Fitting Task Specific Elastic Potential for Robotic Legs

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Problem statement

Efficient legged locomotion requires energy to be stored and released in a cyclic manner. Leg systems can be equipped with mechanical springs as passive energy storage. Considering a periodic locomotion task for

$$M(q)\ddot{q} + C(q,\dot{q})\dot{q} + g(q) = \tau, \tag{1}$$

described by motion $q(t) \in \mathbb{R}^n$ and effort $\tau(t) \in \mathbb{R}^n$. One design criteria for the spring potential $\psi(q)$ is to minimize

$$\int_0^T \left(\tau(t) - \frac{\partial\psi}{\partial q}(q(t))\right)^2 dt. \tag{2}$$

Designing and realizing such potentials is still a central research issue (1). In this work a preliminary result is presented, which allows for an intuitive interpretation.

Introducing constraints

It is assumed that $q(t)$ defines a closed curve in the configuration space without self intersections. Each segment of a kinematic chain can interact with it's predecessor and it's successor, which results in $n-1$ constraint functions of the form

$$q_{i+1} = \delta_i(q_i). \tag{3}$$

Enhancing (3) to

$$q_{i+1} = \delta_i(q_i) + e_{i+1} \tag{4}$$

keeps the DOF of the system constant. Here, $e \in \mathbb{R}^{n-1}$ are deflection coordinates, that will be nonzero when the system leaves the task trajectory. Elastic potential forces of $\psi_i = 1/2K_i e_i^2$, which are defined using the deflections e, will force the system back to the desired trajectory.

Remark: The parameters K_i can be chosen arbitrary to influence the disturbance rejection of the system.

Introducing elastic forces

Similary a chain of force distribution functions can be formulated as:

$$\tau_{i+1} = \phi_i(\tau_i) \tag{5}$$

Assumption: ϕ_i is monotone and $\phi_i(0) = 0$
Fulfilled for e.g. vertical jumping.
Define a_1 as weighted sum of all DoF:

$$a_1 = q_1 + \phi_1^{-1} q_2 + \phi_1^{-1}\phi_2^{-1} q_3 + \cdots + m_1 \tag{6}$$

Remark: a_1 will move when the system is moving along the task trajectory fulfilling the constraints δ_i!
Defining a quadratic potential $\psi_1 = 1/2K_1 a_1^2$ on a_1 will distribute the potential forces over all DoF:

$$\frac{\partial\psi_1}{\partial q_1} = K_1 a_1 = \tau_1 \tag{7}$$

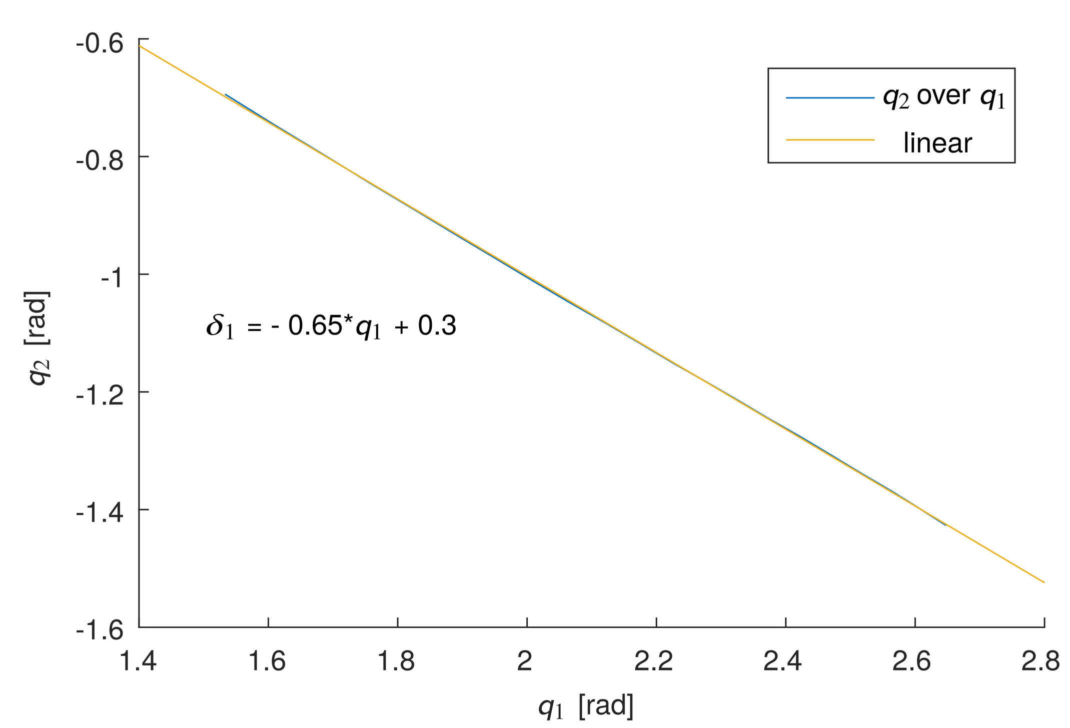
$$\frac{\partial\psi_1}{\partial q_2} = K_1 a_1 \phi_1 = \tau_2 \tag{8}$$

