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**Optimization of the thermoelectric properties of p-type  $\text{Mg}_{2-y}\text{Li}_y\text{Ge}_{1-x}\text{Sn}_x$  and  $\text{Mg}_{2-y}\text{Li}_y\text{Ge}_{1-z}\text{Si}_z$  with  $x, z = 0.1$  and  $0.2$ .**

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## **Abstract**

According to recent investigations on p-type  $\text{Mg}_2X$  ( $X = \text{Si}, \text{Sn}, \text{Ge}$ ), p-type  $\text{Mg}_2\text{Ge}$  is found to be far superior to the p-type binaries while having thermoelectric properties comparable to the best solid solutions of p-type  $\text{Mg}_2(\text{Si}, \text{Sn})$ . The unexpectedly good properties are supposedly due to a non-rigid band structure with a temperature dependent interband separation. Further optimization can be expected by alloying Si or Sn into the Ge site to lower the thermal conductivity thereby increasing the figure of merit. Here, solid solutions of p-type  $\text{Mg}_2\text{Ge}_{1-x}\text{Sn}_x$  and p-type  $\text{Mg}_2\text{Ge}_{1-z}\text{Si}_z$  with  $x, z = 0.1$  and  $0.2$  are successfully synthesized via ball milling. The thermal conductivities

are significantly reduced throughout the whole temperature range by around 30% due to the alloying effect. The electronic properties of all Ge-rich samples show similar temperature dependent behavior as p-type  $\text{Mg}_2\text{Ge}$ . For the  $\text{Mg}_2(\text{Ge},\text{Sn})$  systems, a compensation of reduced thermal conductivity and decreased carrier mobility are found, leading to  $zT$  values comparable to p-type  $\text{Mg}_2\text{Ge}$ . Whereas for the Si containing samples, a thermoelectric figure of merit  $zT$  of  $0.49 \pm 0.07$  at 675 K for  $z = 0.1$  is achieved, the highest reported so far for the p-type  $\text{Mg}_2(\text{Ge},\text{Si})$  systems and a  $zT_{\text{avg}}$  which is 30% higher than that of binary  $\text{Mg}_2\text{Ge}$ .

Keywords: Synthesis, ball milling, p-type  $\text{Mg}_2\text{Ge}_{1-z}\text{Si}_z$  solid solutions, p-type  $\text{Mg}_2\text{Ge}_{1-x}\text{Sn}_x$  solid solutions, thermoelectric properties, microstructure

## 1. Introduction

In the present day and time, global climate change has become a topic spoken in every corner of the world. Due to the increasingly alarming impacts of fossil fuels on the environment, a significant reduction of greenhouse gases defines the energy challenge in the current world. Apart from direct approaches involving harvesting wind and solar energy, strategies to improve energy usage efficiency are also being developed, including the recovery and direct conversion of waste heat to electrical energy using thermoelectric (TE) materials. The advantages of thermoelectric modules over other heat recovery technologies include the absence of moving parts, high reliability, and low maintenance costs <sup>1</sup>.

Thermoelectric performance is characterized by the dimensionless figure of merit,  $zT = \frac{S^2 \sigma T}{\kappa}$  where  $S$  is the Seebeck coefficient,  $\sigma$  is the electrical conductivity,  $\kappa$  is the thermal conductivity and  $T$  is the temperature. The figure of merit  $zT$  can be optimized by increasing the thermoelectric

power factor ( $S^2\sigma$ ) and decreasing  $\kappa$ . The TE power factor can be maximized by an optimization of the carrier concentration ( $p$ ) (linked to both  $S$  and  $\sigma$ )<sup>2</sup> and advanced strategies like band structure engineering<sup>3-6</sup>. In contrast, the thermal conductivity can be brought down by strategies such as alloying<sup>4, 7-8</sup>.

Among various thermoelectric materials<sup>8-13</sup>,  $\text{Mg}_2X$  ( $X = \text{Si, Sn, and Ge}$ ) and its solid solutions have attracted attention due to their non-toxicity, material abundance, and good environmental compatibility. The n-type  $\text{Mg}_2(\text{Si, Ge, Sn})$  materials are known for their excellent thermoelectric properties<sup>4, 7, 14-15</sup>, while the corresponding p-type is found to have inferior properties<sup>16-19</sup>. This is mainly due to the less favorable properties of the valence bands compared to the conduction bands<sup>16-17, 20</sup>. However, to produce efficient thermoelectric modules, it is important to optimize both p-type and n-type materials. Recent systematic investigations identify the optimum properties for p-type  $\text{Mg}_2(\text{Si, Sn})$  with  $zT \approx 0.5$  at 700 K and further improvement, e.g., by refined synthesis approaches is challenging<sup>16, 21</sup>.

Recently, p-type  $\text{Mg}_2\text{Ge}$  was found to have  $zT$  values comparable to optimized p-type  $\text{Mg}_2\text{Si}_{1-x}\text{Sn}_x$  ( $zT \approx 0.5$ )<sup>20</sup> and the highest  $zT$  among the p-type binaries reported in the literature<sup>16, 22-24</sup>. The better thermoelectric properties are attributed to a non-rigid band structure with a temperature dependent interband separation. The second band contributes progressively to the transport properties as the temperature increases thus leading to a higher power factor compared to p-type  $\text{Mg}_2\text{Si}$  and  $\text{Mg}_2\text{Sn}$ <sup>20</sup>. Further improvement of the properties is plausible by alloying and forming  $\text{Mg}_2\text{Ge}$  based solid solutions for a reduction of lattice thermal conductivity similar to  $\text{Mg}_2(\text{Si, Sn})$ <sup>16</sup>.

Here, in this study, the syntheses of p-type  $\text{Mg}_2\text{Ge}_{1-x}\text{Sn}_x$  and  $\text{Mg}_2\text{Ge}_{1-z}\text{Si}_z$  with  $x, z = 0.1$  and  $0.2$  is carried out via high energy ball milling using Li as a dopant. The results reveal that the addition of Si and Sn into the  $\text{Mg}_2\text{Ge}$  system reduces the thermal conductivity. However, with decreasing Ge content, we observe an increasing secondary phase content that might be detrimental for the carrier mobility. Furthermore, for alloying with  $\text{Mg}_2\text{Si}$  we find a systematic decrease of the achievable carrier concentration, limiting the power factor. The best thermoelectric properties were found for  $z = 0.1$  with a  $zT_{\text{max}}$  comparable to that of  $\text{Mg}_2\text{Ge}$  but an average figure of merit which is approximately 30% higher.

## 2. Materials and methods

$\text{Mg}_{2-y}\text{Li}_y\text{Ge}_{1-x}\text{Sn}_x$  and  $\text{Mg}_{2-y}\text{Li}_y\text{Ge}_{1-z}\text{Si}_z$  with  $x, z = 0.1$  and  $0.2$  and  $y = 0.01, 0.02, 0.03$  and  $0.05$  were synthesized via high energy ball milling and direct current sinter pressing. In addition,  $y = 0.01$  and  $0.05$  were prepared for the  $z = 0.1$ . To obtain a homogenous fine powder for the sintering process, mechanical alloying using a SPEX 8000D series Mixer/Mill was carried out for a total of 3 to 4 hours, depending on the composition. A total of 10 grams of the elements, Mg turnings (Merck), Si ( $< 6\text{mm}$ , Chempure), Sn ( $< 71\text{ }\mu\text{m}$ , Merck), Ge (polycrystalline 99%), and Li granules (with purity  $> 99.5\%$ ) were weighted according to nominal composition and loaded into a stainless steel jar along with stainless steel balls (purchased from SPEX) in an Argon glove box. During the milling step, the powders were checked every hour to get any agglomerates off the jar walls. At the final step, if any agglomerates were present, the powders were milled again for a time period of 6 to 18 minutes. No massive mechanical force must be applied to remove the agglomerates since this can result in the burning of the powders, possibly due to the presence of Lithium and Germanium together<sup>20</sup>. The whole procedure was done inside the glovebox including the removal of agglomerations from the jar walls after 1 hour of milling. For the completion of

reaction to obtain the desired phase with good densification, the powder was loaded in a graphite die of diameter 12.7 mm and sintered using a direct current sintering press (DSP 510 SE) from Dr. Fritsch GmbH at 973 K for  $\text{Mg}_2\text{Ge}_{1-z}\text{Si}_z$  and 923 K for  $\text{Mg}_2\text{Ge}_{1-x}\text{Sn}_x$  under vacuum ( $10^{-5}$  bar) with a holding time of 10 minutes, and under a sintering pressure of 66 MPa with a heating rate 1 K/s<sup>20, 25</sup>. The densities of the pellets were measured using the Archimedes method, and the relative densities of the obtained pellets were found to be > 97% of the theoretical values. The samples with  $z = 0.2$  exhibit poor chemical stability under ambient conditions and developed a green oxide layer on the surface of the pellets complicating thermoelectric transport measurements. The pellets were therefore kept inside the glove box or sealed vacuum bags and taken outside only directly before the measurements.

