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Minimizing Water Consumption of a CSP Plant, by Using an Optimization Algorithm for Cooling Operation

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Abstract. Today's thermal power plant cooling systems aim at reaching high steam cycle efficiencies in all operating points while keeping the water consumption as low as possible. For this purpose, cooling systems are becoming more and more complex, such that finding an optimal operation strategy for the cooling components is no longer a trivial problem. To tackle this problem, this paper introduces a cooling operation optimization algorithm. The algorithm is applicable to a black box model of the power plant including the cooling system. Thereby it can easily be applied as online optimizer for existing cooling systems or to investigate different cooling options for different CSP plant configurations during a planning phase. Additionally, the weight of water consumption and electricity production can be adjusted for each optimization run. In this work, we present the algorithm itself by providing a methodology to transform the optimization problem into an oriented decision tree and to solve the resulting problem with a stochastic solution approach developed from an Ant Colony Optimization algorithm. Furthermore, the operability of the algorithm is validated using a performance model of a 50 MW parabolic trough plant with a hybrid cooling system, consisting of a wet cooling tower and a cold thermal energy storage.

INTRODUCTION

Do Concentrated Solar Power (CSP) plants always perform well at sites with high direct solar irradiation? On one hand the performance of CSP plants strongly depends on ambient conditions, especially the solar direct normal irradiation (DNI). Therefore, CSP plants are often located in sunny and arid regions. On the other hand, CSP plants also rely on the availability of water. Water is used for various tasks in a CSP plant but often scarce in arid regions with high DNI values. When a CSP plant uses a conventional wet cooling system, the operation of the wet cooling tower (WCT) is accountable for more than 90 % of the overall water consumption [1]. Hence, the water consumption can be decreased significantly by modifying the cooling system. But the transition from wet to dry cooling leads to a higher levelized cost of electricity (LCOE) [1] and lower steam cycle efficiencies due to higher condensation temperatures [2]. To reach higher steam cycle efficiencies than a dry cooled system and consume less water than a wet cooled system [3], hybrid cooling systems combine different cooling mechanisms in one or more components [4]. Common hybrid cooling systems are for example an air-cooled condenser in parallel with a small WCT or a deluged air-cooled condenser [3]. Theoretical investigations also suggest to utilize a cold thermal energy storage (cTES) in combination with wet or dry cooling components [5, 6, 7]. Regardless of the considered design, hybrid cooling systems are not yet common practice in CSP plants. Still, they show promising potential, especially for plant sites with limited access to water resources.

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The hybridization of cooling technologies leads to an increasing complexity of cooling systems. The number of components and therefore the number of manipulative variables and possible operating states increases. Additionally, the performance of the steam cycle and the water consumption of the cooling system strongly depend on the volatile ambient conditions, especially the ambient temperature and humidity. To operate or design new cooling systems, an operation strategy for the cooling components is needed. With the increasing complexity of the cooling systems, finding an optimal operation strategy becomes a non-trivial problem. To tackle this problem, the cooling system's operation can be mathematically optimized.

When optimizing the cooling system's operating plan, two possible optimization strategies are conceivable: a deterministic solution approach using a mathematical model of the power plant or a stochastic solution approach applicable to a black box model of the investigated system. While deterministic solution approaches can result in a short computing time, they are often bounded to a specific set of equations and therefore to a specific problem. Such methods are used to optimize the operation of conventional cooling systems using only one cooling mechanism, maximizing the net electricity output or the power cycle efficiency [8]. When investigating the optimal operation of hybrid cooling systems, the aim is to minimize the water consumption while maximizing the net electricity production. Additionally, hybrid cooling systems vary in the components and cooling mechanisms utilized, such that a general description in one deterministic optimization problem is hardly possible. Therefore, a black box optimization approach can be developed, applicable to optimize the operation of different hybrid cooling system configurations independent of the technologies used. To also enable the operability of the algorithm for future developments, it is important that hybrid cooling systems with cTES can also be optimized. To the best of the authors' knowledge, no algorithm meeting these demands has been developed so far. Therefore, this work introduces a flexibly applicable black box algorithm minimizing the water consumption and maximizing the net electricity production of a power plant with conventional or hybrid cooling system by adapting the cooling operation.

