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WHIRL FLUTTER STABILITY ASSESSMENT USING ROTOR TRANSFER MATRICES

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Abstract: The aeroelastic instability phenomenon called whirl flutter is of increased interest for the flutter assessment of aircraft, as a new generation of propeller aircraft is being developed. For the modelling of whirl flutter, a description of the motion induced forces on the propeller is necessary. Classical theory uses analytically derived coefficients for this purpose, but with some approximations. This paper presents a new method, called the transfer matrix method, to describe these transfer functions from hub motion to hub forces in the frequency domain and increase fidelity of whirl flutter assessment. Instead of deriving the transfer functions, they are identified from a time domain model of the isolated propeller, for which several fidelity levels already exist. For propellers in axial flow it comes down to two time domain pulse-perturbations of the hub DOF to identify the spectrum of transfer matrices. The usage of this method is demonstrated in the paper using simple propeller models and results are compared to the classical method as well as coupled time domain simulations. Results show a very good agreement with both references, but also the potential benefits of including blade flexibility and more complex aerodynamics into the whirl flutter assessment in the frequency domain.

1 INTRODUCTION

Whirl flutter is an aeroelastic instability caused by the coupling of a rotating propeller or rotor with its elastic support, e.g. a flexible pylon or wing. The whirling motion of such a dynamic system, which are caused by the gyroscopic torques, can become unstable due to the motion-induced aerodynamic forces on the propeller. This instability leads to growing amplitudes in the whirling motion and can cause the failure of the support structure, posing a risk to flight safety. The occurrence of whirl flutter must therefore be precluded in the whole flight envelope [1].

Historically, whirl flutter has been a design criterion for e.g. turboprop transport aircraft [2]. With a new generation of propeller powered aircraft on the horizon, which may be powered e.g. by electric motors, whirl flutter and its prediction becomes a research focus in aircraft aeroelasticity (again). Especially for the design of next generation turboprop aircraft, accurate numerical prediction methods are necessary to improve the accuracy already in the design phase and de-risk the occurrence of whirl flutter. Whirl flutter stability can contradict certain design criteria such as cabin vibrations and others due to limitation e.g. in shock mount stiffness. More accurate numerical simulations can help to reduce margins and lead to an overall better aircraft design.

As whirl flutter is caused by the motion-induced unsteady aerodynamic forces on the propeller (e.g. during pitch and yaw motion), its prediction needs a description of these forces and torques. The goal is therefore to accurately predict the response of the propeller to perturbations at its hub. For this, different approaches have been used in the past, including analytical derivatives, time domain simulation models and even wind tunnel tests.

First descriptions for propeller forces in perturbed conditions were found by Ribner [3], in this case describing the propellers reaction to sheared inflow (for the usage in loads and flight mechanics). Although these aerodynamic coefficients could also be used in whirl analysis, the understanding and modelling of whirl flutter really took of only after two accidents with the Lookheed Electra [2] in the 1960s. Following investigations culminated e.g. in the groundbreaking work by Houbolt and Reed [4], which gives a comprehensive method to calculate the (unsteady) motion-induced aerodynamic forces using an easy to use strip theory approach still in use today. Blands and Bennetts [5] following experimental work validated the theory of Houbolt and Reed with wind tunnel measurements of propeller coefficients and stability boundaries. The next step towards making this method a standard method in whirl flutter analysis was taken by Rodden and Rose [6] by demonstrating how to include the coefficients calculated by the Houbolt/Reed method into MSC Nastran finite element models. A Fortran program to calculate these derivatives is still being shipped with MSC Nastran. Cecrdle later showed several examples [7–9] of applying this theory in combination with Nastran and collected them in a comprehensive book [10].

The method and work described so far has mainly found application in the regime of turboprop aircraft stability analysis. A second stream of whirl flutter related work was induced by research connected to the development of tilt-rotor aircraft. Configurations like these feature large rotors usually on the wing-tips, making whirl flutter an important design risk. Due to their layout and control the rotors are much more flexible than turboprop propellers, so even early publications focus on modelling blade flexibility or hinged blades [11, 12]. In his technical report, Johnson [13] also used a separate analytical modelling of the rotor (using Equations of motion and a generic coupling via the hub DOF) to couple his rotor model with different pylon/wing models. Current investigations make use of so called rotorcraft comprehensive codes for numerical modelling of the complex rotor systems and their aerodynamics [14, 15]. These codes are specialized on the modelling of the rotor including the control system and usually allow for time domain analysis as well as the linearization of the equations of motion. The time domain models used here can feature complex aerodynamic models up to viscous CFD [16]. Comprehensive codes have also been applied to study aircraft [17, 18], but are mainly focused on helicopter and tilt-rotor applications.

Attempts have been made to bring the more sophisticated rotor modelling capabilities (especially with respect to aerodynamics) into frequency domain whirl analysis. In [19], a process to use unsteady vortex lattice method propeller models to identify coefficients like those in the Houbolt/Reed method using quasisteady sinusoidal pitch angle perturbations of the propeller hub is described. The authors in [20] compared the influence of different aerodynamic modelling on propeller whirl flutter, using the approach in [21] to extract reduced order models (ROM) for the unsteady rotor aerodynamics. Serafini et al. [22] and Gori et al. [23] describe an expanded approach, reducing a complex aeroelastic helicopter rotor model including nonlinear blade dynamics and panel aerodynamics in the time domain to a ROM using only the rotor hub DOF. They use time domain perturbations with different frequencies to sample the transfer function of the rotor hub forces with respect to hub motion in the frequency domain.

These frequency domain transfer functions are then approximated with rational functions to obtain a time domain state-space ROM, which can be used e.g. for control design.
The basic idea of [23] will be used and expanded in this paper to bring the accuracy of time domain propeller models into frequency domain aircraft flutter analysis and bridge the gap between accurate whirl analysis in time domain (e.g. using comprehensive codes) and full-aircraft flutter analysis.

2 METHODS

The main objective for whirl flutter calculations is describing a propeller's transfer behaviour from hub displacements to hub forces. These six motion-induced forces and torques are the main driver for whirl flutter. In case the whirl flutter assessment shall take place in the frequency domain (similar or in combination to classical flutter analysis), the problem comes down to finding a description of the propeller forces F_{prop} depending on the Laplace variable s, the hub motion X_{hub} and operational parameters such as the airspeed V, the rotational speed Ω and the air density ρ. This transfer behaviour can in general be nonlinear with regard to every of these dependencies, such as in Eq. 1

$$F_{prop}(s, X_{hub}, V, \Omega, \rho...) = \begin{pmatrix} F_x \\ F_y \\ F_z \\ M_x \\ M_y \\ M_z \end{pmatrix} \tag{1}$$