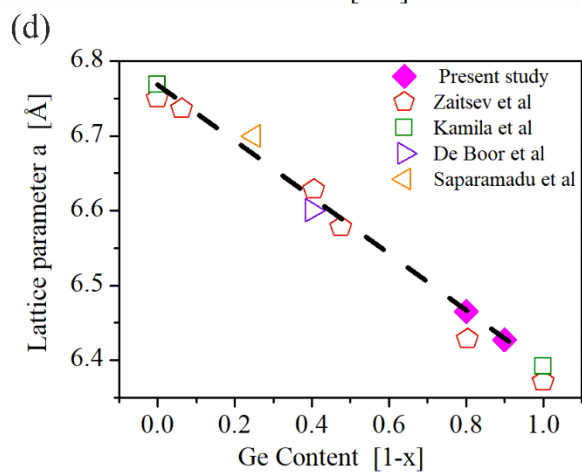
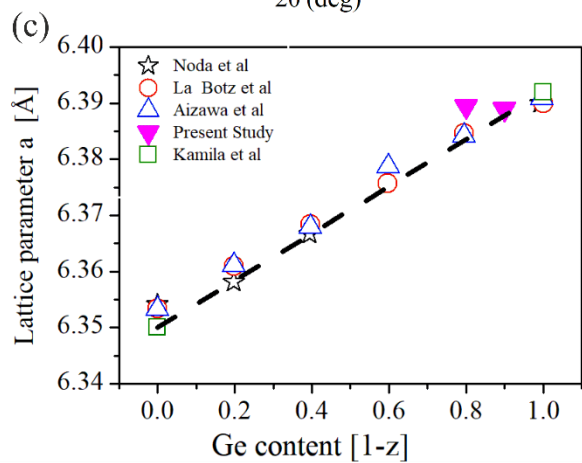
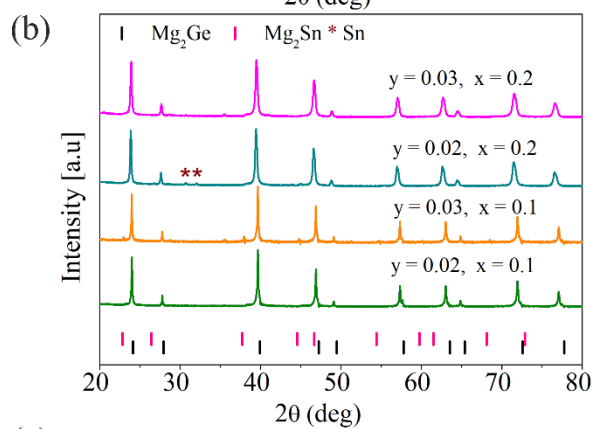
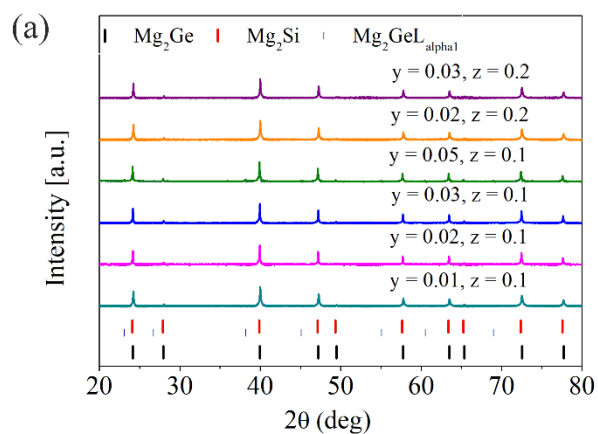
The temperature dependent Seebeck coefficient and electrical conductivity were measured using an in house developed measurement system with an error bar of  $\pm 5\%$ <sup>26-27</sup>. The thermal diffusivity ( $\alpha$ ) was measured using Netzsch LFA 427 (Laser Flash Apparatus) and Netzsch LFA 467 HT Hyperflash setups. The obtained values are used to calculate the thermal conductivity with an error bar  $\pm 8\%$  by using the relation  $\kappa = \alpha C_p \rho$ , where  $\rho$  and  $C_p$  correspond to the density of sample and linear heat capacity.  $C_p$  is calculated by using the Dulong-Petit approximation,  $C_V^{\text{DP}}: C_p = C_V^{\text{DP}} + \frac{9E_t^2 T}{\beta_T \rho}$  where  $E_t \approx 1.6 \times 10^{-5} - 2 \times 10^{-6} z \text{ K}^{-1}$ ,  $E_t \approx 1.6 \times 10^{-5} + 2 \times 10^{-6} x \text{ K}^{-1}$  and  $\beta_T \approx 1.7 \times 10^{-11} \text{ Pa}$ ,  $\beta_T \approx 1.7 \times 10^{-11} + 7.029 \times 10^{-12} x \text{ Pa}$ , are the linear coefficients of thermal expansion and linear isothermal compressibilities, respectively for  $\text{Mg}_{2-y}\text{Li}_y\text{Ge}_{1-z}\text{Si}_z$  and  $\text{Mg}_{2-y}\text{Li}_y\text{Ge}_{1-x}\text{Sn}_x$ <sup>16, 20, 28</sup>. Thermal cycling data for two selected samples is shown in Figure S2. The samples can show a minor change within the first measurement cycle but are stable afterwards, in line with earlier observations on Li-doped  $\text{Mg}_2(\text{Si}, \text{Sn}, \text{Ge})$  samples<sup>16, 20, 25</sup>. The transport properties of all samples which are presented here are the cooling data measured during thermal cycling. For the compositional analysis, X-ray diffraction (XRD) data was obtained using a Bruker's D8-Advanced diffractometer in the  $2\theta$  range of  $20^\circ$  to  $80^\circ$  while using Cu  $K\alpha$  radiation. Rietveld analysis was performed using TOPAS<sup>29</sup> on the XRD patterns. Moreover, Scanning Electron

Microscopy was done using a Zeiss Ultra 55 along with Energy dispersive spectroscopy (EDX) to obtain information on the local composition from different areas of the sample. Hall measurements were performed using an in-house facility with a van der Pauw configuration under a varying magnetic field of max.  $\pm 0.5$  T with an estimated uncertainty of 10%<sup>30</sup>. The Hall carrier concentration ( $p_H$ ) was calculated from the Hall coefficient ( $R_H$ ) assuming a single carrier type using the relation  $p_H = \frac{1}{R_H e}$  where  $e$  denotes an electron's charge. This serves as a rough estimate for the carrier concentration, even though two hole types likely contribute to the transport<sup>20</sup> and the assumption of a SPB model is therefore not strictly valid, see discussion in<sup>31</sup>. This also implies that the obtained mobilities are effective values.

### 3. Results

#### 3.1 Microstructure

The XRD patterns are indexed to the anti-flourite cubic crystal structure with space group Fm-3m (ICSD collection code #81735 for  $Mg_2Ge$ ). The XRD patterns depict the main phase peaks with some secondary phases but no traces of  $MgO$  (ICSD collection code #158103) for both  $z = 0.1$  and  $z = 0.2$  as shown in **Figure 1a**.



**Figure 1** XRD patterns of the pellets (a)  $\text{Mg}_{2-y}\text{Li}_y\text{Ge}_{1-z}\text{Si}_z$  and (b)  $\text{Mg}_{2-y}\text{Li}_y\text{Ge}_{1-}$  with  $x, z = 0.1$  and  $0.2$  for  $y = 0.02$  and  $0.03$ .  $y = 0.01$  and  $0.05$  is for  $z = 0.1$ . Note that in (a) also the peaks corresponding to the  $L_{\alpha 1}$  of  $\text{Mg}_2\text{Ge}$  from a tungsten source are indicated. (c) and (d) Comparison of the respective lattice parameters with literature<sup>19-20, 25, 32-36</sup> with the black dashed line correspond to Vegard's law.