Hereinafter, we provide a description of the development of the optimization problem, the transformation of the resulting problem into an oriented decision tree as well as the solution for the transformed optimization problem using a stochastic optimization approach derived from an Ant Colony Optimization (ACO) algorithm. Furthermore, the operability and performance of the algorithm is investigated using a performance model of a 50 MW parabolic trough plant with a hybrid cooling system consisting of a WCT and a cTES.

OPTIMIZATION PROBLEM

To optimize the operation of a cooling system, the question arises as to how each component of the cooling system should be operated at any given time depending on the possible cooling load and the ambient conditions. The aim of the optimization is to minimize the water consumption while maximizing the net electricity production. These objectives are often contrary to each other. To bring both objectives together in one objective function, a possibility to value the importance of each objective needs to be introduced. By assuming a discrete quasistatic modeling approach, the cooling system's operation can be described as a discontinuous series of stationary operating states. Nevertheless, instead of optimizing the operation in each of the time steps independently, we want the objective function to consider the water consumption and net electricity production over a certain time period, e.g. 24 hours. In this way it is possible to investigate cooling systems with time dependent properties like storage components or restrictions to the daily or weekly water use. Equation (1) represents the resulting objective function for the optimization problem. It describes a maximization approach, containing a sum over all time steps t of the optimization time horizon t_N. For each time step the net produced electricity $W_{el,net}{}^t$ with its weight $k_{el}{}^t$ as well as the consumed water $V_w{}^t$ with a negative sign and its weight $k_w{}^t$ are considered in the sum. Therefore, the value of the objective function increases with increasing net electricity production and decreases with increasing water consumption, both considered over the entire time horizon of the optimization t_N. The importance of the electricity production in relation to the water consumption influences the resulting operating plan and can be adjusted using the weights $k_{el}{}^t$ and $k_w{}^t$.

$$\max_{\boldsymbol{x}} \; f(\boldsymbol{x},\boldsymbol{y}) = \sum_{t=0}^{t_N}\left[k_{el}^t * W_{el,net}^t(\boldsymbol{x},\boldsymbol{y}) - k_w^t * V_w^t(\boldsymbol{x},\boldsymbol{y})\right] \tag{1}$$

The net produced electricity $W_{el,net}{}^t$ and the water consumption $V_w{}^t$ can be calculated depending on the decision variables of the optimization problem $\boldsymbol{x}$ and the external inputs $\boldsymbol{y}$. External inputs refer to information and data needed for the calculation, e. g. ambient conditions or direct normal irradiation for each time step. During the optimization procedure, we try to find optimal values for the decision variables $\boldsymbol{x}$ in each time step to maximize the objective function (1). The possible decision variables vary depending on the investigated cooling system. Typical decision variables can be the cooling water flow to the surface condenser or the fan speed of an air-cooled condenser module, if it is variable. Equations describing the net produced electricity $W_{el,net}^{t}(\boldsymbol{x}, \boldsymbol{y})$ and water consumption $V_{w}^{t}(\boldsymbol{x}, \boldsymbol{y})$ in each time step can be defined depending on the investigated cooling system and power plant. To evaluate the search space for the optimization problem and define feasible solutions, the objective function is usually extended by equality and inequality constraints. The definition of these constraints depends on the investigated cooling system and power plant as well. Therefore, we do not provide detailed equations here. Typical constraints assure that the condenser pressure or the cooling water mass flow do not exceed their operating limits.

