Analysis Methods for Ground Resonance in Partial Ground Contact

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ABSTRACT

Soft-in-plane rotor systems are susceptible to a self-induced vibration phenomenon called ground resonance. This dynamic instability results from lag motions of the rotor blades coupling with airframe degrees of freedom while the helicopter is in ground contact. As an addition to previous studies of nonlinear landing gear effects, this work presents the Matrix Pencil Method as a useful additional tool for signal analysis of perturbed nonlinear systems. Contrary to simple logarithmic decrements of decaying time-series, the Matrix Pencil Method allows additional insight into the underlying structure. This makes the method interesting for ground resonance. Additionally, the Lyapunov Exponent Method is introduced to highlight and analyze nonlinear effects in helicopter substitute models.

INTRODUCTION

Ground resonance is a self-induced vibration phenomenon which causes aeroelastic instability. It occurs while the helicopter is in ground contact. It is a dynamic instability resulting from lag motions of the rotor blades coupling with airframe degrees of freedom (Ref. 1). If the lead-lag motion of the rotor blade is transformed from the rotating system into the non-rotating system, it leads to two eigenfrequency components. One is progressive with the absolute value $|\Omega+\omega_\zeta|$ and the other one is a regressive component with the value $|\Omega-\omega_\zeta|$. Ω denotes the rotation velocity and ω_ζ as the lead-lag frequency in the rotating system (Ref. 1). For soft-in-plane rotor systems ground resonance is critical since the regressive lead-lag frequency can be in a range close to the fuselage eigenfrequencies. The collective and differential lead-lag motion ζ_0 and ζ_d, the progressive lead-lag motion ζ_{prog} and the regressive lead-lag motion in the non-rotating system ζ_{reg} are shown in Figure 1. The critical coupling with two hub frequencies in x and y directions are circled with a red-dotted line. These frequencies overlap and the resulting vibrations can cause large-scale damage to the helicopter.

Since the landing gear elasticity largely determines the dynamic behavior of helicopter airframes in the low-frequency range, the study of landing gear properties was a main focus of ground resonance research. Additionally, the landing gear's contact to the ground during slope landings or in operational conditions with non-primed landing zones were studied (Ref. 2), (Ref. 3). In such landings only partial skid contact can occur and influences the dynamic stability of the helicopter. This includes landings in rocky terrain or pits. The large variability of ground properties and contact conditions yields a wide range of friction or damping characteristics and, consequently, of system behaviors. While a full skid contact increases the eigenfrequencies of the helicopter fuselage and therefore leads to a larger stability margin (Ref. 4), soft underground conditions can add additional damping to the system. Based on the work in (Ref. 3) these counteracting effects have to be studied in greater detail. If ground resonance occurs, a pilot is advised to perform immediate take-off and abort the landing, which can be unacceptable in certain mission profiles like rescue operations. Contrary to classical ground resonance, which has been an active research topic for many years (Refs. 1, 18), there has been considerably less focus on such more exotic landing conditions. This means, that, in addition to suitable simulation models, analysis methods have to be deployed to study ground resonance for such landing scenarios. These methods need to be suited for nonlinear dynamics and should be able to handle a variety of contact definition and ground models while giving as much insight into the helicopter-ground model as possible. Preferably these methods should be able to be employed for simulation and for practical tests as those necessary to ensure the safety of the helicopter under all conditions as requested by the European Aviation Safety Agency (Ref. 5). Previous studies of partial ground contact showed the necessity for analysis methods that can evaluate time simulation data of nonlinear models. Therefore, the addition of the Matrix Pencil Method as a method for ground resonance studies in combination with detailed models for partial skid gear contact, is the focus of the presented paper. The method is suitable for large, complex and inherently nonlinear systems (Ref. 6). The proposed work will compare and combine the results of the Matrix Pencil Method as given in (Ref. 7) with those derived by Lyapunov exponents, a method suitable to determine the stability of nonlinear dynamic systems as pointed out by (Ref. 12). As necessary, the Matrix Pencil Method is adapted, meaning additional filter functions are added to counter high-frequency interference.

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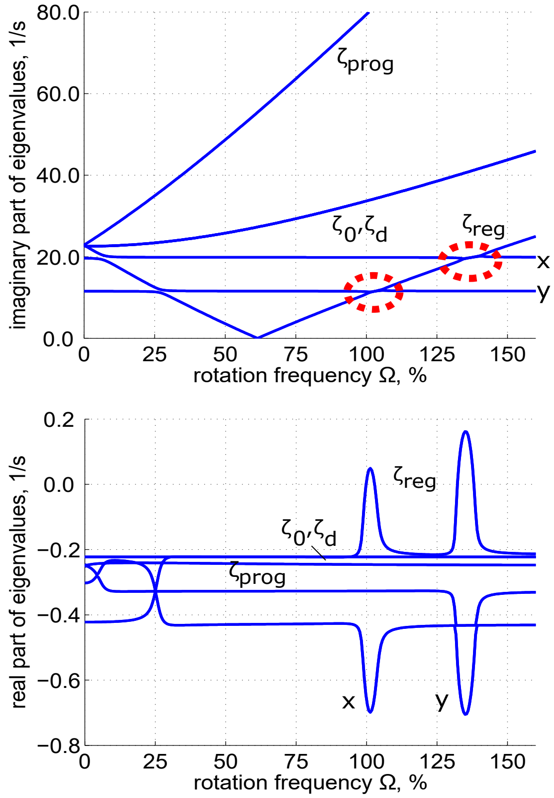


Figure 1: Eigenvalues in the nonrotating system dependent on the rotation frequency; collective, differential blade motion ζ_0 and ζ_d; regressive and progressive lead-lag motion ζ_{ref} and ζ_{prog}, hub motion in x and y directions.

