Quantum annealer for network flow minimization in InSAR images

Soronzonbold Otgonbaatar[1] and Mihai Datcu, Fellow IEEE[2]
[1,2]German Aerospace Center (DLR), Oberpfaffenhofen, Germany,
[1]University of Siegen, Germany,
[2]University POLITEHNICA of Bucharest (UPB), Romania.

Abstract

Quantum Annealer (QA) is well-suited for a certain class of optimization problems which can be expressed as a Quadratic Unconstrained Binary Optimization (QUBO) problem. A QUBO problem belongs to the family of Integer Programming problems which are called the NP-hard optimization problems. Feasible solutions of such problems can be found by using classical optimization techniques. However, studies claim that QA can find a feasible global solution that is faster than a classical annealer for QUBO problems. Hence, it appears promising to program and use the QA-to-QUBO approach for Earth Observation. In search of the QUBO problem in the domain of Earth Observation, we examined several Inteferometric Synthetic Aperture Radar (InSAR) applications and identified a residue connection problem in the phase unwrapping procedure. In particular, we consider the residue connection problem with multiples of 2π as a QUBO problem. For this practical problem, we studied how to formulate this QUBO problem, and we examined the challenges to program the D-Wave quantum annealer, in particular, embedding the QUBO problem into our QA architecture with a so-called Pegasus topology, and the annealing parameter settings in the D-Wave quantum annealer. We then analysed the parameter effects on finding the global minimum of the residue connection problem. From these results, we derived and enhanced our insight for programming future quantum annealers; for instance, choosing real-world problems in Earth Observation, conceiving the embedding procedure, and the tuning of the annealing parameters.

1 Introduction

Quantum Annealer is a novel computing technology promising to provide solutions for hard-optimization or sampling problems. A specific type of QA is the D-Wave quantum annealer which has a specific graph topology called *Pegasus* and *Chimera* [1]. The vertices of these graph topologies represent the physical variables, and the edges represent the interaction strengths among these physical variables. However, not all vertices are connected to all others through the edges; in other words, the interaction among the physical variables is restricted. Further, D-Wave quantum annealer promises to solve only a QUBO-like problem, and the variables of a QUBO problem are logical variables. To solve a QUBO problem via the D-Wave quantum annealer, we need to program the D-Wave quantum annealer; the programming consists of two steps: *embedding* and the setting of the *annealing parameters*. The *embedding* is a procedure to map the QUBO problem to the D-Wave quantum annealer topology, namely, our Pegasus topology; in particular, mapping (embedding) the logical variables to the corresponding physical variables. After embedding the QUBO problem, the D-Wave quantum annealer uses a meta-heuristic (quantum annealing) process for finding a global minimum solution of the QUBO problem; the quantum annealing process is parametrized by its *annealing parameter* [2]. Hence, the *embedding* procedure and the *annealing parameter* setting affect the problem solutions. Several studies already dealt with the embedding of QUBO problems to the physical variables of Chimera graphs, dealing with both academic [3]-[5] as well as with real-world applications, [6], [7]. There are even indications that D-Wave quantum annealer may solve certain QUBO problems faster than classical methods by exploiting quantum effects such as tunnelling and entanglement [8].

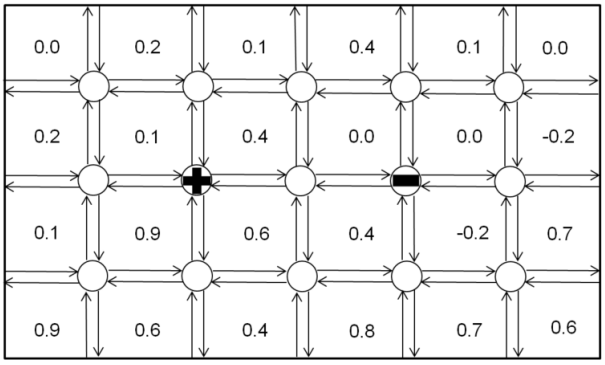

Figure 1 An example of 2D wrapped phase values and their residues. Empty vertices have zero **0** residues. Phases of 2π are normalized to 1.

The QUBO problems are well-known in mathematics, physics, and industrial optimization problems due to the complexity of obtaining a good global optimum. In the field of mathematics, [3], this author identified NP-hard problems and formulated them as QUBO problems to find their global minimum solution by exploiting QA. In the field of graph theory, [5], other author selected certain graph problems $G = (V, E)$ to find their solution by using

the quantum annealing process on QA. Such graph problems were to find minimum cuts and maximum cliques of the graph. In the field of the high energy physics, [9], the wrapped measurement data can be modelled as a QUBO problem, and a quantum annealing process is then employed for un-wrapping the data on a D-Wave quantum annealer. These diverse research findings motivated us to this work. However, so far, no advantage for real-world optimization problems could be demonstrated.

