A Comparison of Function Bases For Polarization Coherence Tomography in Forest Scenarios

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Abstract

Polarization Coherence Tomography (PCT) addresses the TomoSAR inversion by approximating the unknown vertical reflectivity profile by a weighted series of functions constituting a basis. The individual weights can be estimated from the available interferometric coherences. This concept becomes particularly suitable for space borne TomoSAR implementations over forest volumes in which the reflectivity profiles have to be estimated relying on a small number of acquisitions (or interferometric coherences) and/or non-uniform baseline distributions. However, the choice of an adequate (orthogonal) basis suitable for forest scenarios is still an open point. In this work, we compare different alternatives on a real data set acquired in an airborne campaign over a temperate forest.

1 Introduction

Synthetic Aperture Radar Tomography (TomoSAR) estimates the profile of the backscattered power (also reflectivity) $P(z,\boldsymbol{\omega})$ of a distribution of scatterer as a function of the height z and polarization $\boldsymbol{\omega}$. The estimation relies on the availability of a set of interferometric coherences $\gamma(\kappa_Z,\boldsymbol{\omega})$ collected at different vertical wavenumbers κ_Z, directly proportional to the baseline lengths [1]. TomoSAR algorithms invert the Fourier relationship [1]:

$$\gamma(\kappa_Z,\boldsymbol{\omega}) = \frac{\int P(z,\boldsymbol{\omega})\, e^{j\kappa_Z z}\, dz}{\int P(z,\boldsymbol{\omega})\ dz}. \qquad (1)$$

The dependence on $\boldsymbol{\omega}$ is dropped in the following for easiness of notation.

Forest volume scatterers are characterized by continuous and extended vertical distributions of the backscattered power. In these conditions, the TomoSAR inversion becomes underdetermined even for richer acquisitions (e.g., more than 10 coherences available) [2], [3]. For this reason, the selection of the TomoSAR estimation algorithm in forest scenarios is still a critical element [3]. The direct inversion of the Fourier relationship (1) is primarily (but not only) affected by the baselines distribution. Irregular distributions typically results into stronger sidelobes and interpretation ambiguities. Still in the category of the model-free algorithms, the well-known Capon beamformer [4] offers super-resolution (i.e., beyond the Rayleigh limit imposed by the largest κ_Z) and sidelobe reduction [4]. The Capon estimator together with the recently proposed Compressive Sensing techniques [2] are today the state-of-the-art approaches for the reconstruction of vertical profiles from TomoSAR data. Yet, both of them are not optimized to deal with volume scatterers. Any performance improvement of the Capon spectral estimator is achieved at the expense of radiometric linearity and accuracy, while violations of the sparsity constraints used by Compressive Sensing can result into artifacts in the estimated profiles [2], [3].

The estimation of $P(z)$ becomes particularly challenging when addressed in terms of a space borne mission implementation [5]. In particular, temporal decorrelation and/or orbital considerations reduce drastically the number of interferometric coherences available for inversion. The application of conventional TomoSAR algorithms to such framework leads to a sometime significant performance loss [5], [6].

A low-dimensional parameterization of $P(z)$ in terms of geometrical and scattering properties allows to obtain determined inversion problems even with a low number of coherences. However, simple models provide e.g. accurate forest height estimates [6], but the reconstruction of the full $P(z)$ could turn into interpretation ambiguities. Hybrid approaches approximate $P(z)$ by means of a weighted sum of a series of (orthogonal) basis functions, as originally proposed within the so-called Polarization Coherence Tomography (PCT) approach [7], [8]. $P(z)$ is decomposed as:

$$P(z) = \sum_n a_n P_n(z) \qquad (2)$$

where $\{a_n\}$ are the unknown real-valued weights and $\{P_n(z)\}$ are the basis functions. By substituting (2) in (1), a linear inversion can be carried out to estimate $\{a_n\}$ from a reduced set of interferometric coherences [7], [8]. This approach results particularly appealing as (*i*) in principle a suitably large number of weights can be estimated with a rather small number of coherences (in one or more polarization channels), and (*ii*) the estimation of the weights is simple from an inversion point of view.