METHOLOGY AND SIMULATION

Based on the study of partial ground contact during take-off by Kessler and Reichert (Ref. 8) and slope landing studies by Dieterich (Ref. 2), the partial skid contact was the focus of previous ground resonance research. Dedicated models for the investigation of the influence of contact area and different contact models were elaborated in a previous CEAS publication by the authors (Ref. 3). These models allow the detailed simulation of time-variant contact conditions, including damping and friction effects.

Structural Model

In this work, two structural models are used. The first one is used to validate the approach, the second to extrapolate it to a detailed helicopter model. The first model is a simplified two-degree-of-freedom landing gear model based on the one presented in (Ref. 8). In this model, the helicopter is coupled with a nonlinear landing gear and reduced to its heave and pitching degrees of freedom. In this representation the helicopter is assumed rigid but excited by forces on the rotor hub. It is schematically represented in Figure 2.

Mathematically spring-damper force can be expressed as

$$F_{f,r} = \frac{1}{2} \cdot k \cdot z_{f,r} + \varepsilon \cdot z_{f,r}^3 + d \cdot \dot{z_{f,r}}. \tag{1}$$

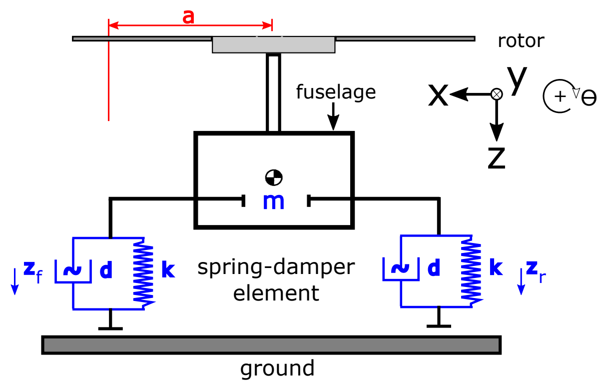


Figure 2: Simplified helicopter model with nonlinear landing gear.

where the deflection at the front element is described by $z_f = z - a\,\Theta$, with a being the horizontal distance to the airframe center of gravity and Θ as the pitch angle of the airframe relative to the ground. The rear deflection is given by $z_r = z + a\,\Theta$. The hardening and softening of the spring-damper element is described by the cubic term. The factor ε describes the nonlinear spring behavior, increasing or decreasing the stiffness depending on the chosen parameter. The landing gear stiffness was chosen as $k = 3.7 \cdot 10^5\, N/m$, the damper constant as $d = 1.1 \cdot 10^6\, N \cdot s/m$ and the airframe mass as $m = 1906.4\, kg$ to resemble a Bo105, since this helicopter model is well known.

The second is a Multibody Simulation Model (MBS model) of an EC135 with a detailed FEM landing gear, complex contact configurations, a rigid fuselage and rotor with four hinged, rigid blades.

The helicopter MBS model on the left side in Figure 4 is used. It consists of an elastic rotor model attached to a rigid fuselage model. Each blade was modeled as a rigid blade with spring elements at the blade hub connection to ensure a first lag mode corresponding to the real EC135 blade. Instead of a reduced lead-lag damper, calculations were conducted using higher rotation frequencies. The overall elasticity of the entire airframe mostly results from the landing gear flexibility. The fuselage flexibility can be neglected. In the following, the fuselage is considered rigid. Its properties like mass, inertia and geometric dimensions are chosen in reference to the EC135. The simulation of the landing gear model was based upon a complete finite element model (Ref. 13). The multibody simulation uses a modal representation with a truncated number of modes. Using component synthesis method, the landing gear model was reduced to 20 significant modes. This is a standard approach in multibody programs like SIMPACK to include flexible bodies and shown in 3. It requires a clear definition of markers on which contact laws and forces are applied. Connections to the fuselage were simulated by elastic bushing elements to ensure that the modal representation of the landing gear attached to a rigid fuselage shows the same landing gear eigenfrequencies as the original model.

Contact to the ground is simulated by polygonal contact elements (PCM-elements) in SIMPACK. They base body surfaces on polygon meshes derived from the underlying FE-

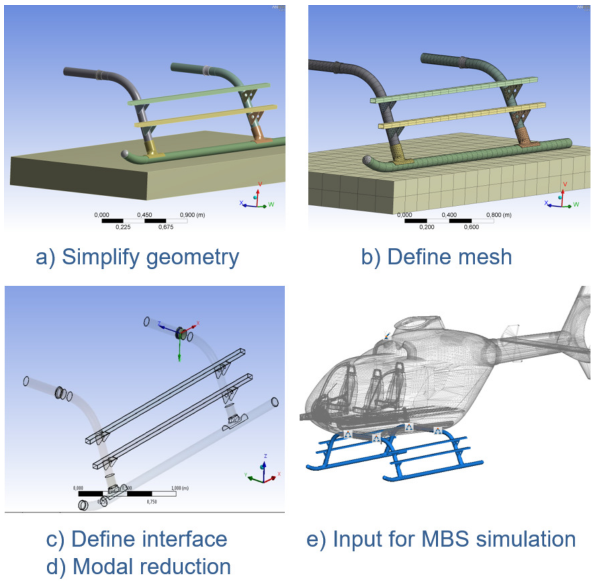


Figure 3: Workflow to include complex flexible bodies into a MBS simulation.

mesh or attached CAD-files. The contact force determination relies on the elastic foundation model. As described in (Ref. 11) these elements can be deployed to model time-variant, multi-point contact conditions. A use case is shown on the right side of Figure 4. These contact elements are used to improve the contact simulation in comparison to standard spring-damper elements.