Therefore, this work aims to investigate to programming of a D-Wave quantum annealer for QUBO problems in Earth Observation. As a real-world problem in Earth Observation, we chose the residue connection problem with multiples of 2π in the phase unwrapping procedure and posed this problem as the QUBO problem. The residue connection problem with multiples of 2π is solvable with several classical optimization techniques, for instance, Network Flow Minimization (NFM) algorithm, [10], [11], in which their authors contributed and used well-known optimization algorithms for finding the global minimum of the objective function for the residue connection (Fig. 1). We even can pose this problem simply and easily as a QUBO problem. Hence, it validates to employ the quantum annealing process on a D-Wave quantum annealer.

Therefore, in this paper, we exhibit a way to formulate the objective function of the residue connection problem as a QUBO problem, and we then embed this QUBO problem to the Pegasus topology. Finally, we run the quantum annealing process with different settings of the annealing parameters to assess the performance of the D-Wave quantum annealer.

1.1 Basics of the residues in the phase unwrapping procedure

Synthetic Aperture Radar (SAR) is a coherent imaging system operating in the microwave domain. Interferometric SAR (InSAR) techniques exploit the information of the phases of two complex-valued SAR signals acquired from different positions in order to construct an "Interferogram"; hence, the phases of two complex-valued SAR signals are called the *interferometric phases* which allow us to estimate the height of the observed scene [12], [13]. The interferometric phases of two complex-valued signals are measured, and their differences are wrapped into the region $[-\pi, \pi]$ called *wrapped phase* [14].

For a $1D$ signal, the true or *unwrapped phases* are then obtained by means of adding multiples of 2π to the wrapped phase values if the absolute value of a wrapped phase difference (i.e., its gradient) is less than $-\pi$. For a $2D$ signal, instead, the same concept as for $1D$ signals does not provide a satisfactory result for the unwrapped phases due to signal noise, and hence, this concept would cause some error propagation for the unwrapped phases. The signal noises are identified as the *residues* which are results of integration of the wrapped phase gradients along 2×2 square grids (Fig. 1); the residues take values of $\{-1, 0, +1\}$. Hence, the wrapped phases of a $2D$ signal are non-conservative and path-dependent [10]. Furthermore, commonly used phase unwrapping procedures are based on the assumption that the wrapped phase gradients $\Delta\psi(n)$ known experimentally are equal to unwrapped phase gradients $\Delta\phi(n)$ at most of the instances, and according to this assumption, the unwrapped phase gradients can be reconstructed from

$$\Delta\phi(n) = \Delta\psi(n) + 2\pi\Delta k(n), \tag{1}$$

where $\Delta\phi(n), \Delta\psi(n) \in [-\pi, +\pi]$ with n representing a discretized location of phase values on the 2D plane, and $k(n) \in \mathbb{N}$ are integer variables. If the integer variables $k(n)$ are known then we can integrate Eq. (1) to reconstruct the unwrapped phases. To avoid residues to be enclosed in the integration path of the wrapped phases, the residues need to be connected, and a number of the connected residues has to be neutral to ensure the path invariance along the integration path. Otherwise, an imbalanced number of the residues distorts the path integral of the wrapped phase gradients (Fig. 1). Hence, the residue connection problem can be expressed as a minimization problem,

$$\min_{\{x_1^+, x_1^-, x_2^+, x_2^-\}} \left[\sum_{i=0}^{N-2} \sum_{j=0}^{M-1} c_1(i,j)[x_1^+(i,j) + x_1^-(i,j)] + \sum_{i=0}^{N-1} \sum_{j=0}^{M-2} c_2(i,j)[x_2^+(i,j) + x_2^-(i,j)] \right], \tag{2}$$

such that

$$\begin{aligned} & x_1^+(i,j+1) - x_1^-(i,j+1) - x_1^+(i,j) + x_1^-(i,j) \\ & - x_2^+(i+1,j) + x_2^-(i+1,j) + x_2^+(i,j) - x_2^-(i,j) \\ & = -\frac{1}{2\pi}(\psi_1(i,j+1) - \psi_1(i,j) - \psi_2(i+1,j) + \psi_2(i,j)), \\ & \vec{x}_1^+ \geq 0, \vec{x}_1^- \geq 0, \vec{x}_1^+, \vec{x}_1^- \in \mathbb{N}, \\ & \vec{x}_2^+ \geq 0, \vec{x}_2^- \geq 0, \vec{x}_2^+, \vec{x}_2^- \in \mathbb{N}. \end{aligned} \tag{3}$$

