOLUTION UNDER ADDITIONAL SPECS

If in some application a *maximum switching frequency* is specified, such constraint can be enforced by artificially increasing the value originally specified for $t_{off}$, since, because of equation (1), maximum switching frequency for constant modulator input is bounded by $1/(t_{on} + t_{off})$.

If (in addition or not) a *minimum switching frequency* is specified, then one can let $T_k$ saturate at a chosen maximum frequency period, $T_{max}$, which will result in a lower limit on $|f(v_c)|$ for which the modulator will work as originally specified. This limit is

$$| f(v_c) |_{min} = \frac{Ut_{on}}{T_{\max}}$$

For lower values of $|f(v_c)|$ the result at actuator output will be $u_m = \text{sign}[f(v_c)](Ut_{on}/T_{\max})$. This, however, is not a drawback of the pulse modulation scheme proposed, but an intrinsic limitation imposed by the minimum switching frequency specification.

When a *maximum on-time* $t_{max}$ is specified for the actuator, the modulation curve for $\gamma_k$ is chosen to saturate at $t_{max}$. In this case the maximum mean value achievable at actuator output is

$$u_m = \frac{Ut_{max}}{t_{max} + t_{off}} = U_{\max} < U \ ,$$

and $f$ will be implementable (in the sense of the problem statement in section 2) only if $f$: ℝ → $[-U_{max}; U_{max}]$.

If $f$ has a *dead band*, a frequent feature in rocket and satellite control systems ([2], [12] - [14], [16]), then $\gamma_k = 0$ is assigned as long as the value of $f$ is zero. Figure 4 shows a typical $\gamma_k$ modulation curve for this case.

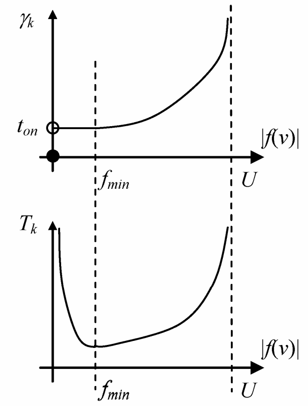



FIG 4. Modulation curves for $T_k$ and $\gamma_k$ ($f$ with dead band).

If the actuator in use is such that *pulse duration is fixed*, say at $t_{on}$, then the modulation scheme degenerates to pure pulse frequency modulation and one must choose the following modulation characteristics:

$$\gamma_k \equiv t_{on}$$

$$T_k = \frac{Ut_{on}}{| f(v) |}$$

In practical cases, it may happen that a certain minimum rest $t_{off1}$ is specified for successive pulses in the *same* direction and a different minimum rest $t_{off2}$ is specified for pulses in *opposite* directions. In this case, the pulse modulation scheme must be applied with a context dependent value selection for $t_{off}$ to provide feasible use of actuators. Since sign inversion at modulator output is usually a sporadic occurrence during a transient, such as the initial response to a large attitude error, performance will be similar to that of a system with $t_{off} = t_{off1}$.

## 5. EXAMPLE

As an illustration example, the single axis attitude regulation of a rigid body satellite using on-off rocket actuators is considered along the lines in [2] and references therein. The block diagram of the controlled system is given in Figure 5, where $\phi$ designates the attitude angle, $I$ is the inertia moment, $K_1$ and $K_2$ are controller gains to be designed. Two integrator blocks model the rigid body dynamics, with a state space form given by

$$\dot{x} = Ax + Bu$$

$$A = \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix}, B = \begin{bmatrix} \frac{1}{I} \\ 0 \end{bmatrix}, C = \begin{bmatrix} 0 & 1 \end{bmatrix}, D = 0 \ .$$

This is called the nominal system model. In Figure 5 it is complemented with features not taken into account in the cited references, such as thrust build-up dynamics and thruster switching restrictions. When switching on a thruster, the nominal thrust value will not be available instantaneously. A model for the underlying dynamics can be identified from the thruster's step response. In the block diagram in Figure 5, the transfer function

$$H(s) = \frac{4800}{(s+60)(s+80)}$$

represents thrust build-up dynamics, which will be viewed as a perturbation of the nominal system, since in practice $H(s)$ may vary with time and operation conditions. The thrusters produce a torque of 2 [Nm] when switched "on", subject to the following switching-time restrictions: a minimum on-time $t_{on}$ = 0.01 [s] and a minimum time-between-pulses of $t_{off}$ = 0.05 [s]. The inertia moment is assumed to be $I$ = 1000 [kg m$^2$].