The structural model has been improved as the previous version used a high value for the ground stiffness leading to a large restorable force and a seemingly unnatural "bounce´´ of the helicopter. The new ground property represents concrete. For soft undergrounds, it is planned to use parameters based upon crash tests. However, the right simulation of underground characteristics is a topic for future studies.

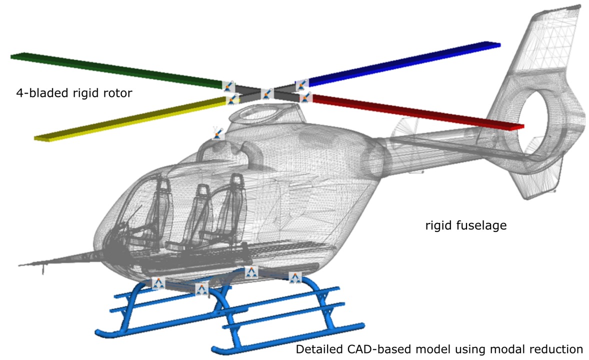
The structural model has been improved to include the gear motion of the helicopter during ground resonance, which was previously modeled incorrectly.

Aerodynamics

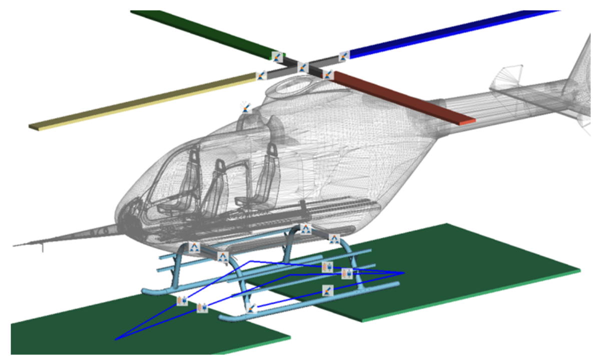
For the aerodynamic simulation, the multibody structural model in SIMPACK is coupled with the rotor airloads model of the rotor dynamics code (Ref. 14). Aerodynamic forces are calculated with twenty elements for each main rotor blade in linear aerodynamics description and distributed as a constant ring area. For trimmed conditions the SIMPACK trim mode is activated.

Contact Simulation

The helicopter landing and the initialization of a ground resonance condition is simulated in five steps that correspond to the sequence of events of a typical landing. First, the helicopter is hovering close to the ground in a trimmed condition. Then the helicopter is lowered to the ground and contact



(a) Helicopter model EC135



(b) EC135 with skid contact at four areas

Figure 4: Helicopter model EC135 with sub-models and contact configuration

is initiated. Since contact conditions and the behavior during contact can vary the end of the first phase is not defined based on the presence of contact or contact forces exceeding a certain threshold, but rather with a fixed duration of 20 seconds after the initiation of this phase. At the end of the 20 seconds, the time responses of the helicopter are analyzed to determine whether the model shows increasing divergent behavior. In that case, then the simulation is not pursued further as the desired ground resonance was already successfully triggered. Should the system remain stable two counteracting 500 N tip loads are applied to the first and third blades for the duration of 0.2 seconds, making it an impulse like excitation. The estimation for the force values is an engineer's guess, based on expected forces at the rotor hub for hard landings. It has to be said, that this is a significant simplification that is made to keep the simulation effort manageable, since this model and its time signals will mainly be used to illustrate the analysis approach of its time signals. As pointed out in section (Ref. 20), the excitation amplitude can affect the system behavior, which is—to certain extend—in accordance with previous studies. It was shown in (Ref. 3) that two effects counteract each other in regard to ground resonance. On the one hand the shift of nacelle frequencies due to ground contact and on the other hand the influence of soft underground, which leads to additional damping. Speaking in terms of the simplified model, this is corresponding to a reduced excitation amplitude. After the impulse excitation a second twenty-second simulation phase is performed. The dynamic response of the helicopter to the applied perturbation exhibits its internal dynamics and should capture the oscillatory character of the system.

The delay time followed by an artificial external impulse aiming to excite the lead-lag motion is a simplification of real landing conditions, where chaotic behavior, an uplifting landing gear and loss of ground contact could occur. If ground contact is lost at any time during the simulation, the results obtained after the loss of ground contact must be discarded for all further analyses, since this is a true chaotic behavior. The detection of such a condition and the acquisition of critical system parameters leading to it, would be the end result of such a simulation run. For the contact conditions itself, this work is limited to standard full contact. However, the contact representation itself is the same as for patch-like contacts in previous publications (Ref. 3). Such limitations of landing configurations and the selection of a subset of possible landing conditions are necessary to limit the simulation effort, but is not a restriction for the analysis approach itself. More in-depth studies of contact varieties with these methods are left for future work.

Analysis Methods

As will be shown in the next section, some excited nonlinear systems show sudden, different behavior depending on the current frequency and amplitude of the excitation. For the analysis of varying excitation amplitudes, Lyapunov exponents are used. For the sweep over the excitation frequency the Matrix Pencil Method is used, which can be categorized as a type of perturbation method. This "grid-type´´ analysis suite is tested in this work for exemplary test cases.