$x_1^+(i,j), x_1^-(i,j)$ are the flow along vertices (i-1,j) $\leftrightarrow$ (i,j), $x_2^+(i,j), x_2^-(i,j)$ are the flow along vertices (i,j-1) $\leftrightarrow$ (i,j), and $c_1(i,j)$, and $c_2(i,j)$ are the unit cost of the flows (Fig. 1). The global minimum of this objective function can be found efficiently by using the NFM algorithm [10], [11].

1.2 The QUBO problem formulation

We consider less densely populated residues with multiples of 2π such that the $\{x_1^+, x_1^-, x_2^+, x_2^-\}$'s of the objective function in Eq. (2) take the binary values $\{0, 1\}$; here, the residues with multiples of 2π mean that the integer variables only take the values one or zero while, for the residues with more than multiples of 2π, the integer variables can be any integer numbers. For either case, we can express the objective function, Eq. (2), as the QUBO problem;

$$H(\mathbf{x}) = \sum_{i,j} x_i Q_{ij} x_j, \tag{4}$$

where the logical variables $\mathbf{x} \in \{0,1\}^n$, and Q_{ij} includes a bias term h_i and the interaction strength of the logical variables J_{ij}.

For the residue connection problem with multiples of 2π, we can easily embed it into the Pegasus topology and assess the performance of the D-Wave quantum annealer with respect to a conventional algorithm. In this case, each residue value, $b(i,j) \in \{-1,0,+1\}$, must be equal to the total edge value, namely the conservation of flow at the vertex $b(i,j)$ (Fig. 1). The constraint Eq. (3) then remains as before

$$\begin{aligned}&\underbrace{[x_1^+(i,j+1) - x_1^-(i,j+1)] + [-x_1^+(i,j) + x_1^-(i,j)]}_{x_1} \\ &+ \underbrace{[-x_2^+(i+1,j) + x_2^-(i+1,j)] + [x_2^+(i,j) - x_2^-(i,j)]}_{x_2} \\ &= b(i,j), \quad x_1^{\pm}, x_2^{\pm} \in \{0,+1\},\end{aligned} \tag{5}$$

here $b(i,j) = -\frac{1}{2\pi}(\psi_1(i,j+1) - \psi_1(i,j) - \psi_2(i+1,j) + \psi_2(i,j))$. Further, we can formulate the QUBO form for the residue connection problem by means of expressing Eq. (2) and Eq. (5) in a quadratic form as

$$\begin{aligned}&\min_{\{x_1^+,x_1^-,x_2^+,x_2^-\}} \Bigg[\sum_{i=0}^{N-2} \sum_{j=0}^{M-1} c_1(i,j)[x_1^+(i,j) + x_1^-(i,j)] \\ &+ \sum_{i=0}^{N-1} \sum_{j=0}^{M-2} c_2(i,j)[x_2^+(i,j) + x_2^-(i,j)] \\ &+ \lambda \sum_{(i,j)\in S} [x_1 + x_2 - b]^2 \Bigg],\end{aligned} \tag{6}$$

where we suppose that $\lambda = 1$ and

$$\begin{aligned}&\sum_{(i,j)\in S} [x_1 + x_2 - b]^2 = \\ &= \sum_{(i,j)\in S} 2x_1x_2 - 2b(x_1 + x_2) + x_1 + x_2.\end{aligned}$$

Eq. (6), is already in the form of the QUBO problem described by Eq. (4), and in the end, we have the QUBO formulation for the residue connection problem with multiples of 2π; in particular, the residues have the multiples of 2π.

Furthermore, we can derive the QUBO problem with the residues having more than multiples of 2π by using the fixed-bit encoding of the integer variable $x_i^* \in \{x_1^+, x_1^-, x_2^+, x_2^-\}$'s,

$$x_i^* = \sum_{k=0}^{n-1} 2^k q_{n\times i+k}^*, \quad x_i^* \in \mathbb{N}, q_{n\times i+k}^* \in \{0,+1\}, \tag{7}$$

where n stands for the number of bits, and "$*$" represents "$+$" or "$-$".

