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Enhancing Classical Impedance Control Concepts while Ensuring Transferability to Flexible Joint Robots

Bachelor's thesis

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Abstract

In robotics the concept of impedance control enables a controlled and therefore safe interaction behavior between robot and human. The objective of this bachelor's thesis was to evaluate several configurations of the enhanced Elastic Structure Preserving Impedance (ESπ) controller. The enhanced controllers reduce the maximal required time-derivative order of the link position by one, compared to classical approaches.

Four different configurations, representing different mass-spring-damper designs, were transformed in a three degrees-of-freedom form, simulated and numerically optimized for improved noise damping and disturbance rejection behavior. The H_∞ method was used.

The configuration ESπ 2 showed the best noise damping in simulation. The results on the testbed confirm a robust implementation, improved noise damping and reduced control effort for disturbance rejection over the initial controller.

Contents

List of Figures

List of Tables

Glossary

List of Abbreviations

DLR	Deutsches Zentrum für Luft- und Raumfahrt (German Aerospace Center)
DoF	Degrees of freedom
ESπ	Elastic Structure Preserving Impedance
FFT	Fast Fourier transformation
MISO	Many input, single output
SISO	Single input, single output

Signals

η	Transformed deflection motor coordinate
τ_{ext}	Disturbance input, external forces
θ	Motor coordinate
$\tilde{\tau}_{ext}$	Relative disturbance input
$\tilde{q}_d$	Filtered reference input
e	Control error $(q_d - q)$
n	Noise input

q	Plant output, link position
q_d	Reference input, desired link position
q_m	Position of the virtual mass m
q_n	Measured plant output, measured link position $(q+n)$
u	Control output

Transfer Functions

D	Disturbance transfer function
G	Plant
G_d	Disturbance model of the plant
K	General controller; also the stiffness of the flexible joint and plant
K_d	Disturbance control filter
K_r	Reference control filter
K_y	Feedback controller for 2-DoF and 3-DoF controllers
L	Loop transfer function
N	Noise transfer function
R	Tracking transfer function for 2-DoF and 3-DoF controllers
R_V	Tracking transfer function of the reference model
R_{Mech}	Tracking transfer function of the mechanical substitution model
S	Sensitivity transfer function
T	Tracking transfer function
Z	Impedance transfer function
Z_V	Impedance transfer function of the reference model

Z_{Mech}	Impedance transfer function of the mechanical substitution model

Virtual Control Parameters

D_q	Virtual link damping
D_η	Virtual motor inertia damping
K_m	Virtual additional control stiffness
K_q	Virtual control stiffness
m	Virtual mass (only ESπ 2 and 4)

Physical Plant Parameters

B	Plant's motor inertia
K	Plant's spring stiffness
M	Plant's link inertia

Other Parameters and Symbols

D_V	Damping of the reference model
$K1$	Control term 1 for the enhanced ESπ models
$K2$	Control term 2 for the enhanced ESπ models
K_N	Gain for the noise weight
K_V	Stiffness of the reference model
M_S	Maximum peak gain of the sensitivity function
M_T	Maximum peak gain of the complementary sensitivity function
X	Ratio of K_m and K_q for the optimization
γ^2	Correlation function
ω_c	Cut-off frequency of the loop transfer function
ω_c^*	Cut-off frequency for the noise weight

ω_{180}	Frequency with a phase of 180 in the loop transfer function
ζ	Damping ratio
w_N	Noise weight
GM	Gain margin
PM	Phase margin

1 Introduction

In the last decades robots working close to humans have become more widespread. This made safe human-robot interaction and collaboration much more relevant. Controlling the robot's interaction with its environment became a key concept. The environment is usually the source of disturbances, or deviations from the controlled robot's motion. Therefore, if the robot's response to disturbances can be controlled, a safe interaction with the environment can be assured.

This motivated the approach of impedance control, introduced by Hogan (1984) [1]. The basic concept is to describe the dynamic interaction with the environment as an interconnection between two physical systems (robot and environment). The robot should assume the behavior of an impedance. Hence, the robot reacts to the deviations from its motion with a force, that is defined by the controller. The force is defined by a function of the position and velocity of the manipulator.

Position control, on the other hand, aims to minimize the deviation from the robots path. This can be dangerous, if the source of disturbance is e. g. a human clamped between the robot and another object.

Furthermore, with impedance control, the controller is designed to interact with the environment as a passive system (i. e. a system that is dissipating energy). Stramigioli and Folkertsma (2017) state, that if a robot is interacting with an unknown environment it must be a passive system to be stable [2, p. 149]. The passivity-based controlled robot behaves like a spring-mass-damper system.

Flexible joint robots add an extra mechanical spring between link and motor to realize a more compliant behavior and increase the mechanical robustness against impacts. However, the extra compliance also introduces unwanted oscillatory dynamics into the link.

Elastic Structure Preserving Impedance (ESπ) control overcame this drawback by controlling the link behavior through virtual visco-elastic elements and therefore controlling the link-side impedance. A virtual control input at the link was attained by a mathematical transformation of the plant system, in a new set of coordinates [3]. ESπ control is used in the DLR anthropomorphic Hand Arm System David (see Figure 1.1) [4].

ESπ control has the drawback, that the third time derivative, i. e. the jerk, of the link coordinate is needed. The jerk cannot be measured and is therefore calculated with the model equations of the plant. This thesis is evaluating enhanced ESπ controllers which omit the jerk, by using a second virtual mass and spring, internal to the controller. In theory, these enhanced concepts should also achieve better noise damping and reduced control effort for disturbance control.

The control parameters were numerically optimized to achieve improved noise damping and disturbance rejection behavior, while using the impedance deviation from a reference model as a constraint. The reference model ESπ V is an intuitive second-order spring-mass-damper system. It is used by the operator to set the impedance behavior of the controller.

In the simulations, the improvement of the disturbance behavior at higher frequencies is significantly better with the enhanced controllers, than the initial controller. High disturbance input frequencies are no longer amplified towards infinity. That applies for all new models and makes them more robust against hard impacts.

Although the different configurations among themselves had similar damping properties, ESπ 2 showed better results among the enhanced controllers. ESπ 2 was evaluated on a testbed and the frequency response to sensor noise and disturbances was measured. Noise damping improved and the control effort due to impacts was lower compared to the initial ESπ controller. This matches the findings of the simulations.

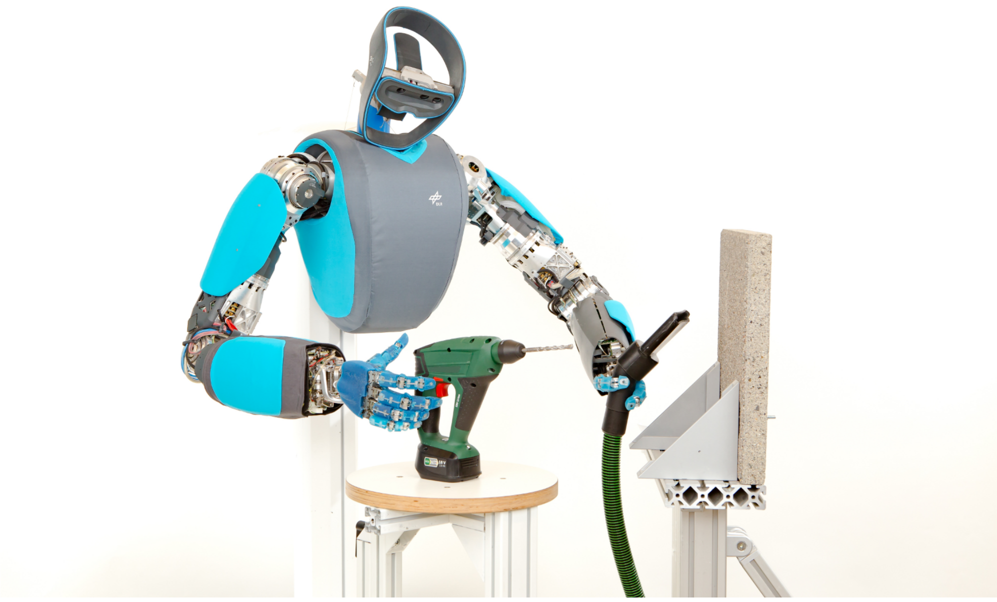


Figure 1.1: Anthropomorphic robot David (DLR).

2 Theoretical Background

This chapter explains the fundamentals of control theory, impedance control and frequency analysis.

2.1 Classical Feedback Control

Figure 2.1 describes the classical feedback control loop of a single input, single output (SISO) controller. The transfer functions K and G represent the controller and the plant. The disturbance model is G_d. It describes how disturbances affect the plant output. The controller has the control error e as input and control output u as output. The reference input and plant output are called q_d and q. Additionally, the noise and disturbance input are reflected by n and τ_{ext} and the measured plant output by q_n. The complete list of control parameters is given by Table 2.1.

K:	Transfer function of the controller
G:	Transfer function of the plant
G_d:	Disturbance transfer function of the plant
q_d:	Reference input
n:	Noise input
τ_{ext}:	Disturbance input
$\tilde{\tau}_{ext}$:	Relative disturbance input
e:	Control error
u:	Control output
q:	Plant output
q_n:	Measured plant output

Table 2.1: SISO control parameters.

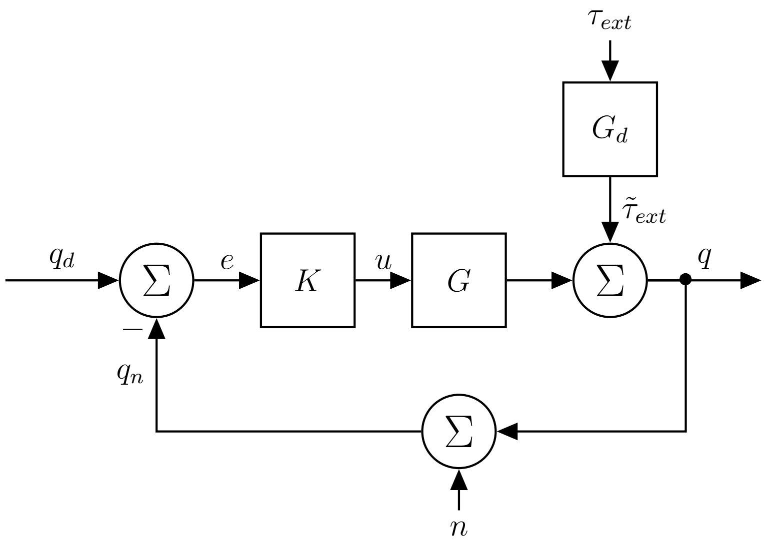


Figure 2.1: Classical feedback control loop.

The open-loop transfer function L is defined as $K * G$. It is used for to calculate the closed-loop transfer functions and for loop shaping (see Section 2.1.1 for details). In a classical control loop these can be calculated by using the following formula (adopted from [5, p. 25]).

$$\frac{Output}{Input} = \frac{"direct\ connection"}{1 + L} \tag{2.1}$$

This formula yields the following closed-loop transfer functions:

$$S = \frac{q}{\tilde{\tau}_{ext}} = \frac{1}{1 + L}\ , \tag{2.2}$$

$$T = \frac{q}{q_d} = \frac{L}{1 + L} = L\,S\ , \tag{2.3}$$

$$N = \frac{u}{n} = \frac{-K}{1 + L} = -K\,S\ , \tag{2.4}$$

$$D = \frac{u}{\tau_{ext}} = \frac{-G_d\,K}{1 + L} = -G_d\,K\,S\ . \tag{2.5}$$

The sensitivity of the control loop is called S. It describes the relative sensitivity of the plant's output to a relative disturbance input, called $\tilde{\tau}_{ext}$. The sensitivity transfer function is mainly used for performance evaluation (more in Section 2.1.2). The complementary sensitivity T determines the controlled plant's tracking behavior.

The effects of noise and disturbances on the control output u are described by N and D.

2.1.1 Loop Shaping

Good control performance is characterized by fast command following and disturbance rejection. This requires a high control output and therefore a large magnitude of the loop transfer function L. In contrast, stability and noise damping deteriorate, because high control gains are necessary for a high control output. Intuitively, a more sensitive controller is more susceptible to noise. Fortunately, L is usually a strictly proper transfer function. Consequently, it is strictly decreasing with higher frequencies, on which sensor noise is usually present.

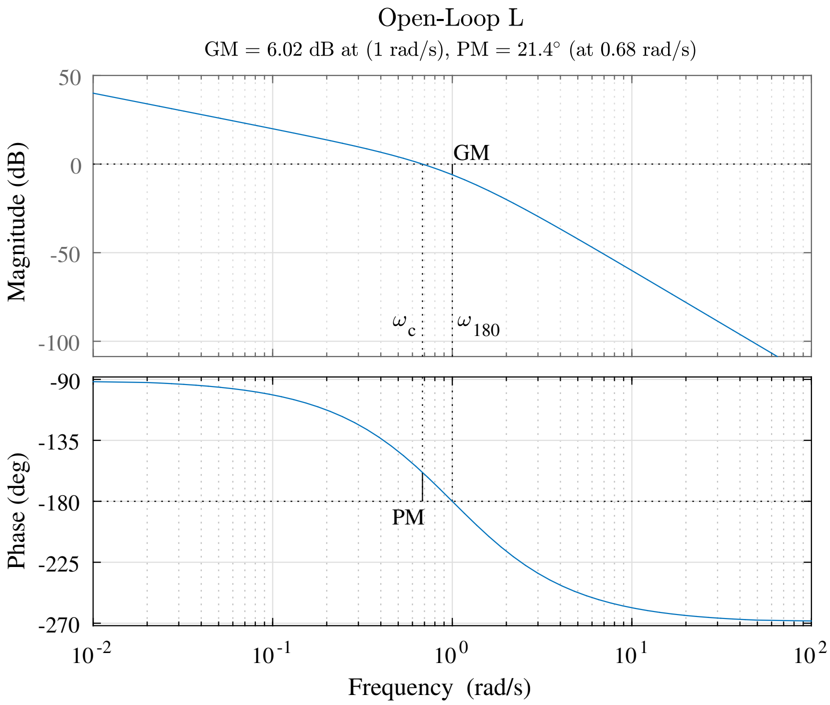


Figure 2.2: Gain and phase margins of a controlled plant $G = \frac{1}{s^4+5s^3+7s^2+3s}$ with a PI controller $K = 3 + s$. The resulting open-loop is $L = \frac{1}{s^3+2s^2+s}$. The PM is too small for the thumb rules for robustness of PM $> 30°$. The GM is sufficient.

After the cutoff frequency ω_c, $|L|$ falls below 1. Additionally, ω_{180} is defined by $\angle L = -180°$. The bandwidth between ω_c and ω_{180} is called the crossover region. The slope of L is called the roll-off rate. It is a factor of $20\,dB$ per decade.

A roll-off rate of -1 during crossover and a larger roll-off rate after ω_{180} will indicate adequate noise damping. Furthermore, Skogestad and Postlethwaite (2007) propound that a robust controlled system needs a gain margin (GM) of at least $6\,dB$ and a phase margin (PM) of at least 30° [5, p. 35f.]. The GM and PM can be measured in the bode diagram of L (cf. Figure 2.2).

Taken together, loop shaping aims for an L with a large loop gain for low frequencies and a small loop gain to high frequencies, while regarding a high enough GM and PM. Usually a function of the control error[1] is minimized and L is used to guarantee robustness.

2.1.2 Shaping Closed-Loop Transfer Functions

Apart from loop shaping, robustness can be evaluated by analyzing the closed-loop transfer functions S and T. More specifically their maximum peaks: M_S and M_T. If $M_S < 6\,dB$ and $M_T < 2\,dB$, the system should be robust and fulfill a sufficient GM and PM [5, p. 38]. Figure 2.3 depicts this method. Analyzing S is sufficient in most cases to determine robustness. In short, $S > 1$ results in amplified disturbances and $T > 1$ in amplified reference inputs.

The maximum peak analysis is a central part of the mixed sensitivity method described by Kwakernaak (2002) [6]. This approach, also known as H_∞ optimization, optimizes the H_∞ norm of the weighted S and T functions. The H_∞ norm of a SISO transfer function is defined as:

$$\left\| G(s) \right\|_\infty = \sup_\omega |G(j\,\omega)| = \lim_{p\to\infty} \left(\int_{-\infty}^{\infty} |G(j\,\omega)|^p \, d\omega \right)^{\frac{1}{p}} . \tag{2.6}$$

Accordingly, $\|w_S\,S\|_\infty$ and $\|w_T\,T\|_\infty$ yields the maximum peak of $w_S\,S$ and $w_T\,T$ in the amplitude spectrum. Good performance and noise damping can be achieved by minimizing then. However, a proper weigh selection is required.

[1] A common practice is using the integral of the control error multiplied by time as the objective function.

The weights w_S and w_T can set an upper bound for M_S and M_T. They can also reflect a sufficient roll-off rate and desired bandwidth. Often the noise transfer function N is integrated in the mixed sensitivity optimization as well, to limit the effects of noise on the control output u. An appropriate method of weight selection is described in [5, p. 61ff.]. The H_∞ optimized controller is achieved by minimizing the H_∞ norm of the vector:

$$H_\infty = \left\| \begin{matrix} w_S\, S \\ w_T\, T \\ w_N\, N \end{matrix} \right\|_\infty = \sup_{\bar{\sigma}} |\vec{H}(j\,\omega)| \; . \tag{2.7}$$

The H_∞ norm of a vector or matrix yields their maximum singular value $\bar{\sigma}$.

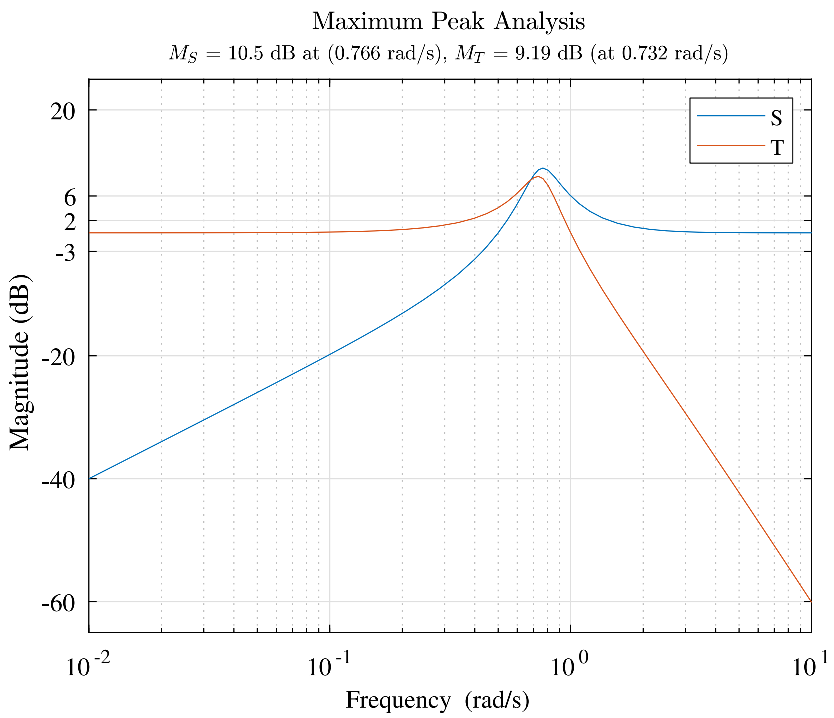


Figure 2.3: Maximum peak gains of the sensitivity and complementary sensitivity function of a controlled plant $G = \frac{1}{s^4+5s^3+7s^2+3s}$ with a PI controller $K = 3 + s$. The controller doesn't satisfy the established rules of thumb of $M_S < 6\,dB$ and $M_T < 2\,dB$.

2.1.3 Pole-Zero Stability Analysis

In linear system theory, the stability of a system can be analyzed by the position of its poles. A system with poles on the right half plane is unstable. In contrast, a system with poles on the imaginary axis is undamped and marginally stable. Further, a system with poles on the left half plane is stable. Additionally, the damping of a system can be calculated by the position of its poles. [5, p. 138f.]

2.1.4 Two Degrees-of-Freedom Controller

A different SISO controller design is the two degrees-of-freedom (DoF) controller, presented in Figure 2.4. The controller has two distinct inputs (q_d and e) rather than one (e). Consequently, the controller is split into K_r and K_y. K_r is manipulating the reference input and K_y is handling the effects of disturbances. This design is often used to shape the tracking behavior of the plant ($q_d \rightarrow \tilde{q}_d$). The new control law is defined in (2.8). A practical example of a 2-DoF controller is presented in [5, p. 56f.].

$$u \;=\; K_y\ (\tilde{q}_d - q_n) \;=\; K_y\ (K_r\, q_d - q_n) \tag{2.8}$$

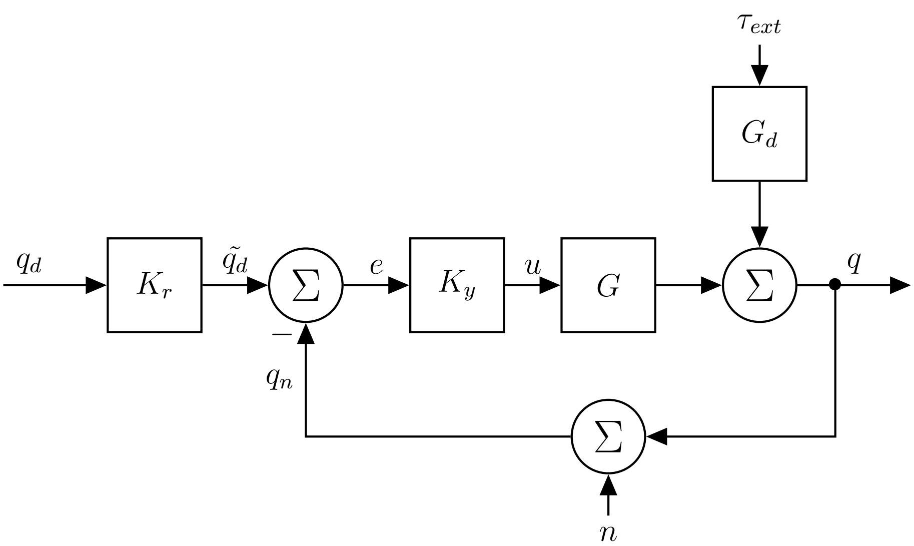


Figure 2.4: Two degrees-of-freedom controller (adapted from Figure 2.22 in [5, p. 56]).

The closed-loop transfer functions S, T, N and D are identical to the 1-DoF controller (with consideration of changing K to K_y and q_d to $\tilde{q}_d$). The tracking behavior now deviates from T. The new tracking transfer function R is defined as:

$$R = \frac{q}{q_d} = K_r T \; . \tag{2.9}$$

2.2 Impedance Control

When two physical systems interact with each other and exchange energy, one is taking the roll of an admittance and the other one as an impedance. Mechanical admittance is a systems velocity output over an imposed force input. Impedance is its reciprocal:

$$Z = \frac{F}{v} \; . \tag{2.10}$$

While not all systems are able to move, or exert a force, they can always be pushed upon and take the roll of an admittance. This applies, for instance, to a rigid environment. Therefore, the environment should take the roll of an admittance and the robot as an impedance. Controlling the robot's impedance is to control the force, the robot exerts to an imposed velocity.

Impedance control is used to control the interaction behavior of a robot.

2.3 Compliance Control

Stramigioli and Folkertsma (2017) explain, that a simpler form of impedance control is compliance control [2, p. 164]. The robot's inertia remains unchanged. A simple form of a compliance controller is the PD controller. It was first introduced to robotics by Takegaki and Arimoto (1981) and since then implemented in many variations [7][8][9].

The force exerted by the controller is defined by:

$$F_{Control} = -K\left(q - q_d\right) - D\dot{q} \; , \tag{2.11}$$

where K and D are the stiffness and damping terms of the controller. The term $q - q_d$ is the negative deviation from the reference position. The PD controller can be represented by a spring-damper system (illustrated in Figure 2.5).

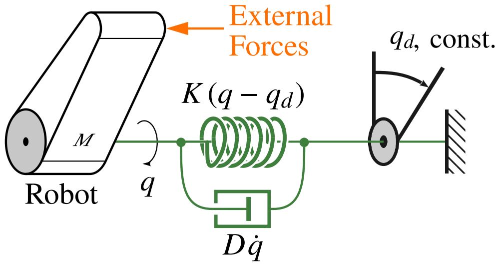


Figure 2.5: Mechanical representation of the PD controller.

In this thesis we only cover the regulation case with the reference input $q_d = const$. For the tracking case, with $q_d \neq const.$, different control strategies are needed.

2.4 Elastic Structure Preserving Impedance Control

Flexible joint robots, with an extra spring between link and motor, have several advantages. The extra spring is an energy storage and can absorb impacts. As a result, the mechanical robustness is increased. Additionally, a robot with more compliant joints is safer for human-robot collaboration. The model of the uncontrolled flexible joint is represented by Figure 2.6 (retrieved from Figure 2a in [3]).

One drawback of flexible joints are their inherent oscillatory properties. The flexible joint in Figure 2.6 has an extra degree of freedom. Hence it is underactuated. In short, the link position q cannot be directly controlled.

This is overcome, by defining a new motor coordinate η, that introduces a second link-side control input $\bar{u}_1$, by using feedforward terms. The resulting system is quasi-fully actuated (cf. Figure 2.7, adapted from Figure 2a in [3]).

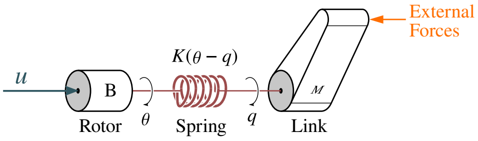
By equating the underactuated and the quasi-fully actuated system, the relationship between the old and new coordinates, the control law for u is revealed.

The formula for the coordinate transformation is deduced by equating the two link dynamics (2.12) and (2.14):

$$\eta \;=\; \theta - \frac{\bar{u}_1}{K}\;. \tag{2.16}$$

The new motor coordinates are called deflection coordinates. They shift the

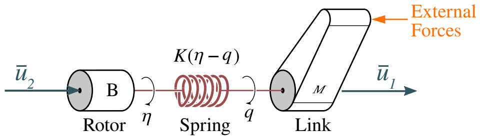
Figure 2.6: Underactuated flexible joint.



$$M\,\ddot{q} = K\,(\theta - q) + \tau_{ext} \quad (2.12)$$

$$B\,\ddot{\theta} + K\,(\theta - q) = u \quad (2.13)$$

Figure 2.7: Quasi-fully actuated flexible joint.



$$M\,\ddot{q} = K\,(\eta - q) + \bar{u}_1 + \tau_{ext} \quad (2.14)$$

$$B\,\ddot{\theta} + K\,(\eta - q) = \bar{u}_2 \quad (2.15)$$

original motor position by the deflection between θ and q, a torque input $\bar{u}_1$, would impose on the plant. The size of the deflection is inversely proportional to the plant's stiffness K.

The control output needed for the underactuated plant is calculated by equating the two motor dynamics (2.13) and (2.15):

$$B\,\ddot{\theta} + K\,\theta - u \;=\; K\,q \;=\; B\,\ddot{\eta} + K\,\eta - \bar{u}_2\;. \quad (2.17)$$

Inserting the coordinate transformation from formula (2.16) yields:

$$B\,\ddot{\theta} + K\,\theta - u \;=\; B\,\ddot{\theta} - \frac{B}{K}\,\ddot{\bar{u}}_1 + K\,\theta - \bar{u}_1 - \bar{u}_2\;. \quad (2.18)$$

This simplifies to the transformation for the control law:

$$u \;=\; \bar{u}_1 + \bar{u}_2 + \frac{B}{K}\,\ddot{\bar{u}}_1\;. \quad (2.19)$$

The new control input $\bar{u}_1$ is used do induce link side impedance through PD control. The virtual control input $\bar{u}_2$ is used to damp the motor inertia B and render the system fully damped. The control laws for the two virtual control inputs are defined by:

$$\bar{u}_1 \;=\; -K_q\,(q - q_d) - D_q\,\dot{q}\;, \quad (2.20)$$

$$\bar{u}_2 \;=\; -D_\eta\,\dot{\eta}\;. \quad (2.21)$$

$\bar{u}_1$ is the control input on the link-side and $\bar{u}_2$ on the motor-side.

ESπ control is not changing the plant's inertial and compliance properties. Instead it is using them for a model-based control approach. Hence the name "structure preserving control" [3].

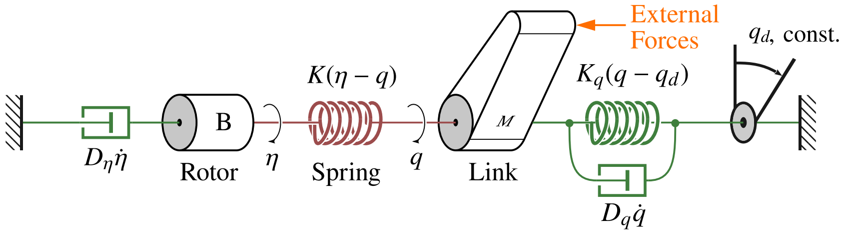


Figure 2.8: Mechanical representation of the ESπ controller (adapted from Figure 2b in [3]).

2.5 Enhanced ESπ Models

For link damping, the link velocity $\dot{q}$ is needed. The transformation (2.19) uses the second derivative of $\bar{u}_1$. Therefore the third derivative (i. e. the jerk) of q, is needed. This makes the controller more complicated to implement as the jerk of a coordinate cannot be measured. In the implementation it is calculated as described in Section 3.4.2.

Stramigioli (1996) used a model-based approach to inject damping without the need of velocity measurement. This is achieved by introducing a virtual mass m internal to the controller [10]. The motivation for the enhanced ESπ controllers is to use this concept to omit one time derivative and therefore the jerk of q as control input.

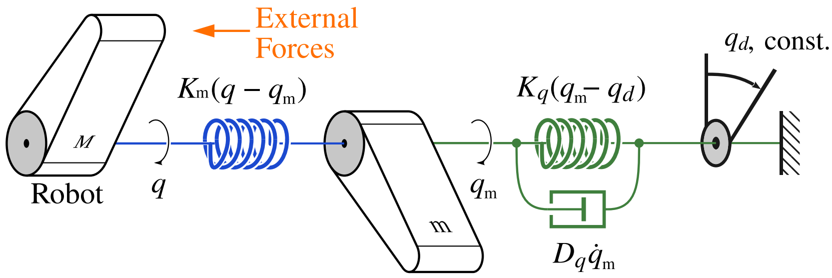


Figure 2.9: Mechanical representation Stramigioli's controller with artificial damping.

ESπ 2 is Stramigioli's controller of Figure 2.9 applied on a flexible joint (see Figure 2.10). ESπ 1 and 3 are models without an extra virtual mass, to evaluate its effect. ESπ 3 and 4 have a different spring configuration. For them, the spring K_q is setting the stiffness of the controller in the static case. For ESπ 1 and 2 the stiffness is determined by the serial interconnection of K_m and K_q. Hereinafter, we will call the initial ESπ controller, of Section 2.4, ESπ 0.

In theory, the enhanced models should be less sensitive for noise, as one derivative is omitted. In addition, control effort for disturbance control should be reduced as the link position q and the desired link position q_d are decoupled by the second virtual spring K_m. Further, omitting $\dot{\tau_{ext}}$, should make the system less responsive to high frequency disturbances. This is improves stability with hard impacts.