OPTIMIZATION ALGORITHM

The cooling operation optimization algorithm presented in this paper is based on a stochastic solution approach using a black box model of the cooling system and power plant. Hereby, the algorithm can be applied to different power plant and cooling system configurations independently of the energy source, power cycle or cooling system used. Additionally, the algorithm can deal with performance maps as well as detailed models with a high level of complexity. In order to apply the stochastic solution approach effectively, the presented optimization problem needs to be transformed into an oriented decision tree. Then the stochastic solution approach based on an ACO algorithm is introduced to solve the transformed optimization problem.

Transforming the Optimization Problem

The transformation of the optimization problem into an oriented decision tree contains two steps. In a first step the decision variables and constraints are transformed into one operating state variable, such that the algorithm decides on the value of the operating state variable for each time step to maximize the objective function (1). For each value of the operating state variable, a set of operation decisions needs to be defined in the model of the power plant and cooling system used for the optimization. Additionally, each definition needs to include measures that are taken if one of the constraints of the optimization problem is harmed, to assure that the constraints are always fulfilled. In this way, the operating state of the cooling system is defined in each time step depending only on the value of the operating state variable and the ambient conditions as well as the possible cooling load. It is crucial for the application of the algorithm that the black box model used can take the operating state variable as input for each time step and apply the operating decisions and additional measures to the cooling system depending on the value of the variable. An operating plan is then defined as a list of tuples containing a value for the operating state variable for every time step of the optimization time horizon. The search space of the algorithm contains all operating plans that can be created by applying a value for the operating state variable in each time step. By transforming the decision variables and constraints into one operating state variable, the search space is reduced and the optimization problem is simplified.

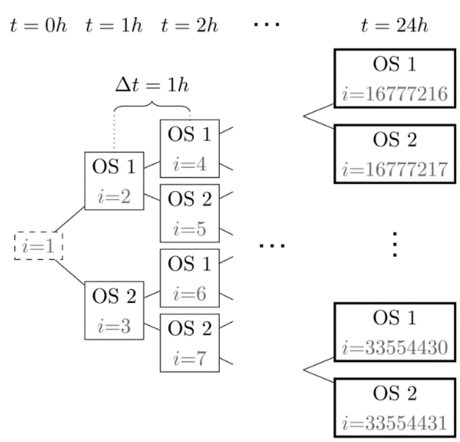


FIGURE 1. Oriented decision tree resulting from an optimization problem with two possible values for the operating state variable, an optimization time step of 1 hour and an optimization time horizon of 24 hours.

In a second step the simplified optimization problem is transferred into an oriented decision tree, where each finished path represents a possible operating plan for the cooling system. The tree nodes represent the state of the cooling system in each time step and store the actual value of the operating state variable. The time steps can be seen as vertical planes containing a node for each possible state the cooling system can be in during this time step. We assume that the state of the cooling system always depends on all previous decisions, such that each node can only be reached by a one-to-one combination of previous nodes. Therefore, different nodes with the same value of the operating state variable appear on the same time step plane representing different cooling system states due to different previous decisions. Each node is uniquely defined by an identification number i. The identification numbers are allocated to the nodes in increasing order from top to bottom in each time step plane and from left to right over the time step planes. Figure 1 shows an exemplary decision tree generated from an optimization problem with two possible values for the operating state variable (OS 1, OS 2), an optimization time horizon of 24 hours and a time step of 1 hour. Following the numbering order explained before, the decision tree can be generated using equations (2) – (5), derived from correlations defined in the field of combinatorics. The number of time steps n_t inside the time horizon of the optimization t_N can be determined with (2). Depending on the number of possible decisions per time step n_d, the number of possible operating plans n_p can be calculated with (3). The possible decisions correspond to the possible values the operating state variable can accept. Equation (4) is used to evaluate the number of nodes needed to formulate the optimization problem as decision tree. With equation (5) a set I_{next} filled with the identification numbers of the possible following nodes for the next time step can be calculated when being in node i. Using this equation, finding a new operating plan is possible without calling possible next nodes from a version of the entire decision tree stored in memory. By only storing information of nodes that are already investigated, the memory usage can be reduced.