Matrix Pencil Method

To account for nonlinear behavior due to partial ground contact and to better analyze time simulation data the Matrix Pencil Method is used. The helicopter model is set in a given landing configuration like a full contact as seen on the right of Figure 4. As described in the previous paragraph, once the system has "settled in" after 20 seconds, a sudden excitation impulse is applied. With the rotation of the rotor, this causes a circular motion of the rotor center. The time signal response of dedicated markers at the nacelle, the tail boom and the landing gear can be processed. The markers were chosen to capture the dominant fuselage modes. The measurement duration of 20 seconds was chosen, since no change in system behavior was observed for the conducted tests. The was verified by examining said marker signals. The simulation duration was not extended, although theoretically nonlinear effect might appear at far-off times (Ref. 8), as it was deemed unpractical. Landings are followed up by additional actions. The marker selected is in the non-rotating hub frame, which is acceptable due to the rigid helicopter nacelle. This marker showed the most "clean" signals. The distance over the rigid nacelle working as a lever to the center of gravity and the landing gear. Additionally, the vibration duration was acceptably long for signal analysis. The Matrix Pencil Method promises to derive an approximated model of the helicopter with an output-only approach and further information about frequencies and eigenvalue compared to simple decay ratios. The basic idea of the Matrix Pencil Method is to extract modal information from the system's response to a perturbation to fit the actual waveform to a predefined waveform of the following type:

$$y(t) = x(t) + n(t) \approx \sum_{i=1}^{M} R_i e^{s_i \cdot t} + n(t) \quad (2)$$

with $y(t)$ as the observed time response, $x(t)$ as the signal and $n(t)$ as noise in the system. The formulation of the waveform contains the Pencil Parameter M, the residues of complex amplitudes R and the complex eigenvalue s_i defined as

$$s_i = -\alpha_i + j \cdot \omega_i\,, \quad (3)$$

with the damping factors α_i and the angular frequencies ω_i. For discrete time data this can be written as

$$y(kT_s) = x(kT_s) + n(kT_s) \approx \sum_{i=1}^{M} R_i \cdot z_i^k \quad (4)$$

This equation is formulated for k sampling points, the sampling period T_s and the roots z_i as:

$$z_i = e^{s_i \cdot T_s} = e^{(-\alpha_i + j \cdot \omega_i) \cdot T_s} \quad (5)$$

In detail the algorithm for the Matrix Pencil Methods consists of the following steps.

1. After an external excitation, the free decay linear response of the system is sampled to form a discrete pattern data set. Then the discrete data sets are organized in a form of a Hankel matrix.

2. The Hankel matrix undergoes a Singular Value Decomposition (SVD). Singular values and vectors corresponding to the noise subspace are discarded, basically filtering the given signal. The threshold determining, if a singular values σ_c will be discarded, can be determined based on the largest singular value σ_{max} and a given accuracy value p_{accu}. This definition, given as:

$$\frac{\sigma_c}{\sigma_{max}} < 10^{p_{accu}} \quad (6)$$

   leads to M significant singular values or poles, with M being called the Pencil Parameter. As described by (Ref. 10) varying the number of poles around the determined one and comparing them, reveals the actual, characteristic poles of the signal. In praxis this means that despite a good overall fit, the result for frequency or amplitude will only be good for those singular values.

3. The SVD leaves a truncated Matrix $\hat{Y}_\tau$. From the truncated matrix two sub-matrices, defined as $\hat{Y}_a$ and $\hat{Y}_b$, are generated, leading to an eigenvalue problem from which eigenfrequencies and damping values are calculated. From the eigenvalue problem the eigenfrequencies and damping values are calculated.

4. In an additional step, once the pencil parameter M and the roots z_i are known the complex residues R_i can be determined.

In order to improve the signal analysis and as an addition for practical applications a fourth-order low-pass Butterworth-Filter was added to the analysis routine. One advantage of the Matrix Pencil Method is, that it not only gives an estimation for the frequency but also for the amplitude. So, it can be linked to all parameters affecting the system behavior. This method was selected because it offers a more comprehensive way of system analysis with its simplification of the given system. Its approximation of frequencies, amplitudes and damping provides deeper insights into the system. The marker whose time signals were analyzed were chosen to capture the dominant fuselage eigenmodes. For decreasing damping values of these signals, it indicates that the energy of the overall response becomes strongly dominated by another mode, instead of just detecting a decrease of the overall response as this was the case for logarithmic decrements. It can be used in conjunction with other methods.

Lyapunov Exponents Method

An alternative to the usage of time simulation in combination with the Matrix Pencil Method are Lyapunov exponents. As pointed out by Tamer and Maserati (Ref. 15), the Lyapunov Characteristic Exponents can be applied without the need for a steady-state or the assumption of a periodic system for rotorcraft aeroelastic stability. Lyapunov Characteristic Exponents were already applied to ground resonance studies of dissimilar lead-lag dampers and helicopters with nonlinear lead-lag dampers. They are defined as a logarithmic growth rate of a perturbation of the original system. For a time-continuous dynamical system, consider the behavior around a point $x(t)$ in phase space and a nearby trajectory $x(t)+\delta(t)$, where $\delta(t)$ is a small deviation of the trajectory. Tracking how $\delta(t)$ changes over time the Lyapunov exponent can be defined as

$$|\delta(t)| \approx |\delta(0)|e^{\lambda_{Lyapunov}} \tag{7}$$

Similar to the largest eigenvalue of a matrix, the largest Lyapunov exponent dominates the behavior of a system. An exponent larger zero indicates that the corresponding trajectory diverges exponentially, meaning it is unstable. For periodic motion the largest Lyapunov exponent is zero. If it is smaller than zero, the trajectory converges to zero. For the calculation of Lyapunov Characteristic Exponent in calculations with constant time step see (Ref. 12). The Lyapunov Exponent Method extended for time series by Wolf and Rosenstein (Ref. 19) is used to determine the Lyapunov Exponent map for the simplified landing gear model. This extension to the classical method is accepted in the scientific community but needs to be validated for the use case at hand. This method is a more practical application of Lyapunov Exponents as advised for helicopter stability analysis (Ref. 12).