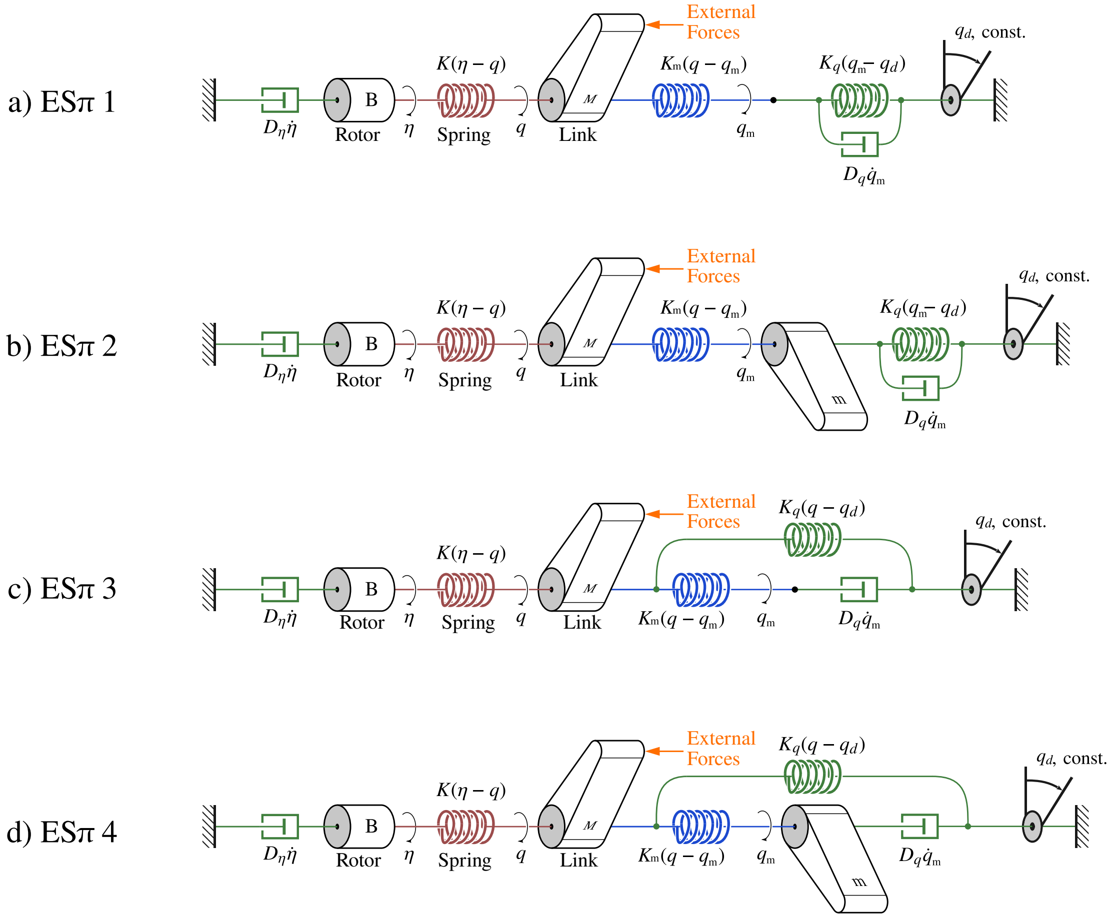


Figure 2.10: Mechanical representation of the enhanced ESπ models.

2.6 Dual Channel Frequency Analysis

We try to verify the results on a testbed and measure the resulting transfer functions. The frequency response function of a system can be attained, by measuring the input $e(t)$ and output signal $a(t)$ and divide their Fourier transformations as shown in (2.22). For discrete systems usually the Fast Fourier Transformation (FFT) is used.

$$H(s) \;=\; \frac{A(s)}{E(s)} \;=\; \frac{FFT(a(t))}{FFF(e(t))} \tag{2.22}$$

The input signal should contain all frequencies up to the Nyquist frequency (e. g. bandwidth limited white noise or impulse). Further, for a more accurate result, the H_1 method is used. The mean of N measurements calculated and each signal is multiplied with the complex conjugate of the input spectrum [11, p. 239 ff.]. The H_1 frequency response is calculated via:

$$H_1(s) \;=\; \frac{\sum_{n=1}^{N}\left(A_n(s)\,E_n{}^*(s)\right)}{\sum_{n=1}^{N}\left(E_n(s)\,E_n{}^*(s)\right)}\;. \tag{2.23}$$

The coherence function γ^2 provides information on how much the output signal was produced by the input signal, or by measurement error. It weights the consistency of the phase difference between the input and output signal. γ^2 can have a range of $0 \le \gamma^2 \le 1$, with full coherence (i. e. sound results) at $\gamma^2 = 1$ [11, p. 230 ff.]. The coherence function is attained by calculating

$$\gamma^2 \;=\; \frac{\left|\sum_{n=1}^{N}\left(A_n(s)\,E_n{}^*(s)\right)\right|^2}{\sum_{n=1}^{N}\left(A_n(s)\,A_n{}^*(s)\right)\,*\,\sum_{n=1}^{N}\left(E_n(s)\,E_n{}^*(s)\right)}\;. \tag{2.24}$$

To avoid spectral leakage, a Hann (Hanning) window is used for the noise signals and an exponential window for the disturbance signals. The Hann window, for the signal length N, is defined as:

$$w_H(n) \;=\; 0.5\left(1-\cos\left(\frac{2\,\pi\,n}{N}\right)\right),\; 0 \,\le\, n \,\le N\;. \tag{2.25}$$

The exponential window, with τ being the time constant, is defined as:

$$w_E(n) \;=\; e^{-\frac{n}{N\,\tau}},\; 0 \,\le\, n \,\le N\;. \tag{2.26}$$

3 Methodology

In the following, the plant and the new ESπ models will be analyzed. The mathematical derivations and the control law of the new models is explained. The models are transformed in a 3-DoF controller notation, to analyze their noise and disturbance behavior. Furthermore, the method of numerical optimization for low noise is set out.

3.1 Plant Model Analysis

The model of the underactuated plant of Figure 2.6 is used. This model represents the plant of the testbed with the following parameters:

K:	374	$N\,m\,rad^{-1}$	Stiffness of the flexible joint
M:	1.00	$kg\,m^2$	Link inertia
B:	0.598	$kg\,m^2$	Motor inertia

Table 3.1: Physical plant parameters.

The coordinates θ and q are coupled by the dynamic equations of the plant. Therefore, by performing a Laplace transform, θ in the equation for the plants motor dynamic (2.13) can be substituted with the dynamic equation of the link (2.12). Setting the disturbance input $\tau_{ext} = 0$ yields the plant's transfer function:

$$G = \frac{q}{u} = \frac{K}{B\,M\,s^4 + (B\,K + K\,M)\;s^2}\;. \tag{3.1}$$

Setting the plant's input $u = 0$ yields the disturbance model. It describes how external torques affect the link position:

$$G_d = \frac{q}{\tau_{ext}} = \frac{B\,s^2 + K}{B\,M\,s^4 + (B\,K + K\,M)\;s^2}\;. \tag{3.2}$$

Figure 3.1 depicts the bode diagrams of G and G_d. The resonance spikes at 24.9 $rad\,s^{-1}$ and 31.8 $rad\,s^{-1}$ are noteworthy. They originate from the undamped oscillations of M in G, and additionally of M and B, in G_d. Theoretically, they produce infinite gains at their resonance frequency. This will be reflected in S (see Section 4.1.2).

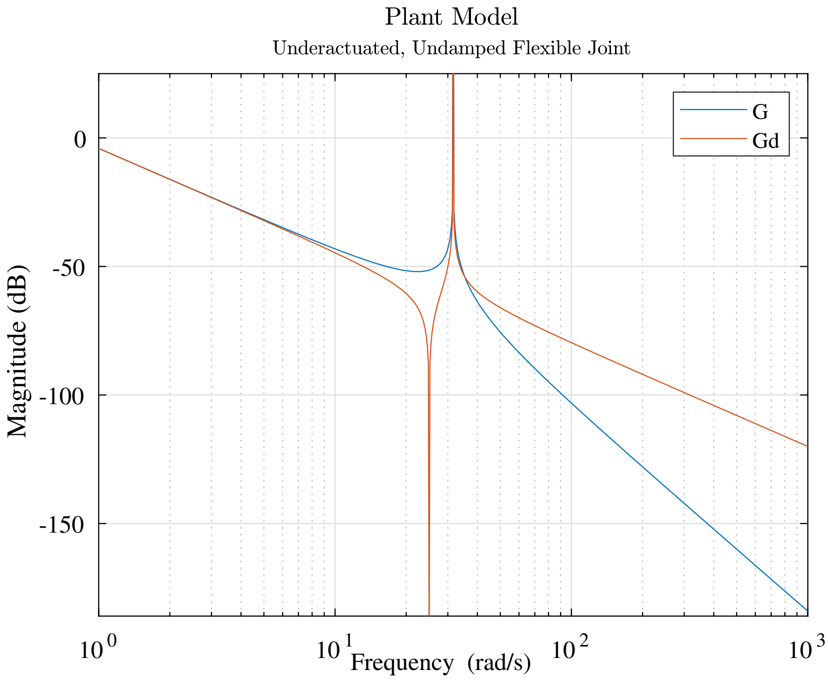


Figure 3.1: Bode diagram of the plant and disturbance model.

Damping the plant (cf. Figure 3.2) results in damped spikes (cf. Figure 3.3).[1] Nevertheless, the undamped plant is used for the model-based control approach. For further simplicity, model uncertainty is also neglected.

[1] A damping ration of $\zeta = 0.01$ was used.

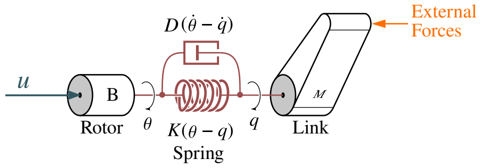


Figure 3.2: Mechanical representation of the underdamped plant.

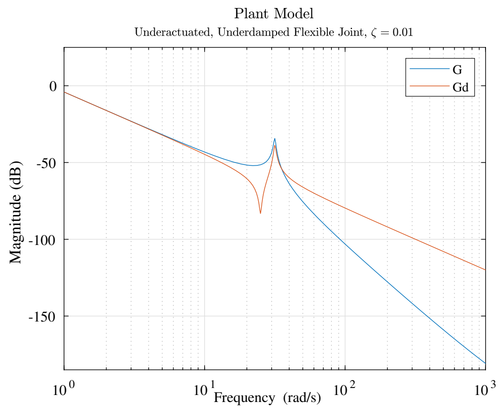


Figure 3.3: Bode diagram of the underdamped plant.

3.2 ESπ V Reference Model

ESπ V (illustrated in Figure 3.4 and described by the model equation (3.3)) is a simple, second order spring-mass-damper configuration, similar to a PD controller. In addition, it is an intuitive model to set a desired impedance behavior by choosing K_V and D_V. It will be used for comparison with the new ESπ models. Ensuring only a small deviation of the new model's impedance from the reference impedance is an essential part of the optimization method (described in Section 3.6.1).

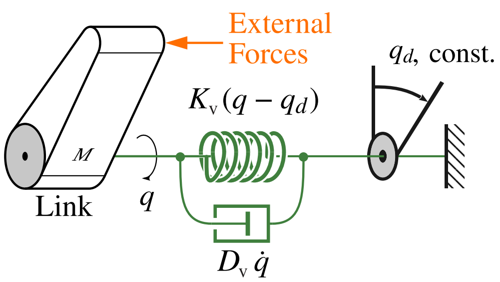


Figure 3.4: Mechanical representation of the ESπ V reference model.

$$M\ddot{q} = -K_V\ (q - q_d) - D_V\dot{q} + \tau_{ext} \tag{3.3}$$

The tracking transfer function R describes the correlation between the reference input q_d and link output q. It is attained by setting $\tau_{ext} = 0$ in the model equation

(3.3) and performing a Laplace transformation:

$$R_V = \frac{q}{q_d} = \frac{K_V}{M\, s^2 + D_V\, s + K_V} \; . \tag{3.4}$$

The impedance behavior Z_V defines the force τ_{ext} the robot exerts as a reaction of the velocity $\dot{q}$ imposed on by the environment. The impedance transfer function is attained by setting $q_d = 0$:

$$Z_V = \frac{\tau_{ext}}{\dot{q}} = \frac{M\, s^2 + D_V\, s + K_V}{s} \; . \tag{3.5}$$

Table 3.2 shows the selected parameters for the reference model. The damping ratio is set to $\zeta = 0.7$. This reduces control effort and the effect of noise. As a result, the model is slightly underdamped.

K_V:	200	$N\,m\,rad^{-1}$	Stiffness
D_V:	$0.7 * 2\sqrt{K_V\, M}$	$N\,m\,rad^{-1}\,s^{-1}$	Damping, with $\zeta = 0.7$

Table 3.2: Parameters of the reference model ESπ V.

Figure 3.5 shows the response of ESπ V to a step in reference. The underdamped ζ contributes to a slight overshoot. Figure 3.6 depicts the frequency response of Z_V and R_V. Intuitively, the model has the lowest impedance at its resonance frequency at 14.1 $rad\,s^{-1}$.

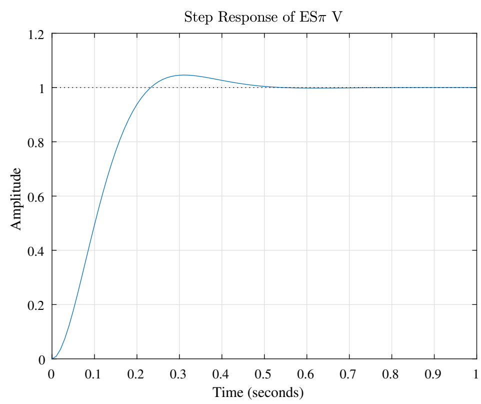


Figure 3.5: Step response of ESπ V.

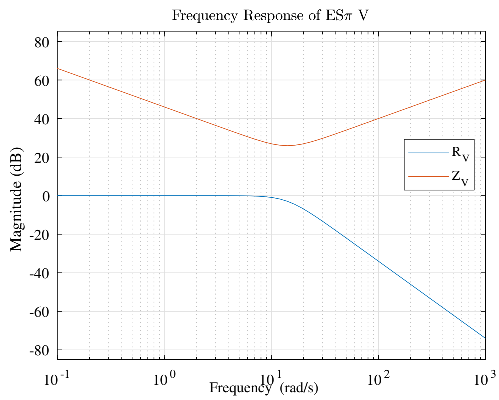


Figure 3.6: Frequency response of the reference and impedance behavior of ESπ V.

3.3 Control Law for ESπ 2

The following derivation is analogous to the other ESπ models. The mechanical substitution model (depicted in Figure 2.10b) is defined by the following system of differential equations:

$$\begin{aligned} M\,\ddot{q} &= K\,(\eta - q) - K_m\,(q - q_m) + \tau_{ext}\ , && (3.6)\\ B\,\ddot{\eta} &= -K\,(\eta - q) - D_\eta\,\dot{\eta}\ , && (3.7)\\ m\,\ddot{q_m} &= K_m\,(q - q_m) - K_q\,(q_m - q_d) - D_q\,\dot{q_m}\ . && (3.8) \end{aligned}$$

To solve the equation system all equations are Laplace transformed. Further, The second system equation (3.7) replaces η in the first (3.6). The third equation (3.8) replaces q_m. Like in Section 3.2, setting $\tau_{ext} = 0$ will yield the tracking transfer function R_{Mech} and setting $q_d = 0$ the impedance transfer function Z_{Mech}.

R_{Mech} and Z_{Mech} are used for comparison with the achieved closed-loop transfer functions. These must be algebraically identical, as we use a model-based approach.[2]

The control law for ESπ 2 is derived by equating the mechanical substitution model with the quasi-fully actuated plant. Consequently, the link dynamics (3.6) and (2.14) will yield the link-side control input $\bar{u_1}$, and through the motor dynamics (3.7) and (2.15) the motor-side control input $\bar{u_2}$ is attained. Accordingly:

$$\begin{aligned} \bar{u_1} &= -K_m\,(q - q_m)\ , && (3.9)\\ \bar{u_2} &= -D_\eta\,\dot{\eta}\ . && (3.10) \end{aligned}$$

The variable q_m is calculated via (3.8).

[2]They are transfer functions of sixth order, with many parameters. Unfortunately, that makes them, alongside with several others, too long to be displayed in this thesis. They are presented in the appendix.

3.4 Implementation of the Enhanced ESπ Models

Figure 3.7 shows the Simulink model of the ESπ 2 implementation[3] for the testbed. First, the coordinate of the virtual mass q_m is calculated. Additionally, a plant model is used for model-based design. The implementation is designed for the use on hardware. θ and q are measured by position sensors and derived numerically for full-state feedback.

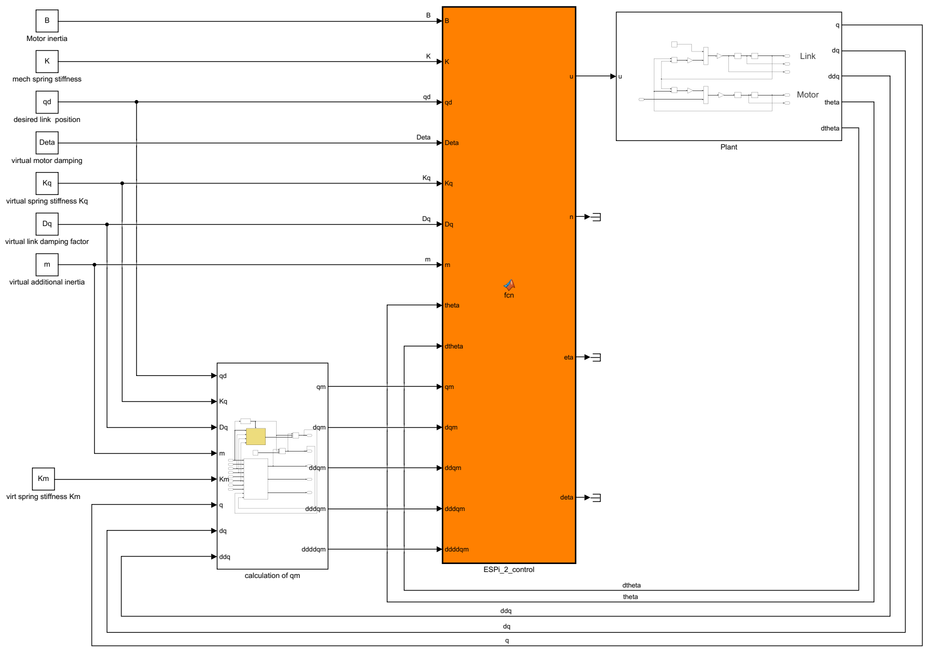


Figure 3.7: Simulink model of the implementation of ESπ 2.

3.4.1 Setting the Initial conditions for q_m

To ensure a smooth initialization of the controller, the virtual springs K_m and K_q have to exert an equal force on the virtual coordinate q_m. This ensures, that q_m

[3]The implementation of the ESπ controller is not part of this thesis.

is not pulled away of q_d after initializing a new reference position. Therefore, the initial condition of q_m for ESπ 1 and 2 must ensure:

$$K_q\ (q_m - q_d) = K_m\ (q - q_m)\ . \tag{3.11}$$

This is the case for

$$q_m init = \frac{K_m\, q + K_q\, q_d}{K_q + K_m}\ . \tag{3.12}$$

For ESπ 3 and 4 a jerk effect can be prevented by setting

$$q_m init = q_d\ . \tag{3.13}$$

Furthermore, no damping forces should be present during initialization. Therefore the initial velocity of q_m is set to zero. The initial conditions are set, if the controller is enabled or the reference position q_d is changed.

3.4.2 Calculating the Higher Derivatives by Estimating the External Forces

The higher derivatives of q can be calculated with the untransformed plant dynamic equations of Section 2.4:

$$\ddot{q} \;=\; \frac{K\,(\theta - q) + \tau_{ext}}{M}\ , \tag{3.14}$$

$$\dddot{q} \;=\; \frac{K\,(\dot{\theta} - \dot{q}) + \dot{\tau_{ext}}}{M}\ . \tag{3.15}$$

The external torque τ_{ext} and its time derivative are needed. De Luca, Albu-Schaffer, Haddadin and Hirzinger (2006) showed, that a filtered version of the external torque can be estimated by observing the robot's momentum and therefore only using position and velocity signals [12].

3.5 Three Degrees-of-Freedom Transformation

A central part of this thesis is to transform the ESπ controller into a conventional form to which the classical SISO control methods apply. To this end, the closed-loop transfer functions can be calculated and shaped. A SISO controller has a control law with only one variable to be controlled. We choose the link position q.

3.5.1 Control Law for the Underactuated Plant

The virtual coordinate q_m, according to the third equation of the mechanical substitution system (3.8), is substituted into the equation for the control law for the link-side input (3.9). This formulates the control law for $\bar{u_1}$ as a function of q_d and q:

$$\bar{u_1} \;=\; \frac{K_m\,K_q}{m\,s^2 + D_q\,s + K_m + K_q}\;q_d \;-\; \frac{K_m\,m\,s^2 + D_q\,K_m\,s + K_m\,K_q}{m\,s^2 + D_q\,s + K_m + K_q}\;q\;. \tag{3.16}$$

By choosing the parameters of Table 3.3 $\bar{u_1}$ simplifies to:

$$\bar{u_1} \;=\; K1\,q_d \;-\; K2\,q\;. \tag{3.17}$$

For the different ESπ models, only $K1$ and $K2$ change.

ESπ 0:	$K1 = K_q$	$K2 = D_q\,s + K_q$
ESπ 1:	$K1 = \frac{K_m\,K_q}{D_q\,s+K_m+K_q}$	$K2 = \frac{D_q\,K_m\,s+K_m\,K_q}{D_q\,s+K_m+K_q}$
ESπ 2:	$K1 = \frac{K_m\,K_q}{m\,s^2+D_q\,s+K_m+K_q}$	$K2 = \frac{K_m\,m\,s^2+D_q\,K_m\,s+K_m\,K_q}{m\,s^2+D_q\,s+K_m+K_q}$
ESπ 3:	$K1 = K_q$	$K2 = \frac{(D_q\,K_m+D_q\,K_q)\,s+K_m\,K_q}{D_q\,s+K_m}$
ESπ 4:	$K1 = K_q$	$K2 = \frac{(K_m\,m+K_q\,m)\,s^2+(D_q\,K_m+D_q\,K_q)\,s+K_m\,K_q}{m\,s^2+D_q\,s+K_m}$

Table 3.3: Control terms $K1$ and $K2$ for ESπ 0-4.

The control law for the motor-side control input $\bar{u_2}$ (3.10), is using the virtual motor coordinate η. By substituting η, according to the equation for the coordinate transformation (2.16), the control law becomes a function of θ, q and q_d.

Further, substituting θ with the equation for the link dynamic of the untransformed plant yields the control law as a function of q_d, q and τ_{ext}:

$$\bar{u_2} = \frac{D_\eta \, K1 \, s}{K} \, qd + \left(-\frac{D_\eta \, M \, s^3}{K} - \frac{D_\eta \, (K + K2) \, s}{K} \right) q + \frac{D_\eta \, s}{K} \, \tau_{ext} \; . \tag{3.18}$$

The control output u is calculated via the transformation for the control law (2.19) and simplifies to:

$$\begin{aligned} u = & \left(\frac{B \, K1}{K} \, s^2 + \frac{D_\eta \, K1}{K} \, s + K1 \right) q_d \\ & - \left(\frac{D_\eta \, M}{K} \, s^3 + \frac{B \, K2}{K} \, s^2 + \frac{D_\eta \, K + D_\eta \, K2}{K} \, s + K2 \right) q \\ & + \frac{D_\eta}{K} \, s \, \tau_{ext} \; . \end{aligned} \tag{3.19}$$

Thus, the control law can be defined with only one controlled coordinate: q.[4] The derivatives of q_d are omitted in the implementation because $q_d = const$. Nevertheless, they are kept in the simulation model for comparison with R_{Mech} and Z_{Mech}.

3.5.2 3-DoF Controller

Equation (3.19) can be simplified by defining three control parameters K_v, K_y and K_τ:

$$u = K_v \, q_d - K_y \, q + K_\tau \, \tau_{ext} \; . \tag{3.20}$$

Choosing

$$K_r = \frac{K_v}{K_y} \; , \tag{3.21}$$

$$K_d = \frac{K_\tau}{K_y} \tag{3.22}$$

[4]The disturbance input τ_{ext} has been transformed to a second control input. Therefore, the resulting control loop is strictly speaking not a SISO system. This is discussed in Chapter 5.

will yield the control law for the 3-DoF controller (with $q_n = q + n$):

$$u \;=\; K_y \; (K_d \, \tau_{ext} + K_r \, q_d - q_n) \;\; . \qquad (3.23)$$

The resulting control loop is illustrated in Figure 3.8. K_d is an extra control variable for the disturbance input. It originates from substituting the plant dynamic (2.12). Additionally, τ_{ext} is affecting the plant output directly via G_d. The 3-DoF controller can be exemplified by an interconnection of two 2-DoF controllers. The transfer functions K_r and K_y control the link position and K_d and K_y are handling disturbance rejection.

A similar controller was proposed by Chapel and Su (1991). Additional torque measurement was used for impedance control. The 3-DoF controller was optimized using the H_∞ method [13].

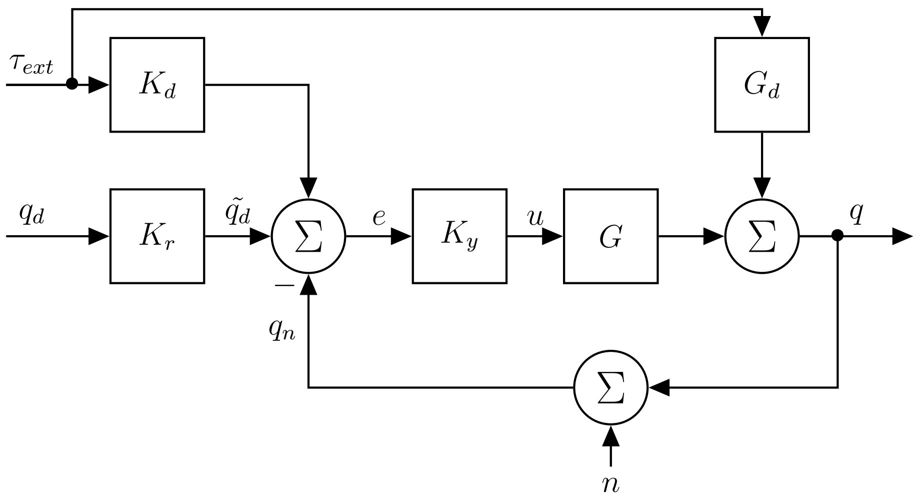


Figure 3.8: Three degrees-of-freedom controller.

An important point is, that τ_{ext} is not measured in the implementation. Instead, it is estimated as described in section 3.4.2.

3.5.3 3-DoF Closed-Loop Transfer Functions

With $\tau_{ext} = 0$, the 3-DoF controller becomes a 2-DoF controller. The transfer functions S, T, N and R are as defined as in Section 2.1. The disturbance and impedance transfer functions D and Z are derived as follows.

The 3-DoF control law (3.23) with $n = 0$ and $q_d = 0$ will yield:

$$\begin{aligned} u &= K_y\,(K_d\,\tau_{ext} - q) \\ u &= K_y\,K_d\,\tau_{ext} - K_y\,G\,u - K_y\,G_d\,\tau_{ext} \\ u\,(1 + K_y\,G) &= \tau_{ext}\,(K_y\,K_d - K_y\,G_d) \end{aligned}$$

with $L = K_y\,G$ and $S = (1+L)^{-1}$

$$\begin{aligned} \frac{u}{\tau_{ext}} &= K_y\,K_d\,S - K_y\,G_d\,S \\ D = \frac{u}{\tau_{ext}} &= K_y\,S\,(K_d - G_d)\ . \end{aligned} \tag{3.24}$$

Additionally:

$$\begin{aligned} q &= G_d\,\tau_{ext} + L\,e \\ q &= G_d\,\tau_{ext} + L\,(K_d\,\tau_{ext} - q) \\ q\,(1+L) &= G_d\,\tau_{ext} + K_d\,L\,\tau_{ext} \end{aligned}$$

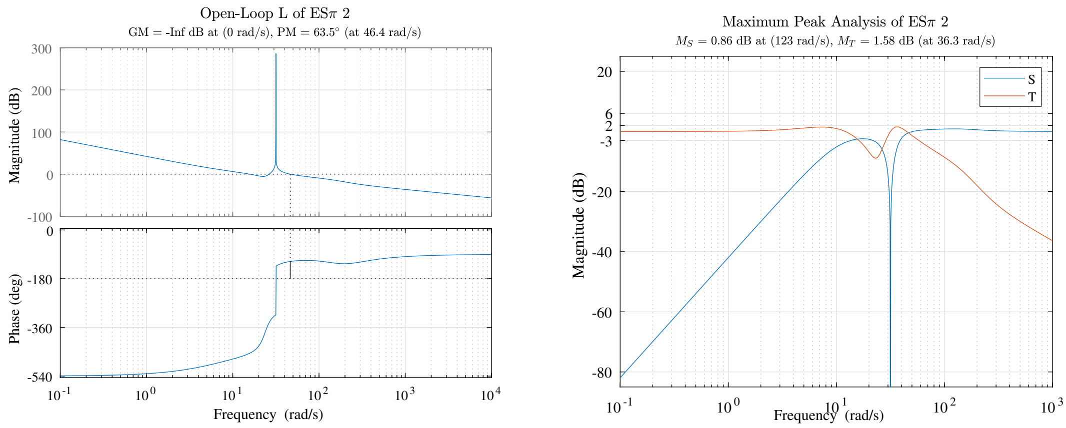
with $T = L\,S$

$$\frac{q}{\tau_{ext}} = G_d\,S + K_d\,T$$

$$Z = \frac{\tau_{ext}}{\dot{q}} = (G_d\,S\,s + K_d\,T\,s)^{-1}\ . \tag{3.25}$$

3.6 ESπ Optimization

Figure 3.9a shows L of ESπ 2. The phase of L is unsuitable to measure the GM and PM by the method laid out in Section 2.1.1 (e. g. $\angle L$ does not cross the -180° mark from above). The closed-loop transfer functions S and T, on the other hand have a conventional shape[5] (cf. Figure 3.9b).



(a) Bode diagram of the open-loop transfer function.

(b) Maximum peak gains of the sensitivity and complementary sensitivity function.

Figure 3.9: Loop shaping and maximum peak analysis of ESπ 2. Parameters: $D_\eta = 0.3 * 2\sqrt{K\,B}$; $K_q = 220$; $D_q = 0.7 * 2\sqrt{K_q\,M}$; $K_m = 10\,K_q$ and $m = 0.1\,M$.

[5]The spike downwards in sensitivity originates from the undamped poles of the plant. This is analyzed in detail in Section 4.1.2.

3.6.1 Impedance Constraints

The virtual parameters of the ESπ models are optimized, to achieve better noise damping and reduced control effort for disturbance rejection. The reference impedance behavior Z_V defines the optimization constraints. Consequently, the optimized ESπ models are only allowed to deviate $\pm\, 6\, dB$ from Z_V (cf. Figure 3.10).