The impurity peaks for  $y = 0.05$  are  $\text{Mg}_2\text{GeL}_{\alpha 1}$ ; this is due to tungsten contamination that evaporates from the cathode and gets deposited on the copper anode<sup>37</sup>. No elemental Si is observed in the XRD patterns; however, due to the similarities between the XRD patterns of the binaries  $\text{Mg}_2\text{Si}$  (ICSD collection code #192556) and  $\text{Mg}_2\text{Ge}$  and the solid solutions, it is not possible to deduce for  $\text{Mg}_2(\text{Ge},\text{Si})$  if the synthesis was successful from the XRD pattern alone. The XRD patterns for  $x = 0.1$  and  $x = 0.2$  also depict the main phase peaks with some secondary phases but no traces of  $\text{MgO}$  as shown in **Figure 1(b)**. Elemental impurities of Sn (ICSD collection code #106072) were found at  $2\theta = 30.72^\circ$  and  $2\theta = 32.13^\circ$  for  $x = 0.2$  and  $y = 0.02$ . The presence of elemental impurities suggests a possible need to increase the milling time and the observation of  $\text{Mg}_2\text{Sn}$  may indicate that  $\text{Mg}_2\text{Ge}$  is thermodynamically more favorable as compared to  $\text{Mg}_2\text{Sn}$ . Rietveld refinement was performed on the pellet XRD data with the TOPAS software. The secondary phases and the refined lattice parameters are given in Table 1. The lattice constants were estimated to be  $a = 6.388 - 6.391 \text{ \AA}$  and  $a = 6.389 \text{ \AA}$  for  $z = 0.1$  and  $0.2$  respectively and  $a = 6.427 \text{ \AA}$  and  $a = 6.465 \text{ \AA}$  for  $x = 0.1$  and  $0.2$ , respectively; in **Figure 1** mean of lattice constants of samples with different  $y$  are shown. For  $z = 0.1$  and  $x = 0.1, 0.2$ , the estimated lattice constants are not far off from the literature values, whereas with  $z = 0.2$  a larger discrepancy is observed. This is possibly due to the presence of several secondary phases in the systems.

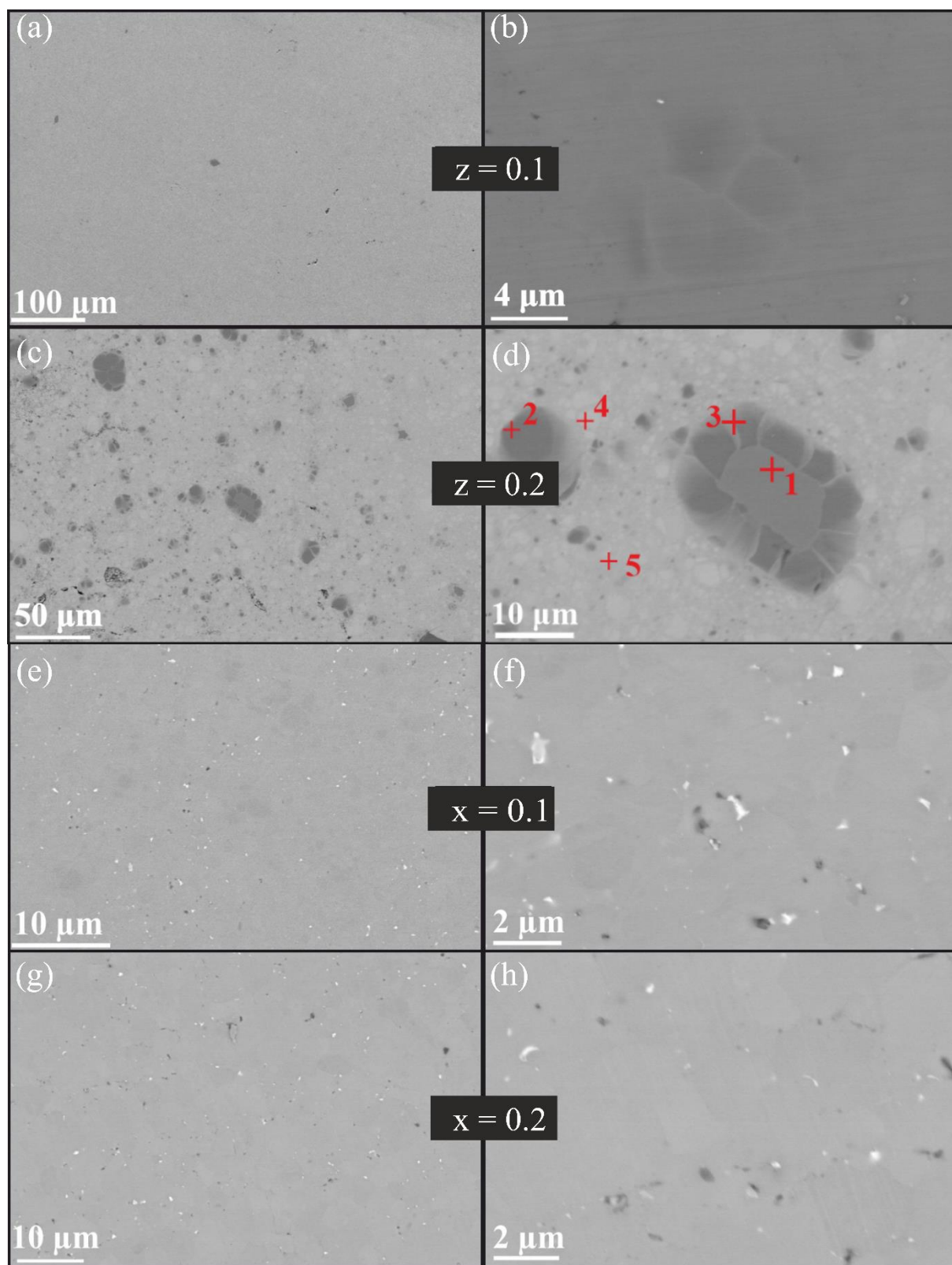
For further analysis Rietveld refinement using TOPAS was carried out and a goodness of fit in the range of 2 – 4 was obtained for all samples (Table S1). Secondary phases corresponding to  $\text{Mg}_2\text{Ge}$ ,  $\text{Mg}_2\text{Si}$ ,  $\text{Mg}_2\text{Ge}_{0.46}\text{Si}_{0.16}$ , and  $\text{Mg}_2\text{Ge}_{0.88}\text{Si}_{0.09}$  were found for  $z = 0.2$ . For all other samples the



Rietveld refinement estimated lattice constants close to those expected for the nominal composition.

Table 1 Rietveld refinement data for  $\text{Mg}_{2-y}\text{Li}_y\text{Ge}_{1-z}\text{Si}_z$  and  $\text{Mg}_{2-y}\text{Li}_y\text{Ge}_{1-x}\text{Sn}_x$  with  $x, z = 0.1$  and  $0.2$  for  $y = 0.02$  and  $0.03$ .  $y = 0.01$  and  $0.05$  is only for  $z = 0.1$ .

