GNSS Acquisition Performance of Short Spreading Codes

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Outline

- What are short spreading codes? Why are they interesting for acquisition?
- Part 1: statistical acquisition performance models for short codes
- Part 2: signal design – selecting a code length

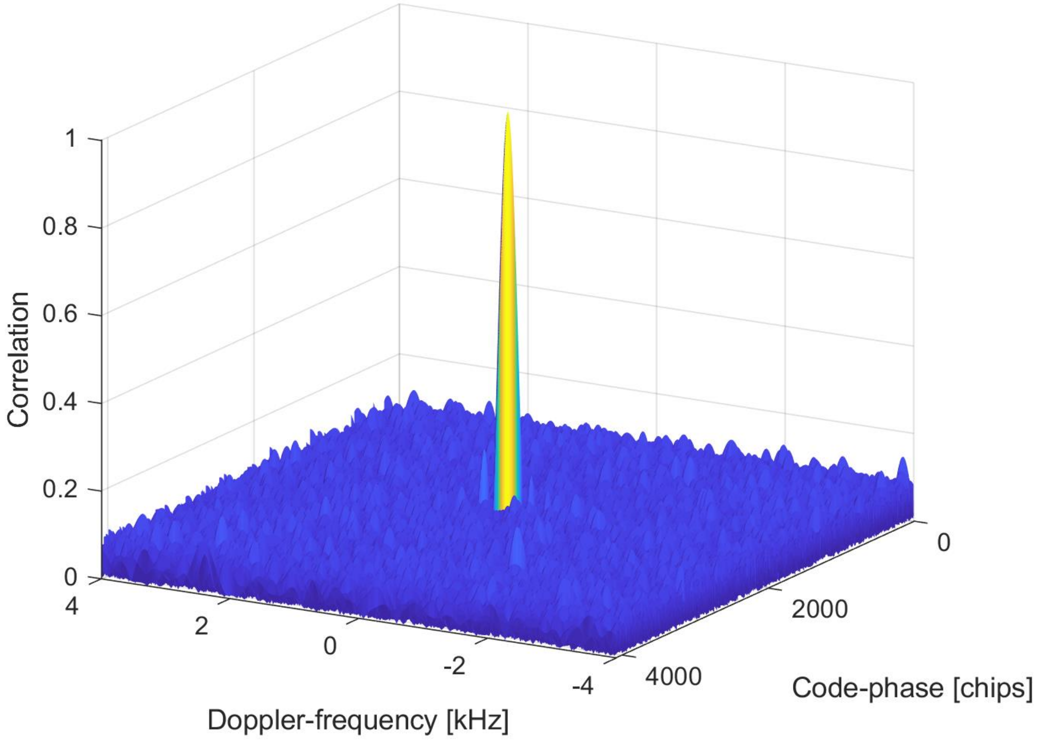
Signal Acquisition is a resource-hungry process

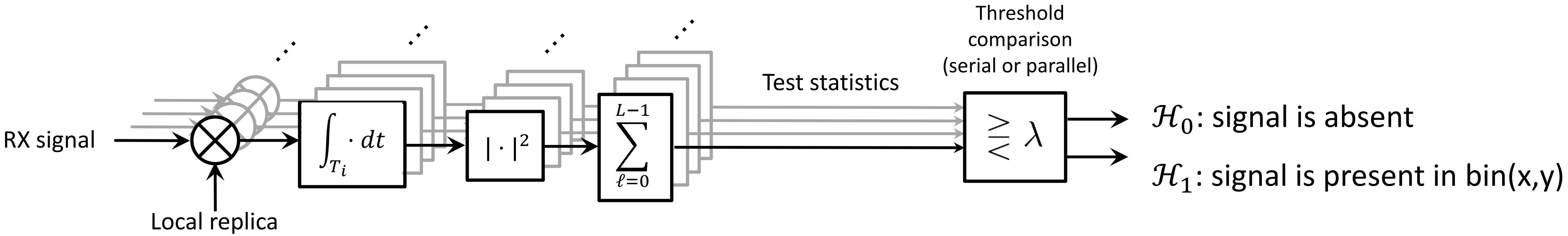
- 2-D search grid of code-phase/Doppler-freq.