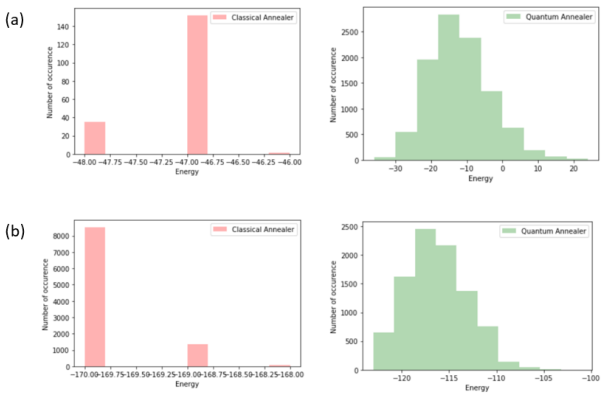

Figure 2 The minimum results of the QUBO problem (minimum energy) on the classical annealer and the D-Wave quantum annealer. (a) Minimum energies for 3×3 and (b) 7×5 residue problems.

However, we consider only the QUBO problem of Eq. (6) for a real-world data set to examine the performance of the D-Wave quantum annealer; in particular, *Cape Verde Volcano, Africa*, as a real-world Sentinel-1 data set. In the next section, we embed the QUBO problem for *Cape Verde Volcano* into the Pegasus topology and run the quantum annealing process with different annealing parameters on the D-Wave quantum annealer.

2 The D-Wave quantum annealer and our experiment

The company D-Wave systems offers a specific kind of a quantum annealer. This quantum annealer, called D-Wave quantum annealer, anneals two-state quantum particles to find their most probable state. The most probable state is equivalent to minimizing the QUBO problem defined by Eq. (4). However, the interaction strength J_{ij} is characterized by the specific topology *Pegasus* on the D-Wave quantum annealer. Hence, the QUBO formulation for a practical problem needs to be embedded into the Pegasus topology. When addressing the QUBO formulation for Cape Verde Volcano contained in Sentinel-1 data, we considered the two cases, namely 3×3 and 7×5 residues, such that

$$\begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}, \quad \begin{bmatrix} 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ -1 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & -1 & 0 \\ 0 & 0 & -1 & 0 & 0 \end{bmatrix}. \tag{8}$$

Here, the number of the local variables of the QUBO problem, given by Eq. (6), is "48" for the first case, while the second case uses "164". We attempted to find the global solution of the QUBO problem for both cases on a classical annealer and the D-Wave quantum annealer. Then we benchmarked the results of the D-Wave quantum annealer with respect to the classical annealer. To program the D-

Wave quantum annealer for obtaining the global solution of Eq. (6), we followed a two-step approach:

1. *Embedding*: Embedding of the logical variables into their corresponding physical variables (in Pegasus topology). In this first step, we used a method named *minor-embedding* offered by the D-Wave quantum annealer. We represented a single logical variable by at most five physical variables.

2. *Annealing parameters*: These are the annealing time, the number of reads, and the chain strength. The annealing time is the time to run the quantum annealing process. We set this time to a value ranging from $20\mu s$ to $190\mu s$, the number of reads to a value from 1000 to 10000. Additionally, the chain strength is the interaction strength which was set to 50. We even tried a method called *reverse annealing* which starts the quantum annealing process at the known local solution of the problem of interest.

We obtained the results for the both residue cases given above by programming the D-Wave quantum annealer, and we have validated these results by inserting them into the constraint of Eq. (5). Our results demonstrate that, in the first case of the 3×3 residues, the D-Wave quantum annealer finds a better solution than the classical annealer. However, in the latter case of the 7×5 residues, the classical annealer finds a better solution than the D-Wave quantum annealer. We plotted the possible minima of the QUBO problem (minimum energies) for both cases of (8), as a demonstration (Fig. 2). For these scenarios, we can conclude that the D-Wave quantum annealer outperforms the classical annealer, while the number of the local variables in the QUBO problem is very limited.

3 Conclusion and Discussion

This work is a first attempt to enhance insights into a quantum annealer, and perceive some challenges to program the D-Wave quantum annealer for future Earth Observation quantum technology. To leverage the D-Wave quantum annealer to obtain a better solution of the optimization problem, we chose a well-known optimization problem in Earth observation, and mapped it to a QUBO problem; in particular, the residue connection problem of the phase unwrapping procedure in the InSAR.
For the chosen residue connection problem with multiples of 2π, we assessed and programmed the D-Wave quantum annealer with different annealing parameters. The results obtained from the D-Wave quantum annealer were benchmarked with respect to a classical annealer. It turned out that the D-Wave quantum annealer performs better than the selected classical annealer while the number of local variables is very small. In further studies, we will investigate other real-world problems of remote sensing and Earth Observation which shall benefit greatly from a QUBO formulation.

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