Nominal composition	Lattice constant (Å)	Secondary phases
$\text{Mg}_{1.99}\text{Li}_{0.01}\text{Ge}_{0.9}\text{Si}_{0.1}$	6.389	-
$\text{Mg}_{1.98}\text{Li}_{0.02}\text{Ge}_{0.9}\text{Si}_{0.1}$	6.388	-
$\text{Mg}_{1.97}\text{Li}_{0.03}\text{Ge}_{0.9}\text{Si}_{0.1}$	6.388	-
$\text{Mg}_{1.95}\text{Li}_{0.05}\text{Ge}_{0.9}\text{Si}_{0.1}$	6.391	-
$\text{Mg}_{1.98}\text{Li}_{0.02}\text{Ge}_{0.8}\text{Si}_{0.2}$	6.390	$\text{Mg}_2\text{Ge}$ (7%)
		$\text{Mg}_2\text{Si}$ (22.7%)
$\text{Mg}_{1.97}\text{Li}_{0.03}\text{Ge}_{0.8}\text{Si}_{0.2}$	6.389	$\text{Mg}_2\text{Ge}_{0.46}\text{Si}_{0.16}$ (27.2%)
		$\text{Mg}_2\text{Si}$ (9.8%)
$\text{Mg}_{1.98}\text{Li}_{0.02}\text{Ge}_{0.9}\text{Sn}_{0.1}$	6.428	
$\text{Mg}_{1.97}\text{Li}_{0.03}\text{Ge}_{0.9}\text{Sn}_{0.1}$	6.426	
$\text{Mg}_{1.98}\text{Li}_{0.02}\text{Ge}_{0.8}\text{Sn}_{0.2}$	6.465	$\text{Sn}$ (1.2%)
$\text{Mg}_{1.97}\text{Li}_{0.03}\text{Ge}_{0.8}\text{Sn}_{0.2}$	6.465	



**Figure 2** SEM-BSE images of (a-b)  $\text{Mg}_{1.97}\text{Li}_{0.03}\text{Ge}_{0.9}\text{Si}_{0.1}$ , (c-d)  $\text{Mg}_{1.98}\text{Li}_{0.02}\text{Ge}_{0.8}\text{Si}_{0.2}$ , (e-f)  $\text{Mg}_{1.98}\text{Li}_{0.02}\text{Ge}_{0.9}\text{Sn}_{0.1}$ ,

(g-h)  $\text{Mg}_{1.98}\text{Li}_{0.02}\text{Ge}_{0.8}\text{Sn}_{0.2}$  with left and right showing low and high magnification images, respectively. The marked points are given in Table 2.

**Table 2** EDX point analysis for  $\text{Mg}_{2-y}\text{Li}_y\text{Ge}_{1-z}\text{Si}_z$  with  $z = 0.2$  and  $y = 0.02$

Number	Mg (at%)	Ge (at%)	Si (at%)	Phase (approximately)
1	0.5	0.1	99.4	Si
2	66.6	0.1	33.3	$\text{Mg}_2\text{Si}$
3	68.2	7.4	24.4	$\text{Mg}_2\text{Ge}_{0.3}\text{Si}_{0.7}$
4	70.8	28.4	0.8	$\text{Mg}_2\text{Ge}$
5	71.1	24.2	4.7	$\text{Mg}_2\text{Ge}_{0.9}\text{Si}_{0.1}$

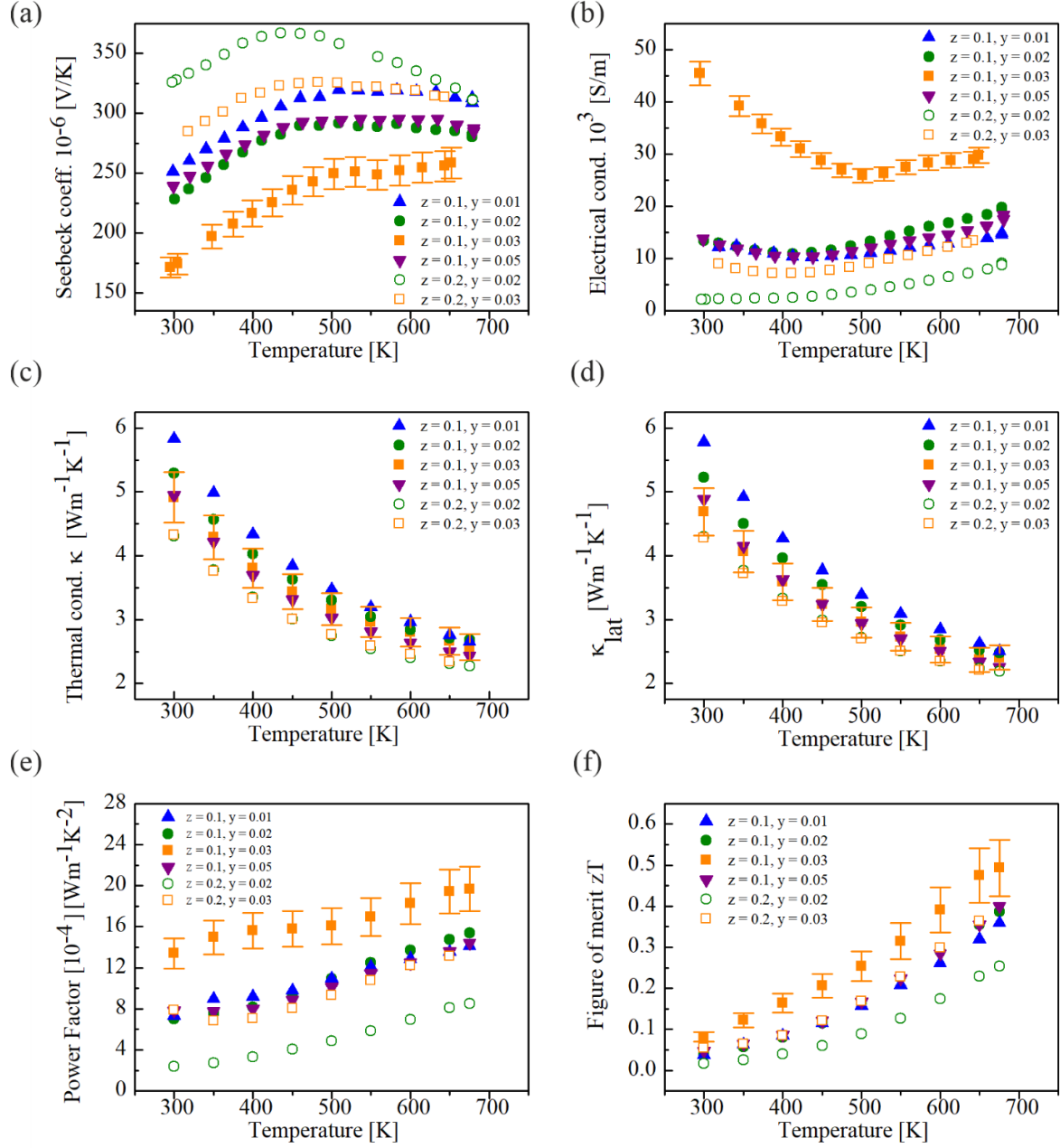
The SEM images depict homogeneous distribution of precipitates for  $z = 0.1$ ,  $x = 0.1$  and  $0.2$ , whereas with  $z = 0.2$  some secondary phases are observed. Through EDX point analysis, the lighter regions (3, 4, and 5 in **Figure 2(d)**) were identified as  $\text{Mg}_2(\text{Si}, \text{Ge})$  and the darker regions (e.g. 1 and 2) corresponded to Si and  $\text{Mg}_2\text{Si}$ , as shown in Table 2. The white particles in images (f) and (h) were found to be Sn, in agreement with the XRD results.

The grain size was estimated using ImageJ to be  $4.35 \mu\text{m}$  for  $z = 0.1$  and  $4.48 \mu\text{m}$  for  $x = 0.2$  respectively. At least 20 grains were measured in order to estimate a reliable average on the grain size. The grain size measured for  $z = 0.1$  and  $x = 0.2$  is in the range of  $4 - 5 \mu\text{m}$  which is the typical grain size obtained for the  $\text{Mg}_2\text{X}$  material systems and therefore presumably representative for all our samples<sup>14, 20, 38</sup>.

### 3.2 Thermoelectric properties of p-type $\text{Mg}_{2-y}\text{Li}_y\text{Ge}_{1-z}\text{Si}_z$

**Table 3** List of  $\text{Mg}_{2-y}\text{Li}_y\text{Ge}_{1-z}\text{Si}_z$  samples with nominal composition, carrier concentration, Hall mobility and lattice constant. The carrier concentration and Hall mobility are calculated as effective quantities using a single parabolic band (SPB) model.