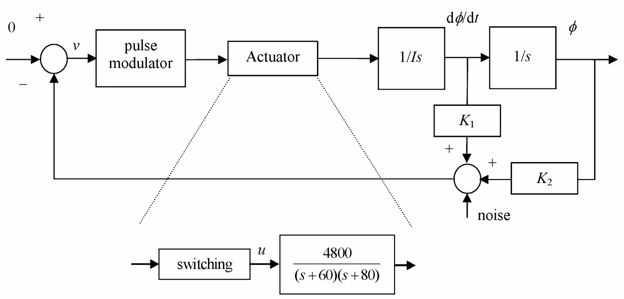



FIG 5. Control loop

Because of its robustness properties, linear quadratic regulator design [4] was used to determine controller gains for the example problem. The cost function

$$J = \int_0^\infty (x'Qx + Ru^2)dt$$

penalises attitude error and fuel consumption. Some experimentation with different values leads to the following choice of $Q$ and $R$:

$$Q = \begin{bmatrix} 20 & 0 \\ 0 & 0.1 \end{bmatrix}, R = 0.001\,.$$

These weights yield state feedback gains $K_1$ = 200 and $K_2$ = 10 and result in nominal attitude regulation response without overshoot. Furthermore, for initial attitude errors of 10 degrees control values stay within actuator range, i.e. below 2.0. The control law resulting from the minimisation of the performance index $J$ was implemented in the control loop given in Figure 5. In order to quasi-linearise the actuator $f(v)$ was chosen as:

$$f(v) = \begin{cases} 2\text{sign}(v) & ,|v| > 2 \\ v & ,2 \geq |v| > 0.04 \\ 0 & ,|v| \leq 0.04 \end{cases}$$

Additionally, a dead band of 0.04 was adopted to keep actuators switched off for small attitude rates and errors. Such use of dead bands is a common procedure in attitude control systems to allow for fuel economy [12], [16].

The characteristics of the pulse modulator curves were chosen according to the approach and considerations discussed in sections 3 and 4, i.e.

$$\gamma_k = \begin{cases} \dfrac{0.05\,|f(v)|}{2-|f(v)|} & \text{if } |f(v)| \geq \dfrac{1}{3} \\ 0.01 & \text{otherwise.} \end{cases}$$

and

$$T_k = \frac{2\gamma_k}{|f(v)|}$$

In Figure 6 the responses of the linear double integrator system and the fourth order pulse modulation control system of Figure 5 to an initial attitude error of 11.25 degrees are compared. All other initial states were set to zero. Consolidated measurement noise (see noise injection point in Figure 5) used in the simulation was Gaussian with a variance of $10^{-4}$. Feedback uses attitude angles measured in [rd] and attitude rates in [rd/s].

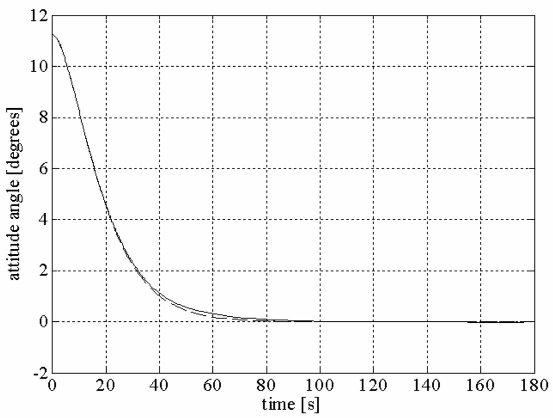
As expected, the responses given in Figure 6 are very similar and the small difference is due to the presence of the dead band, the model perturbation $H(s)$ and the sensor noise in the system of Figure 5. However, very large initial conditions may lead to the incapacity of the modulator to produce an output such that $u_m = v$. This will happen for initial conditions that demand $v(0) > 2$, which is $10\phi(0) + 200\dot{\phi}(0) > 2$, considering the relationships given in Figure 5. In such cases system transient response will be similar to that of the nominal controlled system with saturated actuator.