- Extend spreading code (=PRN code) length ➔ more code bins
- Extend coherent integration time ➔ more Doppler bins

- Generation of test statistics costs memory/energy/time

- Statistical detection problem with possible errors:
  - False alarm (satellite is actually not in-view)
  - Missed detection (satellite is not detected in the correct bin)



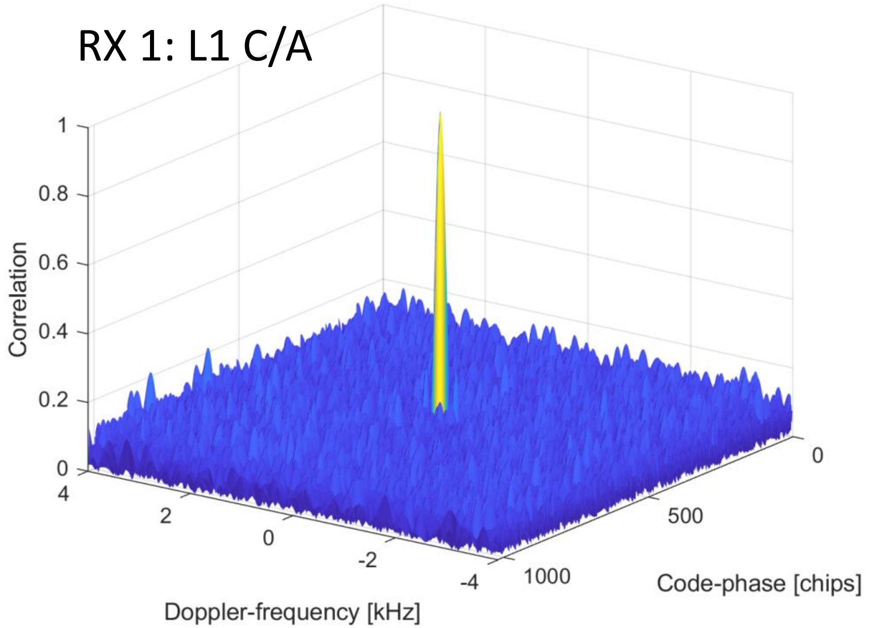


Some examples

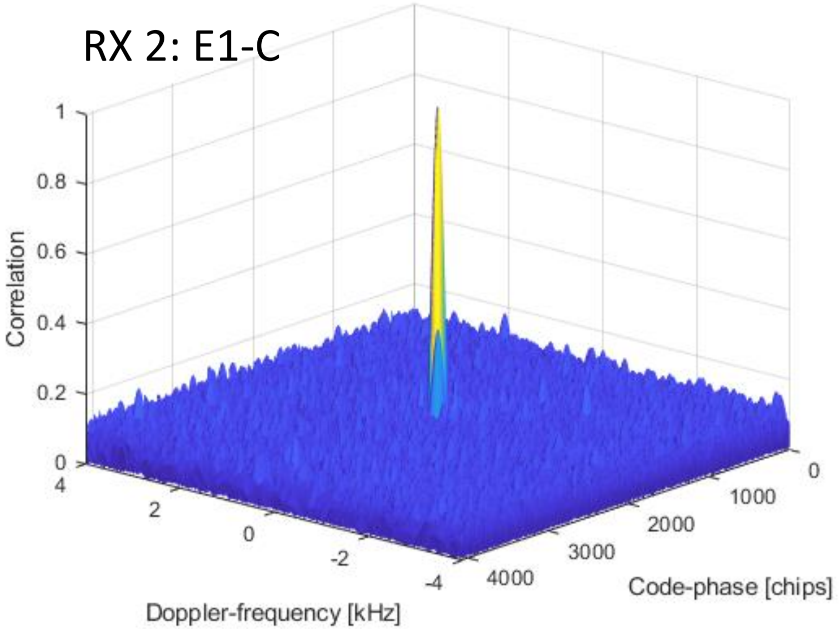
*) assuming 40 correlations per ms **) non-assisted, -158.5 dBW / 7x -153 dBW IF