Composition	$p_{\text{H}} \times 10^{19} (\text{cm}^{-3})$	$\mu_{\text{H}} (\text{cm}^2/\text{Vs})$	$\rho (\text{g}/\text{cm}^3)$
$\text{Mg}_{1.99}\text{Li}_{0.01}\text{Ge}_{0.9}\text{Si}_{0.1}$	2.6	29.1	2.97
$\text{Mg}_{1.98}\text{Li}_{0.02}\text{Ge}_{0.9}\text{Si}_{0.1}$	3.3	25.7	2.97
$\text{Mg}_{1.97}\text{Li}_{0.03}\text{Ge}_{0.9}\text{Si}_{0.1}$	5.8	48.2	2.92
$\text{Mg}_{1.95}\text{Li}_{0.05}\text{Ge}_{0.9}\text{Si}_{0.1}$	3.0	28.6	2.96
$\text{Mg}_{1.98}\text{Li}_{0.02}\text{Ge}_{0.8}\text{Si}_{0.2}$	1.5	9.2	2.82
$\text{Mg}_{1.97}\text{Li}_{0.03}\text{Ge}_{0.8}\text{Si}_{0.2}$	1.6	39.1	2.85



**Figure 3** Temperature dependent (a) Seebeck coefficient, (b) electrical conductivity, (c) thermal conductivity, (d) lattice thermal conductivity, (e) power factor, and (f) figure of merit of  $\text{Mg}_{1-y}\text{Li}_7\text{Ge}_{1-z}\text{Si}_2$  with  $z = 0.1$  and  $0.2$  and  $y = 0.02$ , and  $0.03$ . In addition,  $y = 0.01$  and  $0.05$  were for the  $z = 0.1$ .

In general, all samples show positive Seebeck coefficients corresponding to dominant hole conduction in the materials (**Figure 3a**). For  $z = 0.1$ , generally we observe that the Seebeck coefficient decreases as the Li content increases and that the trend is opposite for the electrical conductivity which is possibly due to the increase in carrier concentration and mobility as verified by the Hall measurement, except for  $y = 0.05$ . The Seebeck coefficient increases with temperature and almost becomes constant at around 500 K. For the electrical conductivity, we observe a decrease followed by a sharp increase as temperature increases. The rising in electrical conductivity initiates around 425 K for the  $y = 0.01$  and  $y = 0.02$  samples whereas in the case of  $y = 0.03$ , the rising is not as sharp as for the other samples and occurs at a higher temperature. The thermal conductivity and the lattice thermal conductivity show similar trends. The lattice thermal conductivity was estimated from  $\kappa_{\text{lat}} + \kappa_{\text{bi}} = \kappa - L\sigma T$ , where the Lorenz number ( $L$ ) was estimated by employing a SPB model:  $L = \left(\frac{k_b}{e}\right)^2 \frac{3F_0(\eta)F_2(\eta) - 4F_1^2(\eta)}{F_0(\eta)^2}$ . We argue that  $\kappa_{\text{bi}} \approx 0$  due to the relatively large band gap of  $\text{Mg}_2\text{Ge}$  (and Ge-rich  $\text{Mg}_2\text{X}$ ) and the arguments given in <sup>20</sup>. The influence of the primary thermoelectric properties can be seen in the trends observed for the power factor as the highest power factor value of  $19.7 \times 10^{-4} [\text{Wm}^{-1}\text{K}^{-2}]$  at 675 K was obtained for the  $y = 0.03$  sample due to the significantly higher electrical conductivity. Thus, due to the high-power factor, the maximum  $zT$  is calculated to be  $0.49 \pm 0.07$  at 675 K.

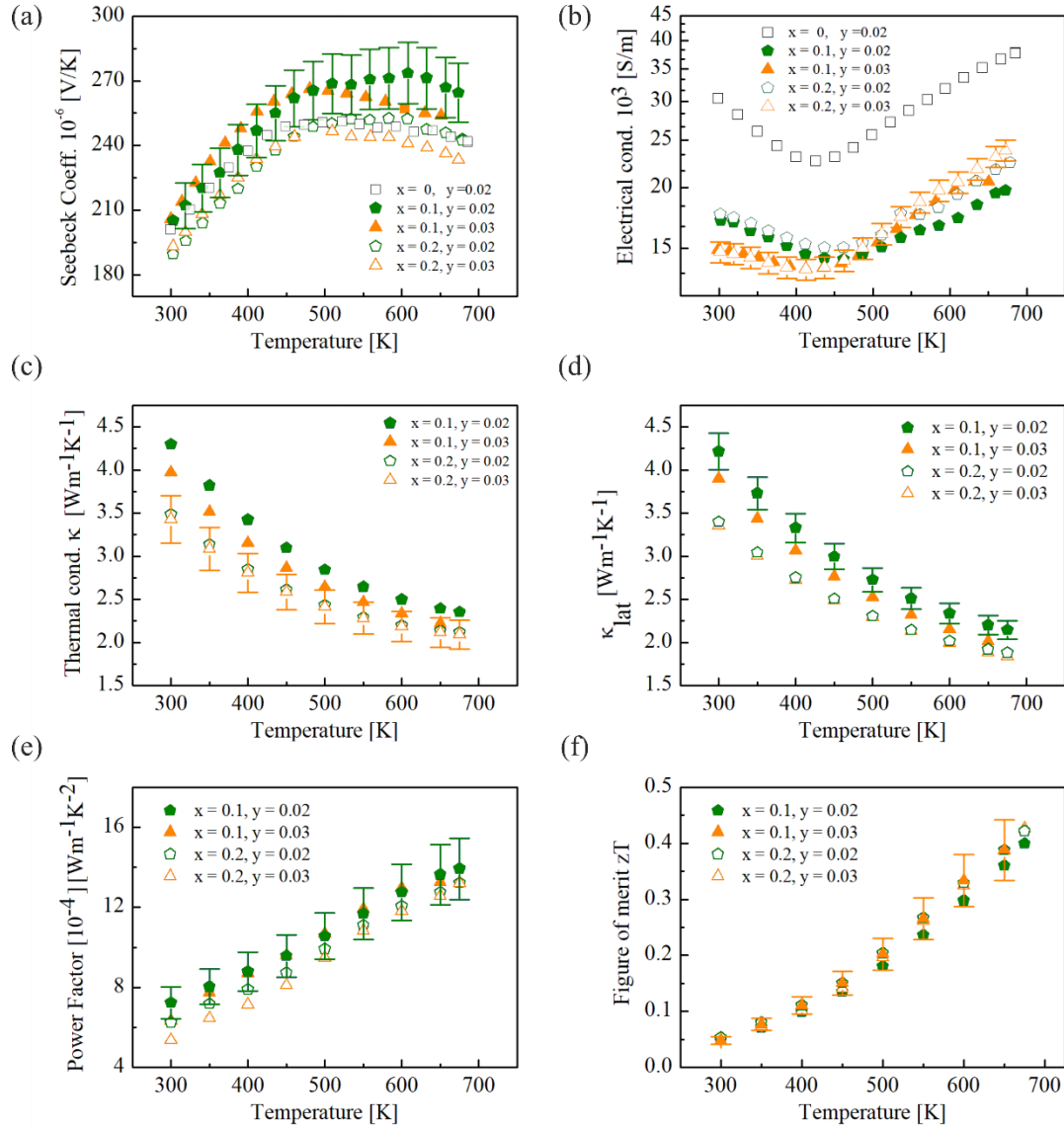
For  $z = 0.2$  too, we observe that the Seebeck coefficient decreases as the Li content increases and that the trend is opposite for the electrical conductivity. Measurement of the transport properties of the  $z = 0.2$  set of samples yielded a lot of complications with high resistivity being observed during the measurements and all the samples were re-measured. The samples were found to be thermally unstable and were also found to undergo oxidation during the measurements. The

Seebeck coefficient increases until 450 K and then decreases for  $y = 0.02$  while it levels off at around 450 K for  $y = 0.03$ . The upturn point in electrical conductivity shifts towards lower temperatures as the carrier concentration is increased ( $y = 0.02$  at around 450 K and  $y = 0.03$  at around 425 K).

### 3.3 Thermoelectric properties of p-type $\text{Mg}_{2-y}\text{Li}_y\text{Ge}_{1-x}\text{Sn}_x$

**Table 4** List of  $\text{Mg}_{2-y}\text{Li}_y\text{Ge}_{1-x}\text{Sn}_x$  samples with nominal composition, carrier concentration, Hall mobility and lattice constant. The carrier concentration and Hall mobility are calculated as effective quantities using the SPB model.