Control performance will degrade if the controller is optimized for low noise as a fast controller is also more sensitive for noise. The impedance constraints serve also as a constraint to ensure a satisfactory disturbance rejection and regulation performance. The constraints of $\pm\, 6\, dB$ showed good performance, i. e. reasonable M_S and M_T.

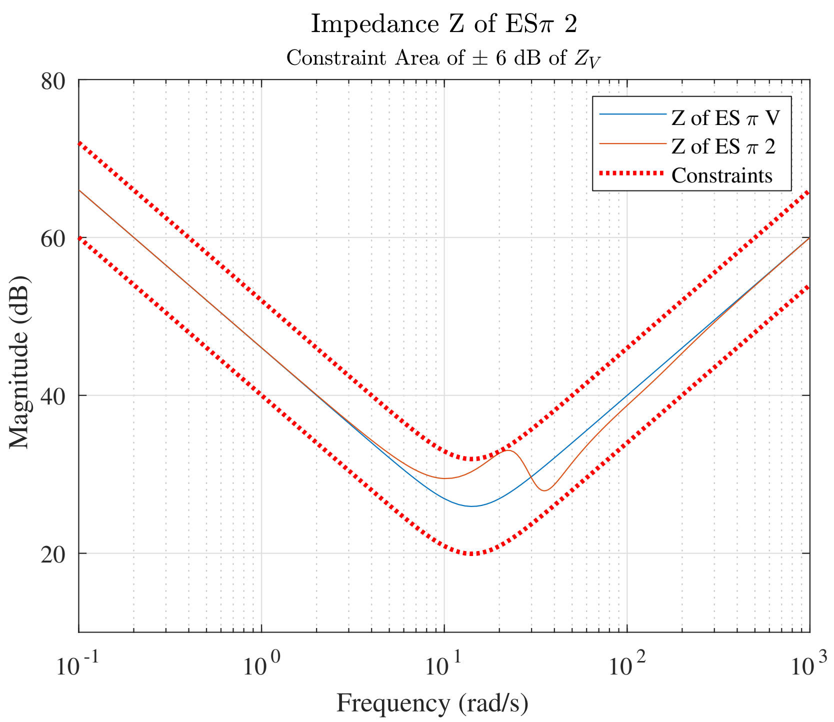


Figure 3.10: ESπ 2 and ESπ V in the impedance constraint area for the optimization, with start parameters.

3.6.2 Parameters for the Optimization

The optimization parameters are reduced by choosing D_η and D_q as follows:

$$
\begin{aligned}
D_\eta &= 0.3 * 2\sqrt{K\,B}\ , && (3.26)\\
D_q &= 0.7 * 2\sqrt{K_q\,M}\ . && (3.27)
\end{aligned}
$$

The link damping D_q depends on the virtual spring stiffness K_q. Consequently, D_q will change during the optimization as well. The chosen interrelation of the parameters has shown good control performance in practice.

For ESπ 1 and 2, the compliance in the in quasi-stationary case is dominated by the serial interconnection of K_m and K_q. Optimizing the ratio $X = \frac{K_m}{K_q}$ further limits the parameters. Additionally, the compliance should correspond to K_V. This will ensure that $Z \rightarrow Z_V$ for low frequencies. Therefore, by defining

$$\frac{1}{K_m} + \frac{1}{K_q} = \frac{1}{K_V} \qquad (3.28)$$

with

$$X = \frac{K_m}{K_q} \qquad (3.29)$$

yields:

$$
\begin{aligned}
K_m &= K_V\ (X+1)\ , && (3.30)\\
K_q &= K_V\,\frac{X+1}{X}\ . && (3.31)
\end{aligned}
$$

Consequently, $K_q = K_V$ is chosen for ESπ 3 and 4.

ESπ 1 and 2:	$X = 10$
ESπ 3 and 4:	$K_m = 10\,K_q$
Additionally for ESπ 2 and 4:	$m = 0.1\,M$

Table 3.4: Start parameters for the optimization.

With the parameters of Table 3.4, and the additional physical plant parameters of Table 3.1, all virtual control parameters can be calculated. As start parameters, the parameters Stramigioli (1996) recommended were chosen [10].

3.6.3 Optimization Method

The objective functions N and D are illustrated in Figure 3.11. The H_∞ method needs proper transfer functions to find the maximum peak gains. Unfortunately, N is not proper. Hence, it is increasing with higher frequencies. A weight w_N, with a cutoff frequency ω_c^* is defined by (3.32), to shape N to be proper. K_N is shifting N in the same range as D. Noise above 100 $rad\, s^{-1}$ is unlikely to affect the plant output, as those frequencies can not be produced by the motor. Therefore the parameters of Table 3.5 are chosen.

Taken together, minimizing the maximum peaks of D and $w_N\, N$ will minimize the effects of noise. M_S and M_T is neglected in the optimization. Instead, the impedance constraints limit robustness degradation and M_S and M_T are verified afterwards.

$$w_N = \frac{K_N}{(s+\omega_c^*)^3} \tag{3.32}$$

ω_c^*:	$100\, rad\, s^{-1}$
K_N:	150 [no unit]

Table 3.5: Parameters for the weight of the noise transfer function w_N.

The H_∞ norm of the vector of $w_N\, N$ and D is calculated with the Matlab function `hinfnorm()`. Further, the Matlab numeric optimization function `fmincon()` was used.[6] An interior-point algorithm is used to find:

$$\min\left(H_\infty\right) = \min\left(\left\| \begin{matrix} w_N\, N \\ D \end{matrix} \right\|_\infty\right) . \tag{3.33}$$

[6]There is also a Matlab function for H_∞ optimization: `hinfsyn()`. This algorithm is the conventional H_∞ optimizer and is used to optimize for fast control performance with acceptable robustness. Conversely, we optimize for low noise and use the reference impedance as constraint. The Matlab code for the used optimization method is in the appendix.

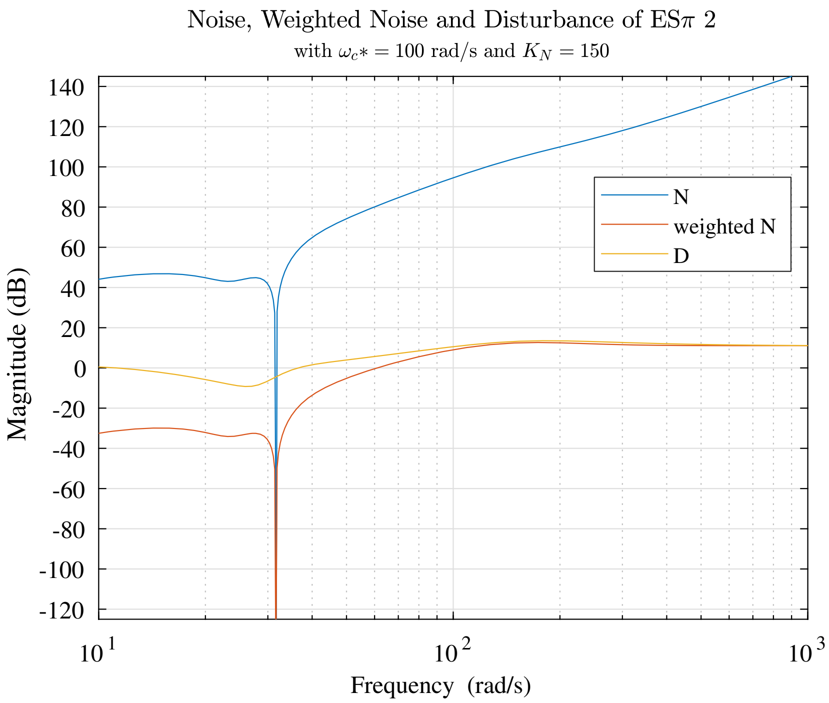


Figure 3.11: Noise and disturbance transfer functions of ESπ 2 with start parameters and weights of $\omega_c^* = 100\, rad\, s^{-1}$ and $K_N = 150$.

4 Experiments and Results

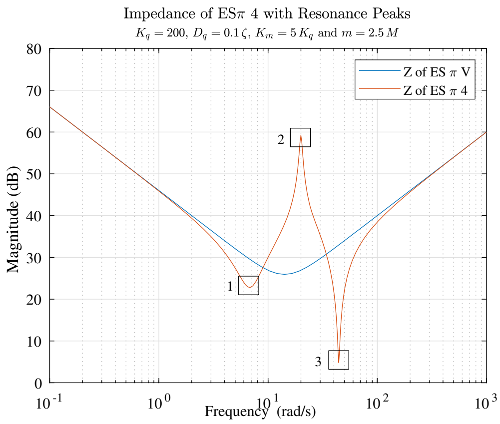
The following chapter presents the optimization results and compares the optimized models ESπ 1 to 4 with ESπ 0. The best candidate is chosen with respect to impedance behavior and noise damping. ESπ 2 was tested on the test bed.

4.1 Simulation and Optimization Results

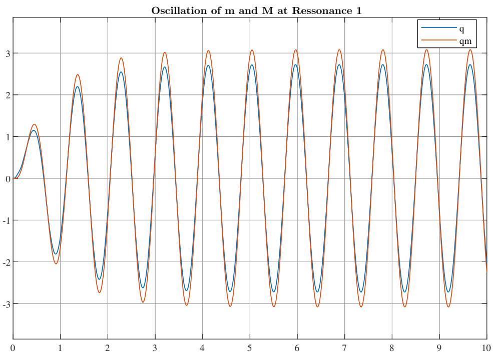
In order to determine the model with the best properties, the simulation results were analyzed. Additionally, they were compared with the initial ESπ 0 controller.

4.1.1 Resonance Frequencies

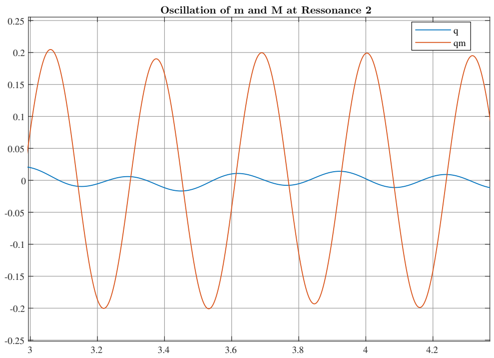
All ESπ models have higher order transfer functions than the ESπ reference model. The extra virtual mass and springs generate additional resonance frequencies. At these frequencies M and m oscillate either in or out of phase. This results in peaks of low and high impedance. Figure 4.1a depicts the resonance frequencies of the impedance function of ESπ 4. Additionally, Figures 4.1b to 4.1d show the oscillations of M and m. At these frequencies, the impedance easily violates the constraints, if sufficient damping is not satisfied. In the following graphs, D_q and K_q were tuned down and m up. This will amplify the oscillations.



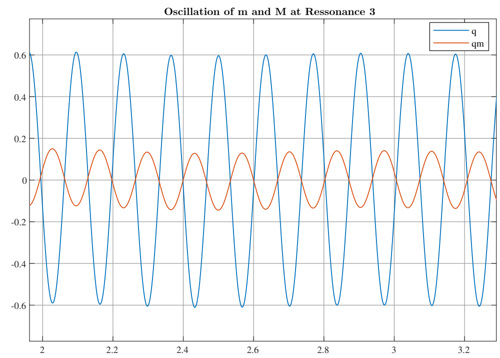
(a) Resonance peaks is impedance of ESπ 4



(b) Resonance 1: M and m oscillate in phase.



(c) Resonance 2: M and m oscillate 90° out of phase.



(d) Resonance 3: M and m oscillate 180° out of phase.

Figure 4.1: Resonance frequencies of ESπ 4 and oscillations of the coordinates q and q_m of the inertia M and m. With the parameters $D_q = 0.1 * 2\sqrt{K_q\,M}$; $K_m = 5\,K_q$ and $m = 2.5\,M$.

The fourth resonance frequency is the resonance of the motor inertia B. Tuning B up to 5 and D_η down to 0.1 ζ will exemplify the effect. At its resonance frequency B and τ_{ext} oscillate 180° out of phase (cf. Figure 4.3). B is acting as a harmonic absorber, canceling the input frequency of τ_{ext} and resulting in high impedance (cf. Figure 4.2).

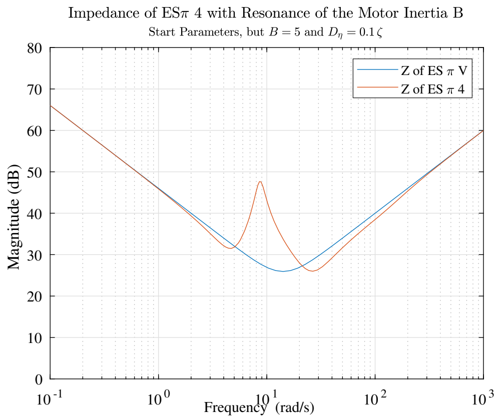


Figure 4.2: Increased impedance at the resonance of the motor inertia B.

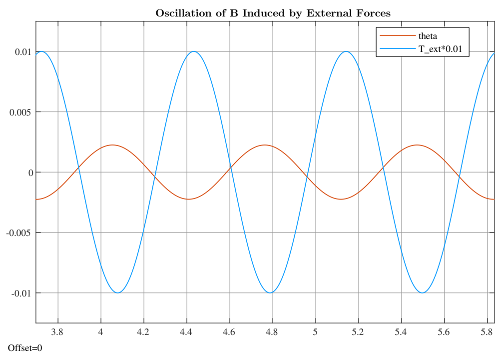


Figure 4.3: Destructive interference of the imposed external forces τ_{ext} and the motor inertia B.

4.1.2 Pole-Zero Cancellation and Stability

The plant model is undamped (cf. Section 3.1) and therefore has poles on the imaginary axis. The controller must compensate, because the simulated mechanical substitution model is fully damped. Hence, the controller will have a high torque output at the resonance frequency of the plant, resulting in high disturbance damping. Consequently, $S \to 0$.

This will reflect in zeros of S on the imaginary axis. To this end, the undamped poles of the plant are canceled by the sensitivity function (illustrated in Figure 4.4) and therefore not present in the tracking transfer function. However, in the physical world control outputs are limited and all oscillations have damping.

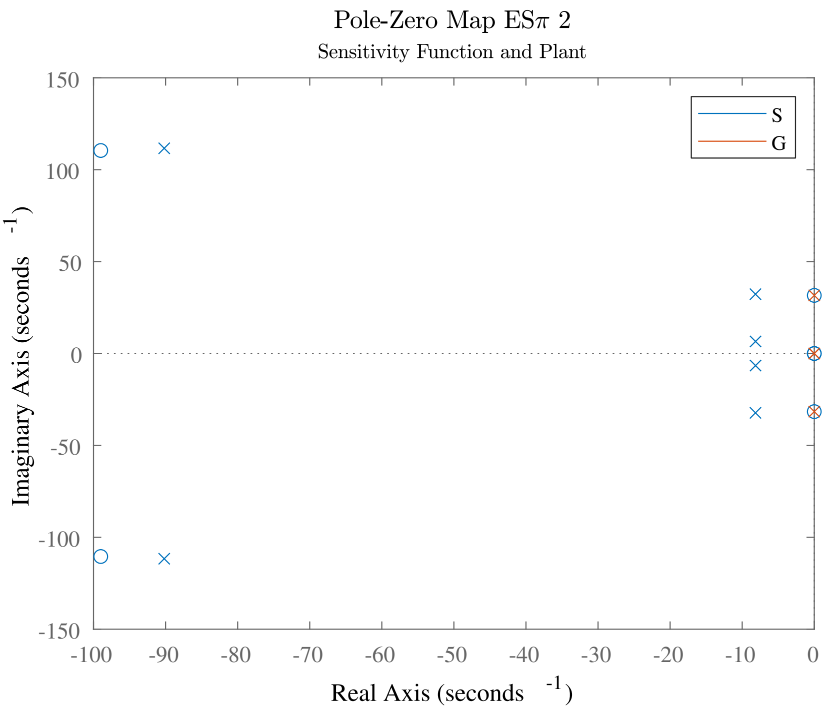


Figure 4.4: Pole-zero cancellation of the undamped poles of the plant G by the sensitivity function S, with model ESπ 2.

4.1.3 Optimized Control Parameters

Table 4.1 presents the optimized parameters. Further, $K_q = K_v$ and $D_q = 0.7 * 2\sqrt{K_q M}$ for ESπ 0, were chosen, for comparison.

$$
\begin{array}{lrcl}
\text{ES}\pi\ 1: & X & = & 5.3980 \\
\text{ES}\pi\ 2: & X & = & 2.7273 \quad m = 0.3898\,M \\
\text{ES}\pi\ 3: & K_m & = & 1154.6 \\
\text{ES}\pi\ 4: & K_m & = & 1000.6 \quad m = 0.0754\,M
\end{array}
$$

Table 4.1: Optimized control parameters with better noise and disturbance damping.

Figures 4.5 and 4.6 show the noise and disturbance improvement of ESπ 2. ESπ 1, 3 and 4 have similar results.

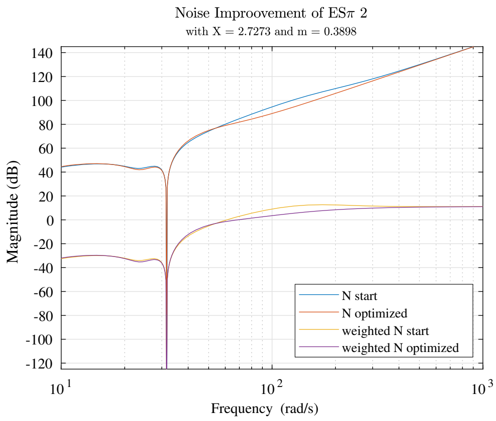


Figure 4.5: Noise improvement of ESπ 2 before and after the optimization.

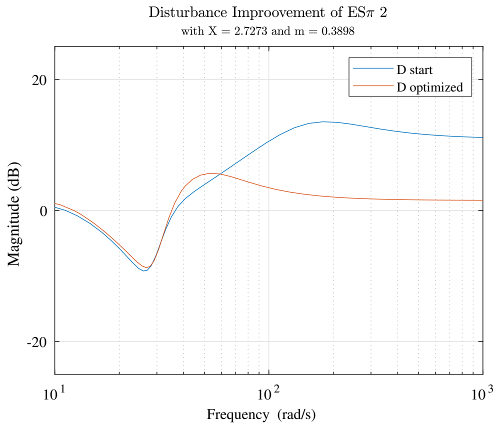


Figure 4.6: Disturbance improvement of ESπ 2 before and after the optimization.

4.1.4 Comparison of the Optimized Models

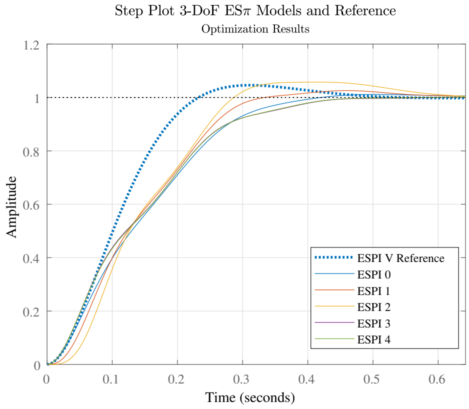
All new models fit in the impedance constraint area. The initial ESπ 0 model is violating the upper constraints slightly. At their resonance frequency at $38.1\,rad\,s^{-1}$ the model's impedances touch the lower constraints and therefore limit the optimization (cf. Figure 4.7b). At the two resonance frequencies the impedance deviates from the reference model. Nevertheless, human interaction with the robot happens at low frequencies. Therefore this effect is unlikely to be noticeable.

The noise improvement over ESπ 0 is $8\,dB$. Moreover, the disturbance improvement is significant (cf. Figure 4.7c and 4.7d). In contrast to ESπ 0, the disturbance response of all new models is proper. A lower disturbance response at high frequencies results in improved robustness against hard impacts.

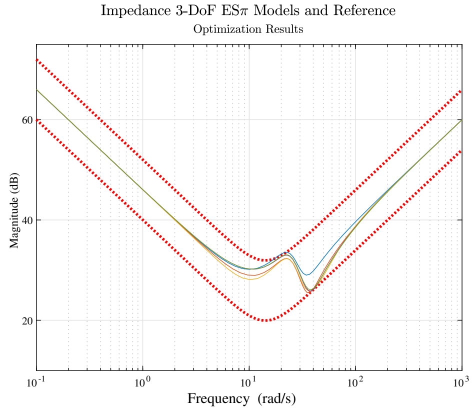
Further, all models are within the robustness criteria $M_S < 6$ dB. The new models violate $M_T < 2$ dB slightly, resulting in a higher overshoot[1](cf. Figure 4.7a). Figure 4.7a also shows the effects of the motor inertia. The step response shows the overlayed oscillation of the motor inertia B.

Although all models show similar results, ESπ 2 shows better noise damping for the disturbance model (cf. Figure 4.7d). Additionally, ESπ 2, has a smaller deviation from the reference impedance, after $5\,rad\,s^{-1}$ (cf. Figure 4.7b). Further, M_T is better than with ESπ 3 and 4.

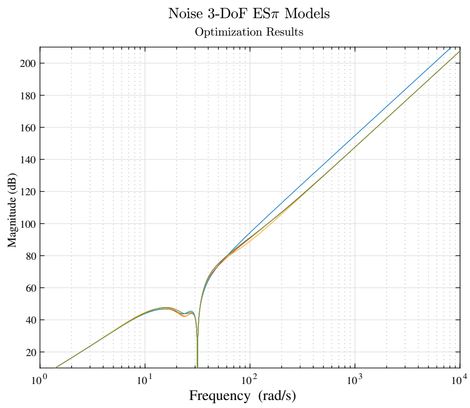
[1]Overshooting can be limited by adjusting the damping ratio of D_q.



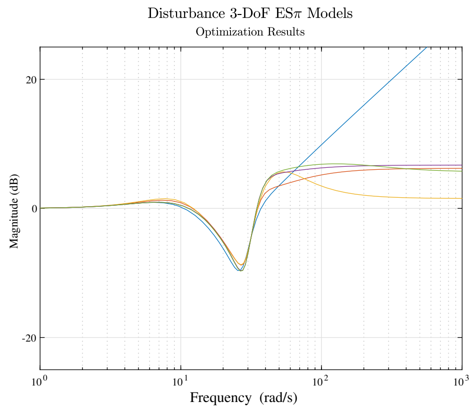
(a) Step responses of the optimized models.



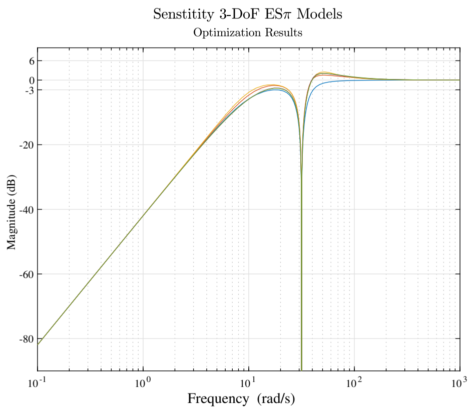
(b) Impedance functions within the constraint area.



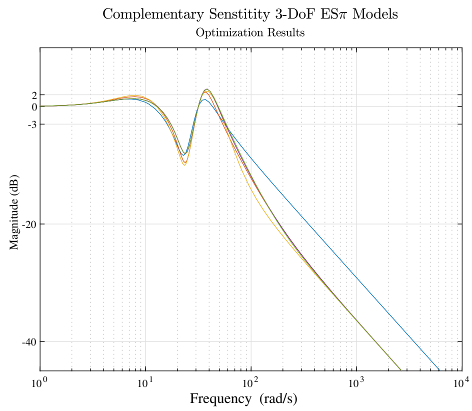
(c) Optimized noise damping.



(d) Optimized disturbance behavior.



(e) Sensitivity functions of optimized models.



(f) Complementary sensitivity functions.

Figure 4.7: Comparison of the optimized models and the initial model.

4.2 Experiments on the Testbed

The testbed of Figure 4.8 is composed of a single elastic joint. It is driven by a brushless DC motor of the DLR LWR III light weight robot. The harmonic drive gear has a ratio of 100. The high gear ratio leads to a high motor inertia of $B = 0.598\ kg\,m^2$. The springs have a stiffness of 400 $N\,m\,rad^{-1}$, but because of the compliance of the motor and gears the actual stiffness is 374 $N\,m\,rad^{-1}$. It was calibrated by measuring the spring deflection and torque of the link.

The movement of the link is in the horizontal axis. Therefore, gravitational forces have no effect on the link motion.

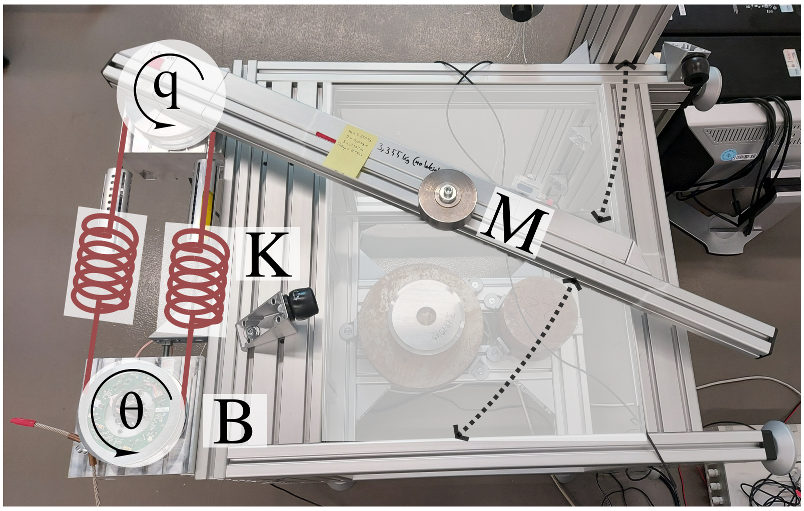


Figure 4.8: Testbed of a single flexible joint.

4.2.1 Setting of the Link Inertia

A link inertia of $M = 1$ was chosen. It represents the highest inertia of David's arm, when it is stretched out. Measuring the inertia of the link J and then adjusting it with a weight is setting M.

By using the link as a pendulum like in Figure 4.9, the inertia can be calculated with its natural frequency ω.

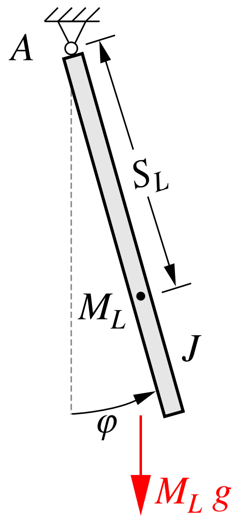


Figure 4.9: Pendulum for measuring the link inertia with its natural frequency.

The sum of angular momentum $\widehat{A}$ around point A yields the differential equation of the pendulum:

$$\widehat{A}: \quad J\,\ddot{\varphi} \;=\; -M_L\,g\,S_L\,\sin\varphi\;. \tag{4.1}$$

The distance to the center of gravity is S_L and the mass of the link M_L. For angles < 10 we can linearize:

$$0 \;=\; \ddot{\varphi} + \frac{M_L\,g\,S_L}{J}\,\varphi\;, \tag{4.2}$$

$$\omega \;=\; \sqrt{\frac{M_L\,g\,S}{J}}\;. \tag{4.3}$$

J can be calculated via:

$$J \;=\; \frac{M_L\,g\,S_L}{\omega^2}\;. \tag{4.4}$$

Mounting a weight M_G at the distance D_G from the rotation axis on the link, manipulates the inertia of the link. With D_G more than five times smaller than the diameter of the weight, M_G can be assumed to be a point of mass. The link inertia M is set with the equation:

$$M \;=\; J \,+\, M_G\,{D_G}^2\;. \tag{4.5}$$

An impulse was used as input signal to measure the disturbance response. To prevent damaging the hinge of the link or the position sensor, the complete impact energy has to be transferred to the angular momentum. This is the case when the link is hit at its center of impact S_C. The new center of gravity of the link with attached weight is S_G.

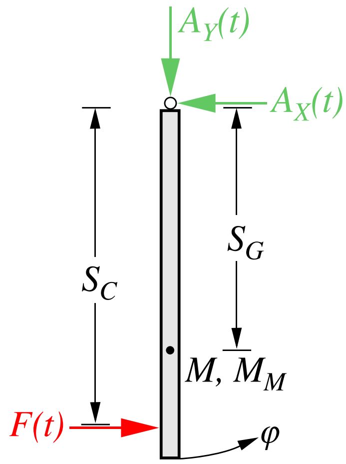


Figure 4.10: Free body diagram for calculating the center of impact of the link.

The impact on the hinge can be calculated with the integrated law of conservation of momentum. The integrated law of angular momentum is represented by $\hat{A}$. The present impulses are $\hat{F}$, $\hat{A}_X$ and $\hat{A}_Y$:

$$\hat{A}: \; M\,\dot{\varphi} = S_C\,\hat{F}\;, \tag{4.6}$$

$$\hat{A}_X = \hat{F} - M_M\,S_G\,\dot{\varphi}\;, \tag{4.7}$$

$$\hat{A}_Y = 0\;. \tag{4.8}$$

Setting $\hat{A}_X = 0$ and solving the equation system for S_C yields the center of impact:

$$\hat{A}_X = \hat{F}\left(1 - \frac{M_M\,S_G\,S_C}{M}\right) = 0\;, \tag{4.9}$$

$$S_C = \frac{M}{M_M\,S_G}\;. \tag{4.10}$$

ω:	4.05	$rad\, s^{-1}$	Natural frequency, measured over 10 periods
S_L:	0.381	m	Center of mass of the link
S_G:	0.395	m	Center of mass of the link with additional weight
D_G:	0.447	m	Distance of the additional weight
S_C:	0.556	m	Center of impact
M_L:	3.355	kg	Mass of the link
M_G:	1.20	kg	Mass of the additional weight
M_M:	4.555	kg	Complete mass of the link $(M_L + M_G)$
g:	9.81	$kg\, m\, s^{-2}$	Gravity constant
J:	0.76	$kg\, m^2$	Measured link inertia
M:	1.00	$kg\, m^2$	Set link inertia

Table 4.2: Measured variables to calculate and set the link inertia and center of impact.

4.2.2 Measuring the Noise Transfer Function N

To measure the noise transfer function N, white noise was injected directly after the sensor signal output. The control output u was recorded.

The controller has a sampling frequency of $F_S = 3\, kHz$ therefore the white noise is limited for frequencies below the Nyquist frequency of 1500 Hz or 9.425 $rad\, s^{-1}$. In addition, u is limited to $\pm$ 100 Nm. A noise power spectral density of $10^{-9}\, W\, Hz^{-1}$ has shown not to violate these constraints and therefore not crop u.

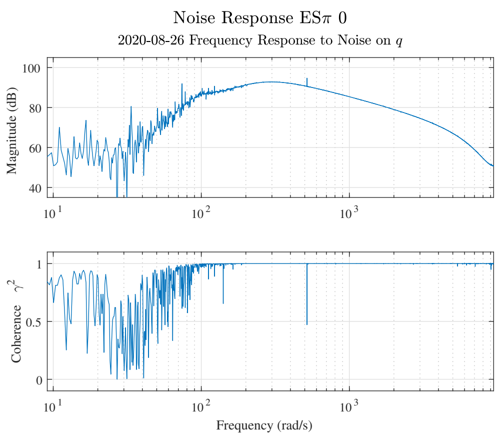


Figure 4.11: Measured frequency response to sensor noise on q of ESπ 0.