$$\frac{\partial\psi_1}{\partial q_3} = K_1 a_1 \phi_1 \phi_2 = \tau_3 \tag{9}$$

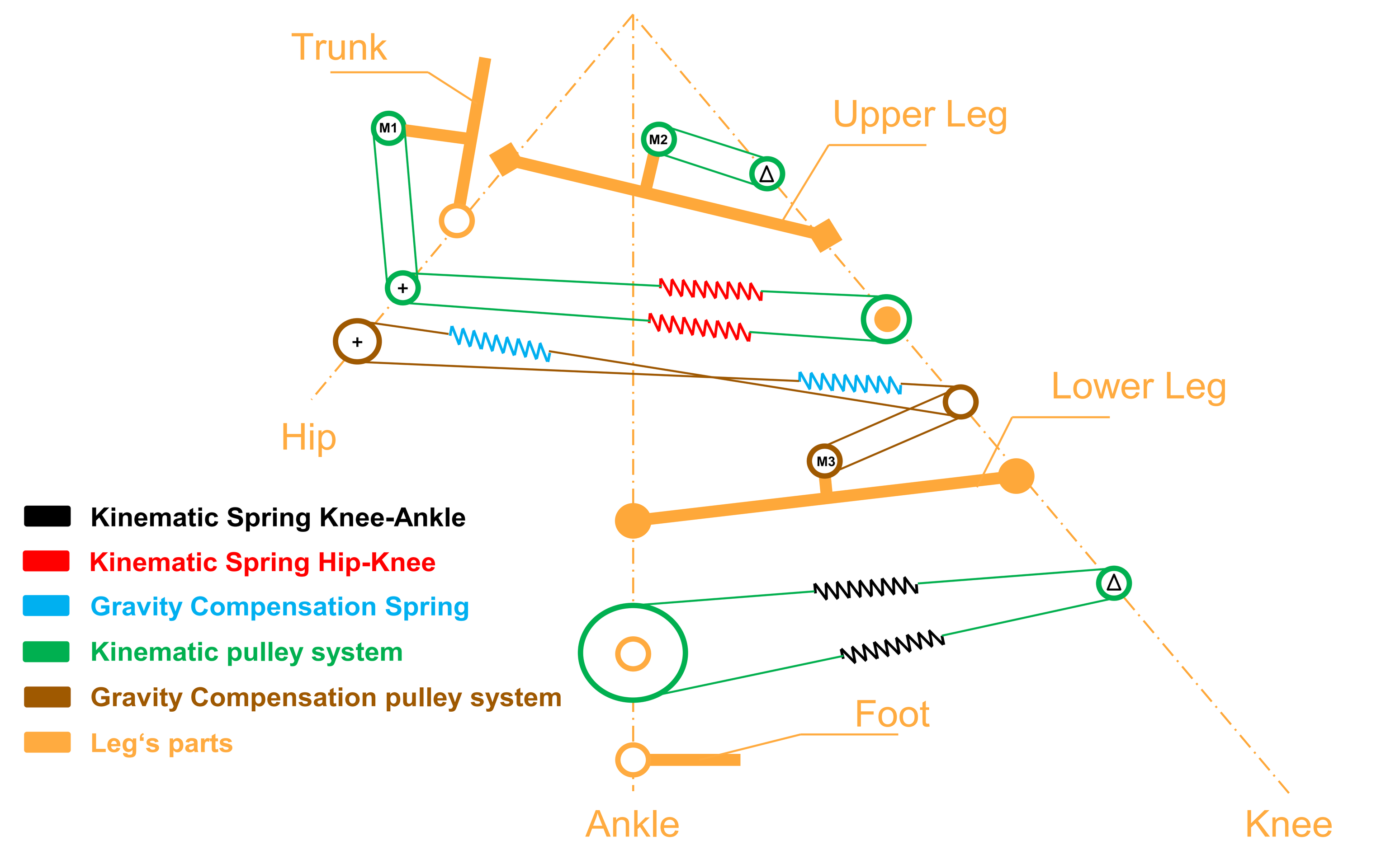
$$\vdots \tag{10}$$

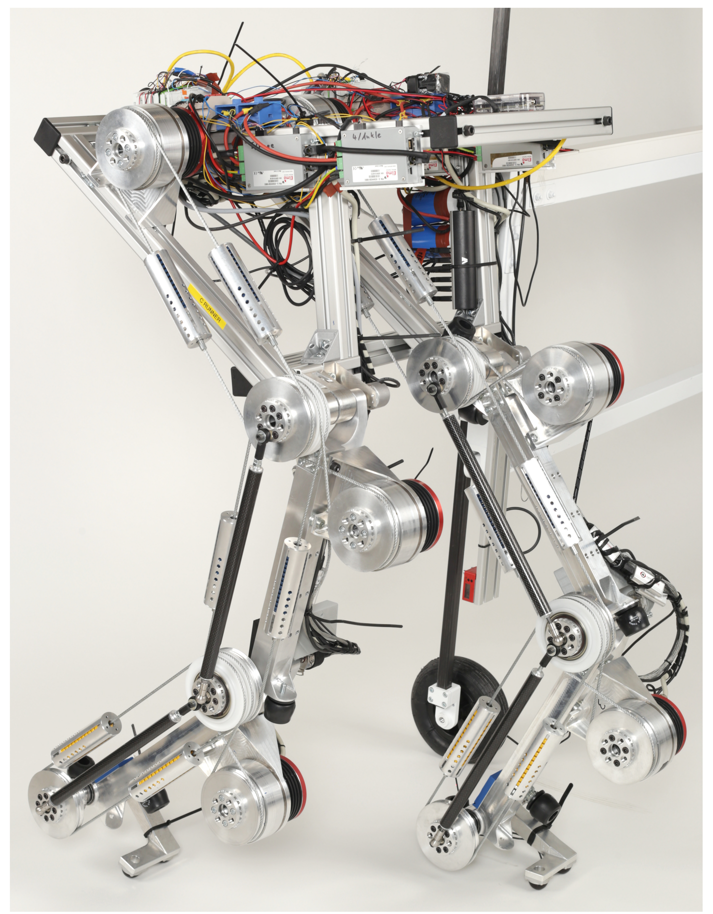
K_1 can be determined from the experimental data by linear regression of $\tau_1(t)/a_1(t)$. Note: The forces induced by ψ_1 will apply the necessary forces on the system to stay on the task (constraint forces) AND those necessary to fulfill the task (task forces)!