Composition	$p_{\text{H}} \times 10^{19} (\text{cm}^{-3})$	$\mu_{\text{H}} (\text{cm}^2/\text{Vs})$	$\rho (\text{g}/\text{cm}^3)$
$\text{Mg}_{1.98}\text{Li}_{0.02}\text{Ge}_{0.9}\text{Sn}_{0.1}$	5.0	21.7	3.10
$\text{Mg}_{1.97}\text{Li}_{0.03}\text{Ge}_{0.9}\text{Sn}_{0.1}$	5.1	18.4	3.18
$\text{Mg}_{1.98}\text{Li}_{0.02}\text{Ge}_{0.8}\text{Sn}_{0.2}$	5.1	21.4	3.14
$\text{Mg}_{1.97}\text{Li}_{0.03}\text{Ge}_{0.8}\text{Sn}_{0.2}$	5.2	18.0	3.12



**Figure 4** Temperature dependent (a) Seebeck Coefficient, (b) electrical conductivity, (c) thermal conductivity, (d) lattice thermal conductivity, (e) power factor, and (f) figure of merit of  $\text{Mg}_{2-y}\text{Li}_y\text{Ge}_{1-x}\text{Sn}_x$  with  $x = 0.1$  and  $0.2$  and  $y = 0.02$  and  $0.03$ .

The Seebeck coefficient displays positive values and for  $x = 0.1$ , we observe that the Seebeck coefficient increases with increasing temperature and decreases at around 600 K for  $y = 0.02$  and 450 K for  $y = 0.03$  whereas for the electrical conductivity we observe a gradual decrease followed by a sharp increase which starts at around 475 K for  $y = 0.02$  and 425 K for  $y = 0.03$  (**Figure 4**). It should also be noted that the upturn observed in the electrical conductivity is much sharper for



$y = 0.03$ . The comparable  $S$  and  $\sigma$  data agree with the relatively similar carrier concentrations and mobility values obtained from the Hall measurements. The temperature dependent Seebeck coefficient and electrical conductivity of  $\text{Mg}_2\text{Ge}_{1-x}\text{Sn}_x$  is similar to the behavior of the Li doped  $\text{Mg}_2\text{Ge}$  (Figure 4a and b).

For  $x = 0.2$ , we notice that the Seebeck coefficient decreases gradually starting at around 500 K for  $y = 0.03$  and 625 K for  $y = 0.02$  while we see a considerable upturn in the electrical conductivity which starts at 425 K for  $y = 0.03$  and 475 K for  $y = 0.02$ . The thermal conductivity and lattice thermal conductivity are in general lower for the  $x = 0.2$  system due to the higher alloy scattering effect and the reduction observed is found to vary from 10% to 20 % throughout the whole temperature range. Thus, owing to the reduction in the thermal conductivity and the higher electrical conductivity observed, the maximum power factor value of  $13 \times 10^{-4} [\text{Wm}^{-1}\text{K}^{-2}]$  at 675 K is achieved for  $x = 0.2$ ,  $y = 0.03$  and thus a maximum  $zT$  of  $0.43 \pm 0.07$  at 675 K is achieved for the same system.

#### 4. Discussion

The XRD and Rietveld refinement results show the main phase and no elemental impurities for  $\text{Mg}_2\text{Ge}_{0.9}\text{Si}_{0.1}$  while a number of secondary phases corresponding to  $\text{Mg}_2\text{Ge}$ ,  $\text{Mg}_2\text{Si}$ ,  $\text{Mg}_2\text{Ge}_{0.88}\text{Si}_{0.09}$ , and  $\text{Mg}_2\text{Ge}_{0.46}\text{Si}_{0.16}$  were obtained for the  $\text{Mg}_2\text{Ge}_{0.8}\text{Si}_{0.2}$ . The SEM images also depict unreacted Si for  $z = 0.2$  indicating an incomplete reaction of Mg, Si, and Ge. Whereas for  $x = 0.1$  and  $0.2$ , white particles corresponding to unreacted Sn are observed with SEM-EDX and Rietveld refinement as well (see supporting information). The presence of unreacted elements and secondary phases is possibly due to the differences in mechanical properties of Mg, Ge, Si, and Sn important for the solid state reaction during the milling process.

From the observation of large elemental Si particles (**Figure 2(c)** and (d)) it can be hypothesized for the  $\text{Mg}_2(\text{Ge},\text{Si})$  system that the  $\text{Mg}_2\text{Ge}$  phase is thermodynamically more favorable than  $\text{Mg}_2\text{Si}$ , thus possibly allowing Si to diffuse into the matrix similar to what was found for  $\text{Mg}_2(\text{Si},\text{Sn})$ <sup>14</sup>. This is supported by the fact that the formation energy of  $\text{Mg}_2\text{Ge}$  is found to be 35% larger than that of  $\text{Mg}_2\text{Si}$  with values corresponding to  $-104.25$  kJ/mol and  $-68.85$  kJ/mol, respectively at 298.15 K, with all values being calculated using Thermo-Calc and the TCAL6 database<sup>39-40</sup>. The calculated values were found to be comparable to those reported in literature<sup>41-42</sup>. Due to sample instability and increased inhomogeneity observed as the Si content increases, it can be concluded that investigating compositions with higher Si content would most likely not produce better results. Finally, it can be deduced that a more suitable microstructure can be expected with a further optimization of the synthesis parameters (milling time), especially for the  $z = 0.2$  system.

With respect to the Sn containing samples, the small amounts of unreacted Sn observed on the  $\text{Mg}_2\text{Ge}_{0.9}\text{Sn}_{0.1}$  and  $\text{Mg}_2\text{Ge}_{0.8}\text{Sn}_{0.2}$  matrix might be due to incomplete diffusion or due to loss of Mg during the sintering step<sup>14, 20, 25</sup> shifting the phase equilibrium towards the Mg poor region where  $\text{Mg}_2(\text{Ge},\text{Sn})$  and Sn coexist. The formation energy of  $\text{Mg}_2\text{Ge}$  is found to be 25% higher than that of  $\text{Mg}_2\text{Sn}$  ( $-76.980$  kJ/mol), thus supporting the fact that the formation of  $\text{Mg}_2\text{Ge}$  is thermodynamically more favorable and Sn might subsequently diffuse into the system; this hypothesis requires verification by experiments performed with longer milling times. Similarly, previous work by Sankhla et al.<sup>14</sup> on the  $\text{Mg}_2(\text{Si},\text{Sn})$  system highlighted the fact that milling for longer times decreases the extent of elemental impurities being present. This would likely lead to a further improvement of the thermoelectric properties as they are correlated with microstructure and phase distribution.