When only considering linear flutter stability, the nonlinear transfer function F_{prop} can be linearised with regard to the hub motion X_{hub}, making it only applicable for small perturbations ΔX_{hub} about a reference state:

$$F_{prop} = \underline{H}_{prop}(s, V, \Omega, \rho)\, \Delta X_{hub} \tag{2}$$

The frequency-dependant 6x6 matrix $\underline{H}_{prop}$ now describes the linear response of the propeller hub forces and moments F_{prop} to perturbations at the hub ΔX_{hub}. This description can then easily be inserted into the aeroelastic equations of motions (here in modal coordinates q using modal transformation with $\underline{\Phi}_{prop}$):

$$\underline{M}_{gen}\ddot{q} + \underline{K}_{gen}q = \left[\underline{\Phi}_{prop}^T \underline{H}_{prop}(s, V, \Omega, \rho)\underline{\Phi}_{prop} + \frac{\rho V^2}{2}\underline{Q}_{hh}(s)\right] q \tag{3}$$

M and K represent the mass and stiffness matrix of the underlying structure (wing, pylon, ...) and $\underline{\Phi}_{prop}$ is used to transform the hub coordinates X_{hub} to modal coordinates. Eq. 3 already includes unsteady aerodynamic forces from other aircraft components (such as wing, nacelle, etc.) using generalized aerodynamic force matrices $\underline{Q}_{hh}$. Solving Eq. 3 for its eigenvalues yields the frequency and damping of the aircraft eigenmodes depending on the operating point (ρ, V, Ω). From the damping the stability of the coupled system can be assessed and possible (whirl) flutter couplings be identified.

The following sections will show two methods to describe H_{prop}: The classical Houbolt/Reed method [4] and the transfer-matrix method, which is the main topic of this paper.
Before, a few assumptions on the propeller have to be made:

- the isolated propeller has to be stable (no blade flutter)
- the propeller has more than two blades to reduce/avoid periodicity

2.1 Houbolt/Reed method

Using the theory developed by Houbolt and Reed [4] the forces and torques due to perturbations in the propeller plane can be calculated by stiffness and damping terms as (c.f. e.g. [24]):

$$\frac{1}{q_\infty 2\pi R^3}\begin{pmatrix} F_y \\ F_z \\ M_y \\ M_z \end{pmatrix} = \underbrace{\begin{bmatrix} 0 & 0 & \frac{C_{y\theta}}{2R} & \frac{C_{y\psi}}{2R} \\ 0 & 0 & \frac{C_{z\theta}}{2R} & \frac{C_{z\psi}}{2R} \\ 0 & 0 & C_{m\theta} & C_{m\psi} \\ 0 & 0 & C_{n\theta} & C_{n\psi} \end{bmatrix}}_{\underline{K}_{prop}} \begin{pmatrix} y \\ z \\ \theta \\ \psi \end{pmatrix} + \underbrace{\begin{bmatrix} -\frac{C_{y\psi}}{2RV} & \frac{C_{y\theta}}{2RV} & \frac{C_{yq}}{2V} & \frac{C_{yr}}{2V} \\ -\frac{C_{z\psi}}{2RV} & \frac{C_{z\theta}}{2RV} & \frac{C_{zq}}{2V} & \frac{C_{zr}}{2V} \\ -\frac{C_{m\psi}}{V} & \frac{C_{m\theta}}{V} & \frac{C_{mq}R}{V} & \frac{C_{mr}R}{V} \\ -\frac{C_{n\psi}}{V} & \frac{C_{n\theta}}{V} & \frac{C_{nq}R}{V} & \frac{C_{nr}R}{V} \end{bmatrix}}_{\underline{D}_{prop}} \begin{pmatrix} \dot{y} \\ \dot{z} \\ \dot{\theta} \\ \dot{\psi} \end{pmatrix} \tag{4}$$

Equation 4 gives therefore a linear description of F_{prop} using stiffness ($\underline{K}_{prop}$) and damping ($\underline{D}_{prop}$) terms.

$$F_{prop}(X, s, V, \Omega) \;=\; \underbrace{\left[\,\underline{K}_{prop}(V,\Omega) \;+\; s\underline{D}_{prop}(V,\Omega)\right]}_{\underline{H}_{prop}}\; X \tag{5}$$

Perturbational force and torque about the propeller axis (X) are neglected in Eq. 4 due to their insignificant contribution to whirl flutter and only perturbation in the propeller plane (y and z translational and θ and ψ angular displacement) considered. The stiffness and damping terms are expressed in coefficient form. [4, 6] give equations for their analytical computation. Not all coefficients are of the same order, some are even considered neglectable [6].

The computation of the coefficients using the method in [4,6] involves a few crucial assumptions (primarily to allow their analytical derivation):

- propeller blades are modelled as rigid
- only perturbational forces are considered, no effect of the steady state is included
- no inflow (e.g. due to thrust) is considered
- aerodynamic forces are computed using a quasisteady strip theory, unsteady aerodynamic effects are only included afterwards using a correction with the local Theodorsen function, but only including the rotational velocity Ω into the reduced frequency

To overcome these assumption, a more sophisticated method to describe F_{prop} is necessary. Of course other analytical descriptions are available in literature (e.g. [13]), but those have other shortcomings and are more suitable e.g. for tilt-rotor applications.

2.2 TM-method

The following subsection will present a more general way of describing and obtaining H_{prop}, called the "Transfer-Matrix-method" (short: "TM-method").

The basic idea is twofold: To replace the analytically derived stiffness and damping matrices by a proper frequency-dependant description $\underline{H}_{prop}(s)$ and to identify this transfer function from time domain rotor models (based on the work in [23]).

The main steps involved in the TM-method are the following:

1. Computation of a time domain steady state of the isolated propeller
2. Perturbation of the steady state using hub motion (e.g. pitch or yaw angle perturbation)
3. Recording of the time response of the propeller hub forces
4. Transformation of those time histories to frequency domain and sampling of the transfer functions from hub motion to forces
5. Coupling with the pylon via a hub node and Eq. 3

One of the main assumptions (next to the linearisation in Eq. 2) is that we can only identify the propeller forces on the imaginary axis, where $s = i\omega$. This assumption is similar to the classical approach in aircraft flutter analysis, where unsteady aerodynamic forces are only computed for zero damping. Technically the results of the eigenvalue analysis of Eq. 3 will only be correct for zero damping (which is fine for stability), but several methods exist in literature to correct for this shortcoming (e.g. [25], [26]). The authors in [23] utilize a complex RMA procedure to approximate H_{prop} with a rational matrix approximation and transform them back into time domain. This is necessary in the context of [23] but not needed in flutter analysis, which is conducted in the frequency domain.

