Recovering unambiguous differential ionospheric phase screens

Francesco De Zan, Giorgio Gomba, Paloma Saporta, and Víctor Navarro Sánchez
German Aerospace Center (DLR), Wessling, Germany

Abstract

Synthetic aperture radar interferograms are affected by ionospheric disturbances. The split-spectrum method exploits the peculiar propagation of the radar waves through the ionosphere and recovers a phase screen that can be used to correct the interferogram to improve the estimation of the geometrical phase. However the recovered screen is ambiguous and it is not possible to reach an absolute phase, which would enable absolute geolocation or absolute motion recovery. We propose splitting the range spectrum in three sub-bands to recover the unambiguous ionospheric difference with a linear problem; we interpret the solution of the linear system as the difference as a demodulation between an ambiguous and an unambiguous estimation of the ionosphere; and we analyse the performance of the proposed technique.

1 Introduction

The recovery of ionospheric phase screens in SAR interferometry relies on the fact that the ionosphere affects phase and group delays of electromagnetic waves in opposite ways [1]. The split-spectrum method is essentially taking the difference between phase delays and groups delays and scaling it to total electron content (TEC) values or interferometric phases. Since the phase delays are ambiguous with the a 2π cycle, also the TEC values and ionospheric phase screens are ambiguous[2]. In L-band the ambiguity interval is just 0.2 TECU. The ambiguity concerns only a single constant in an entire scene or data take: spatial variations within the scene are normally recovered by the phase unwrapping procedure.

The very mechanism that allows ionospheric phase screen recovery also prevents obtaining an absolute phase since groups delays and phase delays are no longer simply coupled, as it is the case with non-dispersive propagation. It is therefore not possible to measure absolute motion with interferometric accuracy in differential SAR interferometry, or absolute positioning of a DEM in across-track SAR interferometry.

The remedy that we suggest in this paper is using three range sub-bands or in general relying on the curvature of the phase dependency on frequency induced by the dispersive ionosphere propagation. This is very similar to what is done with GNSS, although our problem has some specificity and is simpler. It is also similar to what suggested in [3] as a cross-correlation of sub-bands, but we are analysing the problem with more detail here. With the proposed approach it will be possible to recover the absolute phase also for pairs or stacks affected by the ionosphere, and enable the corresponding applications. This is of particular interest for lower frequencies (L-band), where it is also more feasible compared to higher frequencies (C-band, X-band). Once the relative TEC differences in a stack are all unambiguous, with extra knowledge about one single absolute ionosphere in a stack, we could recover all absolute screens.

2 Mathematical derivation

2.1 Linear problem with three unknowns

We can write the interferometric phase as [1]

$$\Delta\Phi = -\frac{4\pi f}{c}\Delta r + \frac{4\pi K}{c f}\Delta\text{TEC} + 2\pi n \tag{1}$$

where we can recognize the usual propagation term proportional to the range difference Δr and the ionospheric propagation term proportional to the TEC difference. The observed phase is naturally ambiguous and therefore there is an unknown number n of entire cycles. In the equation, f is the frequency, c is the speed of light and $K \approx 40.28\ (\text{m}^3/\text{s}^2)$.

By measuring the interferometric phase at three different frequencies (f_0, f_L and f_H), we can build a linear system with three equations and three unknowns:

$$\begin{pmatrix} \Delta\Phi_0 \\ \Delta\Phi_L \\ \Delta\Phi_H \end{pmatrix} = \begin{pmatrix} -\frac{4\pi f_0}{c} & \frac{4\pi K}{f_0 c} & 2\pi \\ -\frac{4\pi f_L}{c} & \frac{4\pi K}{f_L c} & 2\pi \\ -\frac{4\pi f_H}{c} & \frac{4\pi K}{f_H c} & 2\pi \end{pmatrix} \begin{pmatrix} \Delta r \\ \Delta\text{TEC} \\ n \end{pmatrix} \tag{2}$$

We think that we can safely assume to have the same number of missing cycles at the three frequencies. Cross-correlation of the two images should anyway easily detect a phase ramp across the frequencies, enabling relative unwrapping between the sub-bands.

Solving the system for n we obtain:

$$\begin{aligned} 2\pi\hat{n} = & \frac{f_0(f_L+f_H)}{(f_H-f_0)(f_0-f_L)}\Delta\Phi_0 - \frac{f_L(f_0+f_H)}{(f_0-f_L)(f_H-f_L)}\Delta\Phi_L \\ & - \frac{f_H(f_L+f_0)}{(f_H-f_0)(f_H-f_L)}\Delta\Phi_H. \end{aligned} \tag{3}$$

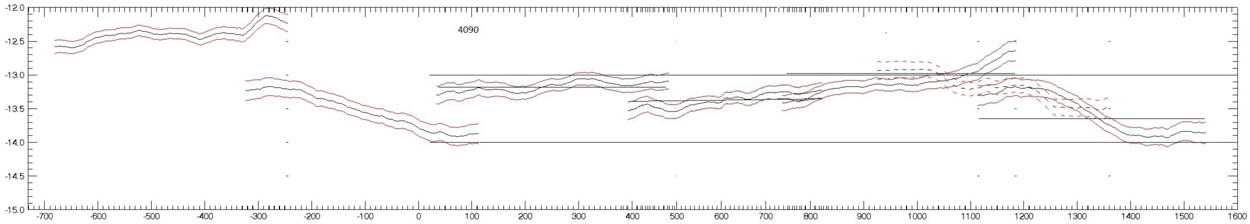

Figure 1 The floating ambiguity $\hat{n}$ estimated on a number of ALOS-2 frames along the same track. We have not been able to clarify yet the discrepancies that can be observed in the overlap areas.