However, the choice of the basis is still an open issue. Indeed, an appropriate basis should provide reliable approximations of reflectivity profiles with a low number of coefficients for different forest types, and at the same time facilitate the interpretation of the estimated profile in terms of physical structure. In this work, different function bases are compared in order to evaluate their appropriateness for forest scenarios. The analysis is carried out using profiles estimated from an airborne data set collected over a temperate forest.

2 Design of a PCT basis from TomoSAR profiles

2.1 Theoretical background

In order to increase the suitability of a function basis for forest scenarios with respect to Legendre or Fourier polynomials used in the conventional PCT formulation [7], [8], in [9] a procedure has been applied to extract a basis from a set of real airborne TomoSAR profiles.

It is assumed that a set of complex coherences $\{\gamma_k\}_{k=1}^{K}$ corresponding to a set of vertical wavenumbers $\{\kappa_Z\}_{k=1}^{K}$ is available. TomoSAR profiles are usually calculated by discretizing the height axis in N_Z samples $\{z_n\}_{n=1}^{N_Z}$ within a height interval of interest, with typically $N_Z \gg K$. The calculated N_Z intensity samples of a generic profile $P(z)$ can be stacked in the column vector $\mathbf{p}$. All the N_P profiles obtained within a set of range-azimuth cells are then collected as columns of a $(N_P \times N_Z)$-dimensional matrix $\mathbf{\Pi}$. If $N_P > N_Z$, from well-known algebra theorems it results that the vertical profiles in $\mathbf{\Pi}$ can be written as a linear combination like in (2) of $N_B \leq N_Z$ linearly independent basis functions with null error [10].

In [9], the direct Fourier inversion of (1) has been selected as TomoSAR algorithm. This presents two advantages. First, either radiometric linearity/accuracy losses or artifacts induced by the presence of volume scatterers are avoided in the profiles. Second, N_B can be determined exactly in the case in which $\{\kappa_Z\}_{k=1}^{K}$ are integer multiples of a minimum quantity [9]. For instance, this is for sure the case of uniformly distributed baselines, for which it immediately results $N_B = 2K + 1$.

Function bases with different characteristics can be extracted from $\mathbf{\Pi}$. One possibility is to build up a basis by looking for N_B linearly independent columns of $\mathbf{\Pi}$ [10]. In [9], the additional condition of minimum correlation among basis functions was added. Alternatively, one can resort to the eigen-decomposition of the covariance matrix $\mathbf{\Pi}\mathbf{\Pi}^T$, where $(.)^T$ denotes the transpose operator. While the eigenvectors are the basis functions, the eigenvalues represent their (average) relative significance in approximating each profile. In this case, the basis function result orthonormal. It is worth noting that, although orthonormality is not strictly required, an othonormal basis minimizes the energy of the reconstruction error at the increase of the number of weights [10].

2.2 A first application to real data

An L-band tomographic data set acquired in 2017 by the DLR's F-SAR airborne platform over the forest of Traunstein (south of Germany) has been used. The available $\{\kappa_Z\}_{k=1}^{K}$ are (nominally) uniformly distributed with $K = 12$. The TomoSAR vertical Rayleigh resolution and ambuguity height range amount to around 6.7 m and 80 m at mid range, respectively. Fourier profiles have been calculated over the whole forest area for multilook cells measuring 15 m in both slant range and azimuth (more than 100 looks) by directly inverting (1). These profiles have been used to build $\mathbf{\Pi}$. A coherence pre-interpolation step has been applied to the data in order to compensate for small deviations of the realized vertical wavenumbers