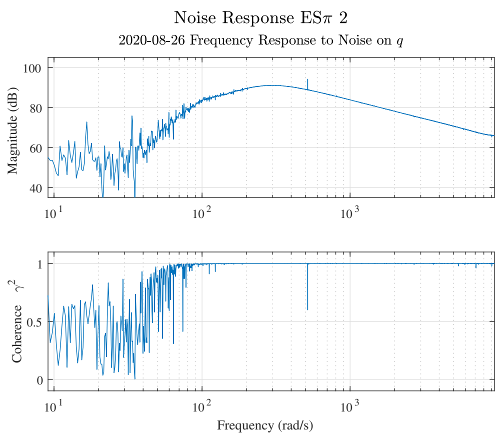


Figure 4.12: Measured frequency response to sensor noise on q of ESπ 2.

Figure 4.11 shows the improvement of ESπ 2 compared to ESπ 0. ESπ 2 is, at its peak, $2\,dB$ lower than ESπ 2. This is lower than the expected $8\,dB$ improvement with the simulation results. Adversely, with frequencies above $5000\,rad\,s^{-1}$ ESπ 2 shows less damping than ESπ 0.

Further, the measured noise transfer functions do not reflect the shape of the simulation results. This is due to the substitution of θ in the simulation model. Theta was not substituted in the implemented model and noise only injected on the signal of the position sensor for q. Further, a derivative filter was used to attain $\dot{q}$, which is damping high frequencies. The implications will be discussed in Chapter 5.

4.2.3 Effect of the Ratio Between the two Virtual Springs on N

The ratio $X = \frac{K_m}{K_q}$ between the two virtual springs has a significant effect on the amplification of sensor noise in the implementation. In Figure 4.13 the ratio is double compared to the optimized ratio. The noise amplification is $5\,dB$ higher.

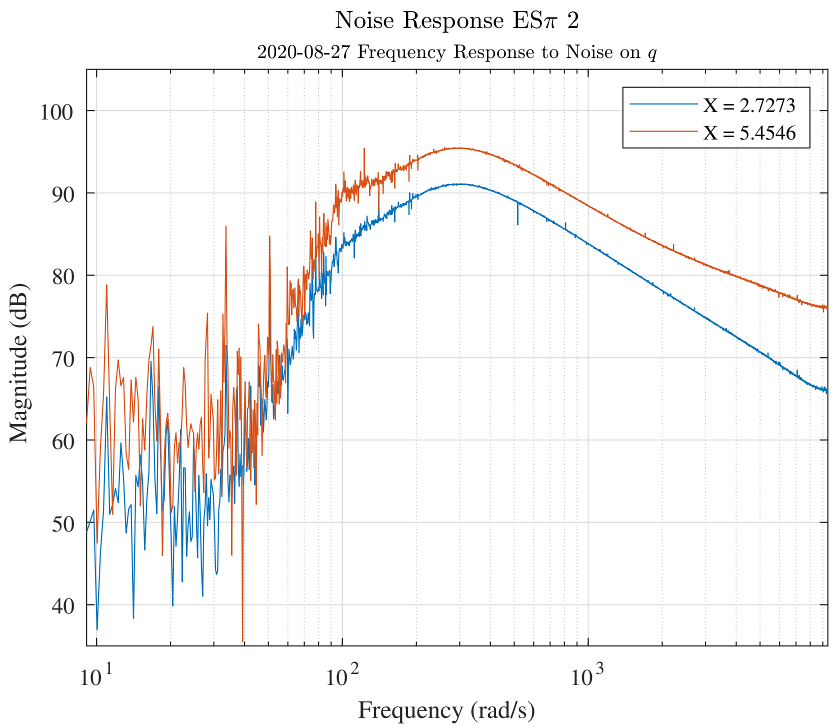


Figure 4.13: Noise amplification of ESπ 2 with X double the optimized ratio.

4.2.4 Measuring the Disturbance Response D

The disturbance response was measured by using an impact at the link as input signal. The impact was measured by a force sensor and multiplied with its lever to calculate the external torque τ_{ext}. Additionally, the control output u was recorded.

To prevent input saturation, the height of the impact has to be small enough to produce a control output below $\pm\, 100\, Nm$. A hammer on a hinge was used to apply the impact. Its deflection was set by a string. Accordingly, the generated input signals were reproducible without saturation.

Figure 4.14 shows the impact mechanism and Figure 4.15 the frequency spectrum of the generated signal. A hard impact with plastic on aluminum was used to generate frequencies up to roughly $1000\, rad\, s^{-1}$. Consequently, we can only measure the disturbance frequency response up to this frequency. This is also reflected by the decreasing signal coherence in the Figures 4.16a and 4.16b.

Figure 4.14: Impact mechanism to measure the disturbance response.

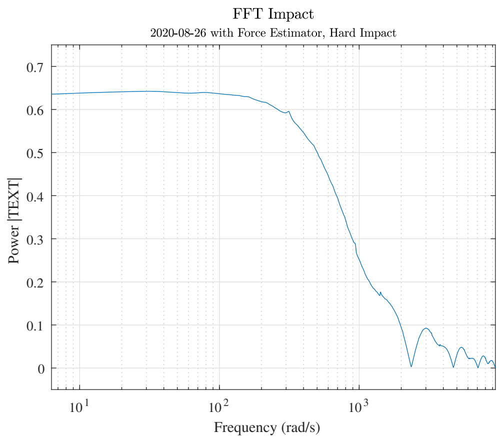
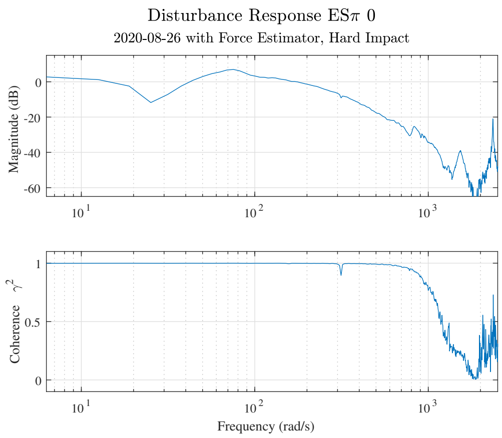
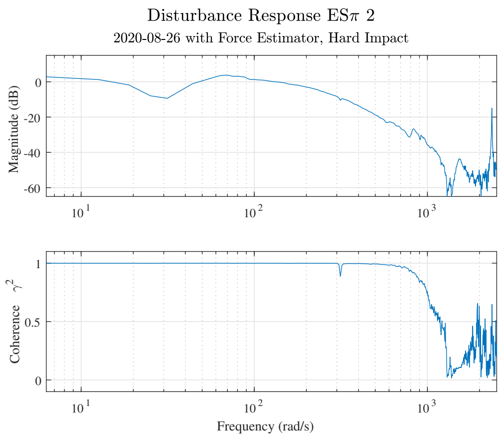


Figure 4.15: Generated impact signal in the frequency spectrum.

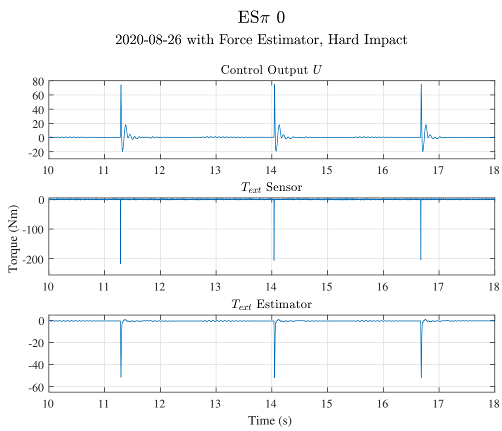
Figure 4.16 shows the improvement of the disturbance response over the initial ESπ 0 controller. Although the improvement in the disturbance response frequency spectrum is only 3 dB (cf. Figure 4.16a and 4.16b), the control effort of ESπ 2 is 12 Nm (16 %) lower, with roughly the same impact torque (cf. Figure 4.16c and 4.16d). Further, ESπ 2 has decreased reverberation.



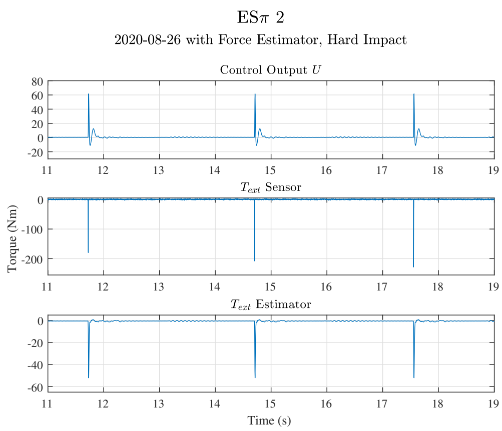
(a) Measured disturbance frequency response of ESπ 0.



(b) Measured disturbance frequency response of ESπ 2.



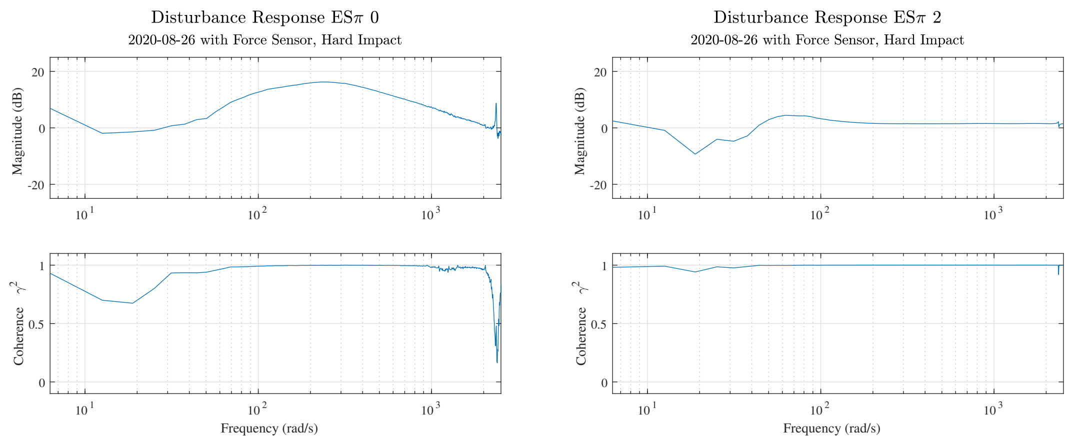
(c) Measured control output of ESπ 0.



(d) Measured control output of ESπ 2.

Figure 4.16: Measured disturbance frequency response and control output of ESπ 0 and 2 with force estimators and a hard impact.

The estimation of the external forces for the calculation of the higher derivatives of the link coordinate q distorts the measurements. In the simulation, the disturbance response of the initial ESπ 0 controller was strictly increasing with higher frequencies, while the enhanced controllers were not. By using a force sensor instead of the force estimators as control input, the improvement of the enhanced ESπ 2 controller over the initial controller can be measured more accurately (cf. Figure 4.17).

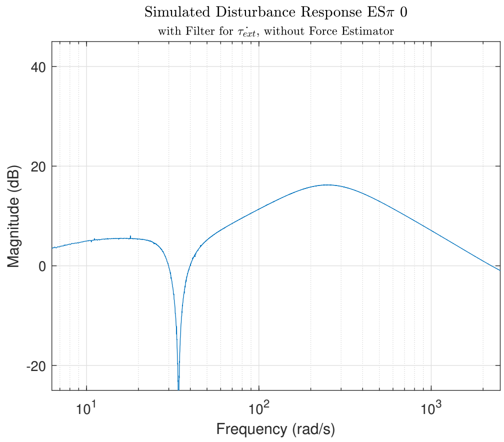


(a) Measured disturbance frequency response of ESπ 0.

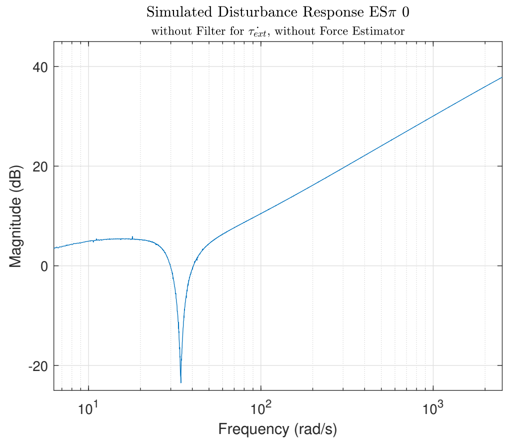
(b) Measured disturbance frequency response of ESπ 2.

Figure 4.17: Measured disturbance frequency response of ESπ 0 and 2 with a force sensor as control input.

A filtered derivative was used to calculate $\dot{\tau_{ext}}$ from the force sensor signal. This prevents further increasing at frequencies above the corner frequency of the filter. The derivative filter cannot be omitted in the implementation for stability reasons. Nevertheless, the simulation results show the theoretical frequency responses of ESπ 0 and the effect of the derivative filter (cf. Figure 4.18).



(a) Simulated disturbance frequency response with derivative filter.



(b) Simulated disturbance frequency response without derivative filter.

Figure 4.18: Simulated disturbance frequency response of ESπ 0 with and without derivative filter for $\dot{\tau_{ext}}$.

5 Discussion and Outlook

We showed that the enhanced ESπ controllers have a lower control effort for the disturbance rejection compared to the initial ESπ 0 controller in the simulations. The simulation results of ESπ 2 were confirmed in the experiments. Although the use of the force estimator is distorting the measured results, the simulated disturbance response could be recreated in the experiments. This was achieved by implementing a force sensor as control input.

The amplification of noise of the link position sensor was, at its peak, slightly lower than with the initial controller. The initial controller, on the other hand, showed superior noise damping above $8000\,rad\,s^{-1}$.

It was not possible to measure the simulated noise transfer function, because we substituted the motor position θ with the plant dynamics, as a function of the link position q, in order to create a SISO controller. In the implementation of the controller θ and q are measured separately. Therefore, injecting noise on q will have a different effect on the controller in the simulations than in the implementation.

The transformation of the ESπ controllers showed to be ineffective, because through the substitution, we attained a second control input representing the external torques τ_{ext}. Strictly speaking, the 3-DoF controller is also a many input, single output (MISO) system. Nevertheless, the system can be analyzed with SISO control theory techniques, because τ_{ext} has only an effect of the disturbance response of the system. Further, the disturbance response can be calculated by setting all other control inputs to zero. A more accurate result can be achieved by using the H_∞ method on a MISO system with θ and q as control input. This MISO system describes the implemented controller more accurately.

Stramigioli (1996) proposed a ratio between the virtual springs of $\frac{K_m}{K_q} = 10$ and a virtual mass of $m = 0.1\,M$ for his controller with artificial damping [10]. This proved to be good for limiting the oscillation of m and the impedance deviation. Nevertheless, we showed, that it is possible to tune this ratio down to 2.7273 and m up to 0.3898 $kg\,m^2$. Tuning the parameters reduces the sensitivity to sensor noise and control effort for disturbance rejection while maintaining a good impedance behavior. Intuitively more compliant virtual springs contribute to better noise and a lower control output with disturbances. Regulation performance and fast disturbance rejection, on the other hand, will deteriorate. Consequently, more compliant K_m yields better noise damping for ESπ 2 for the implementation.

Furthermore, a high ratio showed to have a strong effect on amplifying the noise of the position sensor for the link coordinate and is therefore not recommended for the implementation of ESπ 2.

For future implementations, the control parameters should be optimized by using a simulation model accurately representing the implemented controller. Using θ, q and additionally τ_{ext} (if a force sensor is used) as control inputs for the simulated model is recommended for a more fitting analysis.

ESπ 2 can be implemented without the need of the third derivative of q. Future implementations can also omit the force estimator by measuring $\ddot{q}$ with an acceleration sensor.

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Appendix

>

ESPI Control Terms K1 and K2:

>

ESPI 0:

> $U1 := -Kq\,(q - qd) - s\,Dq\,q$

$$U1 := -Kq\,(q - qd) - s\,Dq\,q \tag{1}$$

> $U1 := collect(simplify(U1), [qd, q], simplify)$

$$U1 := Kq\,qd + (-Dq\,s - Kq)\,q \tag{2}$$

> $K1 := Kq$

$$K1 := Kq \tag{3}$$

> $K2 := simplify(-(-Dq\,s - Kq))$

$$K2 := Dq\,s + Kq \tag{4}$$

> unassign('U1')

>

ESPI 1:

> $U1 := -Km\,(q - qm)$

$$U1 := -Km\,(q - qm) \tag{5}$$

> $Virtual := 0 = Km\,(q - qm) - Kq\,(qm - qd) - Dq\,qm\,s$

$$Virtual := 0 = Km\,(q - qm) - Kq\,(qm - qd) - Dq\,qm\,s \tag{6}$$

> $simplify(isolate(Virtual, qm))$

$$qm = \frac{Km\,q + Kq\,qd}{Dq\,s + Km + Kq} \tag{7}$$

> $qm := \frac{Km\,q + Kq\,qd}{Dq\,s + Km + Kq}:$

> $U1 := collect(simplify(U1), [qd, q], simplify)$

$$U1 := \frac{Km\,Kq\,qd}{Dq\,s + Km + Kq} - \frac{Km\,(Dq\,s + Kq)\,q}{Dq\,s + Km + Kq} \tag{8}$$

> $K1 := collect\left(normal\left(simplify\left(\frac{Km\,Kq}{Dq\,s + Km + Kq}\right), expanded\right), s, recursive, simplify\right)$

$$K1 := \frac{Km\,Kq}{Dq\,s + Km + Kq} \tag{9}$$

> $K2 := collect\left(normal\left(simplify\left(\frac{Km\,(Dq\,s + Kq)}{Dq\,s + Km + Kq}\right), expanded\right), s, recursive, simplify\right)$

$$K2 := \frac{Dq\,Km\,s + Km\,Kq}{Dq\,s + Km + Kq} \tag{10}$$

> unassign('qm','U1','Virtual')

>

ESPI 2:

> $U1 := -Km\,(q - qm)$

$$U1 := -Km\,(q - qm) \tag{11}$$

> $Virtual := m\,qm\,s^2 = Km\,(q - qm) - Kq\,(qm - qd) - Dq\,qm\,s$

$$Virtual := m\,qm\,s^2 = Km\,(q - qm) - Kq\,(qm - qd) - Dq\,qm\,s \tag{12}$$

> $simplify(isolate(Virtual, qm))$

$$qm = \frac{Km\,q + Kq\,qd}{m\,s^2 + Dq\,s + Km + Kq} \tag{13}$$

> $qm := \frac{Km\,q + Kq\,qd}{m\,s^2 + Dq\,s + Km + Kq}:$

> $U1 := collect(simplify(U1), [qd, q], simplify)$

$$U1 := \frac{Km\,Kq\,qd}{m\,s^2 + Dq\,s + Km + Kq} - \frac{(m\,s^2 + Dq\,s + Kq)\,Km\,q}{m\,s^2 + Dq\,s + Km + Kq} \tag{14}$$

> $K1 := collect\left(normal\left(simplify\left(\frac{Km\,Kq}{m\,s^2 + Dq\,s + Km + Kq}\right), expanded\right), s, recursive, simplify\right)$

$$K1 := \frac{Km\,Kq}{m\,s^2 + Dq\,s + Km + Kq} \tag{15}$$

> $K2 := collect\left(normal\left(simplify\left(\frac{Km\,(m\,s^2 + Dq\,s + Kq)}{m\,s^2 + Dq\,s + Km + Kq}\right), expanded\right), s, recursive, simplify\right)$

$$K2 := \frac{Km\,m\,s^2 + Dq\,Km\,s + Km\,Kq}{m\,s^2 + Dq\,s + Km + Kq} \quad (16)$$

> $unassign('qm', 'U1', 'Virtual')$

>

ESPI 3:

> $U1 := -Kq\,(q - qd) - Km\,(q - qm)$

$$U1 := -Kq\,(q - qd) - Km\,(q - qm) \quad (17)$$

> $Virtual := 0 = Km\,(q - qm) - Dq\,qm\,s$

$$Virtual := 0 = Km\,(q - qm) - Dq\,qm\,s \quad (18)$$

> $simplify(isolate(Virtual, qm))$

$$qm = \frac{Km\,q}{Dq\,s + Km} \quad (19)$$

> $qm := \frac{Km\,q}{Dq\,s + Km}$:

> $U1 := collect(simplify(U1), [qd, q], simplify)$

$$U1 := Kq\,qd - \frac{(Dq\,Km\,s + Dq\,Kq\,s + Km\,Kq)\,q}{Dq\,s + Km} \quad (20)$$

> $K1 := Kq$

$$K1 := Kq \quad (21)$$

> $K2 := collect\left(normal\left(simplify\left(\frac{(Dq\,Km\,s + Dq\,Kq\,s + Km\,Kq)}{Dq\,s + Km}\right), expanded\right), s, recursive, simplify\right)$

$$K2 := \frac{(Dq\,Km + Dq\,Kq)\,s + Km\,Kq}{Dq\,s + Km} \quad (22)$$

> $unassign('qm', 'U1', 'Virtual')$

>

ESPI 4:

> $U1 := -Kq\,(q - qd) - Km\,(q - qm)$

$$U1 := -Kq\,(q - qd) - Km\,(q - qm) \quad (23)$$

> $Virtual := m\,qm\,s^2 = Km\,(q - qm) - Dq\,qm\,s$

$$Virtual := m\,qm\,s^2 = Km\,(q - qm) - Dq\,qm\,s \quad (24)$$

> $simplify(isolate(Virtual, qm))$

$$qm = \frac{Km\,q}{m\,s^2 + Dq\,s + Km} \quad (25)$$

> $qm := \frac{Km\,q}{m\,s^2 + Dq\,s + Km}$:

> $U1 := collect(simplify(U1), [qd, q], simplify)$

$$U1 := Kq\,qd - \frac{(Km\,m\,s^2 + Kq\,m\,s^2 + Dq\,Km\,s + Dq\,Kq\,s + Km\,Kq)\,q}{m\,s^2 + Dq\,s + Km} \quad (26)$$

> $K1 := Kq$

$$K1 := Kq \quad (27)$$

> $K2 := collect\left(normal\left(simplify\left(\frac{(Km\,m\,s^2 + Kq\,m\,s^2 + Dq\,Km\,s + Dq\,Kq\,s + Km\,Kq)}{m\,s^2 + Dq\,s + Km}\right), expanded\right), s, recursive, simplify\right)$

$$K2 := \frac{(Km\,m + Kq\,m)\,s^2 + (Dq\,Km + Dq\,Kq)\,s + Km\,Kq}{m\,s^2 + Dq\,s + Km} \quad (28)$$

> $unassign('qm', 'U1', 'Virtual')$

>

>

ESPI 0 Mechanical Substitution Model:

> $Link := M\,q\,s^2 = K\,(eta\,-q)\,-Kq\,(q-qd)\,-Dq\,s\,q+Text$

$$Link := M\,q\,s^2 = K\,(\eta - q) - Kq\,(q - qd) - Dq\,s\,q + Text \tag{1}$$

> $Motor := B\,eta\,s^2 = -K\,(eta-q)\,-Deta\,s\,eta$

$$Motor := B\,\eta\,s^2 = -K\,(\eta - q) - Deta\,s\,\eta \tag{2}$$

>

Tracking Transfer Function:

> $Text := 0$

$$Text := 0 \tag{3}$$

> $isolate(Motor, eta)$

$$\eta = \frac{K\,q}{B\,s^2 + Deta\,s + K} \tag{4}$$

> $\eta := \frac{K\,q}{B\,s^2 + Deta\,s + K}$

$$\eta := \frac{K\,q}{B\,s^2 + Deta\,s + K} \tag{5}$$

> $simplify(isolate(Link, q))$:

> $R\_Mech := collect\left(normal\left(simplify\left(\frac{isolate(Link,\ q)}{qd}\right), expanded\right), s, recursive, simplify\right)$

$$R\_Mech := \frac{q}{qd} = \frac{B\,Kq\,s^2 + Deta\,Kq\,s + K\,Kq}{B\,M\,s^4 + (B\,Dq + Deta\,M)\,s^3 + (B\,K + B\,Kq + Deta\,Dq + K\,M)\,s^2 + (Deta\,K + Deta\,Kq + Dq\,K)\,s + K\,Kq} \tag{6}$$

>

Impedance Transfer Funktion:

> $unassign('Text')$

> $qd := 0$

$$qd := 0 \tag{7}$$

>

> $Z\_Mech := collect\left(normal\left(simplify\left(\frac{Text}{isolate(Link,\ q)\cdot s}\right), expanded\right), s, recursive, simplify\right)$

$$Z\_Mech := \frac{Text}{s\,q} = \frac{B\,M\,s^4 + (B\,Dq + Deta\,M)\,s^3 + (B\,K + B\,Kq + Deta\,Dq + K\,M)\,s^2 + (Deta\,K + Deta\,Kq + Dq\,K)\,s + K\,Kq}{B\,s^3 + Deta\,s^2 + K\,s} \tag{8}$$

> $unassign('Link','Motor','eta','Text','qd')$

>

ESPI 1 Mechanical Substitution Model:

> $Link := M\,q\,s^2 = K\,(\mathrm{eta} - q) - Km\,(q - qm) + Text$

$$Link := M\,q\,s^2 = K\,(\eta - q) - Km\,(q - qm) + Text \tag{1}$$

> $Motor := B\,\mathrm{eta}\,s^2 = -K\,(\mathrm{eta} - q) - Deta\,s\,\mathrm{eta}$

$$Motor := B\,\eta\,s^2 = -K\,(\eta - q) - Deta\,s\,\eta \tag{2}$$

> $Virtual := 0 = Km\,(q - qm) - Kq\,(qm - qd) - Dq\,qm\,s$

$$Virtual := 0 = Km\,(q - qm) - Kq\,(qm - qd) - Dq\,qm\,s \tag{3}$$

Tracking Transfer Funktion:

> $Text := 0$

$$Text := 0 \tag{4}$$

> $isolate(Motor, \mathrm{eta})$

$$\eta = \frac{K\,q}{B\,s^2 + Deta\,s + K} \tag{5}$$

> $\eta := \frac{K\,q}{B\,s^2 + Deta\,s + K}$

$$\eta := \frac{K\,q}{B\,s^2 + Deta\,s + K} \tag{6}$$

> $simplify(isolate(Virtual, qm))$

$$qm = \frac{Km\,q + Kq\,qd}{Dq\,s + Km + Kq} \tag{7}$$

> $qm := \frac{Km\,q + Kq\,qd}{Dq\,s + Km + Kq}$

$$qm := \frac{Km\,q + Kq\,qd}{Dq\,s + Km + Kq} \tag{8}$$

> $simplify(isolate(Link, q)) :$

> $R\_Mech := collect\left(normal\left(simplify\left(\frac{isolate(Link, q)}{qd}\right), expanded\right), s, recursive, simplify\right)$

$$R\_Mech := \frac{q}{qd} = \left(B\,Km\,Kq\,s^2 + Deta\,Km\,Kq\,s + K\,Km\,Kq\right) \Big/ \left(B\,Dq\,M\,s^5 + (B\,Km\,M + B\,Kq\,M + Deta\,Dq\,M)\,s^4 + (B\,Dq\,K + B\,Dq\,Km + Deta\,Km\,M + Deta\,Kq\,M + Dq\,K\,M)\,s^3 + (B\,K\,Km + B\,K\,Kq + B\,Km\,Kq + Deta\,Dq\,K + Deta\,Dq\,Km + K\,Km\,M + K\,Kq\,M)\,s^2 + (Deta\,K\,Km + Deta\,K\,Kq + Deta\,Km\,Kq + Dq\,K\,Km)\,s + K\,Km\,Kq\right) \tag{9}$$

>

Impedance Transfer Funktion:

> $unassign('Text')$

> $qd := 0$

$$qd := 0 \tag{10}$$

> $Z\_Mech := collect\left(normal\left(simplify\left(\frac{Text}{isolate(Link, q) \cdot s}\right), expanded\right), s, recursive, simplify\right)$

$$Z\_Mech := \frac{Text}{s\,q} = \left(B\,Dq\,M\,s^5 + (B\,Km\,M + B\,Kq\,M + Deta\,Dq\,M)\,s^4 + (B\,Dq\,K + B\,Dq\,Km + Deta\,Km\,M + Deta\,Kq\,M + Dq\,K\,M)\,s^3 + (B\,K\,Km + B\,K\,Kq + B\,Km\,Kq + Deta\,Dq\,K + Deta\,Dq\,Km + K\,Km\,M + K\,Kq\,M)\,s^2 + (Deta\,K\,Km + Deta\,K\,Kq + Deta\,Km\,Kq + Dq\,K\,Km)\,s + K\,Km\,Kq\right) \Big/ \left(B\,Dq\,s^4 + (B\,Km + B\,Kq + Deta\,Dq)\,s^3 + (Deta\,Km + Deta\,Kq + Dq\,K)\,s^2 + (K\,Km + K\,Kq)\,s\right) \tag{11}$$

> $unassign('Link','Virtual','Motor','qm','eta','Text','qd')$

>

ESPI 2 Mechanical Substitution Model:

> $Link := M\,q\,s^2 = K\,(\text{eta} - q) - Km\,(q - qm) + Text$

$$Link := M\,q\,s^2 = K\,(\eta - q) - Km\,(q - qm) + Text \tag{1}$$

> $Motor := B\,\text{eta}\,s^2 = -K\,(\text{eta} - q) - Deta\,s\,\text{eta}$

$$Motor := B\,\eta\,s^2 = -K\,(\eta - q) - Deta\,s\,\eta \tag{2}$$

> $Virtual := m\,qm\,s^2 = Km\,(q - qm) - Kq\,(qm - qd) - Dq\,qm\,s$

$$Virtual := m\,qm\,s^2 = Km\,(q - qm) - Kq\,(qm - qd) - Dq\,qm\,s \tag{3}$$

Tracking Transfer Funktion:

> $Text := 0$

$$Text := 0 \tag{4}$$

> $isolate(Motor, \text{eta})$

$$\eta = \frac{K\,q}{B\,s^2 + Deta\,s + K} \tag{5}$$

> $\eta := \frac{K\,q}{B\,s^2 + Deta\,s + K}$

$$\eta := \frac{K\,q}{B\,s^2 + Deta\,s + K} \tag{6}$$

> $simplify(isolate(Virtual, qm))$

$$qm = \frac{Km\,q + Kq\,qd}{m\,s^2 + Dq\,s + Km + Kq} \tag{7}$$

> $qm := \frac{Km\,q + Kq\,qd}{m\,s^2 + Dq\,s + Km + Kq}$

$$qm := \frac{Km\,q + Kq\,qd}{m\,s^2 + Dq\,s + Km + Kq} \tag{8}$$

> $simplify(Link)$

$$M\,q\,s^2 = \frac{1}{(B\,s^2 + Deta\,s + K)\,(m\,s^2 + Dq\,s + Km + Kq)}\Big(-B\,m\,q\,(K + Km)\,s^4 - q\,(K + Km)\,(B\,Dq + Deta\,m)\,s^3 + ((((-B - m)\,K - B\,Kq - Dq\,Deta)\,Km - K\,(B\,Kq + Deta\,Dq))\,q + B\,Km\,Kq\,qd)\,s^2 + ((((-Deta - Dq)\,K - Deta\,Kq)\,Km - Deta\,K\,Kq)\,q + Deta\,Km\,Kq\,qd)\,s - K\,Km\,Kq\,(q - qd)\Big) \tag{9}$$