$$n_t = \frac{t_N}{\Delta t} \tag{2}$$

$$n_p = (n_d)^{n_t} \tag{3}$$

$$n_n = \frac{(n_d)^{(n_t+1)}-1}{n_d-1} \tag{4}$$

$$I_{next} = \{i \, * n_d + r,\ r \in \mathbb{Z}, (-n_d) + 2 \leq r \leq 1\} \tag{5}$$

Introducing the Stochastic Solution Approach

The stochastic solution approach developed for the optimization of the cooling operation is based on the idea of Ant Colony Optimization [9]. The algorithm introduced below evolves from the so-called Max-Min Ant Systems (MMAS) variant [10]. Using a stochastic solution approach, the optimization problem formulated as decision tree is solved by repeatedly evaluating the black box model of the power plant with cooling system. In every evaluation - hereafter referred to as model call – another possible operating plan is investigated. In this way we gain knowledge about the problem and use it to guide our search for the optimal operating plan, such that we can find well performing operating plans without investigating the entire search space. To store, update and evaluate knowledge about the quality of the investigated operating plans and the decisions taken, a weight τ_i is assigned to each tree node i.

Hereinafter the algorithm is explained following Fig. 2. Before the optimization is started, an initializing weight τ_{init} is assigned to all nodes. Subsequently the iterative solution procedure starts. For each iteration, a certain number of model calls K is investigated. In every model call the algorithm navigates through the decision tree, creating a new operating plan. To find well performing operating plans, nodes with high weights are more likely to be investigated. While navigating through the decision tree, a decision on the next value of the operating state variable needs to be taken every time step. Therefore, the possible following nodes - representing the possible next operating state variable values - are defined firstly using (5) and then inactive nodes among them are excluded again. Afterwards a probability for each next node i is calculated depending on the weights τ_i, using equation (6), as introduced in [9]. N represents the set of possible next nodes, α can be used to adjust the influence of relatively high or low node weights. Subsequently the next node is chosen by chance, using the probabilities evaluated before. This decision procedure is repeated creating a certain path through the decision tree until the last time step of the optimization horizon is reached and therefore the model call is finished.

$$p_i = \frac{(\tau_i)^\alpha}{\sum_{j \in N}\left[(\tau_j)^\alpha\right]} \tag{6}$$

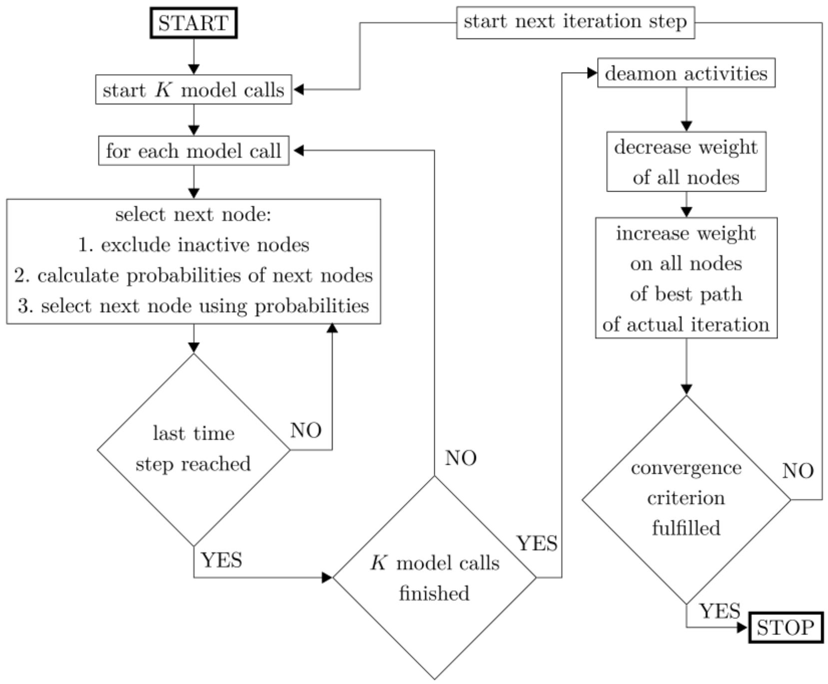


FIGURE 2. Flowchart showing the procedure of the cooling operation optimization algorithm.