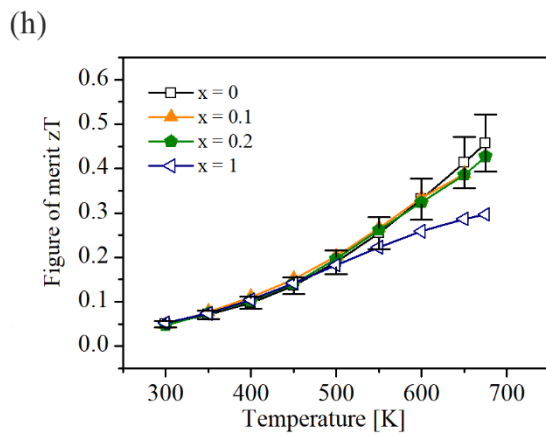
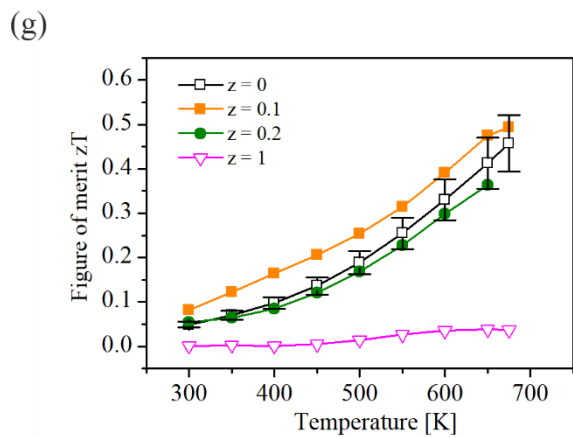
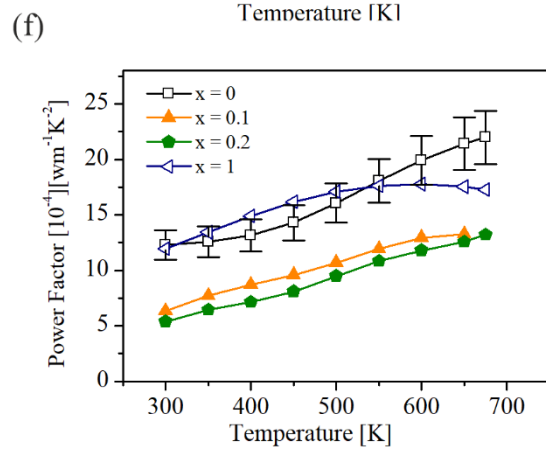
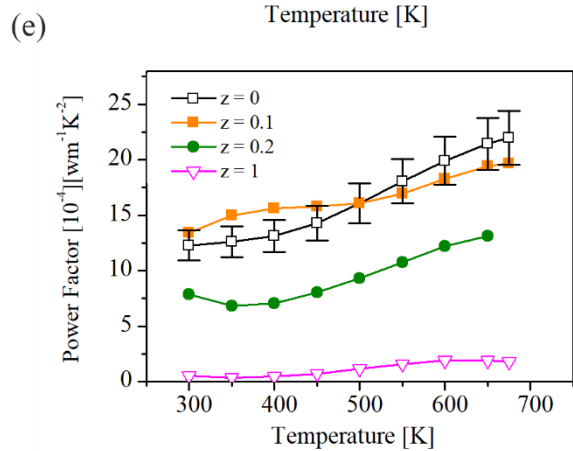
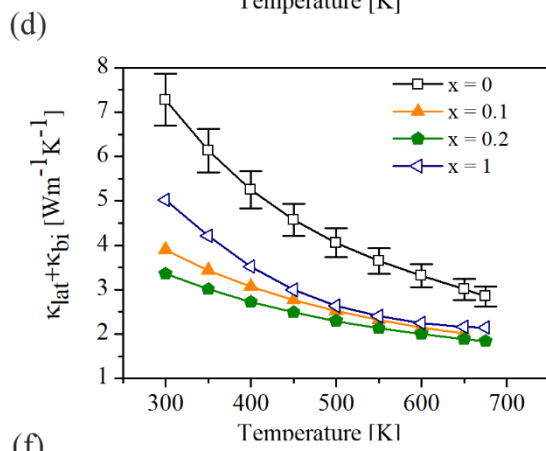
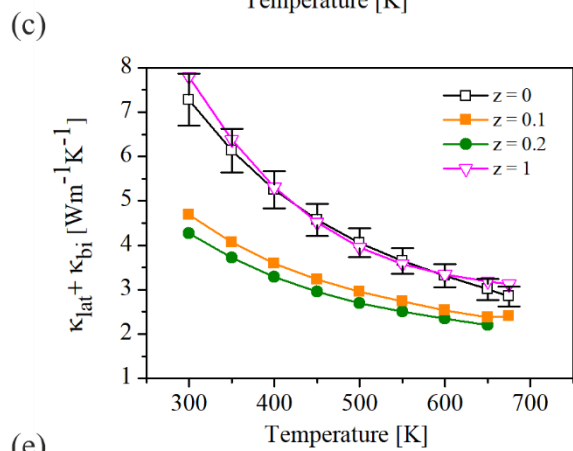
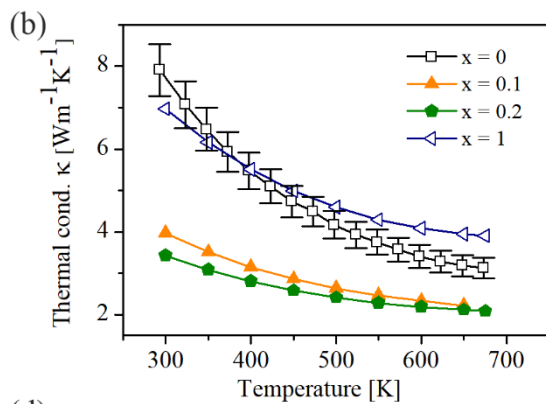
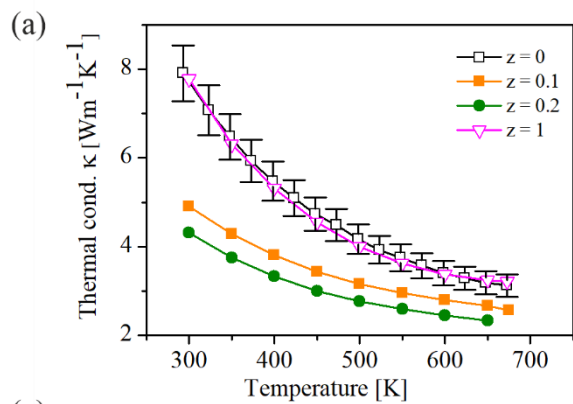
Overall the temperature dependence of the electronic transport properties does not resemble that of typical, highly doped thermoelectric materials <sup>16</sup>. Instead, the characteristic features of a relatively sharp increase in electrical conductivity at low temperature and slight or no decrease of the Seebeck coefficient at high temperatures, match those of binary p-type  $\text{Mg}_2\text{Ge}$  <sup>20</sup> (**Figure 4a** and **b**). For p-type  $\text{Mg}_2\text{Ge}$  this has been explained with a strong temperature dependence of the valence bands and it is plausible that the same mechanism holds here. Thus, the almost constant Seebeck coefficient at higher temperatures and the decrease in electrical conductivity followed by a sharp increase, in Li doped  $\text{Mg}_2\text{Ge}_{1-x}\text{Sn}_x$  and  $\text{Mg}_2\text{Ge}_{1-z}\text{Si}_z$  are possibly due to the movement of the split off (SO) band whose energetic difference decreases with increasing temperature <sup>20</sup>.

The comparison of the behavior of the Seebeck coefficient and electrical conductivity of undoped  $\text{Mg}_2\text{Ge}$  in Figure S3 clearly demonstrates the influence of Li doping on the material systems tested in this work; the increased charge carrier concentration is in full agreement with our earlier work <sup>20</sup>. The undoped  $\text{Mg}_2\text{Ge}$  sample shows indications for mixed conduction and transitions from p-type to n-type above room temperature while Li-doped samples exhibit dominant hole conduction due to an enhanced hole concentration.

From the changes of carrier concentration with increasing  $x$  or  $z$  it can be deduced that the dopant efficiency (measured carrier concentration over Li concentration) is generally relatively low and decreases with increasing Si content and with increasing Li content whereas it seems to remain constant for Sn compositions. The values lie around 0.2 for the binary  $\text{Mg}_2\text{Ge}$  system <sup>20</sup> and  $x = 0.1, 0.2$ , whereas with the addition of Si the dopant efficiency reduces to around 0.1 for  $z = 0.1$  and 0.2. The dopant efficiency for p-type  $\text{Mg}_2\text{Si}$  was found to be even lower <sup>16</sup> and it is unlikely that this changes non-monotonously with composition.

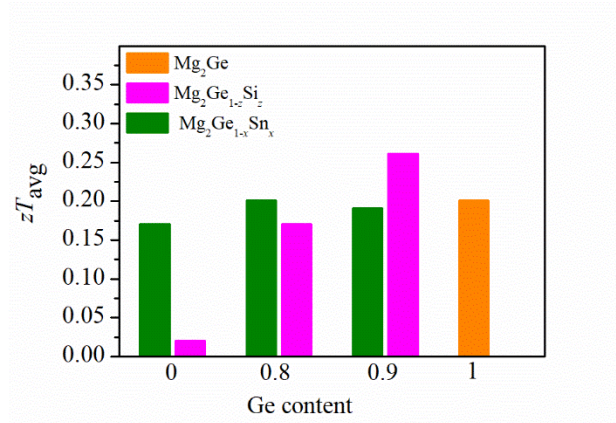
A possible explanation is that Li occupies both Mg and interstitial sites thus serving simultaneously as a p-type and an n-type dopant. Previous report by Gao et al.<sup>18</sup>, suggests that Li can easily occupy both Mg ( $\text{Li}_{\text{Mg}}$ ) substitutional and interstitial sites ( $\text{Li}_{\text{int}}$ ) and it has also been noted by Kumari et al.<sup>43</sup> and Ayachi et al.<sup>44</sup>, that the formation energy is lowest when Li occupies the Mg interstitial positions. These results from calculations are in contradiction to the experimental report by Nieroda et al.<sup>45</sup>, which states that Li occupancy at the Mg site is more favorable. Nevertheless, Li might be self-compensating, explaining the overall low dopant efficiency, while the balance between  $\text{Li}_{\text{Mg}}$  and  $\text{Li}_{\text{int}}$  might depend on the Si:Ge:Sn ratio.