> $R\_Mech := collect\left(normal\left(simplify\left(\frac{isolate(Link, q)}{qd}\right), expanded\right), s, recursive, simplify\right)$

$$R\_Mech := \frac{q}{qd} = (B\,Km\,Kq\,s^2 + Deta\,Km\,Kq\,s + K\,Km\,Kq) \Big/ (B\,M\,m\,s^6 + (B\,Dq\,M + Deta\,M\,m)\,s^5 + (B\,K\,m + B\,Km\,M + B\,Km\,m + B\,Kq\,M + Deta\,Dq\,M + K\,M\,m)\,s^4 + (B\,Dq\,K + B\,Dq\,Km + Deta\,K\,m + Deta\,Km\,M + Deta\,Km\,m + Deta\,Kq\,M + Dq\,K\,M)\,s^3 + (B\,K\,Km + B\,K\,Kq + B\,Km\,Kq + Deta\,Dq\,K + Deta\,Dq\,Km + K\,Km\,M + K\,Km\,m + K\,Kq\,M)\,s^2 + (Deta\,K\,Km + Deta\,K\,Kq + Deta\,Km\,Kq + Dq\,K\,Km)\,s + K\,Km\,Kq) \tag{10}$$

>

Impedance Transfer Funktion:

> $unassign('Text')$

> $qd := 0$

$$qd := 0 \tag{11}$$

> $Z\_Mech := collect\left(normal\left(simplify\left(\frac{Text}{isolate(Link, q)\cdot s}\right), expanded\right), s, recursive, simplify\right)$

$$Z\_Mech := \frac{Text}{s\,q} = (B\,M\,m\,s^6 + (B\,Dq\,M + Deta\,M\,m)\,s^5 + (B\,K\,m + B\,Km\,M + B\,Km\,m + B\,Kq\,M + Deta\,Dq\,M + K\,M\,m)\,s^4 + (B\,Dq\,K + B\,Dq\,Km + Deta\,K\,m + Deta\,Km\,M + Deta\,Km\,m + Deta\,Kq\,M + Dq\,K\,M)\,s^3 + (B\,K\,Km + B\,K\,Kq + B\,Km\,Kq + Deta\,Dq\,K + Deta\,Dq\,Km + K\,Km\,M + K\,Km\,m + K\,Kq\,M)\,s^2 + (Deta\,K\,Km + Deta\,K\,Kq + Deta\,Km\,Kq + Dq\,K\,Km)\,s + K\,Km\,Kq) \Big/ (B\,m\,s^5 + (B\,Dq + Deta\,m)\,s^4 + (B\,Km + B\,Kq + Deta\,Dq + K\,m)\,s^3 + (Deta\,Km + Deta\,Kq + Dq\,K)\,s^2 + (K\,Km + K\,Kq)\,s) \tag{12}$$

> $unassign('Link', 'Virtual', 'Motor', 'qm', 'eta', 'Text', 'qd')$

> $solve(Virtual, qm)$

>

>

ESPI 3 Mechanical Substitution Model:

> $Link := M\,q\,s^2 = K\,(\text{eta} - q) - Kq\,(q - qd) - Km\,(q - qm) + Text$

$$Link := M\,q\,s^2 = K\,(\eta - q) - Kq\,(q - qd) - Km\,(q - qm) + Text \quad (1)$$

> $Motor := B\,\text{eta}\,s^2 = -K\,(\text{eta} - q) - Deta\,s\,\text{eta}$

$$Motor := B\,\eta\,s^2 = -K\,(\eta - q) - Deta\,s\,\eta \quad (2)$$

> $Virtual := 0 = Km\,(q - qm) - Dq\,qm\,s$

$$Virtual := 0 = Km\,(q - qm) - Dq\,qm\,s \quad (3)$$

Tracking Transfer Funktion:

> $Text := 0$

$$Text := 0 \quad (4)$$

> $isolate(Motor, \text{eta})$

$$\eta = \frac{K\,q}{B\,s^2 + Deta\,s + K} \quad (5)$$

> $\eta := \frac{K\,q}{B\,s^2 + Deta\,s + K}$

$$\eta := \frac{K\,q}{B\,s^2 + Deta\,s + K} \quad (6)$$

> $simplify(isolate(Virtual, qm))$

$$qm = \frac{Km\,q}{Dq\,s + Km} \quad (7)$$

> $qm := \frac{Km\,q}{Dq\,s + Km}$

$$qm := \frac{Km\,q}{Dq\,s + Km} \quad (8)$$

> $simplify(isolate(Link, q)):$

> $R\_Mech := collect\left(normal\left(simplify\left(\frac{isolate(Link, q)}{qd}\right), expanded\right), s, recursive, simplify\right)$

$$R\_Mech := \frac{q}{qd} = \left(B\,Dq\,Kq\,s^3 + (B\,Km\,Kq + Deta\,Dq\,Kq)\,s^2 + (Deta\,Km\,Kq + Dq\,K\,Kq)\,s + K\,Km\,Kq\right) \Big/ \left(B\,Dq\,M\,s^5 + (B\,Km\,M + Deta\,Dq\,M)\,s^4 + (B\,Dq\,K + B\,Dq\,Km + B\,Dq\,Kq + Deta\,Km\,M + Dq\,K\,M)\,s^3 + (B\,K\,Km + B\,Km\,Kq + Deta\,Dq\,K + Deta\,Dq\,Km + Deta\,Dq\,Kq + K\,Km\,M)\,s^2 + (Deta\,K\,Km + Deta\,Km\,Kq + Dq\,K\,Km + Dq\,K\,Kq)\,s + K\,Km\,Kq\right) \quad (9)$$

>

Impedance Transfer Funktion:

> $unassign('Text')$

> $qd := 0$

$$qd := 0 \quad (10)$$

> $Z\_Mech := collect\left(normal\left(simplify\left(\frac{Text}{isolate(Link, q)\cdot s}\right), expanded\right), s, recursive, simplify\right)$

$$Z\_Mech := \frac{Text}{s\,q} = \left(B\,Dq\,M\,s^5 + (B\,Km\,M + Deta\,Dq\,M)\,s^4 + (B\,Dq\,K + B\,Dq\,Km + B\,Dq\,Kq + Deta\,Km\,M + Dq\,K\,M)\,s^3 + (B\,K\,Km + B\,Km\,Kq + Deta\,Dq\,K + Deta\,Dq\,Km + Deta\,Dq\,Kq + K\,Km\,M)\,s^2 + (Deta\,K\,Km + Deta\,Km\,Kq + Dq\,K\,Km + Dq\,K\,Kq)\,s + K\,Km\,Kq\right) \Big/ \left(B\,Dq\,s^4 + (B\,Km + Deta\,Dq)\,s^3 + (Deta\,Km + Dq\,K)\,s^2 + K\,Km\,s\right) \quad (11)$$

> $unassign('Link', 'Virtual', 'Motor', 'qm', 'eta', 'Text', 'qd')$

>

>

ESPI 4 Mechanical Substitution Model:

> $Link := M\,q\,s^2 = K\,(\text{eta} - q) - Kq\,(q - qd) - Km\,(q - qm) + Text$

$$Link := M\,q\,s^2 = K\,(\eta - q) - Kq\,(q - qd) - Km\,(q - qm) + Text \tag{1}$$

> $Motor := B\,\text{eta}\,s^2 = -K\,(\text{eta} - q) - Deta\,s\,\text{eta}$

$$Motor := B\,\eta\,s^2 = -K\,(\eta - q) - Deta\,s\,\eta \tag{2}$$

> $Virtual := m\,qm\,s^2 = Km\,(q - qm) - Dq\,qm\,s$

$$Virtual := m\,qm\,s^2 = Km\,(q - qm) - Dq\,qm\,s \tag{3}$$

>

Tracking Transfer Funktion:

> $Text := 0$

$$Text := 0 \tag{4}$$

> $isolate(Motor, \text{eta})$

$$\eta = \frac{K\,q}{B\,s^2 + Deta\,s + K} \tag{5}$$

> $\eta := \frac{K\,q}{B\,s^2 + Deta\,s + K}$

$$\eta := \frac{K\,q}{B\,s^2 + Deta\,s + K} \tag{6}$$

> $simplify(isolate(Virtual, qm))$

$$qm = \frac{Km\,q}{m\,s^2 + Dq\,s + Km} \tag{7}$$

> $qm := \frac{Km\,q}{m\,s^2 + Dq\,s + Km}$

$$qm := \frac{Km\,q}{m\,s^2 + Dq\,s + Km} \tag{8}$$

> $simplify(isolate(Link, q)) :$

> $R\_Mech := collect\left(normal\left(simplify\left(\frac{isolate(Link, q)}{qd}\right), expanded\right), s, recursive, simplify\right)$

$$R\_Mech := \frac{q}{qd} = \big(B\,Kq\,m\,s^4 + (B\,Dq\,Kq + Deta\,Kq\,m)\,s^3 + (B\,Km\,Kq + Deta\,Dq\,Kq + K\,Kq\,m)\,s^2 + (Deta\,Km\,Kq + Dq\,K\,Kq)\,s + K\,Km\,Kq\big) \Big/ \big(B\,M\,m\,s^6 + (B\,Dq\,M + Deta\,M\,m)\,s^5 + (B\,K\,m + B\,Km\,M + B\,Km\,m + B\,Kq\,m + Deta\,Dq\,M + K\,M\,m)\,s^4 + (B\,Dq\,K + B\,Dq\,Km + B\,Dq\,Kq + Deta\,K\,m + Deta\,Km\,M + Deta\,Km\,m + Deta\,Kq\,m + Dq\,K\,M)\,s^3 + (B\,K\,Km + B\,Km\,Kq + Deta\,Dq\,K + Deta\,Dq\,Km + Deta\,Dq\,Kq + K\,Km\,M + K\,Km\,m + K\,Kq\,m)\,s^2 + (Deta\,K\,Km + Deta\,Km\,Kq + Dq\,K\,Km + Dq\,K\,Kq)\,s + K\,Km\,Kq\big) \tag{9}$$

>

Impedance Transfer Funktion:

> $unassign('Text')$

> $qd := 0$

$$qd := 0 \tag{10}$$

> $Z\_Mech := collect\left(normal\left(simplify\left(\frac{Text}{isolate(Link, q)\cdot s}\right), expanded\right), s, recursive, simplify\right)$

$$Z\_Mech := \frac{Text}{s\,q} = \big(B\,M\,m\,s^6 + (B\,Dq\,M + Deta\,M\,m)\,s^5 + (B\,K\,m + B\,Km\,M + B\,Km\,m + B\,Kq\,m + Deta\,Dq\,M + K\,M\,m)\,s^4 + (B\,Dq\,K + B\,Dq\,Km + B\,Dq\,Kq + Deta\,K\,m + Deta\,Km\,M + Deta\,Km\,m + Deta\,Kq\,m + Dq\,K\,M)\,s^3 + (B\,K\,Km + B\,Km\,Kq + Deta\,Dq\,K + Deta\,Dq\,Km + Deta\,Dq\,Kq + K\,Km\,M + K\,Km\,m + K\,Kq\,m)\,s^2 + (Deta\,K\,Km + Deta\,Km\,Kq + Dq\,K\,Km + Dq\,K\,Kq)\,s + K\,Km\,Kq\big) \Big/ \big(B\,m\,s^5 + (B\,Dq + Deta\,m)\,s^4 + (B\,Km + Deta\,Dq + K\,m)\,s^3 + (Deta\,Km + Dq\,K)\,s^2 + K\,Km\,s\big) \tag{11}$$

> $unassign('Link', 'Virtual','Motor','qm','eta','Text','qd')$

>

>

ESPI 0 3-DoF Control Law:

> $u1 := -Kq\,(q-qd) - Dq\cdot q\cdot s$

$$u1 := -Kq\,(q-qd) - Dq\,q\,s \tag{1}$$

> $u2 := -Deta\cdot \text{eta}\cdot s$

$$u2 := -Deta\left(q + \frac{M\,q\,s^2}{K} - \frac{Text}{K} - \frac{-Kq\,(q-qd) - Dq\,q\,s}{K}\right)s \tag{2}$$

> $\text{theta} := q + \frac{M}{K}\cdot q\cdot s^2 - \frac{Text}{K}$

$$\theta := q + \frac{M\,q\,s^2}{K} - \frac{Text}{K} \tag{3}$$

> $\text{eta} := \text{theta} - \frac{u1}{K}$

$$\eta := q + \frac{M\,q\,s^2}{K} - \frac{Text}{K} - \frac{-Kq\,(q-qd) - Dq\,q\,s}{K} \tag{4}$$

> $U := u1 + u2 + \frac{B}{K}\cdot u1\cdot s^2:$

> $U := collect(normal(simplify(U), expanded), [qd, q, s], recursive, simplify)$

$$U := \left(\frac{B\,Kq\,s^2}{K} + \frac{Deta\,Kq\,s}{K} + Kq\right)qd + \left(-\frac{(B\,Dq + Deta\,M)\,s^3}{K} - \frac{(B\,Kq + Deta\,Dq)\,s^2}{K} - \frac{(Deta\,K + Deta\,Kq + Dq\,K)\,s}{K} - Kq\right)q + \frac{Deta\,Text\,s}{K} \tag{5}$$

>

Reference Controller:

> $Text := 0;$

$$Text := 0 \tag{6}$$

> U

$$\left(\frac{B\,Kq\,s^2}{K} + \frac{Deta\,Kq\,s}{K} + Kq\right)qd + \left(-\frac{(B\,Dq + Deta\,M)\,s^3}{K} - \frac{(B\,Kq + Deta\,Dq)\,s^2}{K} - \frac{(Deta\,K + Deta\,Kq + Dq\,K)\,s}{K} - Kq\right)q \tag{7}$$

> $Kv := \frac{B\,Kq\,s^2}{K} + \frac{Deta\,Kq\,s}{K} + Kq:$

> $simplify(Kv)$

$$\frac{Kq\,(B\,s^2 + Deta\,s + K)}{K} \tag{8}$$

> $Ky := -\left(-\frac{(B\,Dq + Deta\,M)\,s^3}{K} - \frac{(B\,Kq + Deta\,Dq)\,s^2}{K} - \frac{(Deta\,K + Deta\,Kq + Dq\,K)\,s}{K} - Kq\right):$

> $Ky := collect(normal(simplify(Ky)), s)$

$$Ky := \frac{(B\,Dq + Deta\,M)\,s^3}{K} + \frac{(B\,Kq + Deta\,Dq)\,s^2}{K} + \frac{(Deta\,K + Deta\,Kq + Dq\,K)\,s}{K} + Kq \tag{9}$$

> $Kv\cdot qd - Ky\cdot q$

$$\left(\frac{B\,Kq\,s^2}{K} + \frac{Deta\,Kq\,s}{K} + Kq\right)qd - \left(\frac{(B\,Dq + Deta\,M)\,s^3}{K} + \frac{(B\,Kq + Deta\,Dq)\,s^2}{K} + \frac{(Deta\,K + Deta\,Kq + Dq\,K)\,s}{K} + Kq\right)q \tag{10}$$

> $Kr := collect\left(normal\left(simplify\left(\frac{Kv}{Ky}\right), expanded\right), s, recursive, simplify\right)$

$$Kr := \frac{B\,Kq\,s^2 + Deta\,Kq\,s + K\,Kq}{(B\,Dq + Deta\,M)\,s^3 + (B\,Kq + Deta\,Dq)\,s^2 + (Deta\,K + Deta\,Kq + Dq\,K)\,s + Kq\,K} \tag{11}$$

>

Complrementary Impedance Controller:

> $qd := 0; unassign('Text')$

$$qd := 0 \tag{12}$$

> U

(13)

$$\left(-\frac{(B\,Dq+Deta\,M)\,s^3}{K}-\frac{(B\,Kq+Deta\,Dq)\,s^2}{K}-\frac{(Deta\,K+Deta\,Kq+Dq\,K)\,s}{K}-Kq\right)q+\frac{Deta\,Text\,s}{K} \quad (13)$$

> $Kt := \frac{Deta\,s}{K}$

$$Kt := \frac{Deta\,s}{K} \quad (14)$$

> $Kd := collect\left(normal\left(simplify\left(\frac{Kt}{Ky}\right), expanded\right), s, recursive, simplify\right)$

$$Kd := \frac{Deta\,s}{(B\,Dq+Deta\,M)\,s^3+(B\,Kq+Deta\,Dq)\,s^2+(Deta\,K+Deta\,Kq+Dq\,K)\,s+Kq\,K} \quad (15)$$

> $Kt\,Text - Ky \cdot q$

$$\frac{Deta\,Text\,s}{K}-\left(\frac{(B\,Dq+Deta\,M)\,s^3}{K}+\frac{(B\,Kq+Deta\,Dq)\,s^2}{K}+\frac{(Deta\,K+Deta\,Kq+Dq\,K)\,s}{K}+Kq\right)q \quad (16)$$

>

Loop-Transfer Function:

> $G := \frac{K}{B\,M\,s^4+(B\,K+M\,K)\,s^2}$:

> $Gd := \frac{B\,s^2+K}{B\,M\,s^4+(B\,K+K\,M)\,s^2}$:

> $L := collect(normal(simplify(G \cdot Ky), expanded), s, recursive, simplify)$

$$L := \frac{(B\,Dq+Deta\,M)\,s^3+(B\,Kq+Deta\,Dq)\,s^2+(Deta\,K+Deta\,Kq+Dq\,K)\,s+Kq\,K}{B\,M\,s^4+(B\,K+K\,M)\,s^2} \quad (17)$$

>

Closed-Loop Transfer Functions:

> $unassign('qd')$

> $S := collect\left(normal\left(simplify\left(\frac{1}{L+1}\right), expanded\right), s, recursive, simplify\right)$

$$S := \frac{B\,M\,s^4+(B\,K+K\,M)\,s^2}{B\,M\,s^4+(B\,Dq+Deta\,M)\,s^3+(B\,K+B\,Kq+Deta\,Dq+K\,M)\,s^2+(Deta\,K+Deta\,Kq+Dq\,K)\,s+Kq\,K} \quad (18)$$

> $T := collect\left(normal\left(simplify\left(\frac{L}{L+1}\right), expanded\right), s, recursive, simplify\right)$

$$T := \frac{(B\,Dq+Deta\,M)\,s^3+(B\,Kq+Deta\,Dq)\,s^2+(Deta\,K+Deta\,Kq+Dq\,K)\,s+Kq\,K}{B\,M\,s^4+(B\,Dq+Deta\,M)\,s^3+(B\,K+B\,Kq+Deta\,Dq+K\,M)\,s^2+(Deta\,K+Deta\,Kq+Dq\,K)\,s+Kq\,K} \quad (19)$$

> $R := \frac{y}{r} = collect(normal(simplify(Kr \cdot T), expanded), s, recursive, simplify)$

$$R := \frac{y}{r} = \frac{B\,Kq\,s^2+Deta\,Kq\,s+K\,Kq}{B\,M\,s^4+(B\,Dq+Deta\,M)\,s^3+(B\,K+B\,Kq+Deta\,Dq+K\,M)\,s^2+(Deta\,K+Deta\,Kq+Dq\,K)\,s+Kq\,K} \quad (20)$$

> $R\_Mech := \frac{q}{qd}$
$= \frac{B\,Kq\,s^2+Deta\,Kq\,s+K\,Kq}{B\,M\,s^4+(B\,Dq+Deta\,M)\,s^3+(B\,K+B\,Kq+Deta\,Dq+K\,M)\,s^2+(Deta\,K+Deta\,Kq+Dq\,K)\,s+K\,Kq}$:

> $A := \frac{s\,q}{Text} = collect(normal(simplify(Gd\,S\,s+Kd\,T\,s), expanded), s, recursive, simplify)$

$$A := \frac{s\,q}{Text} = \frac{B\,s^3+Deta\,s^2+K\,s}{B\,M\,s^4+(B\,Dq+Deta\,M)\,s^3+(B\,K+B\,Kq+Deta\,Dq+K\,M)\,s^2+(Deta\,K+Deta\,Kq+Dq\,K)\,s+Kq\,K} \quad (21)$$

> $Z := A^{-1}$

$$Z := \frac{Text}{s\,q} = \frac{B\,M\,s^4+(B\,Dq+Deta\,M)\,s^3+(B\,K+B\,Kq+Deta\,Dq+K\,M)\,s^2+(Deta\,K+Deta\,Kq+Dq\,K)\,s+Kq\,K}{B\,s^3+Deta\,s^2+K\,s} \quad (22)$$

> $Z\_Mech := \frac{B\,M\,s^4+(B\,Dq+Deta\,M)\,s^3+(B\,K+B\,Kq+Deta\,Dq+K\,M)\,s^2+(Deta\,K+Deta\,Kq+Dq\,K)\,s+K\,Kq}{B\,s^3+Deta\,s^2+K\,s}$:

> $Disturbance := \frac{u}{d} = collect\left(normal\left(simplify\left(\frac{Ky \cdot (Gd-Kd)}{1-L}\right), expanded\right), s, recursive, simplify\right)$

$$Disturbance := \frac{u}{d} = \left(-B^2\,Dq\,s^5+(-B^2\,Kq-B\,Deta\,Dq)\,s^4+(-B\,Deta\,Kq-2\,B\,Dq\,K)\,s^3+(-2\,B\,K\,Kq-Deta\,Dq\,K)\,s^2+(-Deta\,K^2-Deta\,K\,Kq-Dq\,K^2)\,s-K^2\,Kq\right)\Big/\left(-B\,K\,M\,s^4+(B\,Dq\,K+Deta\,K\,M)\,s^3+(-B\,K^2+B\,K\,Kq+Deta\,Dq\,K-K^2\,M)\,s^2+(Deta\,K^2+Deta\,K\,Kq+Dq\,K^2)\,s+K^2\,Kq\right) \quad (23)$$

> $Noise := \frac{u}{n} = collect(normal(simplify(-Ky \cdot S), expanded), s, recursive, simplify)$

$$Noise := \frac{u}{n} = \left(\left(-B^2\,Dq\,M - B\,Deta\,M^2 \right) s^7 + \left(-B^2\,Kq\,M - B\,Deta\,Dq\,M \right) s^6 + \left(-B^2\,Dq\,K - 2\,B\,Deta\,K\,M - B\,Deta\,Kq\,M - 2\,B\,Dq\,K\,M - Deta\,K\,M^2 \right) s^5 + \left(-B^2\,K\,Kq - B\,Deta\,Dq\,K - 2\,B\,K\,Kq\,M - Deta\,Dq\,K\,M \right) s^4 + \left(-B\,Deta\,K^2 - B\,Deta\,K\,Kq - B\,Dq\,K^2 - Deta\,K^2\,M - Deta\,K\,Kq\,M - Dq\,K^2\,M \right) s^3 + \left(-B\,K^2\,Kq - K^2\,Kq\,M \right) s^2 \right) \Big/ \left(B\,K\,M\,s^4 + \left(B\,Dq\,K + Deta\,K\,M \right) s^3 + \left(B\,K^2 + B\,K\,Kq + Deta\,Dq\,K + K^2\,M \right) s^2 + \left(Deta\,K^2 + Deta\,K\,Kq + Dq\,K^2 \right) s + K^2\,Kq \right) \quad \textbf{(24)}$$

>

>

ESPI 1 3-DoF Control Law:

> $u1 := K1\,qd - K2\,q$

$$u1 := K1\,qd - K2\,q \tag{1}$$

> $u2 := -Deta \cdot \mathrm{eta} \cdot s$

$$u2 := -Deta\left(q + \frac{M\,q\,s^2}{K} - \frac{Text}{K} - \frac{K1\,qd - K2\,q}{K}\right)s \tag{2}$$

> $\mathrm{theta} := q + \frac{M}{K}\cdot q\cdot s^2 - \frac{Text}{K}$

$$\theta := q + \frac{M\,q\,s^2}{K} - \frac{Text}{K} \tag{3}$$

> $\mathrm{eta} := \mathrm{theta} - \frac{u1}{K}$

$$\eta := q + \frac{M\,q\,s^2}{K} - \frac{Text}{K} - \frac{K1\,qd - K2\,q}{K} \tag{4}$$

> $U := u1 + u2 + \frac{B}{K}\cdot u1\cdot s^2 :$

> $U := collect(normal(simplify(U), expanded), [qd, q, s], recursive, simplify)$

$$U := \left(\frac{B\,K1\,s^2}{K} + \frac{Deta\,K1\,s}{K} + K1\right)qd + \left(-\frac{Deta\,M\,s^3}{K} - \frac{B\,K2\,s^2}{K} - \frac{Deta\,(K + K2)\,s}{K} - K2\right)q + \frac{Deta\,Text\,s}{K} \tag{5}$$

>

Reference Controller:

> $Text := 0;$

$$Text := 0 \tag{6}$$

> U

$$\left(\frac{B\,K1\,s^2}{K} + \frac{Deta\,K1\,s}{K} + K1\right)qd + \left(-\frac{Deta\,M\,s^3}{K} - \frac{B\,K2\,s^2}{K} - \frac{Deta\,(K + K2)\,s}{K} - K2\right)q \tag{7}$$

> $Kv := \frac{B\,K1\,s^2}{K} + \frac{Deta\,K1\,s}{K} + K1 :$

> $Ky := -\left(-\frac{Deta\,M\,s^3}{K} - \frac{B\,K2\,s^2}{K} - \frac{Deta\,(K + K2)\,s}{K} - K2\right) :$

> $Ky := collect(normal(simplify(Ky)), s) :$

> $Kv\cdot qd - Ky\cdot q$

$$\left(\frac{B\,K1\,s^2}{K} + \frac{Deta\,K1\,s}{K} + K1\right)qd - \left(\frac{Deta\,M\,s^3}{K} + \frac{B\,K2\,s^2}{K} + \frac{(Deta\,K + Deta\,K2)\,s}{K} + K2\right)q \tag{8}$$

> $Kr := collect\left(normal\left(simplify\left(\frac{Kv}{Ky}\right), expanded\right), s, recursive, simplify\right)$

$$Kr := \frac{B\,K1\,s^2 + Deta\,K1\,s + K\,K1}{Deta\,M\,s^3 + B\,K2\,s^2 + (Deta\,K + Deta\,K2)\,s + K\,K2} \tag{9}$$

>

Complrementary Impedance Controller:

> $qd := 0; unassign('Text')$

$$qd := 0 \tag{10}$$

> U

$$\left(-\frac{Deta\,M\,s^3}{K} - \frac{B\,K2\,s^2}{K} - \frac{Deta\,(K + K2)\,s}{K} - K2\right)q + \frac{Deta\,Text\,s}{K} \tag{11}$$

> $Kt := \frac{Deta\,s}{K}$

$$Kt := \frac{Deta\,s}{K} \tag{12}$$

> $Kd := collect\left(normal\left(simplify\left(\frac{Kt}{Ky}\right), expanded\right), s, recursive, simplify\right)$

$$Kd := \frac{s\,Deta}{Deta\,M\,s^3 + B\,K2\,s^2 + (Deta\,K + Deta\,K2)\,s + K\,K2} \tag{13}$$

> $Kt\,Text - Ky\cdot q$

(14)

$$\frac{Deta\,Text\,s}{K} - \left(\frac{Deta\,M\,s^3}{K} + \frac{B\,K2\,s^2}{K} + \frac{(Deta\,K + Deta\,K2)\,s}{K} + K2\right) q \qquad (14)$$

>

Loop-Transfer Function:

> $G := \frac{K}{B\,M\,s^4 + (B\,K + M\,K)\,s^2}$:

> $Gd := \frac{B\,s^2 + K}{B\,M\,s^4 + (B\,K + K\,M)\,s^2}$:

> $L := collect(normal(simplify(G \cdot Ky), expanded), s, recursive, simplify)$

$$L := \frac{Deta\,M\,s^3 + B\,K2\,s^2 + (Deta\,K + Deta\,K2)\,s + K\,K2}{B\,M\,s^4 + (B\,K + K\,M)\,s^2} \qquad (15)$$

Controler and Filters:

> $K1 := \frac{Kq \cdot Km}{Dq\,s + Kq + Km}$:

> $K2 := \frac{Km \cdot Dq \cdot s + Km \cdot Kq}{Dq \cdot s + Kq + Km}$:

> $L := collect(normal(simplify(L), expanded), s, recursive, simplify)$

$$L := \big(Deta\,Dq\,M\,s^4 + (B\,Dq\,Km + Deta\,Km\,M + Deta\,Kq\,M)\,s^3 + (B\,Km\,Kq + Deta\,Dq\,K + Deta\,Dq\,Km)\,s^2 + (Deta\,K\,Km + Deta\,K\,Kq + Deta\,Km\,Kq + Dq\,K\,Km)\,s + K\,Km\,Kq\big) \Big/ \big(B\,Dq\,M\,s^5 + (B\,Km\,M + B\,Kq\,M)\,s^4 + (B\,Dq\,K + Dq\,K\,M)\,s^3 + (B\,K\,Km + B\,K\,Kq + K\,Km\,M + K\,Kq\,M)\,s^2\big) \qquad (16)$$

> $Kr := collect(normal(simplify(Kr), expanded), s, recursive, simplify)$

$$Kr := \big(B\,Km\,Kq\,s^2 + Deta\,Km\,Kq\,s + K\,Km\,Kq\big) \Big/ \big(Deta\,Dq\,M\,s^4 + (B\,Dq\,Km + Deta\,Km\,M + Deta\,Kq\,M)\,s^3 + (B\,Km\,Kq + Deta\,Dq\,K + Deta\,Dq\,Km)\,s^2 + (Deta\,K\,Km + Deta\,K\,Kq + Deta\,Km\,Kq + Dq\,K\,Km)\,s + K\,Km\,Kq\big) \qquad (17)$$

> $Ky := collect(normal(simplify(Ky), expanded), s, recursive, simplify)$

$$Ky := \frac{1}{Dq\,K\,s + K\,Km + K\,Kq}\big(Deta\,Dq\,M\,s^4 + (B\,Dq\,Km + Deta\,Km\,M + Deta\,Kq\,M)\,s^3 + (B\,Km\,Kq + Deta\,Dq\,K + Deta\,Dq\,Km)\,s^2 + (Deta\,K\,Km + Deta\,K\,Kq + Deta\,Km\,Kq + Dq\,K\,Km)\,s + K\,Km\,Kq\big) \qquad (18)$$

> $Kv := collect(normal(simplify(Kv), expanded), s, recursive, simplify)$

$$Kv := \frac{B\,Km\,Kq\,s^2 + Deta\,Km\,Kq\,s + K\,Km\,Kq}{Dq\,K\,s + K\,Km + K\,Kq} \qquad (19)$$

> $Kd := collect(normal(simplify(Kd), expanded), s, recursive, simplify)$

$$Kd := \big(Deta\,Dq\,s^2 + (Deta\,Km + Deta\,Kq)\,s\big) \Big/ \big(Deta\,Dq\,M\,s^4 + (B\,Dq\,Km + Deta\,Km\,M + Deta\,Kq\,M)\,s^3 + (B\,Km\,Kq + Deta\,Dq\,K + Deta\,Dq\,Km)\,s^2 + (Deta\,K\,Km + Deta\,K\,Kq + Deta\,Km\,Kq + Dq\,K\,Km)\,s + K\,Km\,Kq\big) \qquad (20)$$

>

Closed-Loop Transfer Functions:

> $unassign('qd')$

> $S := collect\left(normal\left(simplify\left(\frac{1}{L+1}\right), expanded\right), s, recursive, simplify\right)$

$$S := \big(B\,Dq\,M\,s^5 + (B\,Km\,M + B\,Kq\,M)\,s^4 + (B\,Dq\,K + Dq\,K\,M)\,s^3 + (B\,K\,Km + B\,K\,Kq + K\,Km\,M + K\,Kq\,M)\,s^2\big) \Big/ \big(B\,Dq\,M\,s^5 + (B\,Km\,M + B\,Kq\,M + Deta\,Dq\,M)\,s^4 + (B\,Dq\,K + B\,Dq\,Km + Deta\,Km\,M + Deta\,Kq\,M + Dq\,K\,M)\,s^3 + (B\,K\,Km + B\,K\,Kq + B\,Km\,Kq + Deta\,Dq\,K + Deta\,Dq\,Km + K\,Km\,M + K\,Kq\,M)\,s^2 + (Deta\,K\,Km + Deta\,K\,Kq + Deta\,Km\,Kq + Dq\,K\,Km)\,s + K\,Km\,Kq\big) \qquad (21)$$

> $T := collect\left(normal\left(simplify\left(\frac{L}{L+1}\right), expanded\right), s, recursive, simplify\right)$

$$T := \big(Deta\,Dq\,M\,s^4 + (B\,Dq\,Km + Deta\,Km\,M + Deta\,Kq\,M)\,s^3 + (B\,Km\,Kq + Deta\,Dq\,K + Deta\,Dq\,Km)\,s^2 + (Deta\,K\,Km + Deta\,K\,Kq + Deta\,Km\,Kq + Dq\,K\,Km)\,s + K\,Km\,Kq\big) \Big/ \big(B\,Dq\,M\,s^5 + (B\,Km\,M + B\,Kq\,M + Deta\,Dq\,M)\,s^4 + (B\,Dq\,K + B\,Dq\,Km + Deta\,Km\,M + Deta\,Kq\,M + Dq\,K\,M)\,s^3 + (B\,K\,Km + B\,K\,Kq + B\,Km\,Kq + Deta\,Dq\,K + Deta\,Dq\,Km + K\,Km\,M + K\,Kq\,M)\,s^2 + (Deta\,K\,Km + Deta\,K\,Kq + Deta\,Km\,Kq + Dq\,K\,Km)\,s + K\,Km\,Kq\big) \qquad (22)$$

> $R := \frac{y}{r} = collect(normal(simplify(Kr \cdot T), expanded), s, recursive, simplify)$

(23)

$$R := \frac{y}{r} = \left(B\,Km\,Kq\,s^{2} + Deta\,Km\,Kq\,s + K\,Km\,Kq\right) \Big/ \left(B\,Dq\,M\,s^{5} + (B\,Km\,M + B\,Kq\,M + Deta\,Dq\,M)\,s^{4} + (B\,Dq\,K + B\,Dq\,Km + Deta\,Km\,M + Deta\,Kq\,M + Dq\,K\,M)\,s^{3} + (B\,K\,Km + B\,K\,Kq + B\,Km\,Kq + Deta\,Dq\,K + Deta\,Dq\,Km + K\,Km\,M + K\,Kq\,M)\,s^{2} + (Deta\,K\,Km + Deta\,K\,Kq + Deta\,Km\,Kq + Dq\,K\,Km)\,s + K\,Km\,Kq\right) \tag{23}$$

> $R\_Mech := \frac{q}{qd} = \left(B\,Km\,Kq\,s^{2} + Deta\,Km\,Kq\,s + K\,Km\,Kq\right) \Big/ \left(B\,Dq\,M\,s^{5} + (B\,Km\,M + B\,Kq\,M + Deta\,Dq\,M)\,s^{4} + (B\,Dq\,K + B\,Dq\,Km + Deta\,Km\,M + Deta\,Kq\,M + Dq\,K\,M)\,s^{3} + (B\,K\,Km + B\,K\,Kq + B\,Km\,Kq + Deta\,Dq\,K + Deta\,Dq\,Km + K\,Km\,M + K\,Kq\,M)\,s^{2} + (Deta\,K\,Km + Deta\,K\,Kq + Deta\,Km\,Kq + Dq\,K\,Km)\,s + K\,Km\,Kq\right):$

> $A := \frac{s\,q}{Text} = collect(normal(simplify(Gd\,S\,s + Kd\,T\,s), expanded), s, recursive, simplify)$

$$A := \frac{s\,q}{Text} = \left(B\,Dq\,s^{4} + (B\,Km + B\,Kq + Deta\,Dq)\,s^{3} + (Deta\,Km + Deta\,Kq + Dq\,K)\,s^{2} + (K\,Km + K\,Kq)\,s\right) \Big/ \left(B\,Dq\,M\,s^{5} + (B\,Km\,M + B\,Kq\,M + Deta\,Dq\,M)\,s^{4} + (B\,Dq\,K + B\,Dq\,Km + Deta\,Km\,M + Deta\,Kq\,M + Dq\,K\,M)\,s^{3} + (B\,K\,Km + B\,K\,Kq + B\,Km\,Kq + Deta\,Dq\,K + Deta\,Dq\,Km + K\,Km\,M + K\,Kq\,M)\,s^{2} + (Deta\,K\,Km + Deta\,K\,Kq + Deta\,Km\,Kq + Dq\,K\,Km)\,s + K\,Km\,Kq\right) \tag{24}$$

> $Z := A^{-1}$

$$Z := \frac{Text}{s\,q} = \left(B\,Dq\,M\,s^{5} + (B\,Km\,M + B\,Kq\,M + Deta\,Dq\,M)\,s^{4} + (B\,Dq\,K + B\,Dq\,Km + Deta\,Km\,M + Deta\,Kq\,M + Dq\,K\,M)\,s^{3} + (B\,K\,Km + B\,K\,Kq + B\,Km\,Kq + Deta\,Dq\,K + Deta\,Dq\,Km + K\,Km\,M + K\,Kq\,M)\,s^{2} + (Deta\,K\,Km + Deta\,K\,Kq + Deta\,Km\,Kq + Dq\,K\,Km)\,s + K\,Km\,Kq\right) \Big/ \left(B\,Dq\,s^{4} + (B\,Km + B\,Kq + Deta\,Dq)\,s^{3} + (Deta\,Km + Deta\,Kq + Dq\,K)\,s^{2} + (K\,Km + K\,Kq)\,s\right) \tag{25}$$

> $Z\_Mech := \frac{Text}{s\,q} = \left(B\,Dq\,M\,s^{5} + (B\,Km\,M + B\,Kq\,M + Deta\,Dq\,M)\,s^{4} + (B\,Dq\,K + B\,Dq\,Km + Deta\,Km\,M + Deta\,Kq\,M + Dq\,K\,M)\,s^{3} + (B\,K\,Km + B\,K\,Kq + B\,Km\,Kq + Deta\,Dq\,K + Deta\,Dq\,Km + K\,Km\,M + K\,Kq\,M)\,s^{2} + (Deta\,K\,Km + Deta\,K\,Kq + Deta\,Km\,Kq + Dq\,K\,Km)\,s + K\,Km\,Kq\right) \Big/ \left(B\,Dq\,s^{4} + (B\,Km + B\,Kq + Deta\,Dq)\,s^{3} + (Deta\,Km + Deta\,Kq + Dq\,K)\,s^{2} + (K\,Km + K\,Kq)\,s\right):$

> $Disturbance := \frac{u}{d} = collect\left(normal\left(simplify\left(\frac{Ky\cdot(Gd - Kd)}{1 - L}\right), expanded\right), s, recursive, simplify\right)$

$$Disturbance := \frac{u}{d} = \left(B^{2}\,Dq\,Km\,s^{5} + (B^{2}\,Km\,Kq + B\,Deta\,Dq\,Km)\,s^{4} + (B\,Deta\,Km\,Kq + 2\,B\,Dq\,K\,Km)\,s^{3} + (2\,B\,K\,Km\,Kq + Deta\,Dq\,K^{2} + Deta\,Dq\,K\,Km)\,s^{2} + (Deta\,K^{2}\,Km + Deta\,K^{2}\,Kq + Deta\,K\,Km\,Kq + Dq\,K^{2}\,Km)\,s + K^{2}\,Km\,Kq\right) \Big/ \left(B\,Dq\,K\,M\,s^{5} + (B\,K\,Km\,M + B\,K\,Kq\,M - Deta\,Dq\,K\,M)\,s^{4} + (B\,Dq\,K^{2} - B\,Dq\,K\,Km - Deta\,K\,Km\,M - Deta\,K\,Kq\,M + Dq\,K^{2}\,M)\,s^{3} + (B\,K^{2}\,Km + B\,K^{2}\,Kq - B\,K\,Km\,Kq - Deta\,Dq\,K^{2} - Deta\,Dq\,K\,Km + K^{2}\,Km\,M + K^{2}\,Kq\,M)\,s^{2} + (-Deta\,K^{2}\,Km - Deta\,K^{2}\,Kq - Deta\,K\,Km\,Kq - Dq\,K^{2}\,Km)\,s - K^{2}\,Km\,Kq\right) \tag{26}$$

> $Noise := \frac{u}{n} = collect(normal(simplify(-Ky\cdot S), expanded), s, recursive, simplify)$

$$Noise := \frac{u}{n} = \left(-B\,Deta\,Dq\,M^{2}\,s^{8} + (-B^{2}\,Dq\,Km\,M - B\,Deta\,Km\,M^{2} - B\,Deta\,Kq\,M^{2})\,s^{7} + (-B^{2}\,Km\,Kq\,M - 2\,B\,Deta\,Dq\,K\,M - B\,Deta\,Dq\,Km\,M - Deta\,Dq\,K\,M^{2})\,s^{6} + (-B^{2}\,Dq\,K\,Km - 2\,B\,Deta\,K\,Km\,M - 2\,B\,Deta\,K\,Kq\,M - B\,Deta\,Km\,Kq\,M - 2\,B\,Dq\,K\,Km\,M - Deta\,K\,Km\,M^{2} - Deta\,K\,Kq\,M^{2})\,s^{5} + (-B^{2}\,K\,Km\,Kq - B\,Deta\,Dq\,K^{2} - B\,Deta\,Dq\,K\,Km - 2\,B\,K\,Km\,Kq\,M - Deta\,Dq\,K^{2}\,M - Deta\,Dq\,K\,Km\,M)\,s^{4} + (-B\,Deta\,K^{2}\,Km - B\,Deta\,K^{2}\,Kq - B\,Deta\,K\,Km\,Kq - B\,Dq\,K^{2}\,Km - Deta\,K^{2}\,Km\,M - Deta\,K^{2}\,Kq\,M - Deta\,K\,Km\,Kq\,M - Dq\,K^{2}\,Km\,M)\,s^{3} + (-B\,K^{2}\,Km\,Kq - K^{2}\,Km\,Kq\,M)\,s^{2}\right) \Big/ \left(B\,Dq\,K\,M\,s^{5} + (B\,K\,Km\,M + B\,K\,Kq\,M + Deta\,Dq\,K\,M)\,s^{4} + (B\,Dq\,K^{2} + B\,Dq\,K\,Km + Deta\,K\,Km\,M + Deta\,K\,Kq\,M + Dq\,K^{2}\,M)\,s^{3} + (B\,K^{2}\,Km + B\,K^{2}\,Kq + B\,K\,Km\,Kq + Deta\,Dq\,K^{2} + Deta\,Dq\,K\,Km + K^{2}\,Km\,M + K^{2}\,Kq\,M)\,s^{2} + (Deta\,K^{2}\,Km + Deta\,K^{2}\,Kq + Deta\,K\,Km\,Kq + Dq\,K^{2}\,Km)\,s + K^{2}\,Km\,Kq\right) \tag{27}$$

> $unassign('K1', 'K2')$

>

>

ESPI 2 3-DoF Control Law:

> $u1 := K1\,qd - K2\,q$

$$u1 := K1\,qd - K2\,q \tag{1}$$

> $u2 := -Deta \cdot eta \cdot s$

$$u2 := -Deta\left(q + \frac{M\,q\,s^2}{K} - \frac{Text}{K} - \frac{K1\,qd - K2\,q}{K}\right)s \tag{2}$$

> $\text{theta} := q + \frac{M}{K}\cdot q\cdot s^2 - \frac{Text}{K}$

$$\theta := q + \frac{M\,q\,s^2}{K} - \frac{Text}{K} \tag{3}$$

> $\text{eta} := \text{theta} - \frac{u1}{K}$

$$\eta := q + \frac{M\,q\,s^2}{K} - \frac{Text}{K} - \frac{K1\,qd - K2\,q}{K} \tag{4}$$

> $U := u1 + u2 + \frac{B}{K}\cdot u1\cdot s^2:$

> $U := collect(normal(simplify(U), expanded), [qd, q, s], recursive, simplify)$

$$U := \left(\frac{B\,K1\,s^2}{K} + \frac{Deta\,K1\,s}{K} + K1\right)qd + \left(-\frac{Deta\,M\,s^3}{K} - \frac{B\,K2\,s^2}{K} - \frac{Deta\,(K + K2)\,s}{K} - K2\right)q + \frac{Deta\,Text\,s}{K} \tag{5}$$

>

Reference Controller:

> $Text := 0;$

$$Text := 0 \tag{6}$$

> U

$$\left(\frac{B\,K1\,s^2}{K} + \frac{Deta\,K1\,s}{K} + K1\right)qd + \left(-\frac{Deta\,M\,s^3}{K} - \frac{B\,K2\,s^2}{K} - \frac{Deta\,(K + K2)\,s}{K} - K2\right)q \tag{7}$$

> $Kv := \frac{B\,K1\,s^2}{K} + \frac{Deta\,K1\,s}{K} + K1:$

> $Ky := -\left(-\frac{Deta\,M\,s^3}{K} - \frac{B\,K2\,s^2}{K} - \frac{Deta\,(K + K2)\,s}{K} - K2\right):$

> $Ky := collect(normal(simplify(Ky)), s):$

> $Kv\cdot qd - Ky\cdot q$

$$\left(\frac{B\,K1\,s^2}{K} + \frac{Deta\,K1\,s}{K} + K1\right)qd - \left(\frac{Deta\,M\,s^3}{K} + \frac{B\,K2\,s^2}{K} + \frac{(Deta\,K + Deta\,K2)\,s}{K} + K2\right)q \tag{8}$$

> $Kr := collect\left(normal\left(simplify\left(\frac{Kv}{Ky}\right), expanded\right), s, recursive, simplify\right)$

$$Kr := \frac{B\,K1\,s^2 + Deta\,K1\,s + K\,K1}{Deta\,M\,s^3 + B\,K2\,s^2 + (Deta\,K + Deta\,K2)\,s + K\,K2} \tag{9}$$

>

Complrementary Impedance Controller:

> $qd := 0; unassign('Text')$

$$qd := 0 \tag{10}$$

> U

$$\left(-\frac{Deta\,M\,s^3}{K} - \frac{B\,K2\,s^2}{K} - \frac{Deta\,(K + K2)\,s}{K} - K2\right)q + \frac{Deta\,Text\,s}{K} \tag{11}$$

> $Kt := \frac{Deta\,s}{K}$

$$Kt := \frac{Deta\,s}{K} \tag{12}$$

> $Kd := collect\left(normal\left(simplify\left(\frac{Kt}{Ky}\right), expanded\right), s, recursive, simplify\right)$

$$Kd := \frac{Deta\,s}{Deta\,M\,s^3 + B\,K2\,s^2 + (Deta\,K + Deta\,K2)\,s + K\,K2} \tag{13}$$

> $Kt\,Text - Ky\cdot q$

(14)

$$\frac{Deta\,Text\,s}{K} - \left(\frac{Deta\,M\,s^3}{K} + \frac{B\,K2\,s^2}{K} + \frac{(Deta\,K + Deta\,K2)\,s}{K} + K2\right) q \tag{14}$$

>

Loop-Transfer Function:

> $G := \frac{K}{B\,M\,s^4 + (B\,K + M\,K)\,s^2}$:

> $Gd := \frac{B\,s^2 + K}{B\,M\,s^4 + (B\,K + K\,M)\,s^2}$:

> $L := collect(normal(simplify(G \cdot Ky), expanded), s, recursive, simplify)$

$$L := \frac{Deta\,M\,s^3 + B\,K2\,s^2 + (Deta\,K + Deta\,K2)\,s + K\,K2}{B\,M\,s^4 + (B\,K + K\,M)\,s^2} \tag{15}$$

Controler and Filters:

> $K1 := \frac{Kq \cdot Km}{m\,s^2 + Dq\,s + Kq + Km}$:

> $K2 := \frac{Km \cdot m\,s^2 + Km \cdot Dq \cdot s + Km \cdot Kq}{m\,s^2 + Dq \cdot s + Kq + Km}$:

> $L := collect(normal(simplify(L), expanded), s, recursive, simplify)$

$$L := \big(Deta\,M\,m\,s^5 + (B\,Km\,m + Deta\,Dq\,M)\,s^4 + (B\,Dq\,Km + Deta\,K\,m + Deta\,Km\,M + Deta\,Km\,m + Deta\,Kq\,M)\,s^3 + (B\,Km\,Kq + Deta\,Dq\,K + Deta\,Dq\,Km + K\,Km\,m)\,s^2 + (Deta\,K\,Km + Deta\,K\,Kq + Deta\,Km\,Kq + Dq\,K\,Km)\,s + K\,Km\,Kq\big) \Big/ \big(B\,M\,m\,s^6 + B\,Dq\,M\,s^5 + (B\,K\,m + B\,Km\,M + B\,Kq\,M + K\,M\,m)\,s^4 + (B\,Dq\,K + Dq\,K\,M)\,s^3 + (B\,K\,Km + B\,K\,Kq + K\,Km\,M + K\,Kq\,M)\,s^2\big) \tag{16}$$

> $Kr := collect(normal(simplify(Kr), expanded), s, recursive, simplify)$

$$Kr := \big(B\,Km\,Kq\,s^2 + Deta\,Km\,Kq\,s + K\,Km\,Kq\big) \Big/ \big(Deta\,M\,m\,s^5 + (B\,Km\,m + Deta\,Dq\,M)\,s^4 + (B\,Dq\,Km + Deta\,K\,m + Deta\,Km\,M + Deta\,Km\,m + Deta\,Kq\,M)\,s^3 + (B\,Km\,Kq + Deta\,Dq\,K + Deta\,Dq\,Km + K\,Km\,m)\,s^2 + (Deta\,K\,Km + Deta\,K\,Kq + Deta\,Km\,Kq + Dq\,K\,Km)\,s + K\,Km\,Kq\big) \tag{17}$$

> $Ky := collect(normal(simplify(Ky), expanded), s, recursive, simplify)$

$$Ky := \frac{1}{K\,m\,s^2 + Dq\,K\,s + K\,Km + K\,Kq}\big(Deta\,M\,m\,s^5 + (B\,Km\,m + Deta\,Dq\,M)\,s^4 + (B\,Dq\,Km + Deta\,K\,m + Deta\,Km\,M + Deta\,Km\,m + Deta\,Kq\,M)\,s^3 + (B\,Km\,Kq + Deta\,Dq\,K + Deta\,Dq\,Km + K\,Km\,m)\,s^2 + (Deta\,K\,Km + Deta\,K\,Kq + Deta\,Km\,Kq + Dq\,K\,Km)\,s + K\,Km\,Kq\big) \tag{18}$$

> $Kv := collect(normal(simplify(Kv), expanded), s, recursive, simplify)$

$$Kv := \frac{B\,Km\,Kq\,s^2 + Deta\,Km\,Kq\,s + K\,Km\,Kq}{K\,m\,s^2 + Dq\,K\,s + K\,Km + K\,Kq} \tag{19}$$

> $Kd := collect(normal(simplify(Kd), expanded), s, recursive, simplify)$

$$Kd := \big(Deta\,m\,s^3 + Dq\,Deta\,s^2 + (Deta\,Km + Deta\,Kq)\,s\big) \Big/ \big(Deta\,M\,m\,s^5 + (B\,Km\,m + Deta\,Dq\,M)\,s^4 + (B\,Dq\,Km + Deta\,K\,m + Deta\,Km\,M + Deta\,Km\,m + Deta\,Kq\,M)\,s^3 + (B\,Km\,Kq + Deta\,Dq\,K + Deta\,Dq\,Km + K\,Km\,m)\,s^2 + (Deta\,K\,Km + Deta\,K\,Kq + Deta\,Km\,Kq + Dq\,K\,Km)\,s + K\,Km\,Kq\big) \tag{20}$$

>

Closed-Loop Transfer Functions:

> $unassign('qd')$

> $S := collect\left(normal\left(simplify\left(\frac{1}{L+1}\right), expanded\right), s, recursive, simplify\right)$

$$S := \big(B\,M\,m\,s^6 + B\,Dq\,M\,s^5 + (B\,K\,m + B\,Km\,M + B\,Kq\,M + K\,M\,m)\,s^4 + (B\,Dq\,K + Dq\,K\,M)\,s^3 + (B\,K\,Km + B\,K\,Kq + K\,Km\,M + K\,Kq\,M)\,s^2\big) \Big/ \big(B\,M\,m\,s^6 + (B\,Dq\,M + Deta\,M\,m)\,s^5 + (B\,K\,m + B\,Km\,M + B\,Km\,m + B\,Kq\,M + Deta\,Dq\,M + K\,M\,m)\,s^4 + (B\,Dq\,K + B\,Dq\,Km + Deta\,K\,m + Deta\,Km\,M + Deta\,Km\,m + Deta\,Kq\,M + Dq\,K\,M)\,s^3 + (B\,K\,Km + B\,K\,Kq + B\,Km\,Kq + Deta\,Dq\,K + Deta\,Dq\,Km + K\,Km\,M + K\,Km\,m + K\,Kq\,M)\,s^2 + (Deta\,K\,Km + Deta\,K\,Kq + Deta\,Km\,Kq + Dq\,K\,Km)\,s + K\,Km\,Kq\big) \tag{21}$$

> $T := collect\left(normal\left(simplify\left(\frac{L}{L+1}\right), expanded\right), s, recursive, simplify\right)$

$$T := \big(Deta\,M\,m\,s^5 + (B\,Km\,m + Deta\,Dq\,M)\,s^4 + (B\,Dq\,Km + Deta\,K\,m + Deta\,Km\,M + Deta\,Km\,m + Deta\,Kq\,M)\,s^3 + (B\,Km\,Kq + Deta\,Dq\,K + Deta\,Dq\,Km + K\,Km\,m)\,s^2 + (Deta\,K\,Km + Deta\,K\,Kq + Deta\,Km\,Kq + Dq\,K\,Km)\,s \tag{22}$$

$$+ K\,Km\,Kq\Big) \Big/ \Big(B\,M\,m\,s^6 + (B\,Dq\,M + Deta\,M\,m)\,s^5 + (B\,K\,m + B\,Km\,M + B\,Km\,m + B\,Kq\,M + Deta\,Dq\,M + K\,M\,m)\,s^4 + (B\,Dq\,K + B\,Dq\,Km + Deta\,K\,m + Deta\,Km\,M + Deta\,Km\,m + Deta\,Kq\,M + Dq\,K\,M)\,s^3 + (B\,K\,Km + B\,K\,Kq + B\,Km\,Kq + Deta\,Dq\,K + Deta\,Dq\,Km + K\,Km\,M + K\,Km\,m + K\,Kq\,M)\,s^2 + (Deta\,K\,Km + Deta\,K\,Kq + Deta\,Km\,Kq + Dq\,K\,Km)\,s + K\,Km\,Kq\Big)$$

> $R := \frac{y}{r} = collect(normal(simplify(Kr \cdot T), expanded), s, recursive, simplify)$

$$R := \frac{y}{r} = \Big(B\,Km\,Kq\,s^2 + Deta\,Km\,Kq\,s + K\,Km\,Kq\Big) \Big/ \Big(B\,M\,m\,s^6 + (B\,Dq\,M + Deta\,M\,m)\,s^5 + (B\,K\,m + B\,Km\,M + B\,Km\,m + B\,Kq\,M + Deta\,Dq\,M + K\,M\,m)\,s^4 + (B\,Dq\,K + B\,Dq\,Km + Deta\,K\,m + Deta\,Km\,M + Deta\,Km\,m + Deta\,Kq\,M + Dq\,K\,M)\,s^3 + (B\,K\,Km + B\,K\,Kq + B\,Km\,Kq + Deta\,Dq\,K + Deta\,Dq\,Km + K\,Km\,M + K\,Km\,m + K\,Kq\,M)\,s^2 + (Deta\,K\,Km + Deta\,K\,Kq + Deta\,Km\,Kq + Dq\,K\,Km)\,s + K\,Km\,Kq\Big) \quad \textbf{(23)}$$

> $R\_Mech := \frac{q}{qd} = \Big(B\,Km\,Kq\,s^2 + Deta\,Km\,Kq\,s + K\,Km\,Kq\Big) \Big/ \Big(B\,M\,m\,s^6 + (B\,Dq\,M + Deta\,M\,m)\,s^5 + (B\,K\,m + B\,Km\,M + B\,Km\,m + B\,Kq\,M + Deta\,Dq\,M + K\,M\,m)\,s^4 + (B\,Dq\,K + B\,Dq\,Km + Deta\,K\,m + Deta\,Km\,M + Deta\,Km\,m + Deta\,Kq\,M + Dq\,K\,M)\,s^3 + (B\,K\,Km + B\,K\,Kq + B\,Km\,Kq + Deta\,Dq\,K + Deta\,Dq\,Km + K\,Km\,M + K\,Km\,m + K\,Kq\,M)\,s^2 + (Deta\,K\,Km + Deta\,K\,Kq + Deta\,Km\,Kq + Dq\,K\,Km)\,s + K\,Km\,Kq\Big)$:

> $A := \frac{s\,q}{Text} = collect(normal(simplify(GdS\,s + Kd\,T\,s), expanded), s, recursive, simplify)$

$$A := \frac{s\,q}{Text} = \Big(B\,m\,s^5 + (B\,Dq + Deta\,m)\,s^4 + (B\,Km + B\,Kq + Deta\,Dq + K\,m)\,s^3 + (Deta\,Km + Deta\,Kq + Dq\,K)\,s^2 + (K\,Km + K\,Kq)\,s\Big) \Big/ \Big(B\,M\,m\,s^6 + (B\,Dq\,M + Deta\,M\,m)\,s^5 + (B\,K\,m + B\,Km\,M + B\,Km\,m + B\,Kq\,M + Deta\,Dq\,M + K\,M\,m)\,s^4 + (B\,Dq\,K + B\,Dq\,Km + Deta\,K\,m + Deta\,Km\,M + Deta\,Km\,m + Deta\,Kq\,M + Dq\,K\,M)\,s^3 + (B\,K\,Km + B\,K\,Kq + B\,Km\,Kq + Deta\,Dq\,K + Deta\,Dq\,Km + K\,Km\,M + K\,Km\,m + K\,Kq\,M)\,s^2 + (Deta\,K\,Km + Deta\,K\,Kq + Deta\,Km\,Kq + Dq\,K\,Km)\,s + K\,Km\,Kq\Big) \quad \textbf{(24)}$$

> $Z := A^{-1}$

$$Z := \frac{Text}{s\,q} = \Big(B\,M\,m\,s^6 + (B\,Dq\,M + Deta\,M\,m)\,s^5 + (B\,K\,m + B\,Km\,M + B\,Km\,m + B\,Kq\,M + Deta\,Dq\,M + K\,M\,m)\,s^4 + (B\,Dq\,K + B\,Dq\,Km + Deta\,K\,m + Deta\,Km\,M + Deta\,Km\,m + Deta\,Kq\,M + Dq\,K\,M)\,s^3 + (B\,K\,Km + B\,K\,Kq + B\,Km\,Kq + Deta\,Dq\,K + Deta\,Dq\,Km + K\,Km\,M + K\,Km\,m + K\,Kq\,M)\,s^2 + (Deta\,K\,Km + Deta\,K\,Kq + Deta\,Km\,Kq + Dq\,K\,Km)\,s + K\,Km\,Kq\Big) \Big/ \Big(B\,m\,s^5 + (B\,Dq + Deta\,m)\,s^4 + (B\,Km + B\,Kq + Deta\,Dq + K\,m)\,s^3 + (Deta\,Km + Deta\,Kq + Dq\,K)\,s^2 + (K\,Km + K\,Kq)\,s\Big) \quad \textbf{(25)}$$

> $Z\_Mech := \frac{Text}{s\,q} = \Big(B\,M\,m\,s^6 + (B\,Dq\,M + Deta\,M\,m)\,s^5 + (B\,K\,m + B\,Km\,M + B\,Km\,m + B\,Kq\,M + Deta\,Dq\,M + K\,M\,m)\,s^4 + (B\,Dq\,K + B\,Dq\,Km + Deta\,K\,m + Deta\,Km\,M + Deta\,Km\,m + Deta\,Kq\,M + Dq\,K\,M)\,s^3 + (B\,K\,Km + B\,K\,Kq + B\,Km\,Kq + Deta\,Dq\,K + Deta\,Dq\,Km + K\,Km\,M + K\,Km\,m + K\,Kq\,M)\,s^2 + (Deta\,K\,Km + Deta\,K\,Kq + Deta\,Km\,Kq + Dq\,K\,Km)\,s + K\,Km\,Kq\Big) \Big/ \Big(B\,m\,s^5 + (B\,Dq + Deta\,m)\,s^4 + (B\,Km + B\,Kq + Deta\,Dq + K\,m)\,s^3 + (Deta\,Km + Deta\,Kq + Dq\,K)\,s^2 + (K\,Km + K\,Kq)\,s\Big)$:

> $Disturbance := \frac{u}{d} = collect\left(normal\left(simplify\left(\frac{Ky \cdot (Gd - Kd)}{1 - L}\right), expanded\right), s, recursive, simplify\right)$

$$Disturbance := \frac{u}{d} = \Big(B^2\,Km\,m\,s^6 + (B^2\,Dq\,Km + B\,Deta\,Km\,m)\,s^5 + (B^2\,Km\,Kq + B\,Deta\,Dq\,Km + 2\,B\,K\,Km\,m)\,s^4 + (B\,Deta\,Km\,Kq + 2\,B\,Dq\,K\,Km + Deta\,K^2\,m + Deta\,K\,Km\,m)\,s^3 + (2\,B\,K\,Km\,Kq + Deta\,Dq\,K^2 + Deta\,Dq\,K\,Km + K^2\,Km\,m)\,s^2 + (Deta\,K^2\,Km + Deta\,K^2\,Kq + Deta\,K\,Km\,Kq + Dq\,K^2\,Km)\,s + K^2\,Km\,Kq\Big) \Big/ \Big(B\,K\,M\,m\,s^6 + (B\,Dq\,K\,M - Deta\,K\,M\,m)\,s^5 + (B\,K^2\,m + B\,K\,Km\,M - B\,K\,Km\,m + B\,K\,Kq\,M - Deta\,Dq\,K\,M + K^2\,M\,m)\,s^4 + (B\,Dq\,K^2 - B\,Dq\,K\,Km - Deta\,K^2\,m - Deta\,K\,Km\,M - Deta\,K\,Km\,m - Deta\,K\,Kq\,M + Dq\,K^2\,M)\,s^3 + (B\,K^2\,Km + B\,K^2\,Kq - B\,K\,Km\,Kq - Deta\,Dq\,K^2 - Deta\,Dq\,K\,Km + K^2\,Km\,M - K^2\,Km\,m + K^2\,Kq\,M)\,s^2 + (-Deta\,K^2\,Km - Deta\,K^2\,Kq - Deta\,K\,Km\,Kq - Dq\,K^2\,Km)\,s - K^2\,Km\,Kq\Big) \quad \textbf{(26)}$$