In one iteration step a number of K model calls are investigated. For each model call the objective function is evaluated. Model calls can be parallelized, to save computing time. After finishing the model calls, the deamon activities are applied. They contain rules to minimize the search space by deactivating nodes such that additional constraints can be fulfilled and no operating plan is investigated twice. Additionally, the best operating plan of the actual iteration is determined. Important deamon activities are:

- Deactivate all nodes where all possible following nodes are already investigated (compulsory)
- Deactivate all nodes where the overall water consumption exceeds a certain value (optional)
- Find best operating plan of this iteration step by comparing the value of the objective function for all model calls investigated in this step. Save the operating plan as best path of this iteration. Then determine the overall best operating plan of all iteration steps so far and save it as current solution of the optimization (compulsory)

Following the deamon activities the weights of all nodes are decreased using equation (7), following the ACO algorithm introduced in [9]. ρ is a constant between 0 and 1. By decreasing the weights on all nodes the chance to stagnate in a local optimum is decreased. Then the weights of all nodes on the best path are increased using equations (8) – (11), following the MMAS approach presented in [10]. δ_{opt} is a constant in the range of [0, 1]. The value of δ_{opt} in relation to ρ influences the number of iteration steps the algorithm intensifies the search close to the best path valued with δ_{opt}. As defined in [10] the possible values for the weights have a definition range of [τ_{min}, τ_{max}]. With τ_{max} the algorithm is prevented from converging too fast due to very high weight values. τ_{min} is used to keep the searching area wide and prevent nodes from being excluded from search due to very low weight values.

$$\tau_i^n = (1-\rho)\tau_i^n \tag{7}$$

$$\Delta\tau_i^{k_{best}} = \begin{cases} \delta_{opt} & if\ node\ i\ on\ k_{best} \\ 0 & else \end{cases} \tag{8}$$

$$\tau_i^{n+1} = \tau_i^n + \Delta\tau_i^{k_{best}} \tag{9}$$

$$\tau_i^{n+1} = max(\tau_i^{n+1}, \tau_{min}) \tag{10}$$

$$\tau_i^{n+1} = min(\tau_i^{n+1}, \tau_{max}) \tag{11}$$

With updating the tree node weights, the iteration step n is finished and the next step can be started. The iteration procedure is repeated until the maximum amount of iteration steps $n_{iter,max}$ is reached or $n_{iter,new}$ iteration steps were carried out without finding a new overall best operating plan. To prevent the algorithm from stagnating in local optima, the node weights can be initialized again with τ_{init} after $n_{iter,init}$ iteration steps were carried out without finding a new overall best operating plan. Then we assume that $n_{iter,init}$ is smaller than $n_{iter,new}$. If one of the convergence criterions is fulfilled, the optimization algorithm is finished and the current overall best operating plan is returned as solution to the optimization problem.

VALIDATION AND PERFORMANCE

To validate the operability of the algorithm, it was applied to a performance model of a 50 MW parabolic trough power plant. Simulations were performed using data of a typical meteorological year for a plant site in south Spain.