A big advantage of the TM-method is that $\underline{H}_{prop}(i\omega)$ does not have to be known directly in the frequency domain (it would be complex to derive), but can be identified from time domain simulations. Time domain simulations of isolated rotors are state of the art and range from low-fidelity methods [27] to complex high-fidelity CFD-MBS simulations [28].
[23] demonstrates how a frequency-dependant transfer function for a helicopter rotor can be obtained using several harmonic perturbations. For this, the isolated rotor is perturbed with small harmonic motions at its hub about a steady reference position. The steady state of the hub forces and moments $F_{hub,steady}$ is subtracted from the time history $F_{hub,perturb}$ during the perturbation. The result $\Delta F_{hub,perturb}$ can then be transformed into frequency domain and a transfer function $F_{prop}(i\omega)$ between the input (i.e. the perturbation $\Delta X_{hub,perturb}$) and output spectra can be obtained by division:

$$\underline{H}_{prop}(i\omega) = \frac{\mathcal{F}(\Delta F_{hub,perturb}(t))}{\mathcal{F}(\Delta X_{hub,perturb}(t))} \qquad (6)$$

The perturbation is performed about one DOF at the rotor hub. To capture the full transfer behaviour, all six hub DOF need to be perturbed independently. Each perturbation yields six transfer functions (six force components comprising Fx, Fx, Fz, Mx, My, Mz with regard to the perturbed DOF). Using harmonic hub perturbations has a few downsides though:

- To capture the full spectrum in a certain frequency range, the response to several different perturbations $\Delta X_{hub,perturb}$ has to be calculated and increases the computational cost.

- Because only the response to the harmonic motion is of interest, the transient response in the beginning of the perturbation has to decay first (which can take a while depending on the damping levels) ad requires additional computational time.
- To avoid spectral leakage and aliasing, one has to make sure to capture a whole-number of perturbation periods and to sample the output adequately. This needs several perturbation periods, which increases the computational time especially for low perturbation frequencies.

The benefit of harmonic perturbation is that it can also be applied to systems which are not time-invariant, such as rotors in non-axial flow. This is described in detail in [23]. The turboprop-propellers considered in his work operate usually in (almost) axial flow, making this feature not so relevant.

the frequency-dependant transfer matrix $\underline{H}_{prop}$ can be assembled from several perturbations (n_ω times six hub DOF), describing the transfer behaviour of the propeller hub forces with regard to perturbations at the hub. Each entry in the matrix (cf. Eq. 7) represents a scalar transfer spectrum. For example, $M_{z,\theta}$ would be the yaw moment response due to a pitch angle perturbation.

$$F_{prop} = \begin{pmatrix} F_x \\ F_y \\ F_z \\ M_x \\ M_y \\ M_z \end{pmatrix} = \underbrace{\begin{Bmatrix} F_{xx} & F_{xy} & F_{xz} & F_{x\phi} & F_{x\theta} & F_{x\psi} \\ F_{yx} & F_{yy} & F_{yz} & F_{y\phi} & F_{y\theta} & F_{y\psi} \\ F_{zx} & F_{zy} & F_{xz} & F_{z\phi} & F_{z\theta} & F_{z\psi} \\ M_{xx} & M_{xy} & M_{xz} & M_{x\phi} & M_{x\theta} & M_{x\psi} \\ M_{yx} & M_{yy} & M_{yz} & M_{y\phi} & M_{y\theta} & M_{y\psi} \\ M_{zx} & M_{zy} & M_{xz} & M_{z\phi} & M_{z\theta} & M_{z\psi} \end{Bmatrix}}_{\underline{H}_{prop}} \begin{pmatrix} x \\ y \\ z \\ \phi \\ \theta \\ \psi \end{pmatrix} \tag{7}$$

For rotors or propellers in axial flow, a few simplifications can be implemented to drastically save computational time. First and foremost, because the system is time-invariant and considered linear with regard to small perturbations, a pulse or step perturbation can be used for $\Delta X_{hub,perturb}$, perturbing a broad frequency band at once and allowing the identification of the full spectrum in the frequency band of interest with one single time simulation. Because this identification method uses the transient response and can be adapted quite easily to yield a lot of frequency samples in the band of interest, it is very well suited for the identification of such transfer functions [29].
A second simplification for propellers in axial flow uses the axial symmetry to mirror the transfer functions identified about the y-axis (y-displacement and pitch angle) to those about the z-axis (z-displacement and yaw angle), further cutting down on the computational time.
A third simplification can be used if the structural model below the propeller does not have any DOF in the axial direction (or this direction shall/can be neglected). This allows skipping the perturbations about the global x-axis.

Solution of the equation of motion

Eq. 3 already stated the equations of motions including a generalized (frequency-dependant) transfer function H_{prop} for the propeller forces. Due to the frequency-dependant character, a non-linear solution method for finding the eigenvalues is necessary. Because this is a common problem in aircraft flutter analysis (Q_{hh} in Eq. 3 is also frequency-dependant) several solution methods exist in literature.

- The most well-known method is the iteration-based pk-method [30] and its derivatives such as the g-method [25]. In these methods a starting-frequency is picked, H_{prop} is evaluated at this frequency and the eigenvalues for this particular point are evaluated. The resulting eigenfrequency is then compared with the initial value and an iteration procedure is used to find a converged solution. Advantages of this method are that it is fast, robust and already well known in the field of aircraft flutter analysis. Disadvantages are the necessity for mode-tracking inside the iteration loops (which can get confused by additional poles in H_{prop}, c.f. section 4.1) and that the damping solution outside the actual flutter point can be of (because $H_{prop}(s)$ is approximated by $H_{prop}(i\omega)$). The latter can be compensated by using the g-method, which approximates the behaviour in the Laplace domain using a Taylor expansion (c.f. [25]).
- By reshaping Eq. 3 to a frequency domain transfer function of the coupled system comprising of propeller and underlying structure one can evaluate the transfer spectra of the coupled system on the imaginary axis [31]. These spectra are exact (because $H_{prop}(i\omega)$ is known exactly) and identification or pole-fitting routines (such as vectorfitting [32]) can be used to extract the eigenvalues of the coupled system. The advantages of using this solver (in this paper called "coupled Frequency Response Function" or cFRF-Solver) are that the damping approximation is much better compared to the pk-method (c.f. section 4.2) and it allows for finding poles that are hidden inside $H_{prop}(s)$, for example modes of the flexible propeller blades. Disadvantages are the lack of eigenvector-entries of these DOF, therefore a missing mode-tracking (at least when using vectorfitting) and problems with accuracy for highly damped or closely spaced poles.
- Gori et al. [23] use a rational matrix approximation (RMA) to convert the frequency-dependant $H_{prop}(i\omega)$ into state-space form, hence allowing a direct coupling and the usage of a standard eigenvalue-solver on the coupled state-space-form. Although this makes the eigenvalue solution itself simpler it shifts the complexity to finding a good and robust RMA. This solution sequence is not used in this paper.

Comparing the TM-method to others such as the Houbolt/Reed method or coupled time domain models shows some advantages, making it a very useful tool in aircraft (whirl) flutter analysis:

- The transfer functions H_{prop} includes the full accuracy with respect to physical modelling details of the time domain solver used its identification.
- The character of $H_{prop}(i\omega)$ as a frequency-dependant transfer function makes it suitable for most existing aircraft flutter analysis workflows.
- Because the transfer matrices H_{prop} are identified in physical (hub) coordinates and transformed later into the modal domain it is very convenient to use them with different structural models, reducing computational effort when conducting larger parameter studies on the structural model (payload-fuel cases, sensitivies, ...).