> $Noise := \frac{u}{n} = collect(normal(simplify(-Ky \cdot S), expanded), s, recursive, simplify)$

$$Noise := \frac{u}{n} = \Big(-B\,Deta\,M^2\,m\,s^9 + (-B^2\,Km\,M\,m - B\,Deta\,Dq\,M^2)\,s^8 + (-B^2\,Dq\,Km\,M - 2\,B\,Deta\,K\,M\,m - B\,Deta\,Km\,M^2 - B\,Deta\,Km\,M\,m - B\,Deta\,Kq\,M^2 - Deta\,K\,M^2\,m)\,s^7 + (-B^2\,K\,Km\,m - B^2\,Km\,Kq\,M - 2\,B\,Deta\,Dq\,K\,M - B\,Deta\,Dq\,Km\,M - 2\,B\,K\,Km\,M\,m - Deta\,Dq\,K\,M^2)\,s^6 + (-B^2\,Dq\,K\,Km - B\,Deta\,K^2\,m - 2\,B\,Deta\,K\,Km\,M - B\,Deta\,K\,Km\,m - 2\,B\,Deta\,K\,Kq\,M - B\,Deta\,Km\,Kq\,M - 2\,B\,Dq\,K\,Km\,M - Deta\,K^2\,M\,m - Deta\,K\,Km\,M^2 - Deta\,K\,Km\,M\,m - Deta\,K\,Kq\,M^2)\,s^5 + (-B^2\,K\,Km\,Kq - B\,Deta\,Dq\,K^2 - B\,Deta\,Dq\,K\,Km - B\,K^2\,Km\,m - 2\,B\,K\,Km\,Kq\,M - Deta\,Dq\,K^2\,M - Deta\,Dq\,K\,Km\,M - K^2\,Km\,M\,m)\,s^4 + (-B\,Deta\,K^2\,Km - B\,Deta\,K^2\,Kq$$

$$- B\,Deta\,K\,Km\,Kq - B\,Dq\,K^2\,Km - Deta\,K^2\,Km\,M - Deta\,K^2\,Kq\,M - Deta\,K\,Km\,Kq\,M - Dq\,K^2\,Km\,M)\,s^3 + ($$

$$-B\,K^2\,Km\,Kq - K^2\,Km\,Kq\,M)\,s^2\big) \Big/ \big(B\,K\,M\,m\,s^6 + (B\,Dq\,K\,M + Deta\,K\,M\,m)\,s^5 + (B\,K^2\,m + B\,K\,Km\,M + B\,K\,Km\,m$$

$$+ B\,K\,Kq\,M + Deta\,Dq\,K\,M + K^2\,M\,m)\,s^4 + (B\,Dq\,K^2 + B\,Dq\,K\,Km + Deta\,K^2\,m + Deta\,K\,Km\,M + Deta\,K\,Km\,m$$

$$+ Deta\,K\,Kq\,M + Dq\,K^2\,M)\,s^3 + (B\,K^2\,Km + B\,K^2\,Kq + B\,K\,Km\,Kq + Deta\,Dq\,K^2 + Deta\,Dq\,K\,Km + K^2\,Km\,M$$

$$+ K^2\,Km\,m + K^2\,Kq\,M)\,s^2 + (Deta\,K^2\,Km + Deta\,K^2\,Kq + Deta\,K\,Km\,Kq + Dq\,K^2\,Km)\,s + K^2\,Km\,Kq\big)$$

```
> unassign('K1', 'K2')
>
```

>

ESPI 3 3-DoF Control Law:

> $u1 := K1\,qd - K2\,q$

$$u1 := K1\,qd - K2\,q \tag{1}$$

> $u2 := -Deta \cdot \mathrm{eta} \cdot s$

$$u2 := -Deta\left(q + \frac{M\,q\,s^2}{K} - \frac{Text}{K} - \frac{K1\,qd - K2\,q}{K}\right)s \tag{2}$$

> $\mathrm{theta} := q + \frac{M}{K}\cdot q\cdot s^2 - \frac{Text}{K}$

$$\theta := q + \frac{M\,q\,s^2}{K} - \frac{Text}{K} \tag{3}$$

> $\mathrm{eta} := \mathrm{theta} - \frac{u1}{K}$

$$\eta := q + \frac{M\,q\,s^2}{K} - \frac{Text}{K} - \frac{K1\,qd - K2\,q}{K} \tag{4}$$

> $U := u1 + u2 + \frac{B}{K}\cdot u1\cdot s^2 :$

> $U := collect(normal(simplify(U), expanded), [qd, q, s], recursive, simplify)$

$$U := \left(\frac{B\,K1\,s^2}{K} + \frac{Deta\,K1\,s}{K} + K1\right)qd + \left(-\frac{Deta\,M\,s^3}{K} - \frac{B\,K2\,s^2}{K} - \frac{Deta\,(K + K2)\,s}{K} - K2\right)q + \frac{Deta\,Text\,s}{K} \tag{5}$$

>

Reference Controller:

> $Text := 0;$

$$Text := 0 \tag{6}$$

> U

$$\left(\frac{B\,K1\,s^2}{K} + \frac{Deta\,K1\,s}{K} + K1\right)qd + \left(-\frac{Deta\,M\,s^3}{K} - \frac{B\,K2\,s^2}{K} - \frac{Deta\,(K + K2)\,s}{K} - K2\right)q \tag{7}$$

> $Kv := \frac{B\,K1\,s^2}{K} + \frac{Deta\,K1\,s}{K} + K1 :$

> $Ky := -\left(-\frac{Deta\,M\,s^3}{K} - \frac{B\,K2\,s^2}{K} - \frac{Deta\,(K + K2)\,s}{K} - K2\right) :$

> $Ky := collect(normal(simplify(Ky)), s) :$

> $Kv\cdot qd - Ky\cdot q$

$$\left(\frac{B\,K1\,s^2}{K} + \frac{Deta\,K1\,s}{K} + K1\right)qd - \left(\frac{Deta\,M\,s^3}{K} + \frac{B\,K2\,s^2}{K} + \frac{(Deta\,K + Deta\,K2)\,s}{K} + K2\right)q \tag{8}$$

> $Kr := collect\left(normal\left(simplify\left(\frac{Kv}{Ky}\right), expanded\right), s, recursive, simplify\right)$

$$Kr := \frac{B\,K1\,s^2 + Deta\,K1\,s + K\,K1}{Deta\,M\,s^3 + B\,K2\,s^2 + (Deta\,K + Deta\,K2)\,s + K\,K2} \tag{9}$$

>

Complrementary Impedance Controller:

> $qd := 0; unassign('Text')$

$$qd := 0 \tag{10}$$

> U

$$\left(-\frac{Deta\,M\,s^3}{K} - \frac{B\,K2\,s^2}{K} - \frac{Deta\,(K + K2)\,s}{K} - K2\right)q + \frac{Deta\,Text\,s}{K} \tag{11}$$

> $Kt := \frac{Deta\,s}{K}$

$$Kt := \frac{Deta\,s}{K} \tag{12}$$

> $Kd := collect\left(normal\left(simplify\left(\frac{Kt}{Ky}\right), expanded\right), s, recursive, simplify\right)$

$$Kd := \frac{Deta\,s}{Deta\,M\,s^3 + B\,K2\,s^2 + (Deta\,K + Deta\,K2)\,s + K\,K2} \tag{13}$$

> $Kt\,Text - Ky\cdot q$

(14)

$$\frac{Deta\,Text\,s}{K} - \left(\frac{Deta\,M\,s^3}{K} + \frac{B\,K2\,s^2}{K} + \frac{(Deta\,K + Deta\,K2)\,s}{K} + K2\right) q \tag{14}$$

>

Loop-Transfer Function:

> $G := \frac{K}{B\,M\,s^4 + (B\,K + M\,K)\,s^2}$:

> $Gd := \frac{B\,s^2 + K}{B\,M\,s^4 + (B\,K + K\,M)\,s^2}$:

> $L := collect(normal(simplify(G\cdot Ky), expanded), s, recursive, simplify)$

$$L := \frac{Deta\,M\,s^3 + B\,K2\,s^2 + (Deta\,K + Deta\,K2)\,s + K\,K2}{B\,M\,s^4 + (B\,K + K\,M)\,s^2} \tag{15}$$

Controler and Filters:

> $K1 := Kq$:

> $K2 := \frac{(Dq\,Km + Dq\,Kq)\,s + Km\,Kq}{Dq\,s + Km}$:

> $L := collect(normal(simplify(L), expanded), s, recursive, simplify)$

$$L := \big(Deta\,Dq\,M\,s^4 + (B\,Dq\,Km + B\,Dq\,Kq + Deta\,Km\,M)\,s^3 + (B\,Km\,Kq + Deta\,Dq\,K + Deta\,Dq\,Km + Deta\,Dq\,Kq)\,s^2 + (Deta\,K\,Km + Deta\,Km\,Kq + Dq\,K\,Km + Dq\,K\,Kq)\,s + K\,Km\,Kq\big) \big/ \big(B\,Dq\,M\,s^5 + B\,Km\,M\,s^4 + (B\,Dq\,K + Dq\,K\,M)\,s^3 + (B\,K\,Km + K\,Km\,M)\,s^2\big) \tag{16}$$

> $Kr := collect(normal(simplify(Kr), expanded), s, recursive, simplify)$

$$Kr := \big(B\,Dq\,Kq\,s^3 + (B\,Km\,Kq + Deta\,Dq\,Kq)\,s^2 + (Deta\,Km\,Kq + Dq\,K\,Kq)\,s + K\,Km\,Kq\big) \big/ \big(Deta\,Dq\,M\,s^4 + (B\,Dq\,Km + B\,Dq\,Kq + Deta\,Km\,M)\,s^3 + (B\,Km\,Kq + Deta\,Dq\,K + Deta\,Dq\,Km + Deta\,Dq\,Kq)\,s^2 + (Deta\,K\,Km + Deta\,Km\,Kq + Dq\,K\,Km + Dq\,K\,Kq)\,s + K\,Km\,Kq\big) \tag{17}$$

> $Ky := collect(normal(simplify(Ky), expanded), s, recursive, simplify)$

$$Ky := \frac{1}{Dq\,K\,s + K\,Km}\big(Deta\,Dq\,M\,s^4 + (B\,Dq\,Km + B\,Dq\,Kq + Deta\,Km\,M)\,s^3 + (B\,Km\,Kq + Deta\,Dq\,K + Deta\,Dq\,Km + Deta\,Dq\,Kq)\,s^2 + (Deta\,K\,Km + Deta\,Km\,Kq + Dq\,K\,Km + Dq\,K\,Kq)\,s + K\,Km\,Kq\big) \tag{18}$$

> $Kv := collect(normal(simplify(Kv), expanded), s, recursive, simplify)$

$$Kv := \frac{B\,Kq\,s^2}{K} + \frac{Deta\,Kq\,s}{K} + Kq \tag{19}$$

> $Kd := collect(normal(simplify(Kd), expanded), s, recursive, simplify)$

$$Kd := \big(Deta\,Dq\,s^2 + Deta\,Km\,s\big) \big/ \big(Deta\,Dq\,M\,s^4 + (B\,Dq\,Km + B\,Dq\,Kq + Deta\,Km\,M)\,s^3 + (B\,Km\,Kq + Deta\,Dq\,K + Deta\,Dq\,Km + Deta\,Dq\,Kq)\,s^2 + (Deta\,K\,Km + Deta\,Km\,Kq + Dq\,K\,Km + Dq\,K\,Kq)\,s + K\,Km\,Kq\big) \tag{20}$$

>

Closed-Loop Transfer Functions:

> $unassign('qd')$

> $S := collect\left(normal\left(simplify\left(\frac{1}{L+1}\right), expanded\right), s, recursive, simplify\right)$

$$S := \big(B\,Dq\,M\,s^5 + B\,Km\,M\,s^4 + (B\,Dq\,K + Dq\,K\,M)\,s^3 + (B\,K\,Km + K\,Km\,M)\,s^2\big) \big/ \big(B\,Dq\,M\,s^5 + (B\,Km\,M + Deta\,Dq\,M)\,s^4 + (B\,Dq\,K + B\,Dq\,Km + B\,Dq\,Kq + Deta\,Km\,M + Dq\,K\,M)\,s^3 + (B\,K\,Km + B\,Km\,Kq + Deta\,Dq\,K + Deta\,Dq\,Km + Deta\,Dq\,Kq + K\,Km\,M)\,s^2 + (Deta\,K\,Km + Deta\,Km\,Kq + Dq\,K\,Km + Dq\,K\,Kq)\,s + K\,Km\,Kq\big) \tag{21}$$

> $T := collect\left(normal\left(simplify\left(\frac{L}{L+1}\right), expanded\right), s, recursive, simplify\right)$

$$T := \big(Deta\,Dq\,M\,s^4 + (B\,Dq\,Km + B\,Dq\,Kq + Deta\,Km\,M)\,s^3 + (B\,Km\,Kq + Deta\,Dq\,K + Deta\,Dq\,Km + Deta\,Dq\,Kq)\,s^2 + (Deta\,K\,Km + Deta\,Km\,Kq + Dq\,K\,Km + Dq\,K\,Kq)\,s + K\,Km\,Kq\big) \big/ \big(B\,Dq\,M\,s^5 + (B\,Km\,M + Deta\,Dq\,M)\,s^4 + (B\,Dq\,K + B\,Dq\,Km + B\,Dq\,Kq + Deta\,Km\,M + Dq\,K\,M)\,s^3 + (B\,K\,Km + B\,Km\,Kq + Deta\,Dq\,K + Deta\,Dq\,Km + Deta\,Dq\,Kq + K\,Km\,M)\,s^2 + (Deta\,K\,Km + Deta\,Km\,Kq + Dq\,K\,Km + Dq\,K\,Kq)\,s + K\,Km\,Kq\big) \tag{22}$$

> $R := \frac{y}{r} = collect(normal(simplify(Kr\cdot T), expanded), s, recursive, simplify)$

$$R := \frac{y}{r} = \big(B\,Dq\,Kq\,s^3 + (B\,Km\,Kq + Deta\,Dq\,Kq)\,s^2 + (Deta\,Km\,Kq + Dq\,K\,Kq)\,s + K\,Km\,Kq\big) \big/ \big(B\,Dq\,M\,s^5 + (B\,Km\,M + Deta\,Dq\,M)\,s^4 + (B\,Dq\,K + B\,Dq\,Km + B\,Dq\,Kq + Deta\,Km\,M + Dq\,K\,M)\,s^3 + (B\,K\,Km + B\,Km\,Kq + Deta\,Dq\,K \tag{23}$$

$$+ Deta\, Dq\, Km + Deta\, Dq\, Kq + K\, Km\, M)\, s^2 + (Deta\, K\, Km + Deta\, Km\, Kq + Dq\, K\, Km + Dq\, K\, Kq)\, s + K\, Km\, Kq\big)$$

> $R\_Mech := \frac{q}{qd} = \big(B\, Dq\, Kq\, s^3 + (B\, Km\, Kq + Deta\, Dq\, Kq)\, s^2 + (Deta\, Km\, Kq + Dq\, K\, Kq)\, s + K\, Km\, Kq\big) \Big/ \big(B\, Dq\, M\, s^5 + (B\, Km\, M + Deta\, Dq\, M)\, s^4 + (B\, Dq\, K + B\, Dq\, Km + B\, Dq\, Kq + Deta\, Km\, M + Dq\, K\, M)\, s^3 + (B\, K\, Km + B\, Km\, Kq + Deta\, Dq\, K + Deta\, Dq\, Km + Deta\, Dq\, Kq + K\, Km\, M)\, s^2 + (Deta\, K\, Km + Deta\, Km\, Kq + Dq\, K\, Km + Dq\, K\, Kq)\, s + K\, Km\, Kq\big)$:

> $A := \frac{s\, q}{Text} = collect(normal(simplify(Gd\, S\, s + Kd\, T\, s), expanded), s, recursive, simplify)$

$$A := \frac{s\, q}{Text} = \big(B\, Dq\, s^4 + (B\, Km + Deta\, Dq)\, s^3 + (Deta\, Km + Dq\, K)\, s^2 + K\, Km\, s\big) \Big/ \big(B\, Dq\, M\, s^5 + (B\, Km\, M + Deta\, Dq\, M)\, s^4 + (B\, Dq\, K + B\, Dq\, Km + B\, Dq\, Kq + Deta\, Km\, M + Dq\, K\, M)\, s^3 + (B\, K\, Km + B\, Km\, Kq + Deta\, Dq\, K + Deta\, Dq\, Km + Deta\, Dq\, Kq + K\, Km\, M)\, s^2 + (Deta\, K\, Km + Deta\, Km\, Kq + Dq\, K\, Km + Dq\, K\, Kq)\, s + K\, Km\, Kq\big) \tag{24}$$

> $Z := A^{-1}$

$$Z := \frac{Text}{s\, q} = \big(B\, Dq\, M\, s^5 + (B\, Km\, M + Deta\, Dq\, M)\, s^4 + (B\, Dq\, K + B\, Dq\, Km + B\, Dq\, Kq + Deta\, Km\, M + Dq\, K\, M)\, s^3 + (B\, K\, Km + B\, Km\, Kq + Deta\, Dq\, K + Deta\, Dq\, Km + Deta\, Dq\, Kq + K\, Km\, M)\, s^2 + (Deta\, K\, Km + Deta\, Km\, Kq + Dq\, K\, Km + Dq\, K\, Kq)\, s + K\, Km\, Kq\big) \Big/ \big(B\, Dq\, s^4 + (B\, Km + Deta\, Dq)\, s^3 + (Deta\, Km + Dq\, K)\, s^2 + K\, Km\, s\big) \tag{25}$$

> $Z\_Mech := \frac{Text}{s\, q} = \big(B\, Dq\, M\, s^5 + (B\, Km\, M + Deta\, Dq\, M)\, s^4 + (B\, Dq\, K + B\, Dq\, Km + B\, Dq\, Kq + Deta\, Km\, M + Dq\, K\, M)\, s^3 + (B\, K\, Km + B\, Km\, Kq + Deta\, Dq\, K + Deta\, Dq\, Km + Deta\, Dq\, Kq + K\, Km\, M)\, s^2 + (Deta\, K\, Km + Deta\, Km\, Kq + Dq\, K\, Km + Dq\, K\, Kq)\, s + K\, Km\, Kq\big) \Big/ \big(B\, Dq\, s^4 + (B\, Km + Deta\, Dq)\, s^3 + (Deta\, Km + Dq\, K)\, s^2 + K\, Km\, s\big)$:

> $Disturbance := \frac{u}{d} = collect\left(normal\left(simplify\left(\frac{Ky \cdot (Gd - Kd)}{1 - L}\right), expanded\right), s, recursive, simplify\right)$

$$Disturbance := \frac{u}{d} = \big((B^2\, Dq\, Km + B^2\, Dq\, Kq)\, s^5 + (B^2\, Km\, Kq + B\, Deta\, Dq\, Km + B\, Deta\, Dq\, Kq)\, s^4 + (B\, Deta\, Km\, Kq + 2\, B\, Dq\, K\, Km + 2\, B\, Dq\, K\, Kq)\, s^3 + (2\, B\, K\, Km\, Kq + Deta\, Dq\, K^2 + Deta\, Dq\, K\, Km + Deta\, Dq\, K\, Kq)\, s^2 + (Deta\, K^2\, Km + Deta\, K\, Km\, Kq + Dq\, K^2\, Km + Dq\, K^2\, Kq)\, s + K^2\, Km\, Kq\big) \Big/ \big(B\, Dq\, K\, M\, s^5 + (B\, K\, Km\, M - Deta\, Dq\, K\, M)\, s^4 + (B\, Dq\, K^2 - B\, Dq\, K\, Km - B\, Dq\, K\, Kq - Deta\, K\, Km\, M + Dq\, K^2\, M)\, s^3 + (B\, K^2\, Km - B\, K\, Km\, Kq - Deta\, Dq\, K^2 - Deta\, Dq\, K\, Km - Deta\, Dq\, K\, Kq + K^2\, Km\, M)\, s^2 + (-Deta\, K^2\, Km - Deta\, K\, Km\, Kq - Dq\, K^2\, Km - Dq\, K^2\, Kq)\, s - K^2\, Km\, Kq\big) \tag{26}$$

> $Noise := \frac{u}{n} = collect(normal(simplify(-Ky \cdot S), expanded), s, recursive, simplify)$

$$Noise := \frac{u}{n} = \big(-B\, Deta\, Dq\, M^2\, s^8 + (-B^2\, Dq\, Km\, M - B^2\, Dq\, Kq\, M - B\, Deta\, Km\, M^2)\, s^7 + (-B^2\, Km\, Kq\, M - 2\, B\, Deta\, Dq\, K\, M - B\, Deta\, Dq\, Km\, M - B\, Deta\, Dq\, Kq\, M - Deta\, Dq\, K\, M^2)\, s^6 + (-B^2\, Dq\, K\, Km - B^2\, Dq\, K\, Kq - 2\, B\, Deta\, K\, Km\, M - B\, Deta\, Km\, Kq\, M - 2\, B\, Dq\, K\, Km\, M - 2\, B\, Dq\, K\, Kq\, M - Deta\, K\, Km\, M^2)\, s^5 + (-B^2\, K\, Km\, Kq - B\, Deta\, Dq\, K^2 - B\, Deta\, Dq\, K\, Km - B\, Deta\, Dq\, K\, Kq - 2\, B\, K\, Km\, Kq\, M - Deta\, Dq\, K^2\, M - Deta\, Dq\, K\, Km\, M - Deta\, Dq\, K\, Kq\, M)\, s^4 + (-B\, Deta\, K^2\, Km - B\, Deta\, K\, Km\, Kq - B\, Dq\, K^2\, Km - B\, Dq\, K^2\, Kq - Deta\, K^2\, Km\, M - Deta\, K\, Km\, Kq\, M - Dq\, K^2\, Km\, M - Dq\, K^2\, Kq\, M)\, s^3 + (-B\, K^2\, Km\, Kq - K^2\, Km\, Kq\, M)\, s^2\big) \Big/ \big(B\, Dq\, K\, M\, s^5 + (B\, K\, Km\, M + Deta\, Dq\, K\, M)\, s^4 + (B\, Dq\, K^2 + B\, Dq\, K\, Km + B\, Dq\, K\, Kq + Deta\, K\, Km\, M + Dq\, K^2\, M)\, s^3 + (B\, K^2\, Km + B\, K\, Km\, Kq + Deta\, Dq\, K^2 + Deta\, Dq\, K\, Km + Deta\, Dq\, K\, Kq + K^2\, Km\, M)\, s^2 + (Deta\, K^2\, Km + Deta\, K\, Km\, Kq + Dq\, K^2\, Km + Dq\, K^2\, Kq)\, s + K^2\, Km\, Kq\big) \tag{27}$$

> unassign('K1', 'K2')

>

>

ESPI 4 3-DoF Control Law:

> $u1 := K1\, qd - K2\, q$

$$u1 := K1\, qd - K2\, q \quad (1)$$

> $u2 := -Deta \cdot \text{eta} \cdot s$

$$u2 := -Deta\, \eta\, s \quad (2)$$

> $\text{theta} := q + \frac{M}{K} \cdot q \cdot s^2 - \frac{Text}{K}$

$$\theta := q + \frac{M\, q\, s^2}{K} - \frac{Text}{K} \quad (3)$$

> $\text{eta} := \text{theta} - \frac{u1}{K}$

$$\eta := q + \frac{M\, q\, s^2}{K} - \frac{Text}{K} - \frac{K1\, qd - K2\, q}{K} \quad (4)$$

> $U := u1 + u2 + \frac{B}{K} \cdot u1 \cdot s^2 :$

> $U := collect(normal(simplify(U), expanded), [qd, q, s], recursive, simplify)$

$$U := \left(\frac{B\, K1\, s^2}{K} + \frac{Deta\, K1\, s}{K} + K1\right) qd + \left(-\frac{Deta\, M\, s^3}{K} - \frac{B\, K2\, s^2}{K} - \frac{Deta\, (K + K2)\, s}{K} - K2\right) q + \frac{Deta\, Text\, s}{K} \quad (5)$$

>

Reference Controller:

> $Text := 0;$

$$Text := 0 \quad (6)$$

> U

$$\left(\frac{B\, K1\, s^2}{K} + \frac{Deta\, K1\, s}{K} + K1\right) qd + \left(-\frac{Deta\, M\, s^3}{K} - \frac{B\, K2\, s^2}{K} - \frac{Deta\, (K + K2)\, s}{K} - K2\right) q \quad (7)$$

> $Kv := \frac{B\, K1\, s^2}{K} + \frac{Deta\, K1\, s}{K} + K1 :$

> $Ky := -\left(-\frac{Deta\, M\, s^3}{K} - \frac{B\, K2\, s^2}{K} - \frac{Deta\, (K + K2)\, s}{K} - K2\right) :$

> $Ky := collect(normal(simplify(Ky)), s) :$

> $Kv \cdot qd - Ky \cdot q$

$$\left(\frac{B\, K1\, s^2}{K} + \frac{Deta\, K1\, s}{K} + K1\right) qd - \left(\frac{Deta\, M\, s^3}{K} + \frac{B\, K2\, s^2}{K} + \frac{(Deta\, K + Deta\, K2)\, s}{K} + K2\right) q \quad (8)$$

> $Kr := collect\left(normal\left(simplify\left(\frac{Kv}{Ky}\right), expanded\right), s, recursive, simplify\right)$

$$Kr := \frac{B\, K1\, s^2 + Deta\, K1\, s + K\, K1}{Deta\, M\, s^3 + B\, K2\, s^2 + (Deta\, K + Deta\, K2)\, s + K\, K2} \quad (9)$$

>

Complrementary Impedance Controller:

> $qd := 0; unassign('Text')$

$$qd := 0 \quad (10)$$

> U

$$\left(-\frac{Deta\, M\, s^3}{K} - \frac{B\, K2\, s^2}{K} - \frac{Deta\, (K + K2)\, s}{K} - K2\right) q + \frac{Deta\, Text\, s}{K} \quad (11)$$

> $Kt := \frac{Deta\, s}{K}$

$$Kt := \frac{Deta\, s}{K} \quad (12)$$

> $Kd := collect\left(normal\left(simplify\left(\frac{Kt}{Ky}\right), expanded\right), s, recursive, simplify\right)$

$$Kd := \frac{Deta\, s}{Deta\, M\, s^3 + B\, K2\, s^2 + (Deta\, K + Deta\, K2)\, s + K\, K2} \quad (13)$$

> $Kt\, Text - Ky \cdot q$

$$\frac{Deta\, Text\, s}{K} - \left(\frac{Deta\, M\, s^3}{K} + \frac{B\, K2\, s^2}{K} + \frac{(Deta\, K + Deta\, K2)\, s}{K} + K2\right) q \quad (14)$$

>

Loop-Transfer Function:

> $G := \frac{K}{B\,M\,s^4 + (B\,K + M\,K)\,s^2}$:

> $Gd := \frac{B\,s^2 + K}{B\,M\,s^4 + (B\,K + K\,M)\,s^2}$:

> $L := collect(normal(simplify(G \cdot Ky), expanded), s, recursive, simplify)$

$$L := \frac{Deta\,M\,s^3 + B\,K2\,s^2 + (Deta\,K + Deta\,K2)\,s + K\,K2}{B\,M\,s^4 + (B\,K + K\,M)\,s^2} \quad \textbf{(15)}$$

Controler and Filters:

> $K1 := Kq$:

> $K2 := \frac{(Km\,m + Kq\,m)\,s^2 + (Dq\,Km + Dq\,Kq)\,s + Km\,Kq}{m\,s^2 + Dq\,s + Km}$:

> $L := collect(normal(simplify(L), expanded), s, recursive, simplify)$

$$L := \left(Deta\,M\,m\,s^5 + (B\,Km\,m + B\,Kq\,m + Deta\,Dq\,M)\,s^4 + (B\,Dq\,Km + B\,Dq\,Kq + Deta\,K\,m + Deta\,Km\,M + Deta\,Km\,m + Deta\,Kq\,m)\,s^3 + (B\,Km\,Kq + Deta\,Dq\,K + Deta\,Dq\,Km + Deta\,Dq\,Kq + K\,Km\,m + K\,Kq\,m)\,s^2 + (Deta\,K\,Km + Deta\,Km\,Kq + Dq\,K\,Km + Dq\,K\,Kq)\,s + K\,Km\,Kq\right) \Big/ \left(B\,M\,m\,s^6 + B\,Dq\,M\,s^5 + (B\,K\,m + B\,Km\,M + K\,M\,m)\,s^4 + (B\,Dq\,K + Dq\,K\,M)\,s^3 + (B\,K\,Km + K\,Km\,M)\,s^2\right) \quad \textbf{(16)}$$

> $Kr := collect(normal(simplify(Kr), expanded), s, recursive, simplify)$

$$Kr := \left(B\,Kq\,m\,s^4 + (B\,Dq\,Kq + Deta\,Kq\,m)\,s^3 + (B\,Km\,Kq + Deta\,Dq\,Kq + K\,Kq\,m)\,s^2 + (Deta\,Km\,Kq + Dq\,K\,Kq)\,s + K\,Km\,Kq\right) \Big/ \left(Deta\,M\,m\,s^5 + (B\,Km\,m + B\,Kq\,m + Deta\,Dq\,M)\,s^4 + (B\,Dq\,Km + B\,Dq\,Kq + Deta\,K\,m + Deta\,Km\,M + Deta\,Km\,m + Deta\,Kq\,m)\,s^3 + (B\,Km\,Kq + Deta\,Dq\,K + Deta\,Dq\,Km + Deta\,Dq\,Kq + K\,Km\,m + K\,Kq\,m)\,s^2 + (Deta\,K\,Km + Deta\,Km\,Kq + Dq\,K\,Km + Dq\,K\,Kq)\,s + K\,Km\,Kq\right) \quad \textbf{(17)}$$

> $Ky := collect(normal(simplify(Ky), expanded), s, recursive, simplify)$

$$Ky := \frac{1}{K\,m\,s^2 + Dq\,K\,s + K\,Km} \left(Deta\,M\,m\,s^5 + (B\,Km\,m + B\,Kq\,m + Deta\,Dq\,M)\,s^4 + (B\,Dq\,Km + B\,Dq\,Kq + Deta\,K\,m + Deta\,Km\,M + Deta\,Km\,m + Deta\,Kq\,m)\,s^3 + (B\,Km\,Kq + Deta\,Dq\,K + Deta\,Dq\,Km + Deta\,Dq\,Kq + K\,Km\,m + K\,Kq\,m)\,s^2 + (Deta\,K\,Km + Deta\,Km\,Kq + Dq\,K\,Km + Dq\,K\,Kq)\,s + K\,Km\,Kq\right) \quad \textbf{(18)}$$