NONLINEAR EFFECTS IN GROUND RESONANCE

In order to simplify the analysis of nonlinear ground resonance substitute models based on harmonic excited lag-damper models were used in previous studies (Refs. 3, 8). Since self-induced, nonlinear systems can in principle be compared to the Duffing equation as given in Equation 8, some basic characteristics of nonlinear systems can be showcased, which can then be extrapolated to the simplified landing gear model. The first three terms on the left side and the drive term on the right side represent a harmonic oscillator. The left side is modified by a nonlinear cubic term, which is used in this example as a placeholder for nonlinear terms stemming from the contact conditions. In case of $\alpha > 0$ and $\beta > 0$ this system shows a hardening of the spring damper thus resembling the landing on soft soil which provides exponentially more resistance, the deeper the skid sinks in. The softening case appears during take-off and landing as pointed out by Kessler (Ref. 8).

$$\ddot{x} + \gamma x + \beta x^3 = \Omega_{ref} cos(\omega\, t) \tag{8}$$

The Duffing Equation is a standard example for systems of nonlinear dynamics. Its basic mathematical formulation was used as a prototype for the usage of different methods aiming to understand the behavior of nonlinear systems. Here the Duffing Equation is adapted for the simplified simulation of a landing gear in ground contact.

In the case of ground resonance, the equivalent to harmonic excitation is the imbalanced rotor head and the nonlinear terms are corresponding to nonlinearity from the ground contact conditions in combination with landing gear flexibility. Extrapolating from this substitute equation to the simplified landing gear can showcase some basic characteristics of nonlinear systems, which are of high interest for studies of self-induced vibration systems and can be used to list the requirements and test the analysis methods for ground resonance beyond the classical approach. It is used as additional examples to validate the methods presented in this paper.

To visualize a qualitative change of state in nonlinear systems under the influence of selected system parameters like excitation amplitude, bifurcation diagrams can be used. Bifurcation is a transition in the dynamic states abruptly as a result of a system or control parameter change. The system undergoing bifurcation changes its dynamic behavior affecting the stability of the system. There are several classes of bifurcation from which the supercritical pitchfork bifurcation and the Hopf Bifurcation shall be mentioned here. A supercritical pitchfork bifurcation splits a stable equilibrium condition into two stable and one unstable equilibrium. A behavior which was witnessed in landing and take-off conditions with increased lift to mass ratios (Ref. 8). Another type is the Hopf Bifurcation which just describes the appearance of limit cycles.

In Figure 5 the nonlinear term of the Duffing equation was singled out and plotted in a bifurcation diagram. The blue line shows the calculated Lyapunov exponents. The purple line depicts the approximation based purely on time simulation data.

As can be seen, the value depicts the unstable condition while the slope towards the zero value is a measure for the deteriorating system stability margin.

At $\gamma = 1.5$ and $\gamma = 2.2$ the cubic term runs into a pitchfork bifurcation. The Lyapunov Exponents indicate such a change in dynamic behavior. After the second bifurcation, the system becomes unstable, no equilibrium point can be reached and chaos ensues.

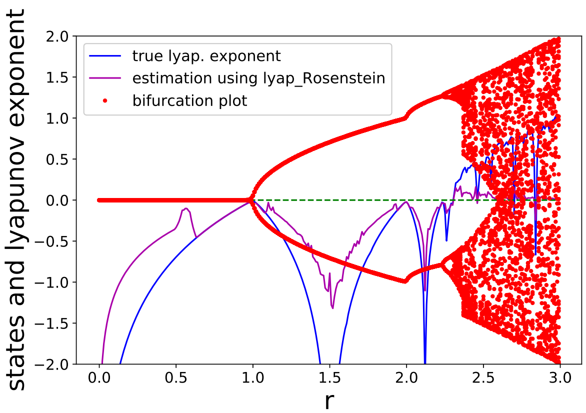


Figure 5: Cubic-term bifurcation map and Lyapunov exponent.

As can be seen in the above plot, a bifurcation (in the orange map) is indicated when the Lyapunov exponent (blue) approaches zero (green line). The onset of deterministic chaos, i.e. the danger of rapid divergence of two initially close dynamics, is indicated by the Lyapunov exponent becoming positive (crossing the zero line). This happens after a double supercritical pitchfork bifurcation. In contrast to that locations in the plot, where the Lyapunov exponent diverges to deep negative values (at times even infinite values) indicate extremely stable equilibrium points in the system.

A phenomenon encounter in previous studies (Ref. 8) was limit cycles. For harmonically excited systems or self-induced ones, this means that a system does not respond with the harmonic frequency, like linear systems but can fall into stable oscillation with sub-harmonic components. In the results section the simplified landing gear model was modified to exhibit such oscillations in order to test the Matrix Pencil Method.

In its most extreme form, the system can exhibit deterministic chaos. For such a system similar initial values do not lead to similar effects in the long run. The system itself is clearly defined with known initial values and deterministic dynamics but the long-term behavior is still unpredictable. According to the Poincare-Bendixson theorem, finite-state, linear systems can never show chaos. But for the problem at hand, this can be the case. Should a system transform to deterministic chaos, then that is the result of the test case. In a chaotic system no meaningful information can be derived from the system.