Cooling System with Cold Thermal Energy Storage and Wet Cooling Tower as Test-Application

The 50 MW parabolic trough power plant is modeled using the hybrid cooling system shown in Fig. 3. The cooling system uses a surface condenser connected to a cTES operated in parallel to a WCT. To validate the optimization results, a reference strategy was developed using two possible values for the operating state variable (OS 1, OS 2) representing the following operating decisions:

- OS 1: if power block is operated use WCT and do not use cTES, if power block is not operated cooling system is in idle mode
- OS 2: if power block is operated use cTES and do not use WCT, if power block is not operated charge cTES with WCT

The cooling system model is connected to a steam cycle model taking the condenser temperature as an input and returning the necessary cooling load. The cooling system model takes the cooling load as input and returns the condenser temperature. The models can be connected by iteratively adjusting the two quantities. Together they form the power block model. The characteristics of the steam cycle model at nominal operation are noted in Fig. 4 (b). Figure 4 (a) shows the gross efficiency and the parasitic power of the steam cycle model depending on the condenser temperature. The values are calculated with a steam cycle model without condenser and other cooling components, always assuming an isothermal and isobar condensation with an outlet steam content of zero. To calculate the parasitic power of the whole power block model, the parasitic power of the cooling system needs to be considered additionally.

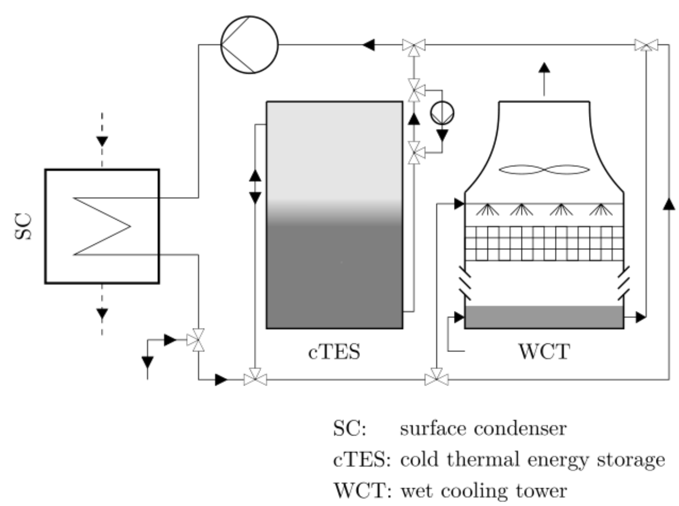
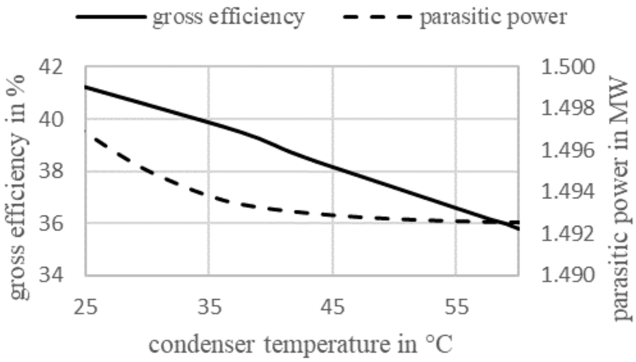


FIGURE 3. Schematic sketch of a hybrid cooling system with a surface condenser connected to a cTES and a WCT in parallel.



Characteristic	Nominal Value
oil inlet temperature	393 °C
oil inlet mass flow	551 kg/s
gross efficiency	39.44 %
gross electric output	50 MW
live steam pressure	105 bar
live steam temperature	380 °C
condensation pressure	0.067 bar
condensation temperature	38.19 °C

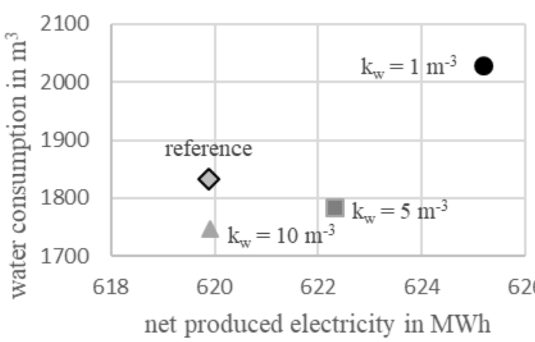
(a) (b)