Graphical solution for δ_1



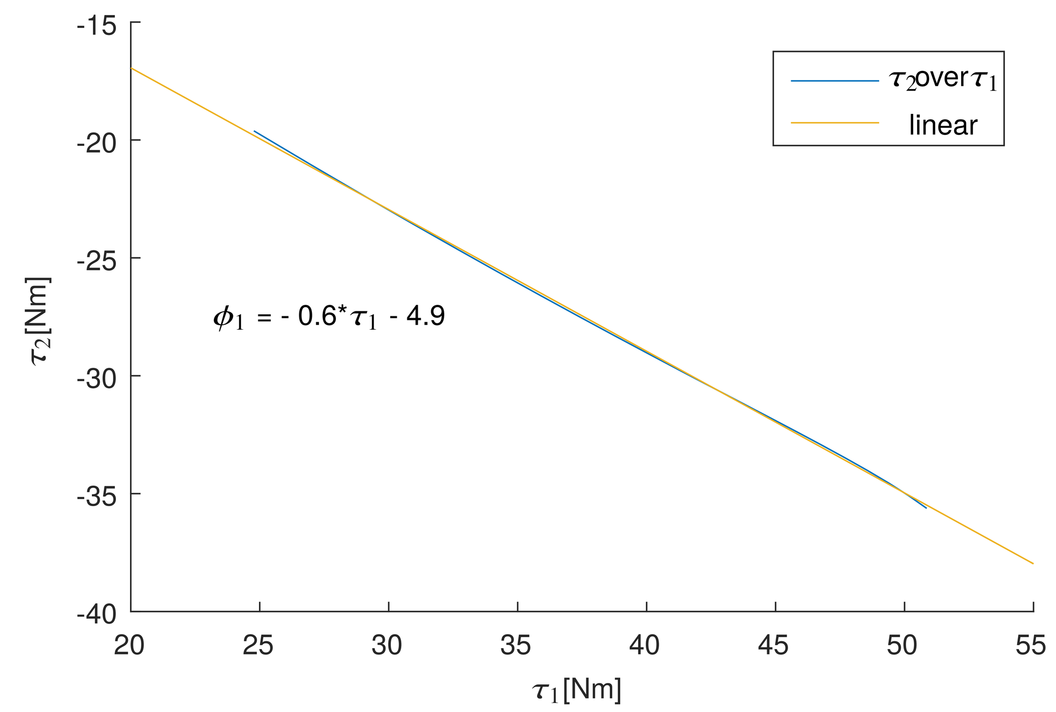
Implementation on a Planar Testbed

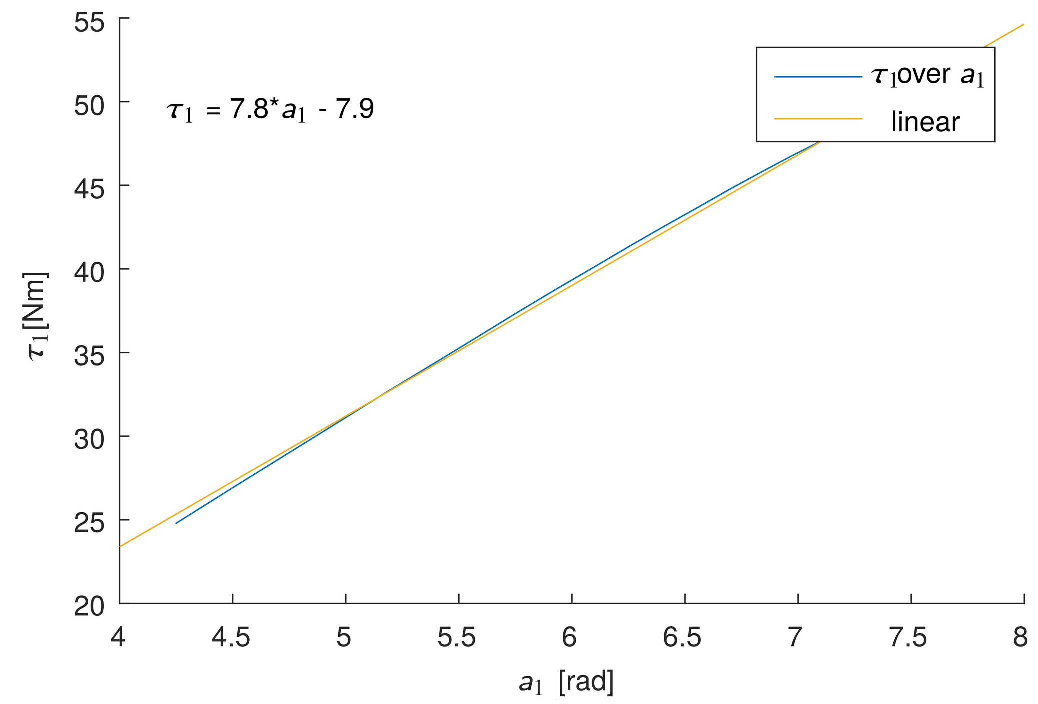




The CRunner System (see (2)).

Graphical solution for ϕ_1 and K_1





Results

For simple implementation we choose $\delta_i(q_i) = rq_i q_i + e_{i+1;0}$ and $\phi_i(\tau_i) = rt_i \tau_i$
From the graphical solution (linear regression) we obtain: rq_1 = -0.65 rt_1 = -0.60
rq_2 = -0.18 rt_2 = -1.24e-6
As rt_1 is nearby zero, we are removing the terms containing this parameter from ψ_1, to obtain a realistic coupling matrix.
We choose: K_2=700 Nm/rad and K_3=600 Nm/rad.
The resulting K matrix from $\frac{\partial^2}{\partial[qae]^2}(\psi_1+\psi_2+\psi_3)$ solves to:

305.36	443.38	0	456.38	0	7.81
443.38	741.96	110.44	700.00	110.44	-13.00
0	110.44	600.00	0	600.00	0
456.38	700.00	0	700.00	0	0
0	110.44	600.00	0	600.00	0
7.81	-13.00	0	0	0	7.81

References

[1] A. Abate, R. L. Hatton, and J. Hurst, "Passive-dynamic leg design for agile robots," in *IEEE International Conference on Robotics and Automation (ICRA)*, May 2015, pp. 4519–4524.

[2] F. Loeffl, A. Werner, D. Lakatos, J. Reinecke, S. Wolf, R. Burger, T. Gumpert, F. Schmidt, C. Ott, M. Grebenstein, and A. Albu-Schäffer, "The dlr c-runner: Concept, design and experiments," in *2016 IEEE-RAS 16th International Conference on Humanoid Robots (Humanoids)*, 2016, pp. 758–765.