- Already existing (and tuned) aircraft flutter models can be altered with a more sophisticated propeller description (instead of remodelling the whole aircraft in a time domain code such as MBS).

2.3 Time domain simulation of the coupled system

A second option besides evaluating the eigenvalues of eq. 3 for assessing the coupled system's whirl flutter stability comprise direct time domain simulations. For this approach, not only the isolated propeller but the full system including the structure below (engine, pylon, wing) needs to be modelled together, e.g. using Multi-Body-Systems. This coupled model can then be perturbed about a reference state and the extracted perturbations evaluated e.g. using Floquet-Theory [33, 34]. This yields frequency and damping curves directly from the coupled system response, so without splitting the model or using frequency domain approximations, making it a good reference solution for comparison. It will be used for this purpose later e.g. in section 4.1. A clear disadvantage is the high computational cost, especially for complex models with many states. The postprocessing gets more complicated in case the model is not fully state-space-based (e.g. with panel or CFD-aerodynamics). Also, every system has to be fully evaluated at every operating point, making it very costly for parameter studies.

After the first part, which presented the theory used in this paper, the second part will demonstrate the usage of the TM-method in a simple whirl flutter model and present some verification studies and examples. It will start with describing the actual simulation models and will continue with the result section giving examples for actual transfer spectra, frequency and damping plots as well as parameter studies.

3 MODELS

To demonstrate the usage of the TM-method, a simple two-degree-of-freedom structural model will be coupled with a propeller model. The structural model as well as the geometry of the propeller and its operating points have been described in [24] as "simple model". The nominal rotor speed is at 23 Hz or 1380 rpm and the nominal airspeed is set to 100 m/s.

To use the TM-method for describing the propeller transfer behaviour, a time domain model of the isolated propeller is necessary. In this case, the MBS-Code Simpack [35] has been used to model the three propeller blades. An unsteady aerodynamic strip theory using the Wagner-function is applied using User-Force-Elements [27]. This modelling approach is very close to the Theodorsen-corrected strip approach in the Houbolt/Reed theory, which can therefore be used as a verification. The TM-method itself allows for much more sophisticated aerodynamic models, but this is out of the scope of this paper.

Fig. 1 shows the Simpack model of the isolated propeller on the left-hand side. The model contains eight equidistantly spaced aerodynamic strips per blade. Depending on the configuration the blades are either modelled as rigid massless bodies (to get the aerodynamic transfer function only) or as bodies with mass and a flapping-hinge at the root to resemble an elastic blade model. On the right-hand side of Fig. 1 the full structural model used for time domain-validation is shown. The propeller model is the same as for the isolated propeller, the simple two DOF pylon structure is added to the model using rotational joints and a point mass in the hub. The characteristics of this pylon are added to the transfer matrices using the approach in Eq. 3.

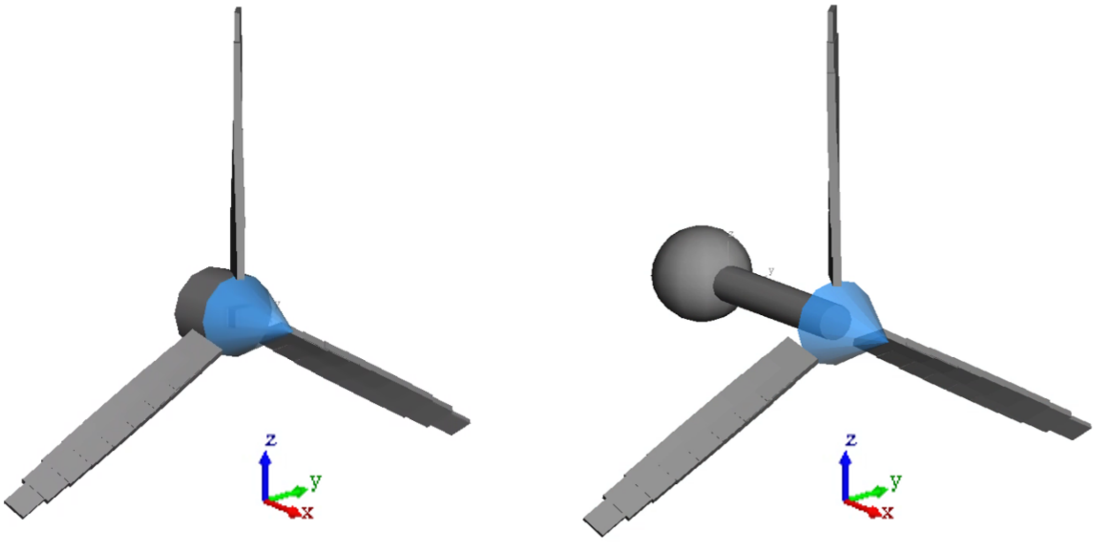


Figure 1: Left: Isolated propeller model in MBS Simpack, Right: Coupled pylon and propeller model for validation purposes

4 RESULTS

The propeller showed in section 3 will now be used to demonstrate the usage of the proposed TM-method and compare the results to the classical Houbolt/Reed method. First this will be done on transfer-function-level, comparing F_{prop} from different methods and models. Coupled stability results will be used to further validate the TM-method by comparing frequency and damping curves with coupled time domain results. The last subsection will then use the TM-method for parameter studies to show applications to models that surpass the limits of the classical Houbolt/Reed method.

4.1 Transfer matrices

The core of frequency domain whirl computations and the basic idea of the TM-method is a (frequency-dependant) transfer function projecting hub motion to propeller hub loads. As pointed out in section 2, the classical Houbolt/Reed method defines those using stiffness and damping terms (c.f. Eq. 5). The TM-method uses identification on time domain models to identify the transfer functions. To compare both approaches, a very simple propeller model is used to resemble the assumptions of the Houbolt/Reed method (rigid blades, no inflow, strip aerodynamics using Theodorsens function). For this model the frequency domain response to pitch angle perturbation is compared. Fig. 2 shows the results for this transfer function, comparing the Houbolt/Reed method (dashed lines) with the identified transfer functions (solid lines). Markers indicate the response component, the diamond e.g. representing the force in z-direction due to pitch angle perturbation $F_{z\theta}$. The transfer functions are complex and frequency-dependant, so they are plotted for different frequencies on the x-axis and split into amplitude and phase.

Analysing the general trends of the lines in Fig. 2 it can be seen that two components ($F_{z\theta}$ and $M_{z\theta}$) are almost independent of frequency and therefore mainly dominated by stiffness contributions. Both are negative stiffness terms (phase is at 180 deg). The other two components are mainly damping-dominated but with (small) stiffness contributions. In general, a good agreement between the Houbolt/Reed-curves and the identified spectra can be observed when comparing the stiffness at zero frequency and the damping (gradient at higher frequencies).