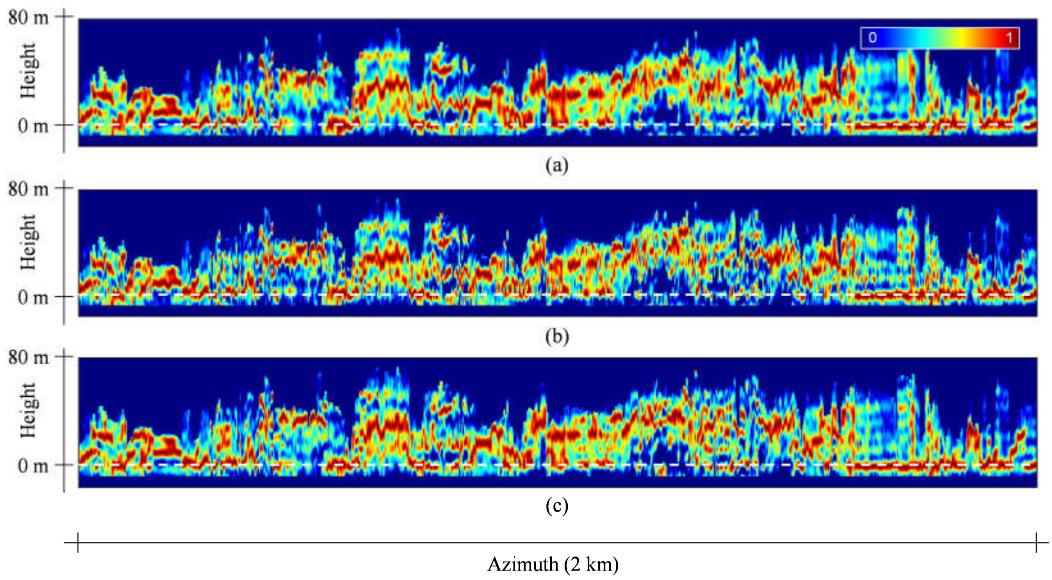

Figure 1 Traunstein forest, representative Fourier profile along azimuth for a fixed mid range coordinate (normalized amplitudes). (a) Original profiles; (b) approximation of the original profiles obtained by using 15 least correlated linearly independent profiles of $\mathbf{\Pi}$; (c) approximation of the original profiles obtained by using 10 eigenvectors of $\mathbf{\Pi}\mathbf{\Pi}^T$. The ground topography has been compensated, therefore the ground scattering occurs at 0 m (indicated by the white dashed line).

from the planned uniform ones [2], [11]. Recalling Section 2.1, in this way the basis dimension results exactly $N_B = 25$. Finally, the TomoSAR processing has been carried out by using heights normalized by the TomoSAR Rayleigh resolution so that the range-azimuth variability of $\{\kappa_Z\}_{k=1}^{K}$ could be compensated. Thus, fixed a criterion, the retrieval of only one basis able to represent the whole forest area was needed [9].

Figure 1(a) shows a representative Fourier profile in the azimuth – height (rescaled in meters) plane obtained by inverting the full ($K = 12$) coherence set. Function bases have been found using the methodologies described in Section 2.1. For each profile, the expansion weights have been calculated by inverting (2) according to a least squares criterion. **Figure 1(b)** shows the approximation obtained by weighting 15 (out of 25) least correlating linearly independent profiles in $\mathbf{\Pi}$. The approximation reported in **Figure 1(c)** has been obtained by weighting 10 eigenvectors of $\mathbf{\Pi\Pi}^T$ corresponding to the 10 largest eigenvalues. By comparing **Figures 1(b)-(c)** with **Figure 1(a)** it is apparent that the original profiles can be well approximated by employing a number of functions significantly lower than the basis dimensionality. This example also demonstrates that, for a fixed approximation error, the type of basis determines the number of needed functions. For this, the (minimum) number of interferometric coherences needed to obtain a determined inversion follows straightforwardly. For instance, the availability of 7 and 5 coherences may suffice for the bases of **Figure 1(b)** and **(c)**, respectively.

3 Outlook

The real data example in Section 2.2 has shown that an appropriate basis function can provide a low dimensional, yet accurate enough approximation of a set of profiles. The considered bases have been built up using real TomoSAR profiles. However, only algebraic optimization criteria have been considered. In the full paper, the possibility to include physical structure information in the basis design will be explored. The performance in profile reconstruction obtained by the different basis choices will be quantitatively addressed. The validity of each basis for different structure types will be evaluated as well.

The comparison among different function bases will then be extended by considering the performance of a PCT inversion with a small number of functions from a limited set of interferometric coherences. Trade-offs in terms of the number of coherences and suitable values of the related κ_Z for a certain inversion performance will be investigated.

4 Literature

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