The Matrix Pencil Method uses sinusoid functions as an estimate for system behavior. While a combination of sinusoidal functions stay stable and do not show chaotic states, they at least can encapsulate basic changes in system behavior due to parameter changes i.e. bifurcations. To showcase this, Figure 6 contains a bifurcation map and Laypunov Exponents of the following form, with the varying system parameter r, indicating that the chosen approach is sensible within the limits of practical applications.

$$x_{t+1} = r \cdot sin(x_t) \tag{9}$$

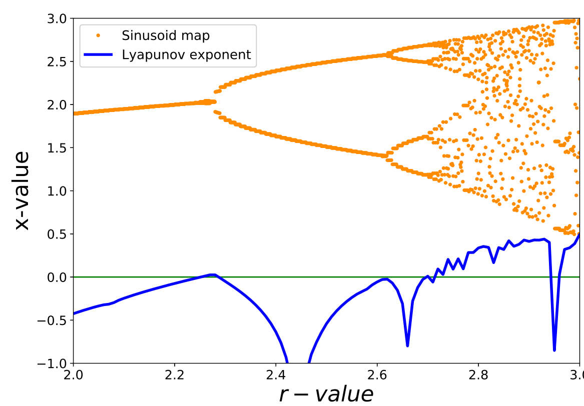


Figure 6: Sinusoid-term bifurcation map and Lyapunov exponent.

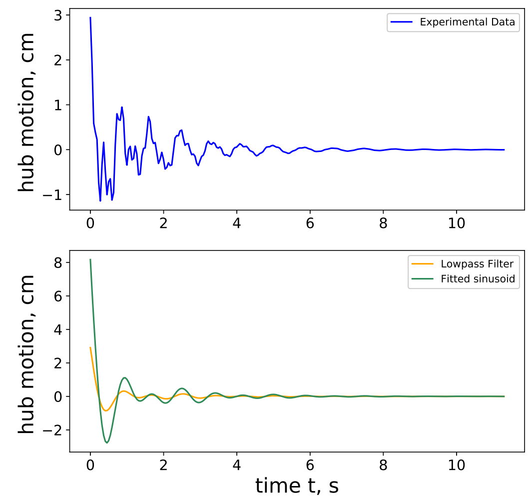
The next section shows that additionally, a system approximation as a sum of sinusoid functions exhibits a more stable behavior than when using logarithmic decrements.

RESULTS

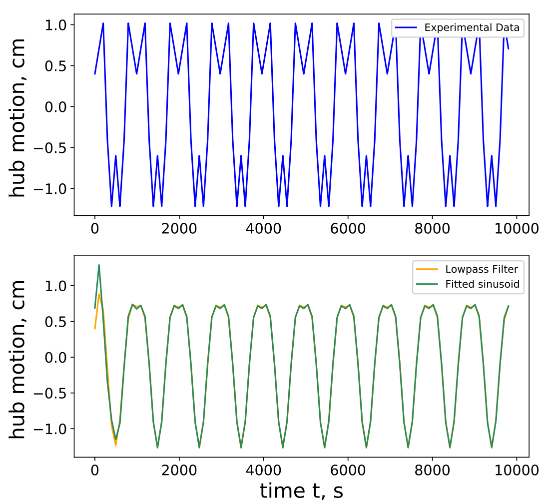
Preliminary studies were conducted with synthetic signals resembling the time simulation data given in previous studies. The left side of Figure 7 shows the unfiltered phase plot of a particular solution for a system in a stable configuration. The corresponding data is shown in Table 1. The figure shows the synthetic sinusoidal test signal with known amplitudes, frequencies, damping and added noise level as a blue line. It is composed of four overlapping sinusoidal signals and given in the form

$$y(t) = \sum_{i=1}^{4} R_i e^{d_{i(t)}} \cdot cos(2\pi \cdot f_i(t)) \tag{10}$$

with amplitudes $R_i = (0.4, 1.0, 0.89, 0.65)$, frequencies $f_i = (80, 120, 250, 560)$ and damping values $d_i = (70, 50, 90, 80)$ and a randomly generated noise values of 3 dB. The generated noise aims to test the robustness of the method.



(a) Helicopter model time simulation data example



(b) Limit cycle signal

Figure 7: Time simulation data example; characteristics of test signal and its reconstruction

Applying the Matrix Pencil Method with an accuracy level of 10^{-3} leads to a Pencil Parameter of $M = 8$. The approximated signal using the amplitude, frequency and phase values from the analysis is given as the green, continuous curve in Figure 7. The corresponding data is shown in Table 2. This "restored" signal reproduces the filtered basic characteristic of the given test signal. Since previous studies showed that limit cycles can occur (Ref. 17), synthetic periodic signals with constant amplitude were generated by forcing the substitute structural model into a limit cycle condition. The signal in Figure 7 functions as a representation of these tests.

For the Limit Cycle Test case, despite the large differences in the amplitude, the characteristics of the signal could be kept and the R2-value = 0.994 indicates is good fit. However, one has to keep in mind, that the goal is not to completely reproduce the signal, but to find a suitable substitution. Compared to the restored signal, higher frequency components are eliminated. As described by (Ref. 10) the varying number of poles which are expected for the signal and comparing them, reveals the actual, characteristic poles of the signal.

Table 1: Comparison of original test signal and approximated signal for time simulation data example.

Amplitudes				
original	0.4	0.65	0.89	1.0
reconstructed	0.4	0.65	0.89	1.0
Ang. Freq. (rad/s)				
original	560.0	250.0	120.0	80.0
reconstructed	562.25	251.00	120.48	80.32
Damping (Ns/m)				
original	80.0	90.0	50.0	70.0
reconstructed	80.32	90.36	50.20	70.28

Table 2: Comparison of original test signal and approximated signal for limit cylcle data example.