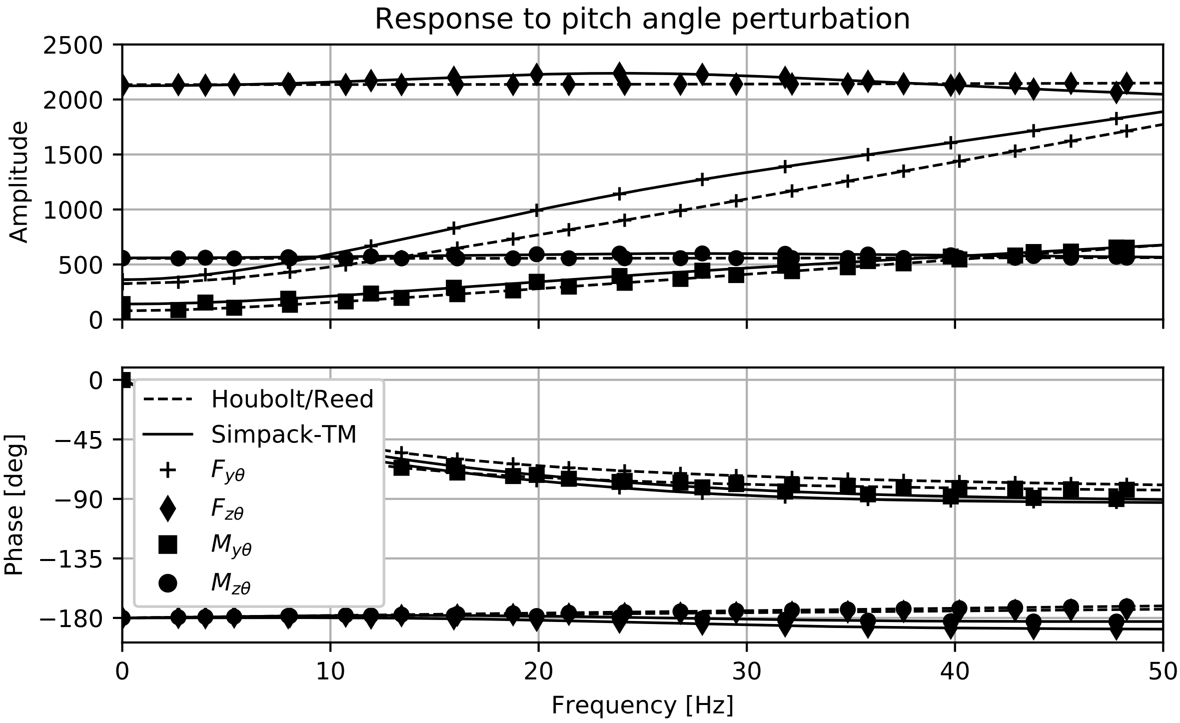


Figure 2: Plot of complex transfer spectra from propeller pitch motion to hub forces with a comparison between Houbolt/Reed method and MBS Simpack propeller model with rigid blades at nominal speed.

This was expected, as the propeller model used for identification resembles the assumptions of the Houbolt/Reed method. Still, differences can be observed, especially in the force-terms ($F_{y\theta}$ and $F_{z\theta}$). These differences are largest around 23 Hz, which is the nominal rotor speed of the propeller. This can be explained by a simplification in the Houbolt/Reed method: Only the rotational speed is used for the reduced frequency and $\Omega \pm \omega \approx \Omega$ is assumed. The TM-method does not make this assumption. If the unsteady aerodynamic model was more complex (e.g. taking dynamic inflow or dynamic stall into account) the TM-method would capture that, while the classical method is limited to the Theodorsen correction.

Fig. 3 shows a more complex example of a frequency domain response spectrum. The propeller is the same as described in section 3, but in contrast to Fig. 2 the blades are hinged at the root (tuned with a spring to a non-rotating flap eigenfrequency at 30 Hz). The transfer spectra therefore show not only the aerodynamic, but also the dynamic response of the propeller to pitch perturbation. This response can clearly be seen at the resonance peaks of the global propeller cyclic modes at 16 Hz and 62 Hz ($\omega_{blade,rot} \pm \Omega$). In this case, the torque terms ($M_{y\theta}$ and $M_{z\theta}$) show a stronger amplification than the force terms. Also, the phase of the transfer functions jumps when crossing one of the resonance peaks, which already gives a hint on a completely different stability behaviour.
Also, Fig. 3 compares the two excitation methods for computing the spectra. While the solid lines represent the spectra identified using one single pulse excitation (with 1100 sample points up to 90 Hz), the markers represent samples identified using harmonic excitation. Both match perfectly well in amplitude and phase, verifying the pulse-approach for propeller applications.

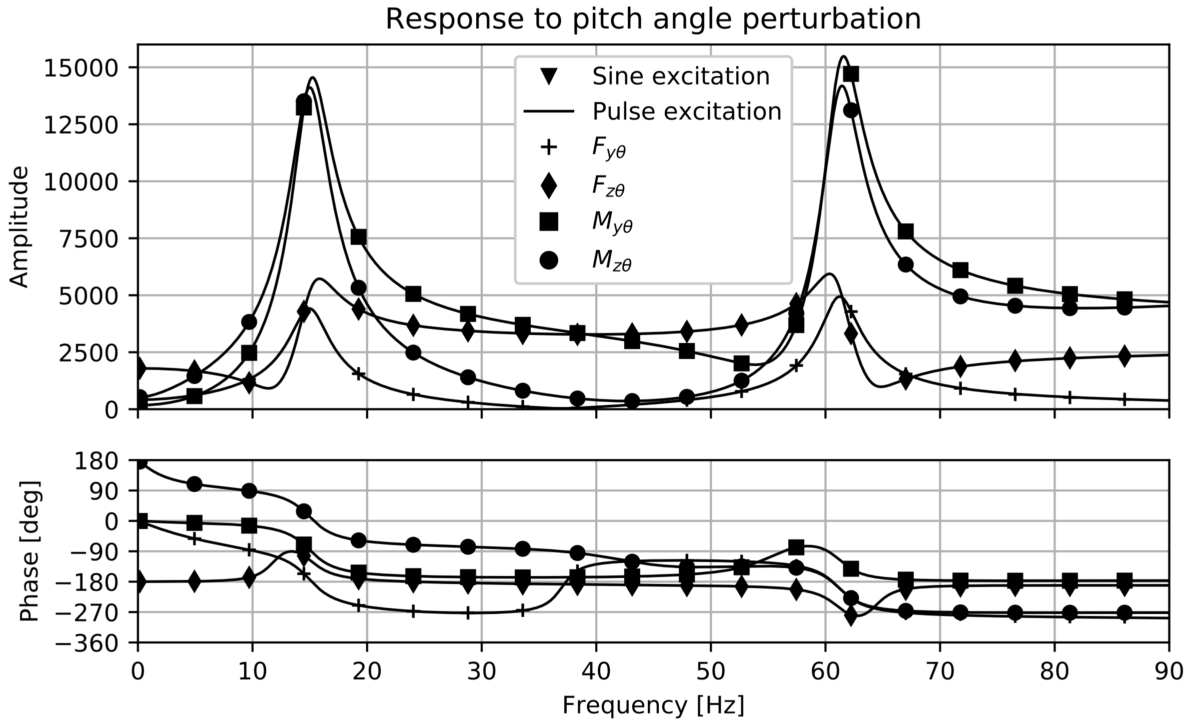


Figure 3: Plot of complex transfer spectra from propeller pitch motion to hub forces with for the MBS Simpack propeller model with flapping blades at nominal rotor speed.