Although the response of the pulse modulated system is similar to that of the linear system in the transient phase, during precision control, i.e. $t$ > 100 [s] in Figure 6a, the modulated system enters a region of persistent, small amplitude motion not very noticeable in Figure 6 because of chosen scales. This is a well known phenomenon for switching attitude control systems (e.g. [1] - [3], [9]). The amplitude of this motion depends on the linear subsystem as well as the modulator parameters. In particular, it can be adjusted iteratively varying the dead band size. A detailed discussion of persistent small amplitude motion in pulse modulated systems is found in [9]. That discussion, however, does not explicitly consider switching restrictions. Nevertheless, it may be used in combination with the modulation curves derived here, because the resulting modulator guarantees non-violation of the restrictions.

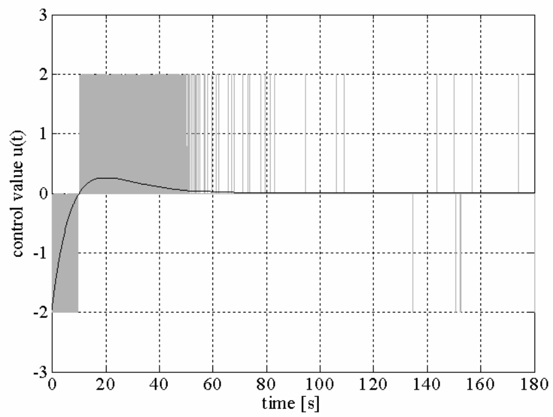
## 6. CONCLUSION

The approach to pulse modulation presented in this paper is suitable for determining the modulation curves in attitude control applications with actuators subject to switching restrictions. Switching restrictions such as maximum and/or minimum switching frequencies, maximum and/or minimum on-times and minimum rest between successive pulses are enforced in a systematic way at design time and simulations are needed only for verification purposes.

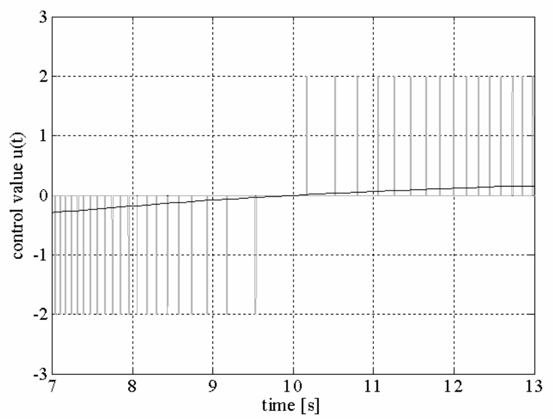
The modulation algorithm is suitable for quasi-continuous implementation and uses two modulation curves, one for the time interval between successive pulses, one for pulse duration. It can implement arbitrary bounded functions between constant modulator input and averaged modulator output.




(a) Solid line: output of nominal regulated second order system; dashed line: output of system from Figure 5.




(b) Solid line: control value for nominal regulated second order system; grey line: “switch output” $u(t)$ in Figure 5.




(c) Zoom into (b)

FIG 6. Responses to an initial attitude error of 11.25 degrees (other initial states are zero).

An example illustrates the design of the modulation curves for a quasi-linear implementation of a linear quadratic attitude regulation law. Simulation was used to verify system performance in the presence of white measurement noise.

## ACKNOWLEDGEMENTS

The second author is on leave from Instituto Tecnológico de Aeronáutica, Brazil, and acknowledges partial support by the Alexander von Humboldt Foundation, Germany, and FAPESP, Brazil.

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