Fitting Task Specific Elastic Potential for Robotic Legs

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Problem statement
Efficient legged locomotion requires energy to be stored and released in a cyclic manner. Leg systems can be equipped with mechanical springs as passive energy storage. Considering a periodic locomotion task for

\[ M(q) \ddot{q} + C(q, \dot{q}) \dot{q} + g(q) = \tau, \]

(1)
described by motion \( q(t) \in \mathbb{R}^n \) and effort \( \tau(t) \in \mathbb{R}^n \). One design criteria for the spring potential \( \psi(q) \) is to minimize

\[ \int_0^T (\tau(t) - \frac{\partial \psi}{\partial q}(q(t)))^2 dt. \]

(2)

Designing and realizing such potentials is still a central research issue (1). In this work a preliminary result is presented, which allows for an intuitive interpretation.

Introducing elastic forces
Similarly a chain of force distribution functions can be formulated as:

\[ \tau_{i+1} = \phi_i(\tau_i) \]

(5)

Assumption: \( \phi_i \) is monotone and \( \phi_i(0) = 0 \)

Fulfilled for e.g. vertical jumping.

Define \( a_1 \) as weighted sum of all DoF:

\[ a_1 = q_1 + q_1 q_2 + q_1^{-1} e_2 q_1 + \ldots + m_1 \]

(6)

Remark: \( a_1 \) will move when the system is moving along the task trajectory fulfilling the constraints \( \delta_i \! \!\! \). Defining a quadratic potential \( \psi_1 = 1/2 K_i e_2^2 \) on \( a_1 \) will distribute the potential forces over all DoF:

\[ \frac{\partial \psi_1}{\partial q_1} = K_1 a_1 = \tau_1 \]

(7)

\[ \frac{\partial \psi_1}{\partial q_2} = K_1 a_1 \phi_1 = \tau_2 \]

(8)

\[ \frac{\partial \psi_1}{\partial q_3} = K_1 a_1 \phi_1 \phi_2 = \tau_3 \]

(9)

\[ \vdots \]

(10)

\( K_1 \) can be determined from the experimental data by linear regression of \( \tau_1(t)/a_1(t) \). Note: The forces induced by \( \psi_1 \) will apply the necessary forces on the system to stay on the task (constraint forces) AND those necessary to fulfill the task (task forces)!

Introducing constraints
It is assumed that \( q(t) \) defines a closed curve in the configuration space without self intersections. Each segment of a kinematic chain can interact with it’s predecessor and it’s successor, which results in \( n-1 \) constraint functions of the form

\[ q_{i+1} = \delta_i(q_i). \]

(3)

Enhancing (3) to

\[ q_{i+1} = \delta_i(q_i) + e_{i+1} \]

(4)

keeps the DOF of the system constant. Here, \( e \in \mathbb{R}^{n-1} \) are deflection coordinates, that will be nonzero when the system leaves the task trajectory. Elastic potential forces of \( \psi_i = 1/2 K_i e^2_i \), which are defined using the deflections \( e \), will force the system back to the desired trajectory.

Remark: The parameters \( K_i \) can be chosen arbitrary to influence the disturbance rejection of the system.

Implementation on a Planar Testbed

Graphical solution for \( \phi_1 \) and \( K_1 \)

For simple implementation we choose \( \delta_i(q_i) = r_1 q_i + e_{i+1,0} \) and \( \phi_i(\tau_i) = r_0 \tau_i \). From the graphical solution (linear regression) we obtain:

\[ r_1 = -0.65 \quad r_0 = 0.60 \quad r_0 = -0.18 \quad r_0 = -1.24a-6 \]

As \( r_0 \) is nearby zero, we are removing the terms containing this parameter from \( \psi_1 \), to obtain a realistic coupling matrix.

We choose: \( K_2 = 700 \text{Nm/ rad} \) and \( K_3 = 600 \text{Nm/ rad} \). The resulting K matrix from

\[ \frac{\partial \psi_1}{\partial \tau} (\psi_2 + \psi_3) \]

solves to:

\[ 305.36 \quad 443.38 \quad 0 \quad 456.38 \quad 0 \quad 7.81 \]

\[ 443.38 \quad 741.96 \quad 110.44 \quad 700.00 \quad 110.44 \quad -13.00 \]

\[ 0 \quad 110.44 \quad 600.00 \quad 0 \quad 600.00 \quad 0 \quad 7.81 \quad -13.00 \quad 0 \quad 0 \quad 7.81 \]

Graphical solution for \( \delta_1 \)

Results

References