4.2 Frequency and damping solutions

After examples for transfer functions have been reviewed, coupled stability results using the TM-method will be compared to the time domain reference solution. Results using different solution approaches (pk-method and system identification, c.f. section 2) will be compared.

Fig. 4 depicts the frequency and damping results of the simple two DOF model shown in Fig. 1. The left plot gives the two eigenfrequencies of the system (low-frequency backward whirl and high-frequency forward whirl) over a variation of rotational speeds. The advance ratio $V/\Omega R$ is kept constant so that the propeller stays in a wind-milling point and the velocity changes accordingly. Transfer matrices have to be calculated for each operating point separately, due to the dependency on both rotational speed and airspeed. The plot to the right shows the corresponding damping trends. The instability at 70 rad/s is a classical backward whirl flutter, the forward whirl increases in damping over the whole domain. Fig. 4 plots the stability results using three different solution methods: Time domain reference (markers) vs. TM-method using system identification (cFRF, solid line) and pk-method (dashed line). The reference solution is computed using the model coupled in MBS (shown on the right in Fig. 1) and frequency and damping are evaluated using Floquet-theory. For the TM-results, only the propeller itself is modelled in MBS and coupled with the structure using stiffness and mass matrices as in Eq. 3. When comparing them to the reference solution some similarities and also differences emerge. At the flutter point, all three methods predict the same frequency and damping. The TM-method can therefore exactly predict the flutter point of the reference solution. In the flutter point the pole lies on the imaginary axis, so the assumption of $H_{prop}(s) \approx H_{prop}(i\omega)$ is exact. Outside the flutter point small deviations can be observed, larger in the dashed line representing the pk-solution. The deviation is mainly in damping and larger for higher absolute damping values, where the poles move away from the imaginary axis and the approximation gets worse. Still, the values are close to the reference solution, especially using the cFRF solver.

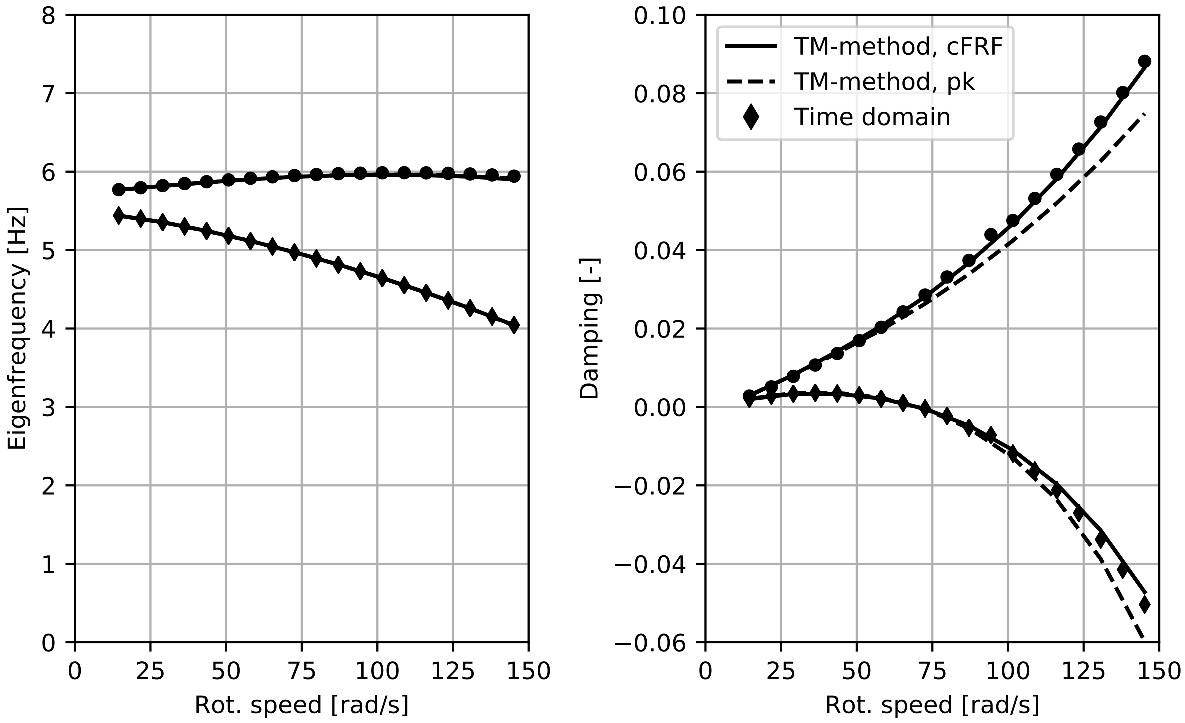
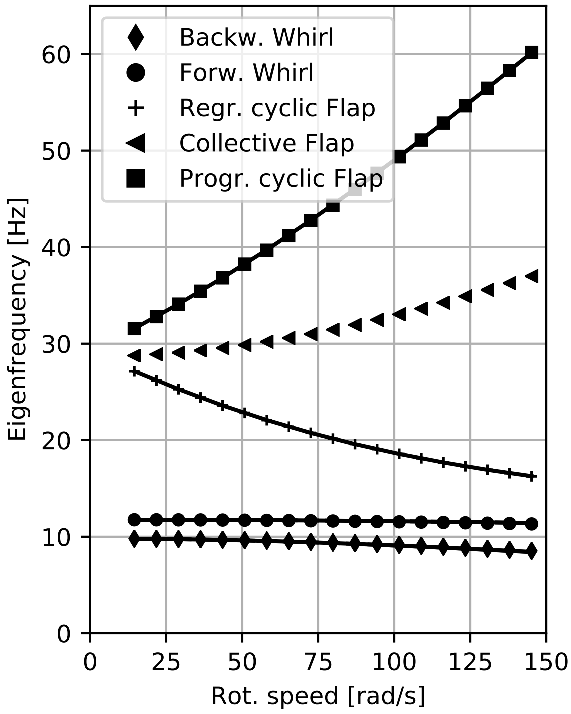


Figure 4: Comparison of frequency and damping results between different solution methods for the simple model with rigid blades.