Amplitude		Angular Freq.	
original	reconstructed	original	reconstructed
1.0	-	2.5133	-
-0.5	0.9974	1.8849	1.8853
-	0.8234	-	1.6098
0.2	0.2898	1.2566	1.2568
-0.3	0.0278	0.6283	0.6283

In the next section the Matrix Pencil Method is used for the complex helicopter model to determine its damping characteristics, which are compared with the ones determined by Multiblade Coordinate Transformation and eigenvalue analysis.

Results of the EC135 model

One of the main goals for the investigations with the complex helicopter model is to improve the computation of decay ratios after sudden lead-lag excitation of the rotor system. Previous attempts to analyze the resulting system from a sudden perturbation were based on the calculation of the logarithmic decrement. However, this method proved to be prone to errors and has the significant disadvantage of assigning one decay ratio to the complete system, although the underlying influences of the system reaction to perturbation can be multiple modes or coupling modes. An example of such a behavior can be seen in Figure 8, which shows the non-rotated hub position time signal in y-direction. This is corresponding to the lateral helicopter direction. The Figure shows two time signals. The first one, colored red in the background shows the signal at 105% nominal rotation speed. It is a clear exponential decaying signal as is expected. The second, gray line shows the system's reaction to excitation at 112%. At a rotation speed near one of the lower frequencies of the nacelle, which were determined to be at 113.5% and 117% of the nominal rotation speed by Multiblade Coordinate Transformation and eigenvalue analysis for this model. This signal shows a larger reaction amplitude. The gray line starts with a higher amplitude and from 7.5 seconds onwards its amplitude is very slightly smaller than the one of the red curve. This shows the increased influence of an underlying frequency. Such effects on the damping behavior and the underlying frequencies can hardly be described by a single parameter like the overall logarithmic decrement.

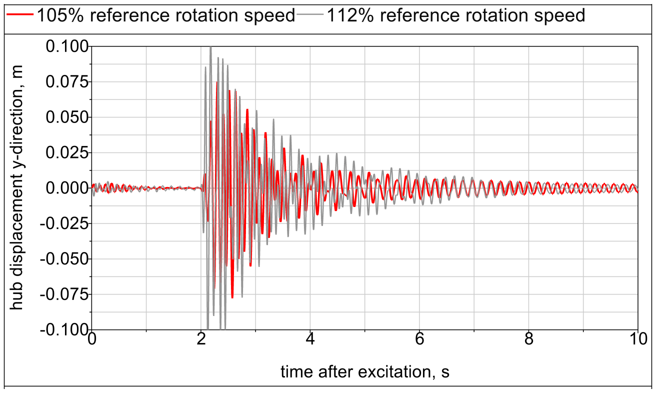


Figure 8: Time signal after excitation for different rotational velocities.

Contrary to that, using the Matrix Pencil Method, decay ratios of corresponding sums of sinusoid functions were determined over a system sweep. In Figure 9 the decay ratio sweep for the complex helicopter model in full ground contact is shown. The resulting dips in the damping curves fit quite reasonably to those expected at the coupling frequencies of the helicopter model.

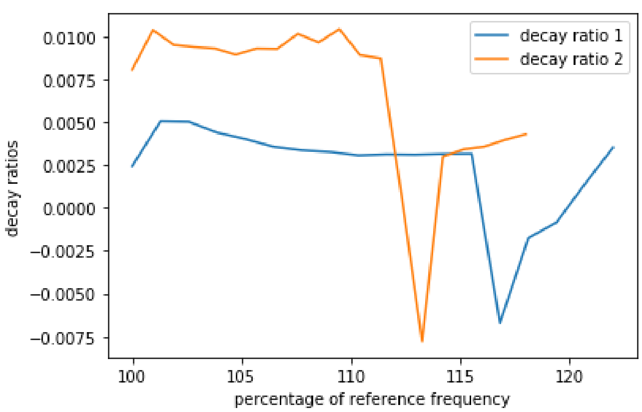
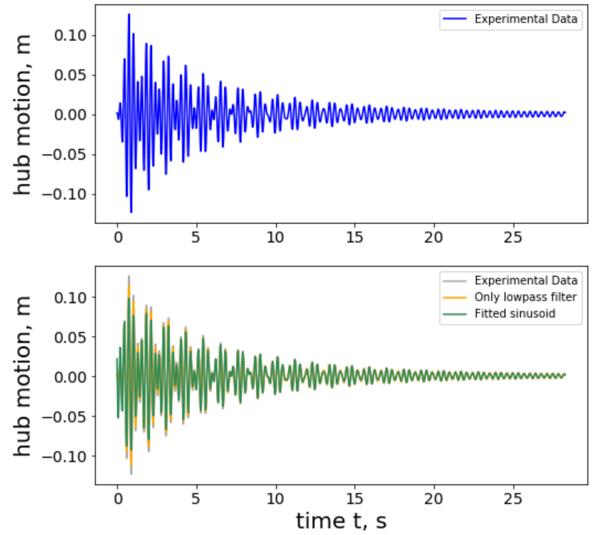


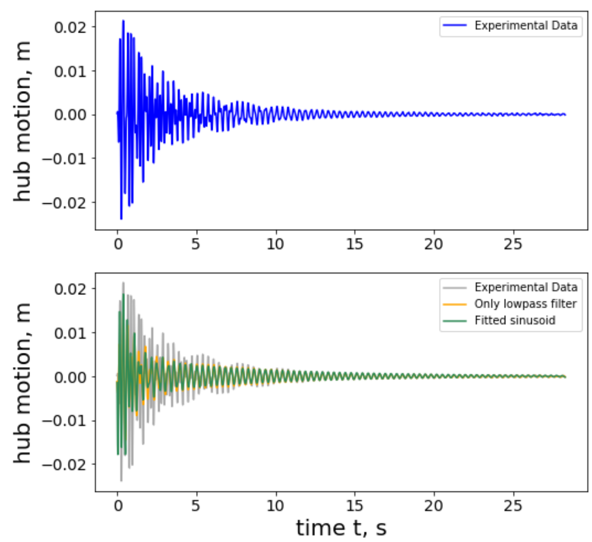
Figure 9: Decay ratios over rotation frequency for helicopter in full contact.

