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FlowFit3: Fast Data Assimilation for Recovering Instantaneous Details of Incompressible Flows based on scattered data

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Abstract

This works presents an evolution of the FlowFit method for reconstructing incompressible flow fields (specifically velocity and pressure) based on scattered velocity and acceleration data. It can thus be used to process the data obtained from Lagrangian Particle Tracking (LPT) in order to recover structures of fluid motion. The goal of this development was to improve both accuracy and speed over its predecessor so this method can be used in experiments with large measurement domains and/or a high number of seeding particles. The method will be explained and assessed in comparison with its predecessor using synthetic data derived from fluid simulations.

FlowFit2 (Gesemann 2016) and FlowFit3 recover the flow fields by recasting the problem as a cost function minimization problem where the solution to this optimization problem represents the velocity and pressure fields that are consistent with the scattered data. But whereas in FlowFit2 we dealt with physical constraints such as mass conservation, momentum conservation and no-slip conditions only by penalizing constraint violations as part of the cost function (penalty method), FlowFit3 uses a mix of techniques to satisfy most of these constraints exactly (divergence & pressure Poisson equation) and greatly reduce other constraint violations (no-slip conditions). In addition, FlowFit3 uses a different set of base functions for a continuous representation of the flow field based on a staggered grid and mixed-order B-splines. This allows the divergence constraint to be satisfied exactly over the whole domain instead of just over a finite set of grid points. To satisfy all the constraints we employ all of the following techniques depending on the constraints:

(1) Substitution: Pressure is derived from velocity so that the Poisson equations for pressure are always satisfied automatically.
(2) Orthogonal Gradient Projection: The cost function's gradient is projected into a divergence-free linear subspace. Therefore, a gradient-based solver for the optimization problem will not be tempted to leave this subspace and the divergence will always stay zero everywhere.
(3) Augmented Lagrangian Method: This is a simplification of the method of Lagrange multipliers for constraint optimization where the Lagrange multipliers are not variables that are optimized but updated after each optimization pass (Hestenes, 1969). We use this to deal with conditions related to immersed boundaries (e.g. no-slip on a wall).

Techniques (1) and (2) both involve solving 3D Poisson problems on a cuboid domain with the same size and same homogeneous Dirichlet boundary condition. They only differ in the filter kernel for the Laplace operator. These types of problems can be very efficiently solved because the Eigen decomposition of the respective equation systems can be efficiently computed using 3D discrete sine transforms (DST), for example, using the popular FFTW library. See figure 1 for a graphical visualization of the cost function.

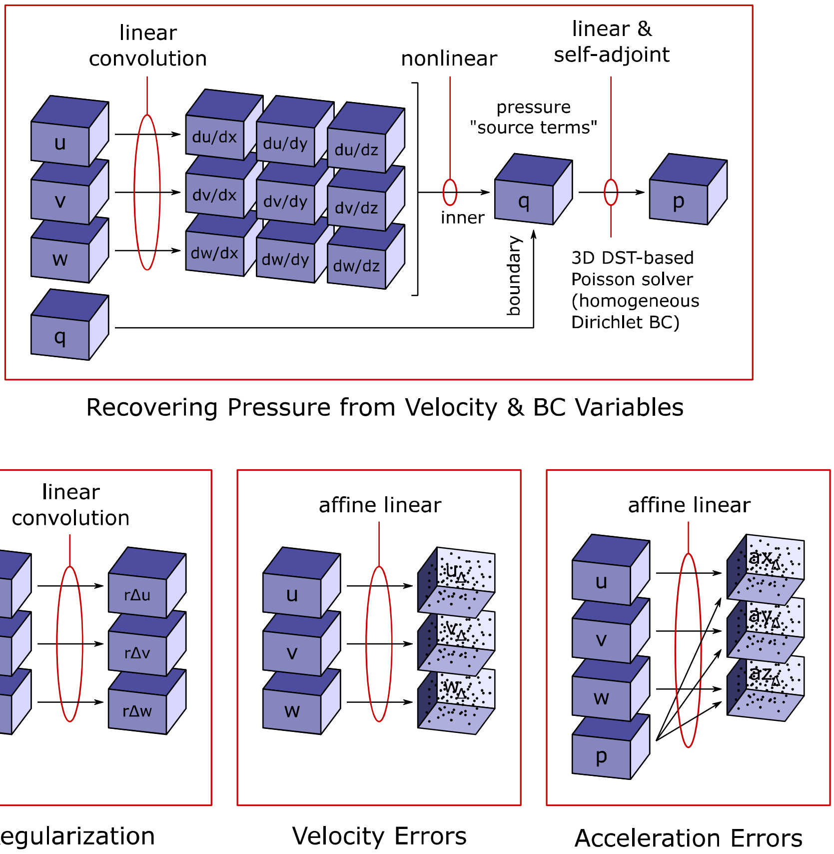

Fig. 1 Graphical representation of the cost function.

References

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