Deviations found in the pk-solution can become larger, if the TM-spectra become more complex. Fig. 5 shows the frequency and damping plots for the simple model and a propeller with flap hinges. Now the solution shows five different modes: forward and backward whirl and the three global rotor modes emerging as a fan at 30 Hz for $\Omega = 0$. The middle of these three lines is the collective mode, where all blades flap with the same phase angle. The two outer lines represent the cyclic modes, where the blades flap with $120\,\mathrm{deg}$ phase angle, resulting in a forward and backward whirl motion of the propeller plane due to blade flapping. As in Fig. 4, the reference time domain solution using Floquet-theory on the coupled model is plotted using markers. The TM-solution using the pk-method is plotted in a dashed line. In this case, the deviation from the damping solution is larger because $H_{prop}(s)$ is much more complex due to the blade flapping poles (c.f. Fig. 3). Also, the pk-solution can only find those poles that are explicitly modelled as DOF in the equation of motion (namely the two whirl modes). The flap modes are hidden in the spectra of H_{prop}. This is were the system identification on the coupled system frequency response function (cFRF) excels, because it does not predict only the damping behaviour much better (because the coupled FRF can be evaluated exactly for $s = i\omega$), but also those flap modes that couple with the underlying pylon structure (the cyclic modes) can be identified, too. Only the collective flap mode cannot be found using the TM-method here, because the underlying system has DOF only in the propeller plane (pitch and yaw), but no axial motion. Therefore there is no dynamic coupling with the collective mode, which would require a fore-aft motion in one of the modes. The frequency and damping prediction is very close to the time domain reference, even for the propeller flap modes.



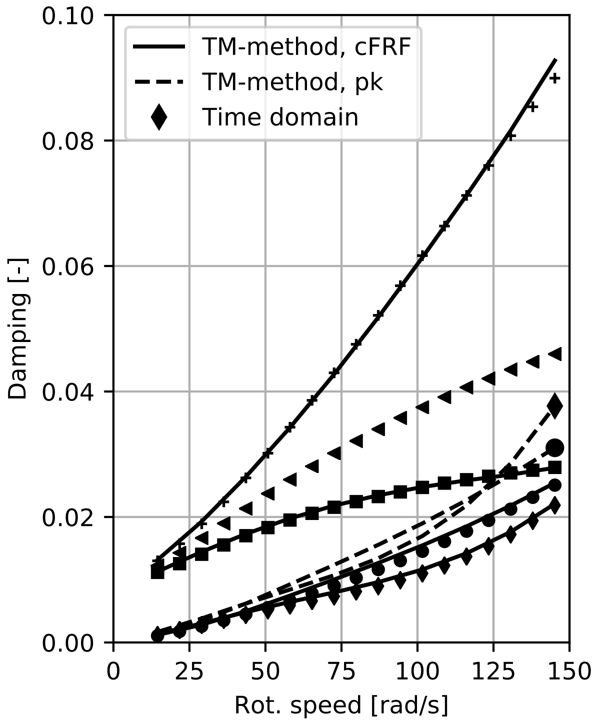


Figure 5: Comparison of frequency and damping results between different solution methods for the simple model with flapping blades.

4.3 Parameter studies

After section 4.2 showed coupled stability results for varying operating points and compared time domain and TM-results, the next section will make use of an advantage of the TM-method and conduct parameter studies on the underlying structure. For this the TM-method is very advantageous because the transfer matrices are calculated in physical coordinates (x, y, z) and transformed to modal coordinates after the identification. Therefore, they can be reused when changing the structural model, making the method very suitable for parameter studies.

Fig. 6 presents the results of such a study on the pitch and yaw stiffness of the simple two DOF model from section 3. The x- and y-axis represent the uncoupled pitch and yaw frequency as a measure for the stiffnesses. The top right area is stable, the bottom left parameter range under the bell curve marks the domain of whirl flutter. Plots like these are very useful to demonstrate the influence of certain parameters on the overall stability of the system, as changes of this parameter enlarge or shrink the bell curve and therefore the range of instability (c.f. [24] for general parameter dependencies). Fig. 6 now compares the stability maps computed with three different methods (Houbolt/Reed, time domain, TM-method) on the same structural model. Again, TM-method and time domain results match very closely. Because in this case only the stability is compared (and not the actual damping level) the pk-solver is used for the TM-results. It has to be mentioned in this case that an evaluation of the coupled system stability in MBS using Floquet requires time domain simulations for every point in the stability map to be evaluated, while the TM-method only requires the perturbation of the isolated propeller for pitch and y-displacement once (c.f. 2). The TM-solution is therefore much faster with regard to computational time. Comparing the stability map using the TM-method with the map computed with the classical Houbolt/Reed method shows a similar result, but the classical method gives a slightly larger instability range (and therefore conservative results). The differences between the two methods have already been explained in the previous description of Fig. 2.

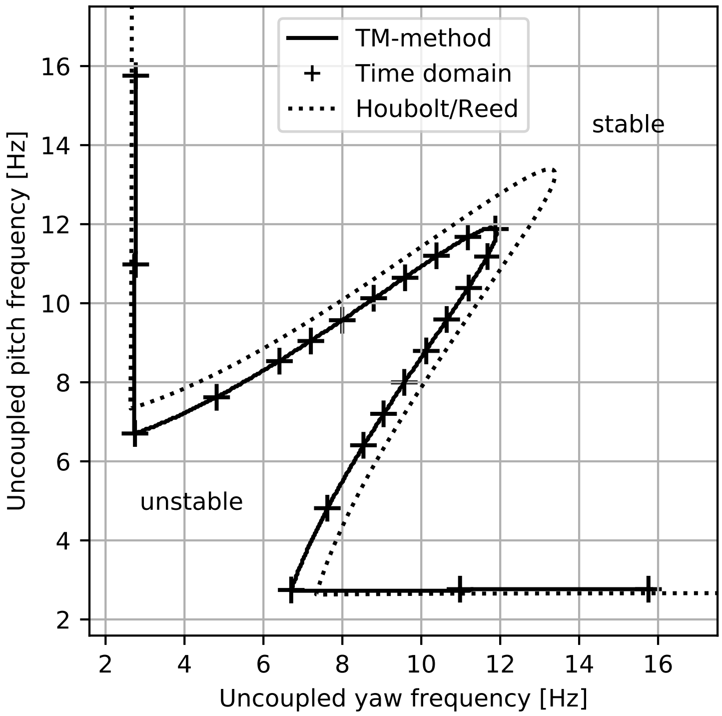


Figure 6: Comparison of whirl flutter stability map between different methods.

One of the benefits of the TM-method compared to the classical method is the ability to account for the influence of steady aerodynamic forces on the stability. Because the TM-spectra also include effects like the tilting of the steady lift and torque vector (and eventual inflow effects if included in the aerodynamic model), the effect of propeller thrust can be studied. Fig. 7 shows again a stability map for the simple model, this time comparing only results from the TM-method, but with two different thrust settings. The zero degree blade incidence curve represents the reference with no thrust and is the same result as in Fig. 6. When increasing the blade incidence angle to 5 degrees, the propeller yields forward thrust. In this case, the whirl flutter stability is affected in a stabilizing manner. This might change for a propeller in pusher configuration [24]. It has to be noted though that the aerodynamic method used in the MBS-model is not suited to properly model a propeller in thrust conditions, because linear airfoil aerodynamics and no inflow is assumed. Still, Fig. 7 demonstrates that the TM-method is able to capture eventual thrust effects.