However, one has to consider that the damping values themselves are those of the approximated models. Only the characteristic of the damping behavior is approximated.

To illustrate the fitting of actual signals, Figure 10 shows two examples. On the left side the vibration response for the nominal rotation speed is shown. Even for such a standard test case, the determination of the decay ratios using the logarithmic decrement can be difficult or require manual adjustments. The Matrix Pencil Method produces better results.



(a) Helicopter model time simulation at nominal rotation speed



(b) Helicopter model time simulation at 112% rotation speed

Figure 10: Characteristics of test signal and its reconstruction

The restoration system almost perfectly fits the original signal. For systems near conditions of coupling frequencies, the fit quality decreases but still captures the overall characteristics. A meaningful logarithmic decrement could only be determined as the signal envelope. Further statements about the underlying system could not be made.

In combination with the Lyapunov Exponent method the influence of restoring forces of the ground contact configuration and the influence of ground damping and hardening during contact and its capacity to dissipate energy from the excited system can be studied. When encountering external influences like hardening or softening underground a basic stability test can be conducted with the Lyapunov Exponents. Additionally, the Matrix Pencil Method allows a complex helicopter model approximation with a simplified sum of sinusoids. The method itself showed robustness and adaptability to nonlinear effects like limit cycles of noised signals.

The simulation results for the simplified model show that the Lyapunov Exponent method can detect a change in the frequencies and damping values at the rotation frequencies at which we expect a coupling between rotor modes and fuselage modes (near (113.5% and 117%) of the nominal rotation speed. Lyapunov Exponent maps and bifurcations diagrams, which are well-known tools for the analysis of nonlinear systems were shown to be a useful addition to the analysis of ground resonance. For the simplified model the Lyapunov Exponent method extended for time series analysis was employed.

The results for the full model show that the Matrix Pencil Method is a useful addition to the simulation of ground resonance with Multi-Body-Systems. The method works with the signals from time simulations of perturbed rotor systems, with contact conditions and detects changes in the damping of the dominant modes. In combination with a drop in the fit quality, this is a measurement for instability. The analysis tool set shown in this paper adds more flexibility for ground resonance analysis and is ready to be used for advanced contact simulations.

CONCLUSION

Two helicopter models for ground resonance stability analysis have been developed. One is a simplified helicopter model with spring-damper elements to showcase basic nonlinear effects and constitutes a testbed for analysis methods, the other is a more complex helicopter model with flexible landing gear and a real contact model with realistic contact conditions. The Matrix Pencil Method for the analysis of nonlinear systems, tested and verified with examples from the literature was applied to the helicopter models for the analysis of ground resonance. First tests with a simplified landing gear model and a full helicopter model show promising results.

One of the advantages of the Matrix Pencil Method and the Lyapunov Exponent method is that the basic information about nonlinearity in the system is kept. In the calculation of a fitting sinusoid estimate one assumes a periodic solution similar to Floquet theory. It is just a fitting of time series data. The original nonlinear information is still available, allowing further analysis or combinations of additional methods. In combination with the quality of the fit, the awareness of this assumption is always given and any configuration not fitting this assumption is clearly visible. Since the Matrix Pencil Method is not restricted to approximating only one parameter as it is the case with decay ratios, it was found more suitable to determine stability margins and give an indication of underlying eigenmodes responsible for ground resonance. The Lyapunov exponent method described in this paper does in general not require the assumption of a linear time-invariant or linear time-periodic solution. It is a true measure of the stability margin of the given system. The Lyapunov Exponent trajectory allows predicting of chaotic behavior to a certain extent. The Lyapunov exponent method for time series presented here is an extension of the classical Lyapunov Exponent Method, which uses directly the mathematical time derivative of the system instead of calculating it from the given time series data. The first results look promising, but this fact should be kept in mind. Nevertheless, considering the complexity of nonlinear systems and the fact that there is no absolute a priori determinable stability factor for all initial conditions, the methods presented in this work are valuable tools. The Lyapunov Exponents from time series and the Matrix Pencil Method for time series allows system evaluations without setting up a dedicated mathematical model and the a priori assumption going into them. The usability of both methods for simulations and practical tests alike, allow an easier comparison between simulation data and practical tests. This, in return, allows the development of improved simulations for ground contact simulations.

OUTLOOK

It is planned to combine the simulations and analysis methods in this paper with shake test of a EC135 landing gear in low configuration. To check the vibration behavior the landing gear will be attached to a frame of a fuselage dummy. The helicopter nacelle will be replaced by a support frame attached to the landing gear. This dummy will contain weights to approximate the inertia properties of a real helicopter similarly as done in the numerical simulations. The construct will be softly suspended in a rig and excited. For various force and moment excitations the structural response will be measured. The dynamic response signals of the measurement sensors will be analyzed through vectormeters and will be compared to the results of the numerical simulation.

Special focus will be given to the attachment stiffness, since preliminary modal analysis with a finite element model showed its major influence on the overall eigenfrequencies.

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