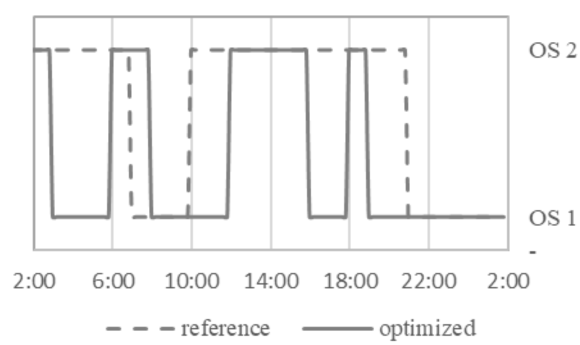
FIGURE 4. Gross efficiency and parasitic power depending on the condenser temperature calculated with the steam cycle model without cooling system at nominal oil inlet state (a) and characteristics of the steam cycle model in nominal operating point (b).

Validation of the Operability

To validate the operability of the algorithm, it was implemented in Python and connected to a performance model of the 50 MW parabolic trough power plant with the hybrid cooling system introduced before. The power plant was investigated following a solar only strategy for a late summer day using the two possible values for the operating state variable (OS 1 and OS 2) to set the operating state of the cooling system. The algorithm was applicated using the following parameters:

$n_{iter,max}=100$; $n_{iter,new} = 25$; $n_{iter,init} = 15$; $\alpha = 1$; $\delta_{opt} = 0.2$; $\rho = 0.1$; $\tau_{init} = 1$; $\tau_{min} = 0.4$; $\tau_{max} = 1$; $k_{el}^{t} = 0.1$





(a) (b)

FIGURE 5. Water consumption and net produced electricity with the reference strategy and optimized strategies for different water weight values (a) and operating plan for the reference strategy and the optimized strategy for a water weight of 5 m^{-3} (b) both optimized and simulated over a late summer day using a performance model of a 50 MW parabolic trough power plant with a surface condenser connected to a WCT and a cTES in parallel.

Figure 5 (a) shows the water consumption and net produced electricity simulated with optimized operating plans for different water weights k_w and with the reference plan. The reference operating plan and the optimized operating plan for a water weight of 5 m^{-3} are depicted Fig. 5 (b). With Fig. 5 (a) it is shown that the algorithm finds operating plans with less water use and higher net produced electricity than the reference plan and that the water use decreases with increasing k_w. Thereby, the algorithm optimizes the operation successfully and it is sensitive to the water value.

Performance of the Algorithm

The optimization problem introduced before with two possible values for the operating state variable, a time step of 1 hour and a time horizon of 24 hours results in a search space of 16 777 216 possible operating plans and a decision tree with 33 554 431 nodes. For 15 optimization runs solving the same optimization problem, the algorithm investigated 33 540 nodes on average and took 48 iteration steps to converge. With 64 model calls per iteration step this corresponds to 3072 model calls. For all optimization runs the algorithm found the same operating plan as solution. Therefore, we can say that the algorithm is operable and stable for the considered problem size. The algorithm cannot guarantee to find the global optimum, but already finds a significantly better solution than the reference plan by only investigating 0.02 % of the search space. The more iteration steps the algorithm is allowed to do, the better the solution can get. Still it is not guaranteed that more iteration steps result in better solutions.

CONCLUSION

This paper introduces a stochastic algorithm to optimize the operation of cooling systems in thermal power plants. The algorithm can be applied to a black box of the power plant including the cooling system. Therefore, different cooling system and power plant configurations can be investigated. The algorithm was applied successfully to a hybrid cooling system with cTES and WCT used in a 50 MW parabolic trough power plant and a problem size in the order of magnitude of 10^7 possible operating plans. The algorithm found an operating plan where the water consumption over 24 h could be decreased by 4.5 % without decreasing the net produced electricity in comparison to the reference plan. As a next step, the algorithm can be applied to other cooling system and power plant configurations and for even larger optimization problems with more possible operating states or a larger time horizon. In this case, additional measures to speed up the algorithm and preventing it from getting stuck in local optima should be considered.

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