Fig. 3 already showed the impact of a blade flapping DOF on the TM-spectra, Fig. 5 demonstrated and validated the use of these spectra for stability evaluations. In addition, Fig. 8 shows the influence of blade flapping frequency on the whirl flutter stability map of the simple two DOF system. For this, three different flapping frequencies have been evaluated and compared to the rigid blade result from Fig. 6. The comparison shows a massive stabilisation of the whirl system due to rigid blade flapping (which was already shown in the past, c.f. [12]). Even for high blade flapping frequencies far away from the actual whirl modes the influence is significant. Although a simple flap hinge is only a very simplified modelling of blade flexibility, Fig. 4 and 8 demonstrate the applicability of the TM-method to such problems. Increasing the modelling depth is now a problem of MBS-modelling, the TM-method then allows to transfer this modelling depth into the flutter evaluation.

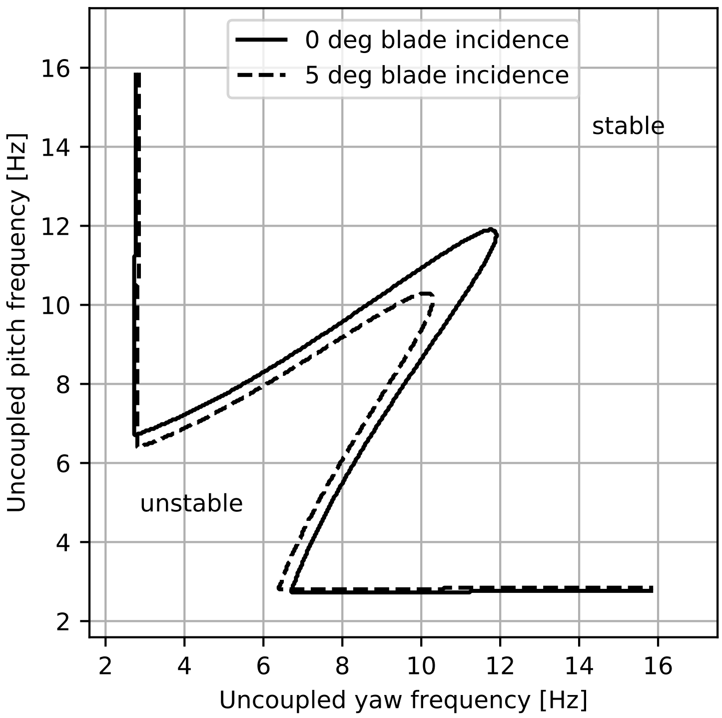


Figure 7: Effect of thrust (due to change in blade incidence angle) on the whirl flutter stability map.

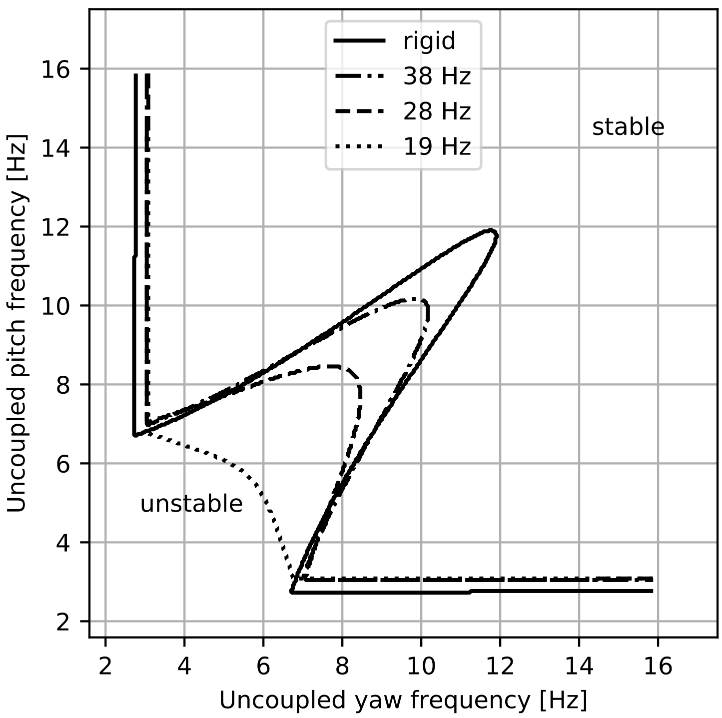


Figure 8: Effect of different blade flapping stiffnesses on the whirl flutter stability map.

5 CONCLUSION AND OUTLOOK

This paper dealt with demonstrating a method to allow for an increase in fidelity level in the frequency domain whirl flutter assessment of turboprop aircraft. The classical approach for this task is the Houbolt/Reed method, which calculates the motion-induced forces and torques caused by the propeller using analytically derived derivatives. To overcome the limitations of this analytical theory, the transfer matrix (TM) method is introduced. It replaces the description of the hub forces by frequency-dependant transfer functions identified from time domain simulations of the isolated propeller. The accuracy and modelling depth of the time domain propeller model is captured and transferred into the frequency domain flutter evaluation using the 6x6 transfer matrix projecting hub motion to hub forces. With this approach effects like complex time domain propeller aerodynamics and blade elasticity can be used in the flutter assessment without the need to model them explicitly in frequency domain. After presenting the theory and approach, example applications to a simple two DOF whirl model are shown and compared to the classical methods as well as time domain reference solutions. While the results of the frequency domain evaluations matched the time domain results very well (which validated the method), some differences with the classical theory were observed. These differences can be explained by the assumptions in the Houbolt/Reed theory. The classical theory proved to be conservative in this case though. As a conclusion, it can be said that the TM-method proved itself as a potent replacement for the classical Houbolt/Reed theory.

The models used in this paper for the examples were fairly simple though with respect to blade aerodynamics and dynamic modelling. The important message should be the demonstration of the method itself, which has almost no limitations with regard to modelling depth of the propeller model. Future work should therefore make use of this method to investigate the influence of propeller modelling on whirl flutter stability. This paper and literature already demonstrated the impact of including propeller blade elasticity on whirl flutter. This should be investigated further including the effect of propeller twist and proper blade modelling as elastic bodies. Also, the effect of thrust was briefly investigated here, but using a fairly simple aerodynamic model. To investigate this further and also answer the question for the necessary modelling depth the impact of more complex aerodynamic methods should be investigated. The TM-method gives the ideal framework to do (and even combine) all these comparisons in a structured manner. The comparisons so far have been conducted using a simple two DOF pylon model, but extension to full aircraft models (with e.g. wing aerodynamics) is straightforward and already prepared in this paper. An open questions remaining is how big the effect of (unsteady) aerodynamic interference between propeller and wing on whirl flutter stability is and how to combine it with the TM-method.

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