> $Kv := collect(normal(simplify(Kv), expanded), s, recursive, simplify)$

$$Kv := \frac{B\,Kq\,s^2}{K} + \frac{Deta\,Kq\,s}{K} + Kq \quad \textbf{(19)}$$

> $Kd := collect(normal(simplify(Kd), expanded), s, recursive, simplify)$

$$Kd := \left(Deta\,m\,s^3 + Deta\,Dq\,s^2 + Deta\,Km\,s\right) \Big/ \left(Deta\,M\,m\,s^5 + (B\,Km\,m + B\,Kq\,m + Deta\,Dq\,M)\,s^4 + (B\,Dq\,Km + B\,Dq\,Kq + Deta\,K\,m + Deta\,Km\,M + Deta\,Km\,m + Deta\,Kq\,m)\,s^3 + (B\,Km\,Kq + Deta\,Dq\,K + Deta\,Dq\,Km + Deta\,Dq\,Kq + K\,Km\,m + K\,Kq\,m)\,s^2 + (Deta\,K\,Km + Deta\,Km\,Kq + Dq\,K\,Km + Dq\,K\,Kq)\,s + K\,Km\,Kq\right) \quad \textbf{(20)}$$

>

Closed-Loop Transfer Functions:

> $unassign('qd')$

> $S := collect\left(normal\left(simplify\left(\frac{1}{L+1}\right), expanded\right), s, recursive, simplify\right)$

$$S := \left(B\,M\,m\,s^6 + B\,Dq\,M\,s^5 + (B\,K\,m + B\,Km\,M + K\,M\,m)\,s^4 + (B\,Dq\,K + Dq\,K\,M)\,s^3 + (B\,K\,Km + K\,Km\,M)\,s^2\right) \Big/ \left(B\,M\,m\,s^6 + (B\,Dq\,M + Deta\,M\,m)\,s^5 + (B\,K\,m + B\,Km\,M + B\,Km\,m + B\,Kq\,m + Deta\,Dq\,M + K\,M\,m)\,s^4 + (B\,Dq\,K + B\,Dq\,Km + B\,Dq\,Kq + Deta\,K\,m + Deta\,Km\,M + Deta\,Km\,m + Deta\,Kq\,m + Dq\,K\,M)\,s^3 + (B\,K\,Km + B\,Km\,Kq + Deta\,Dq\,K + Deta\,Dq\,Km + Deta\,Dq\,Kq + K\,Km\,M + K\,Km\,m + K\,Kq\,m)\,s^2 + (Deta\,K\,Km + Deta\,Km\,Kq + Dq\,K\,Km + Dq\,K\,Kq)\,s + K\,Km\,Kq\right) \quad \textbf{(21)}$$

> $T := collect\left(normal\left(simplify\left(\frac{L}{L+1}\right), expanded\right), s, recursive, simplify\right)$

$$T := \left(Deta\,M\,m\,s^5 + (B\,Km\,m + B\,Kq\,m + Deta\,Dq\,M)\,s^4 + (B\,Dq\,Km + B\,Dq\,Kq + Deta\,K\,m + Deta\,Km\,M + Deta\,Km\,m + Deta\,Kq\,m)\,s^3 + (B\,Km\,Kq + Deta\,Dq\,K + Deta\,Dq\,Km + Deta\,Dq\,Kq + K\,Km\,m + K\,Kq\,m)\,s^2 + (Deta\,K\,Km + Deta\,Km\,Kq + Dq\,K\,Km + Dq\,K\,Kq)\,s + K\,Km\,Kq\right) \Big/ \left(B\,M\,m\,s^6 + (B\,Dq\,M + Deta\,M\,m)\,s^5 + (B\,K\,m + B\,Km\,M + B\,Km\,m + B\,Kq\,m + Deta\,Dq\,M + K\,M\,m)\,s^4 + (B\,Dq\,K + B\,Dq\,Km + B\,Dq\,Kq + Deta\,K\,m + Deta\,Km\,M\right. \quad \textbf{(22)}$$

$+ Deta\,Km\,m + Deta\,Kq\,m + Dq\,K\,M)\,s^3 + (B\,K\,Km + B\,Km\,Kq + Deta\,Dq\,K + Deta\,Dq\,Km + Deta\,Dq\,Kq + K\,Km\,M + K\,Km\,m + K\,Kq\,m)\,s^2 + (Deta\,K\,Km + Deta\,Km\,Kq + Dq\,K\,Km + Dq\,K\,Kq)\,s + K\,Km\,Kq)$

> $R := \frac{y}{r} = collect(normal(simplify(Kr \cdot T), expanded), s, recursive, simplify)$

$$R := \frac{y}{r} = \big(B\,Kq\,m\,s^4 + (B\,Dq\,Kq + Deta\,Kq\,m)\,s^3 + (B\,Km\,Kq + Deta\,Dq\,Kq + K\,Kq\,m)\,s^2 + (Deta\,Km\,Kq + Dq\,K\,Kq)\,s + K\,Km\,Kq\big) \Big/ \big(B\,M\,m\,s^6 + (B\,Dq\,M + Deta\,M\,m)\,s^5 + (B\,K\,m + B\,Km\,M + B\,Km\,m + B\,Kq\,m + Deta\,Dq\,M + K\,M\,m)\,s^4 + (B\,Dq\,K + B\,Dq\,Km + B\,Dq\,Kq + Deta\,K\,m + Deta\,Km\,M + Deta\,Km\,m + Deta\,Kq\,m + Dq\,K\,M)\,s^3 + (B\,K\,Km + B\,Km\,Kq + Deta\,Dq\,K + Deta\,Dq\,Km + Deta\,Dq\,Kq + K\,Km\,M + K\,Km\,m + K\,Kq\,m)\,s^2 + (Deta\,K\,Km + Deta\,Km\,Kq + Dq\,K\,Km + Dq\,K\,Kq)\,s + K\,Km\,Kq\big) \quad \textbf{(23)}$$

> $R\_Mech := \frac{q}{qd} = \big(B\,Kq\,m\,s^4 + (B\,Dq\,Kq + Deta\,Kq\,m)\,s^3 + (B\,Km\,Kq + Deta\,Dq\,Kq + K\,Kq\,m)\,s^2 + (Deta\,Km\,Kq + Dq\,K\,Kq)\,s + K\,Km\,Kq\big) \Big/ \big(B\,M\,m\,s^6 + (B\,Dq\,M + Deta\,M\,m)\,s^5 + (B\,K\,m + B\,Km\,M + B\,Km\,m + B\,Kq\,m + Deta\,Dq\,M + K\,M\,m)\,s^4 + (B\,Dq\,K + B\,Dq\,Km + B\,Dq\,Kq + Deta\,K\,m + Deta\,Km\,M + Deta\,Km\,m + Deta\,Kq\,m + Dq\,K\,M)\,s^3 + (B\,K\,Km + B\,Km\,Kq + Deta\,Dq\,K + Deta\,Dq\,Km + Deta\,Dq\,Kq + K\,Km\,M + K\,Km\,m + K\,Kq\,m)\,s^2 + (Deta\,K\,Km + Deta\,Km\,Kq + Dq\,K\,Km + Dq\,K\,Kq)\,s + K\,Km\,Kq\big) :$

> $A := \frac{s\,q}{Text} = collect(normal(simplify(GdS\,s + Kd\,T\,s), expanded), s, recursive, simplify)$

$$A := \frac{s\,q}{Text} = \big(B\,m\,s^5 + (B\,Dq + Deta\,m)\,s^4 + (B\,Km + Deta\,Dq + K\,m)\,s^3 + (Deta\,Km + Dq\,K)\,s^2 + K\,Km\,s\big) \Big/ \big(B\,M\,m\,s^6 + (B\,Dq\,M + Deta\,M\,m)\,s^5 + (B\,K\,m + B\,Km\,M + B\,Km\,m + B\,Kq\,m + Deta\,Dq\,M + K\,M\,m)\,s^4 + (B\,Dq\,K + B\,Dq\,Km + B\,Dq\,Kq + Deta\,K\,m + Deta\,Km\,M + Deta\,Km\,m + Deta\,Kq\,m + Dq\,K\,M)\,s^3 + (B\,K\,Km + B\,Km\,Kq + Deta\,Dq\,K + Deta\,Dq\,Km + Deta\,Dq\,Kq + K\,Km\,M + K\,Km\,m + K\,Kq\,m)\,s^2 + (Deta\,K\,Km + Deta\,Km\,Kq + Dq\,K\,Km + Dq\,K\,Kq)\,s + K\,Km\,Kq\big) \quad \textbf{(24)}$$

> $Z := A^{-1}$

$$Z := \frac{Text}{s\,q} = \big(B\,M\,m\,s^6 + (B\,Dq\,M + Deta\,M\,m)\,s^5 + (B\,K\,m + B\,Km\,M + B\,Km\,m + B\,Kq\,m + Deta\,Dq\,M + K\,M\,m)\,s^4 + (B\,Dq\,K + B\,Dq\,Km + B\,Dq\,Kq + Deta\,K\,m + Deta\,Km\,M + Deta\,Km\,m + Deta\,Kq\,m + Dq\,K\,M)\,s^3 + (B\,K\,Km + B\,Km\,Kq + Deta\,Dq\,K + Deta\,Dq\,Km + Deta\,Dq\,Kq + K\,Km\,M + K\,Km\,m + K\,Kq\,m)\,s^2 + (Deta\,K\,Km + Deta\,Km\,Kq + Dq\,K\,Km + Dq\,K\,Kq)\,s + K\,Km\,Kq\big) \Big/ \big(B\,m\,s^5 + (B\,Dq + Deta\,m)\,s^4 + (B\,Km + Deta\,Dq + K\,m)\,s^3 + (Deta\,Km + Dq\,K)\,s^2 + K\,Km\,s\big) \quad \textbf{(25)}$$

> $Z\_Mech := \frac{Text}{s\,q} = \big(B\,M\,m\,s^6 + (B\,Dq\,M + Deta\,M\,m)\,s^5 + (B\,K\,m + B\,Km\,M + B\,Km\,m + B\,Kq\,m + Deta\,Dq\,M + K\,M\,m)\,s^4 + (B\,Dq\,K + B\,Dq\,Km + B\,Dq\,Kq + Deta\,K\,m + Deta\,Km\,M + Deta\,Km\,m + Deta\,Kq\,m + Dq\,K\,M)\,s^3 + (B\,K\,Km + B\,Km\,Kq + Deta\,Dq\,K + Deta\,Dq\,Km + Deta\,Dq\,Kq + K\,Km\,M + K\,Km\,m + K\,Kq\,m)\,s^2 + (Deta\,K\,Km + Deta\,Km\,Kq + Dq\,K\,Km + Dq\,K\,Kq)\,s + K\,Km\,Kq\big) \Big/ \big(B\,m\,s^5 + (B\,Dq + Deta\,m)\,s^4 + (B\,Km + Deta\,Dq + K\,m)\,s^3 + (Deta\,Km + Dq\,K)\,s^2 + K\,Km\,s\big) :$

> $Disturbance := \frac{u}{d} = collect\left(normal\left(simplify\left(\frac{Ky \cdot (Gd - Kd)}{1 - L}\right), expanded\right), s, recursive, simplify\right)$

$$Disturbance := \frac{u}{d} = \big((B^2\,Km\,m + B^2\,Kq\,m)\,s^6 + (B^2\,Dq\,Km + B^2\,Dq\,Kq + B\,Deta\,Km\,m + B\,Deta\,Kq\,m)\,s^5 + (B^2\,Km\,Kq + B\,Deta\,Dq\,Km + B\,Deta\,Dq\,Kq + 2\,B\,K\,Km\,m + 2\,B\,K\,Kq\,m)\,s^4 + (B\,Deta\,Km\,Kq + 2\,B\,Dq\,K\,Km + 2\,B\,Dq\,K\,Kq + Deta\,K^2\,m + Deta\,K\,Km\,m + Deta\,K\,Kq\,m)\,s^3 + (2\,B\,K\,Km\,Kq + Deta\,Dq\,K^2 + Deta\,Dq\,K\,Km + Deta\,Dq\,K\,Kq + K^2\,Km\,m + K^2\,Kq\,m)\,s^2 + (Deta\,K^2\,Km + Deta\,K\,Km\,Kq + Dq\,K^2\,Km + Dq\,K^2\,Kq)\,s + K^2\,Km\,Kq\big) \Big/ \big(B\,K\,M\,m\,s^6 + (B\,Dq\,K\,M - Deta\,K\,M\,m)\,s^5 + (B\,K^2\,m + B\,K\,Km\,M - B\,K\,Km\,m - B\,K\,Kq\,m - Deta\,Dq\,K\,M + K^2\,M\,m)\,s^4 + (B\,Dq\,K^2 - B\,Dq\,K\,Km - B\,Dq\,K\,Kq - Deta\,K^2\,m - Deta\,K\,Km\,M - Deta\,K\,Km\,m - Deta\,K\,Kq\,m + Dq\,K^2\,M)\,s^3 + (B\,K^2\,Km - B\,K\,Km\,Kq - Deta\,Dq\,K^2 - Deta\,Dq\,K\,Km - Deta\,Dq\,K\,Kq + K^2\,Km\,M - K^2\,Km\,m - K^2\,Kq\,m)\,s^2 + (-Deta\,K^2\,Km - Deta\,K\,Km\,Kq - Dq\,K^2\,Km - Dq\,K^2\,Kq)\,s - K^2\,Km\,Kq\big) \quad \textbf{(26)}$$

> $Noise := \frac{u}{n} = collect(normal(simplify(-Ky \cdot S), expanded), s, recursive, simplify)$

$$Noise := \frac{u}{n} = \big(-B\,Deta\,M^2\,m\,s^9 + (-B^2\,Km\,M\,m - B^2\,Kq\,M\,m - B\,Deta\,Dq\,M^2)\,s^8 + (-B^2\,Dq\,Km\,M - B^2\,Dq\,Kq\,M - 2\,B\,Deta\,K\,M\,m - B\,Deta\,Km\,M^2 - B\,Deta\,Km\,M\,m - B\,Deta\,Kq\,M\,m - Deta\,K\,M^2\,m)\,s^7 + (-B^2\,K\,Km\,m - B^2\,K\,Kq\,m - B^2\,Km\,Kq\,M - 2\,B\,Deta\,Dq\,K\,M - B\,Deta\,Dq\,Km\,M - B\,Deta\,Dq\,Kq\,M - 2\,B\,K\,Km\,M\,m - 2\,B\,K\,Kq\,M\,m - Deta\,Dq\,K\,M^2)\,s^6 + (-B^2\,Dq\,K\,Km - B^2\,Dq\,K\,Kq - B\,Deta\,K^2\,m - 2\,B\,Deta\,K\,Km\,M - B\,Deta\,K\,Km\,m - B\,Deta\,K\,Kq\,m - B\,Deta\,Km\,Kq\,M - 2\,B\,Dq\,K\,Km\,M - 2\,B\,Dq\,K\,Kq\,M - Deta\,K^2\,M\,m \quad \textbf{(27)}$$

$$- Deta\, K\, Km\, M^2 - Deta\, K\, Km\, M\, m - Deta\, K\, Kq\, M\, m)\, s^5 + (-B^2\, K\, Km\, Kq - B\, Deta\, Dq\, K^2 - B\, Deta\, Dq\, K\, Km$$
$$- B\, Deta\, Dq\, K\, Kq - B\, K^2\, Km\, m - B\, K^2\, Kq\, m - 2\, B\, K\, Km\, Kq\, M - Deta\, Dq\, K^2\, M - Deta\, Dq\, K\, Km\, M$$
$$- Deta\, Dq\, K\, Kq\, M - K^2\, Km\, M\, m - K^2\, Kq\, M\, m)\, s^4 + (-B\, Deta\, K^2\, Km - B\, Deta\, K\, Km\, Kq - B\, Dq\, K^2\, Km$$
$$- B\, Dq\, K^2\, Kq - Deta\, K^2\, Km\, M - Deta\, K\, Km\, Kq\, M - Dq\, K^2\, Km\, M - Dq\, K^2\, Kq\, M)\, s^3 + (-B\, K^2\, Km\, Kq$$
$$- K^2\, Km\, Kq\, M)\, s^2\Big) \Big/ \Big(B\, K\, M\, m\, s^6 + (B\, Dq\, K\, M + Deta\, K\, M\, m)\, s^5 + (B\, K^2\, m + B\, K\, Km\, M + B\, K\, Km\, m + B\, K\, Kq\, m$$
$$+ Deta\, Dq\, K\, M + K^2\, M\, m)\, s^4 + (B\, Dq\, K^2 + B\, Dq\, K\, Km + B\, Dq\, K\, Kq + Deta\, K^2\, m + Deta\, K\, Km\, M + Deta\, K\, Km\, m$$
$$+ Deta\, K\, Kq\, m + Dq\, K^2\, M)\, s^3 + (B\, K^2\, Km + B\, K\, Km\, Kq + Deta\, Dq\, K^2 + Deta\, Dq\, K\, Km + Deta\, Dq\, K\, Kq + K^2\, Km\, M$$
$$+ K^2\, Km\, m + K^2\, Kq\, m)\, s^2 + (Deta\, K^2\, Km + Deta\, K\, Km\, Kq + Dq\, K^2\, Km + Dq\, K^2\, Kq)\, s + K^2\, Km\, Kq\Big)$$

>

```matlab
%% Parameters - Explanation
% M         Link inertia
% B         Motor inertia
% m         Additional virtual inertia
% q         Link coordiante
% theta     Motor coordinate
% eta       VIRTUAL motor coordinate
% qm        Coordinate of the VIRTUAL mass m
% qd        Desired link position
% u         Control input = motor torque
% K         Stiffness of the mechanical spring
% Km        Stiffness of the VIRTUAL link side spring
% Kq        Stiffness of the VIRTUAL spring of the spring damper unit
% Dq        Damping factor of the VIRTUAL damper of the spring damper unit
% Deta      Damping factor of the VIRTUAL motor side damper
% T_ext     External link side torque
%
% Kv        Reference Stiffness
% Dv        Reference Dampening

%% Parameters - Settings
global B M K Kv Dv  %for the optimization functions
global Deta Dq      %for the optimization with reduced parameters
global Gd           %for the disturbance optimization

% Plant
M = 1;
B = 0.5980;
K = 374;

% Reference
Kv = 200;
Dv = 0.7*(2*sqrt(Kv*M));

% Virtual
Deta = 0.3*(2*sqrt(K*B));
Kq = 200;
Dq = 0.7*(2*sqrt(Kq*M));
Km = 10*Kq;
m  = 0.1*M;

% Input
T_ext = 0;
qd    = 1;

%% Initialization fransfer functions
s=tf('s');
% Plant model
    G  = tf([K],[B*M 0 (B*K+K*M) 0 0]);
    Gd = tf([B 0 K],[B*M 0 (B*K+K*M) 0 0]);
% Reference model
    R_V = tf([Kv],[M Dv Kv]);
    Z_V = tf([M Dv Kv],[1 0]);
```

```matlab
%% Initialization of the optimization with reduced parameters
% Run ESPI_init first
% We optimize for low noise at high frequencies
% The constraints are 6 dB deviation of the reference impedance
% The reference impedance behavior Z_V is defined in ESPI_init
% Optimization parameters:
% ESPI 1:[Kq/Km]     ESPI 2:[Kq/Km m]    ESPI 3:[Km]     ESPI 4:[Km m]

%% Init
global version w loghighmag loglowmag
global  KN fN

version = 2;             %ESPI Version 1-4

% Start  [Kq/Km    m]
ESPI_1 = [10];
ESPI_2 = [10 0.1];
% Start  [Km    m]
ESPI_3 = [2000];
ESPI_4 = [2000 0.1];

Xsred = {ESPI_1, ESPI_2, ESPI_3, ESPI_4};
clear ESPI_1 ESPI_2 ESPI_3 ESPI_4
x0 = Xsred{version};

% Bounds:
uB =  6;                %upper constraint impedance in dB
lB = -6;                %lower constraint impedance in dB
ub = [];                %upper parameters bound
lb = [0 0];             %lower parameters bound

% The vector of the frequency rage of the bode calculation.
% The incrementation can be variable.
w  = [0.1:0.1:10 11:1:1000]';

KN = 150;               %Gain
fN = 100;               %Cutoff frequency

% Constraint area
[mag_ref,~,~] = bode(Z_V,w);
 mag_ref      = squeeze(mag_ref);
loghighmag    = 20*log10(mag_ref)+uB;   %upper constraint
loglowmag     = 20*log10(mag_ref)-uB;   %lower constraint


%% Optimization
[x, fval]=fmincon(@ESPI_optim_phi_wND,x0,[],[],[],[],lb,ub,@ESPI_optim_psi_Z)
```

```
function [psi_inequal,psi_equal] = ESPI_optim_psi_Z(x)
%% Constraint function of the ESPI System
% The impedance constraints are set by the reference model ESPI V
global version w loghighmag loglowmag
global B M K Kv Deta

if version == 1
    Kq = Kv*((x(1)+1)/x(1));
    Km = Kv*(x(1)+1);
    Dq = 0.7*2*sqrt(Kq*M);
    Z = tf([(B*Dq*M) (B*Km*M+B*Kq*M+Deta*Dq*M)
(B*Dq*K+B*Dq*Km+Deta*Km*M+Deta*Kq*M+Dq*K*M)
(B*K*Km+B*K*Kq+B*Km*Kq+Deta*Dq*K+Deta*Dq*Km+K*Km*M+K*Kq*M)
(Deta*K*Km+Deta*K*Kq+Deta*Km*Kq+Dq*K*Km) (K*Km*Kq)],[(B*Dq) (B*Km+B*Kq+Deta*Dq)
(Deta*Km+Deta*Kq+Dq*K) (K*Km+K*Kq) 0]);
end
if version == 2
    Kq = Kv*((x(1)+1)/x(1));
    Km = Kv*(x(1)+1);
    m  = x(2)*M;
    Dq = 0.7*2*sqrt(Kq*M);
    Z = tf([(B*M*m) (M*(B*Dq+Deta*m)) (m*(B+M)*K+B*(M+m)*Km+M*(B*Kq+Deta*Dq))
((B*Dq+Deta*m+Dq*M)*K+(B*Dq+Deta*M+Deta*m)*Km+Deta*M*Kq) (((B+M+m)
*Km+B*Kq+Kq*M+Dq*Deta)*K+Km*(B*Kq+Deta*Dq)) (((Deta+Dq)*Km+Deta*Kq)*K+Deta*Km*Kq)
(K*Km*Kq)],[(B*m) (B*Dq+Deta*m) (K*m+Dq*Deta+B*(Km+Kq)) (K*Dq+Deta*(Km+Kq)) (K*
(Km+Kq)) 0]);
end
if version == 3
    Kq = 200;
    Km = x(1);
    Dq = 0.7*2*sqrt(Kq*M);
    Z = tf([(B*Dq*M) (M*(B*Km+Deta*Dq)) ((B*Km+(B+M)*K+B*Kq)*Dq+M*Km*Deta) (Deta*
(K+Km+Kq)*Dq+((B+M)*K+B*Kq)*Km) (K*(Km+Kq)*Dq+Km*Deta*(K+Kq)) (K*Km*Kq)],[(B*Dq)
(B*Km+Deta*Dq) (Deta*Km+Dq*K) (K*Km) 0]);
end
if version == 4
    Kq = 200;
    Km = x(1);
    m  = x(2)*M;
    Dq = 0.7*2*sqrt(Kq*M);
    Z = tf([(B*M*m) (M*(B*Dq+Deta*m)) (B*(M+m)*Km+m*(B+M)*K+B*m*Kq+M*Dq*Deta) (((M+m)
*Deta+B*Dq)*Km+(m*Deta+Dq*(B+M))*K+Kq*(B*Dq+Deta*m)) (((B+M+m)*K+B*Kq+Dq*Deta)*Km+
(Deta*Dq+Kq*m)*K+Deta*Dq*Kq) (((Deta+Dq)*K+Deta*Kq)*Km+K*Dq*Kq) (K*Km*Kq)],[(B*m)
(B*Dq+Deta*m) (B*Km+Deta*Dq+K*m) (Deta*Km+Dq*K) (K*Km) 0]);
end

[mag_Z,~,~] = bode(Z,w);
 mag_Z      = squeeze(mag_Z);
% Vector of the impedance magnitude of the objective system
 logmag_Z   = 20*log10(mag_Z);
% Constraint psi_inequal has to be nagative
psi_inequal = max(max(logmag_Z-loghighmag),max(loglowmag-logmag_Z));
psi_equal=[];
end
```

```
function [wND] = ESPI_optim_phi_wND(x)
%% Objective function of the ESPI System
% The optimization parameters are the reduced virtual control parameters
% The output is an average value of the last quarter of the values of N

global version KN fN
global B M K Kv Deta Gd
s = tf('s');

% Objective model transfer functions
if version == 1
    Kq = Kv*((x(1)+1)/x(1));
    Km = Kv*(x(1)+1);
    Dq = 0.7*2*sqrt(Kq*M);
    L  = tf([(Deta*Dq*M) (B*Dq*Km+Deta*Km*M+Deta*Kq*M) (B*Km*Kq+Deta*Dq*K+Deta*Dq*Km)
(Deta*K*Km+Deta*K*Kq+Deta*Km*Kq+Dq*K*Km) (K*Km*Kq)],[(Dq*B*M) (B*Km*M+B*Kq*M)
(B*Dq*K+Dq*K*M) (B*K*Km+B*K*Kq+K*Km*M+K*Kq*M) 0 0]);
    Ky = tf([(Deta*Dq*M) (B*Dq*Km+Deta*Km*M+Deta*Kq*M) (B*Km*Kq+Deta*Dq*K+Deta*Dq*Km)
(Deta*K*Km+Deta*K*Kq+Deta*Km*Kq+Dq*K*Km) (K*Km*Kq)],[(Dq*K) (K*Km+K*Kq)]);
    Kd = tf([(Dq*Deta) (Deta*Km+Deta*Kq) 0],[(Deta*Dq*M) (B*Dq*Km+Deta*Km*M+Deta*Kq*M)
(B*Km*Kq+Deta*Dq*K+Deta*Dq*Km) (Deta*K*Km+Deta*K*Kq+Deta*Km*Kq+Dq*K*Km) (K*Km*Kq)]);
end
if version == 2
    Kq = Kv*((x(1)+1)/x(1));
    Km = Kv*(x(1)+1);
    m  = x(2)*M;
    Dq = 0.7*2*sqrt(Kq*M);
    L  = tf([(Deta*M*m) (B*Km*m+Deta*Dq*M)
(B*Dq*Km+Deta*K*m+Deta*Km*M+Deta*Km*m+Deta*Kq*M) (B*Km*Kq+Deta*Dq*K+Deta*Dq*Km+K*Km*m)
(Deta*K*Km+Deta*K*Kq+Deta*Km*Kq+Dq*K*Km) (K*Km*Kq)],[(B*M*m) (B*Dq*M)
(B*K*m+B*Km*M+B*Kq*M+K*M*m) (B*Dq*K+Dq*K*M) (B*K*Km+B*K*Kq+K*Km*M+K*Kq*M) 0 0]);
    Ky = tf([(Deta*M*m) (B*Km*m+Deta*Dq*M)
(B*Dq*Km+Deta*K*m+Deta*Km*M+Deta*Km*m+Deta*Kq*M) (B*Km*Kq+Deta*Dq*K+Deta*Dq*Km+K*Km*m)
(Deta*K*Km+Deta*K*Kq+Deta*Km*Kq+Dq*K*Km) (K*Km*Kq)],[(K*m) (Dq*K) (K*Km+K*Kq)]);
    Kd = tf([(Deta*m) (Deta*Dq) (Deta*Km+Deta*Kq) 0],[(Deta*M*m) (B*Km*m+Deta*Dq*M)
(B*Dq*Km+Deta*K*m+Deta*Km*M+Deta*Km*m+Deta*Kq*M) (B*Km*Kq+Deta*Dq*K+Deta*Dq*Km+K*Km*m)
(Deta*K*Km+Deta*K*Kq+Deta*Km*Kq+Dq*K*Km) (K*Km*Kq)]);
end
if version == 3
    Kq = 200;
    Km = x(1);
    Dq = 0.7*2*sqrt(Kq*M);
    L  = tf([(Deta*Dq*M) (B*Dq*Km+B*Dq*Kq+Deta*Km*M)
(B*Km*Kq+Deta*Dq*K+Deta*Dq*Km+Deta*Dq*Kq) (Deta*K*Km+Deta*Km*Kq+Dq*K*Km+Dq*K*Kq)
(K*Km*Kq)],[(B*Dq*M) (B*Km*M) (B*Dq*K+Dq*K*M) (B*K*Km+K*Km*M) 0 0]);
    Ky = tf([(Deta*Dq*M) (B*Dq*Km+B*Dq*Kq+Deta*Km*M)
(B*Km*Kq+Deta*Dq*K+Deta*Dq*Km+Deta*Dq*Kq) (Deta*K*Km+Deta*Km*Kq+Dq*K*Km+Dq*K*Kq)
(K*Km*Kq)],[Dq*K K*Km]);
    Kd = tf([(Deta*Dq) (Deta*Km) 0],[(Deta*Dq*M) (B*Dq*Km+B*Dq*Kq+Deta*Km*M)
(B*Km*Kq+Deta*Dq*K+Deta*Dq*Km+Deta*Dq*Kq) (Deta*K*Km+Deta*Km*Kq+Dq*K*Km+Dq*K*Kq)
(K*Km*Kq)]);
end
if version == 4
    Kq = 200;
    Km = x(1);
```

```
    m  = x(2)*M;
    Dq = 0.7*2*sqrt(Kq*M);
    L  = tf([(Deta*M*m) (B*Km*m+B*Kq*m+Deta*Dq*M) ↙
(B*Dq*Km+B*Dq*Kq+Deta*K*m+Deta*Km*M+Deta*Km*m+Deta*Kq*m) ↙
(B*Km*Kq+Deta*Dq*K+Deta*Dq*Km+Deta*Dq*Kq+K*Km*m+K*Kq*m) ↙
(Deta*K*Km+Deta*Km*Kq+Dq*K*Km+Dq*K*Kq) (K*Km*Kq)],[(B*M*m) (B*Dq*M) ↙
(B*K*m+B*Km*M+K*M*m) (B*Dq*K+Dq*K*M) (B*K*Km+K*Km*M) 0 0]);
    Ky = tf([(Deta*M*m) (B*Km*m+B*Kq*m+Deta*Dq*M) ↙
(B*Dq*Km+B*Dq*Kq+Deta*K*m+Deta*Km*M+Deta*Km*m+Deta*Kq*m) ↙
(B*Km*Kq+Deta*Dq*K+Deta*Dq*Km+Deta*Dq*Kq+K*Km*m+K*Kq*m) ↙
(Deta*K*Km+Deta*Km*Kq+Dq*K*Km+Dq*K*Kq) (K*Km*Kq)],[(K*m) (Dq*K) K*Km]);
    Kd = tf([(Deta*m) (Deta*Dq) (Deta*Km) 0],[(Deta*M*m) (B*Km*m+B*Kq*m+Deta*Dq*M) ↙
(B*Dq*Km+B*Dq*Kq+Deta*K*m+Deta*Km*M+Deta*Km*m+Deta*Kq*m) ↙
(B*Km*Kq+Deta*Dq*K+Deta*Dq*Km+Deta*Dq*Kq+K*Km*m+K*Kq*m) ↙
(Deta*K*Km+Deta*Km*Kq+Dq*K*Km+Dq*K*Kq) (K*Km*Kq)]);
end

S  = minreal(1/(1+L));
N  = minreal(-Ky*S);
D  = minreal(Ky*S*(Kd-Gd));
wN = KN/((s+fN)^3);

wND = hinfnorm([minreal(wN*N) D]);

end
```