2.2 Interpretation of the result

To interpret the result we assume that the lower and upper sub-bands are equally displaced from the central frequency by δ_f. By substituting $f_L = f_0 - \delta_f$ and $f_H = f_0 + \delta_f$, one gets:

$$2\pi\hat{n} = -\frac{2f_0^2 - \delta_f^2}{2\delta_f^2}(\Delta\phi_H - 2\Delta\Phi_0 + \Delta\Phi_L) + \Delta\Phi_0 - \frac{f_0}{2\delta_f}(\Delta\Phi_H - \Delta\Phi_L) \quad (4)$$

This first line represents (twice) the ionospheric phase derived from the curvature of the phase along the frequency. It is an absolute phase, though very noisy since it involves a second derivative. In the second line one can easily recognize the difference between groups phase and group delays. This is the usual split-spectrum ionospheric estimation [2], much more precise but ambiguous, since $\Delta\phi_0$ is itself ambiguous.

In practice the estimation of n consists in demodulating the unambiguous estimation of the ionospheric phase with the ambiguous one. The remainder will be noisy but equal to a constant number of 2π.

3 Feasibility of ambiguity resolution

3.1 Theoretical performance

To solve for the ambiguity n it is necessary that the estimate $\hat{n}$ lies close enough to an integer. We will assume that we can safely recover n if its standard deviation σ_n is less than 0.1, so that the +/-3σ_n interval is comfortably below 1. To reach such an accuracy, it will be necessary to average $\hat{n}$ across many (millions of) independent samples.

Considering Eq. 4, most of the error will come from the first term. With some simple approximations and considering three unequal sub-bands (1/6, 2/3, 1/6 of the available range bandwidth) the standard deviation for $\hat{n}$ is:

$$\sigma_{\hat{n}} = \frac{1}{2\pi}\left(\frac{f_0}{\delta f}\right)^2 \sqrt{6 + 4\cdot 3/2 + 6}\sqrt{\frac{1-\gamma^2}{2\gamma^2 L}}, \quad (5)$$

where γ is the interferometric coherence and L is the number of independent samples on the full resolution images. We are ignoring here that the estimator is slightly biased, so that the actual performance will be a little worse. The bias relates to the fact that the expressions given so far for the phases are monochromatic approximations: in reality one should consider the non-linear variation of the phase within the integration bandwidth.

Notice the quadratic growth with the frequency. This makes it much more difficult to recover the ambiguity for higher frequencies. This might sound counter-intuitive, since higher frequencies are less affected by the ionosphere. However one should consider that we are relying on the curvature of the ionospheric phase screen as a function of frequency, which depends on $1/f^3$!

3.2 Feasibility for typical SAR missions

Looking at typical figures derived from some current SAR missions, it should be possible to recover the ambiguity with the proposed approach for L-band SAR missions with sufficient bandwidth (e.g. 80 MHz).

Satellite	f_0 (GHz)	B_{range} (MHz)	L ($\times 10^6$)	$\sigma_{\hat{n}}$ ($\gamma = 0.4$)
ALOS	1.270	28	107	1.3
		14	16	13
ALOS-2	1.2575	80	390	**0.08**
		28	106	1.2
S-1	5.405	42.8-56.5	417	3.7
TerraSAR-X	9.65	150	350	1.4

Table 1 Theoretical standard deviation of the ambiguity estimate for one scene for different SAR missions. For Sentinel-1 we have taken an optimum combination of the three IW swathes.

4 First demonstration

We have applied the algorithm to ALOS-2/PALSAR-2 data over a region with high coherence. **Figure 1** reports the recovered ambiguity for different frames along the same track. The performance falls short from unambiguously indicating a single integer, however the oscillation is limited and almost in the desired range. The presence of discrepancies between overlapping frames points to systematic errors related to some unidentified processing step and gives some hope for the possibility of improvement. The ALOS-2/PALSAR-2 data were kindly provided by the Japanese Aerospace Exploration Agency (JAXA)

5 Literature

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[2] G. Gomba, A. Parizzi, F. De Zan, M. Eineder and R. Bamler, "Toward Operational Compensation of Ionospheric Effects in SAR Interferograms: The Split-Spectrum Method," in IEEE Transactions on Geoscience and Remote Sensing, vol. 54, no. 3, pp. 1446-1461, March 2016.

[3] R. Brcic, M. Eineder, R. Bamler, F. Meyer, "Ionospheric Effects in SAR Interferometry: An Analysis and Comparison of Methods for their estimation", in International Geoscience and Remote Sensing Symposium, 2011.