We have also observed a minor change in the first measurement cycle which later becomes stable (Figure S2). The effect of the observed secondary phases can be estimated, in particular as these form only non-connected islands. Si can be expected to have a much lower electrical conductivity, possibly coming from a reduction of the mobility of the charge carriers. Elemental Sn is metallic but as the Sn-islands are sub- $\mu\text{m}$  sized, they also probably act as scattering centers. As the dopant efficiency decreases with increasing Si-content in  $\text{Mg}_2(\text{Ge},\text{Si})$ ,  $\text{Mg}_2\text{Si}$  or Si-richer  $\text{Mg}_2(\text{Ge},\text{Si})$  can likewise be expected to behave as lower doped particles, causing mainly a reduction in carrier mobility, similar to what has been previously observed in<sup>38</sup>.



**Figure 5** Thermoelectric properties of  $\text{Mg}_{2-y}\text{Li}_y\text{Ge}_{1-z}\text{Si}_z$  and  $\text{Mg}_{2-y}\text{Li}_y\text{Ge}_{1-x}\text{Sn}_x$  where  $x$  and  $z = 0 - 1$ . Li concentration ( $y = 0.02$  or  $0.03$ ) was taken from the sample with the best overall thermoelectric properties and was  $y = 0.02$  or  $0.03$ . Temperature dependent (a-b) thermal conductivity, (c-d) lattice thermal conductivity, (e - f) power factor and (g-h) figure of merit for different Li concentrations compared with literature from <sup>16, 19-20, 36</sup>.

The effect of solid solution formation and the trends with increasing Si and Sn content can be discerned from **Figure 5(a)** which shows that the thermal conductivity decreases around 30% throughout the whole temperature range for  $z = 0.1$  and  $0.2$  as compared to the binaries, most likely because of more scattering in the system. The addition of Si into the system creates an alloying effect and increases disorder in the system. For the power factor a reduction is expected as the scattering of the holes is also increased due to the alloying <sup>46</sup>. From the data available we find comparable power factors for  $\text{Mg}_2\text{Ge}$  and  $\text{Mg}_2\text{Ge}_{0.9}\text{Si}_{0.1}$  with the  $\text{Mg}_2\text{Ge}$  being superior only at higher temperatures. The combined effect of  $\kappa_{\text{lat}}$  and PF lead to a higher  $zT$  for  $z = 0.1$  compared to  $\text{Mg}_2\text{Ge}$  ( $z = 0$ ) in a large temperature range. In the case of  $\text{Mg}_2(\text{Ge},\text{Sn})$ , we observe that the thermal conductivities decreases by  $\sim 35\%$  as compared to  $\text{Mg}_2\text{Ge}$  ( $z = 0$ ) throughout the whole temperature range. However, as compared to  $\text{Mg}_2\text{Sn}$ , the values are relatively similar for  $T > 500$  K. Furthermore, we observe that the power factor is significantly reduced compared to  $\text{Mg}_2\text{Ge}$  and thus overall, there is no improvement in  $zT$ .



**Figure 6**  $zT_{\text{avg}}$  calculated for between 300 K and 675 K (except for  $z = 0.2$  where  $T_h = 650$  K was employed) along with literature comparison.<sup>16, 20</sup>

While materials are often compared with respect to their maximum figure of merit, the average figure of merit is much closely related to the expected device efficiency ( $\eta_{\text{max}}$ )<sup>47-49</sup>. The average  $zT$  was calculated by integrating the  $zT$  curve from the hot end ( $T_h$ ) to the cold end ( $T_c$ )<sup>47, 49</sup>, where  $T_h = 675$  K and  $T_c = 300$  K for all samples except  $z = 0.2$  and  $x = 0.1$  ( $T_h = 650$  K) (**Figure 6**).

For  $z = 0$  and  $z = 0.1$ ,  $zT_{\text{avg}}$  values of 0.20 and 0.26 were obtained respectively while with  $x = 0.1$  and  $x = 0.2$  systems, we obtain  $zT_{\text{avg}}$  values of 0.19 and 0.20. In principle, we see that there is an increase of ~30% for the  $z = 0.1$  system as compared to the binary  $\text{Mg}_2\text{Ge}$  system whereas for the Sn containing samples we do not observe a significant improvement. We don't expect further improvement for Si-rich compositions in the  $\text{Mg}_2\text{Si}$ - $\text{Mg}_2\text{Ge}$  system due to material instability and low dopant efficiency.

## 5. Conclusion

The first reported syntheses of p-type  $\text{Mg}_{2-y}\text{Li}_y\text{Ge}_{1-z}\text{Si}_z$  and  $\text{Mg}_{2-y}\text{Li}_y\text{Ge}_{1-x}\text{Sn}_x$  with  $x, z = 0.1$  and 0.2 using Li as a dopant were carried out via high energy ball milling and direct current sintering. No  $\text{MgO}$  was observed in all samples, however the presence of elemental Sn and  $\text{Mg}_2\text{Sn}$  was observed for  $x = 0.1$  and 0.2 through XRD, Rietveld refinement, and SEM/EDX studies. In addition, several secondary phases corresponding to Si and Ge rich phases were observed for  $z = 0.2$  through SEM and Rietveld refinement. Our results indicate that milling time should be increased with increasing substitution of Ge by Si/Sn. We further observe unusual temperature dependent electronic transport data, of  $S$  and  $\sigma$ , which are similar to those of binary p-type  $\text{Mg}_2\text{Ge}$ .

Li was identified as suitable p-type dopant but the dopant efficiency was found to decrease with increasing Si content. The thermal conductivity decreases significantly ( $\sim 30\%$ ) by alloying which compensates for the decrease in carrier mobility observed due to alloy scattering. A maximum  $zT$  value of 0.49 was obtained at 675 K for  $z = 0.1$  which currently lies in the range of best values for Ge rich  $\text{Mg}_2(\text{Si}, \text{Ge}, \text{Sn})$  solid solutions. More importantly the average  $zT$  of 0.26 was found to be 30 % higher than that of binary  $\text{Mg}_2\text{Ge}$ . Furthermore, a maximum  $zT$  value of 0.43 and average  $zT$  value of 0.20 were obtained for  $x = 0.2$  which is comparable to that of binary  $\text{Mg}_2\text{Ge}$ .

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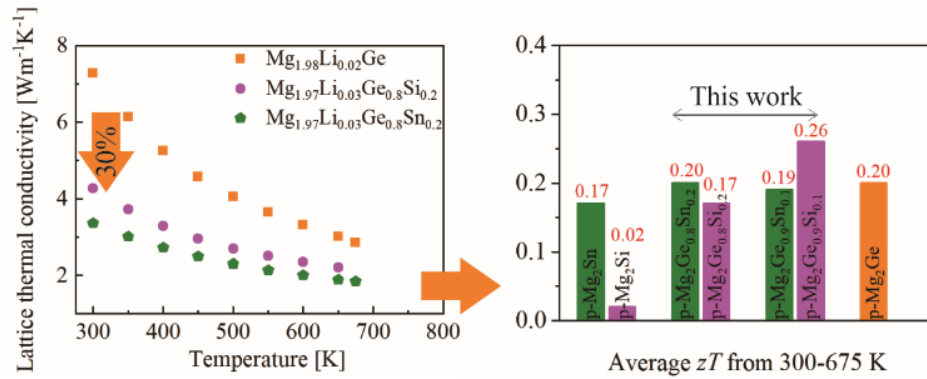
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**“Optimization of the thermoelectric properties of p-type  $\text{Mg}_{2-y}\text{Li}_y\text{Ge}_{1-x}\text{Sn}_x$  and  $\text{Mg}_{2-y}\text{Li}_y\text{Ge}_{1-z}\text{Si}_z$  with  $x, z = 0.1$  and  $0.2$ .**

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Alloying of Si or Sn into the Ge site of p-type  $\text{Mg}_2\text{Ge}_{1-x}\text{Sn}_x$  and  $\text{Mg}_2\text{Ge}_{1-z}\text{Si}_z$  with  $x, z = 0.1$  and  $0.2$  leads to a 30% decrease in the lattice thermal conductivity. Furthermore, a 30% increase in average  $zT$  values are achieved as compared with p-type  $\text{Mg}_2\text{Ge}$  binary.