Signal	Coherent integration	Doppler bins	PRN code length (chips)	Code bins per chip	Overall bins	Required time *	Data (or overlay) bit rate	Reliability $P_{DET}(P_{FA})$ **
GPS L1 C/A	4 ms	**32** (8 kHz x 4ms)	**1023**	**x1** BPSK(1)	**= 32736**	0.82 s	50 Hz	82% (5%)
Galileo E1-C	4 ms	**32**	**4092**	**x3** BOC(1,1)	**= 392832**	9.82 s	250 Hz	66% (5%)

RX 1: L1 C/A



RX 2: E1-C



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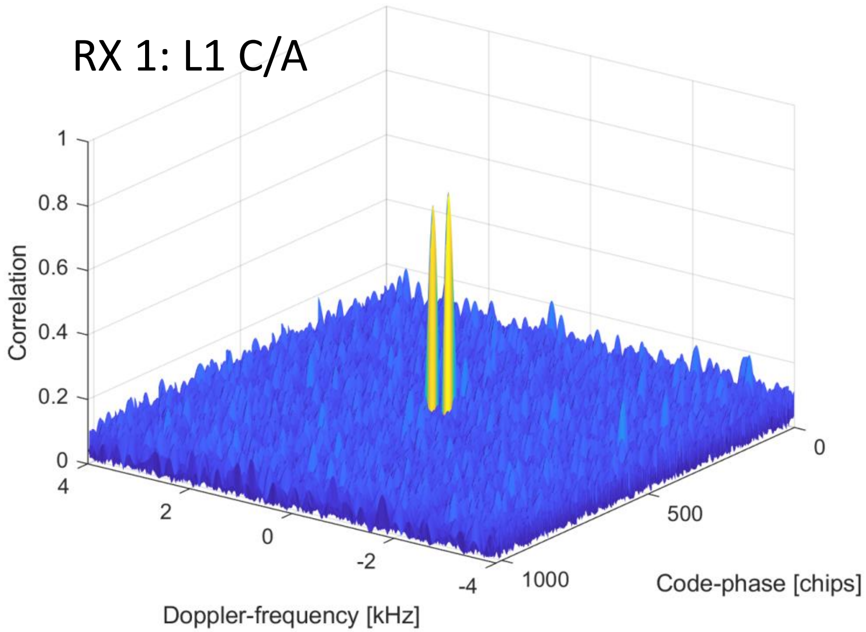
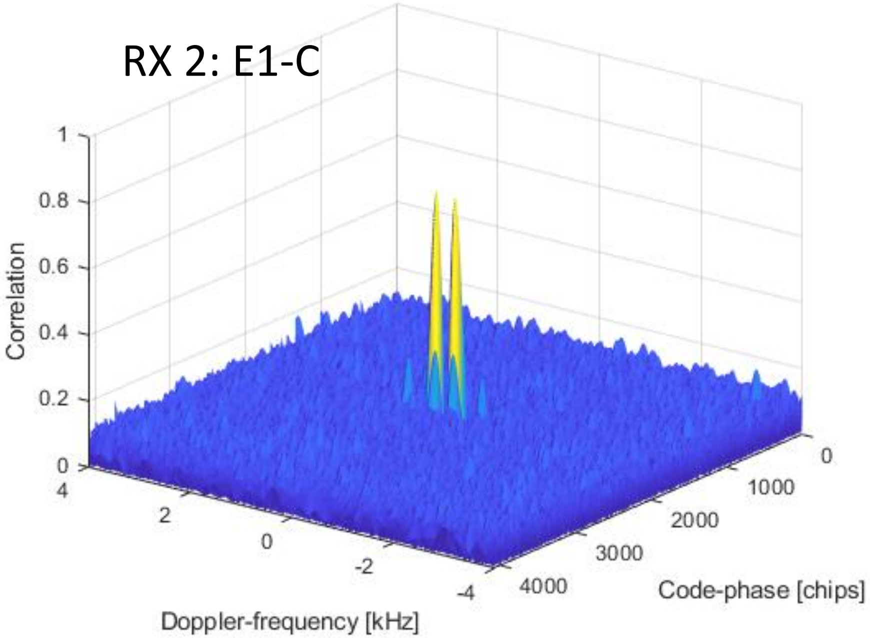
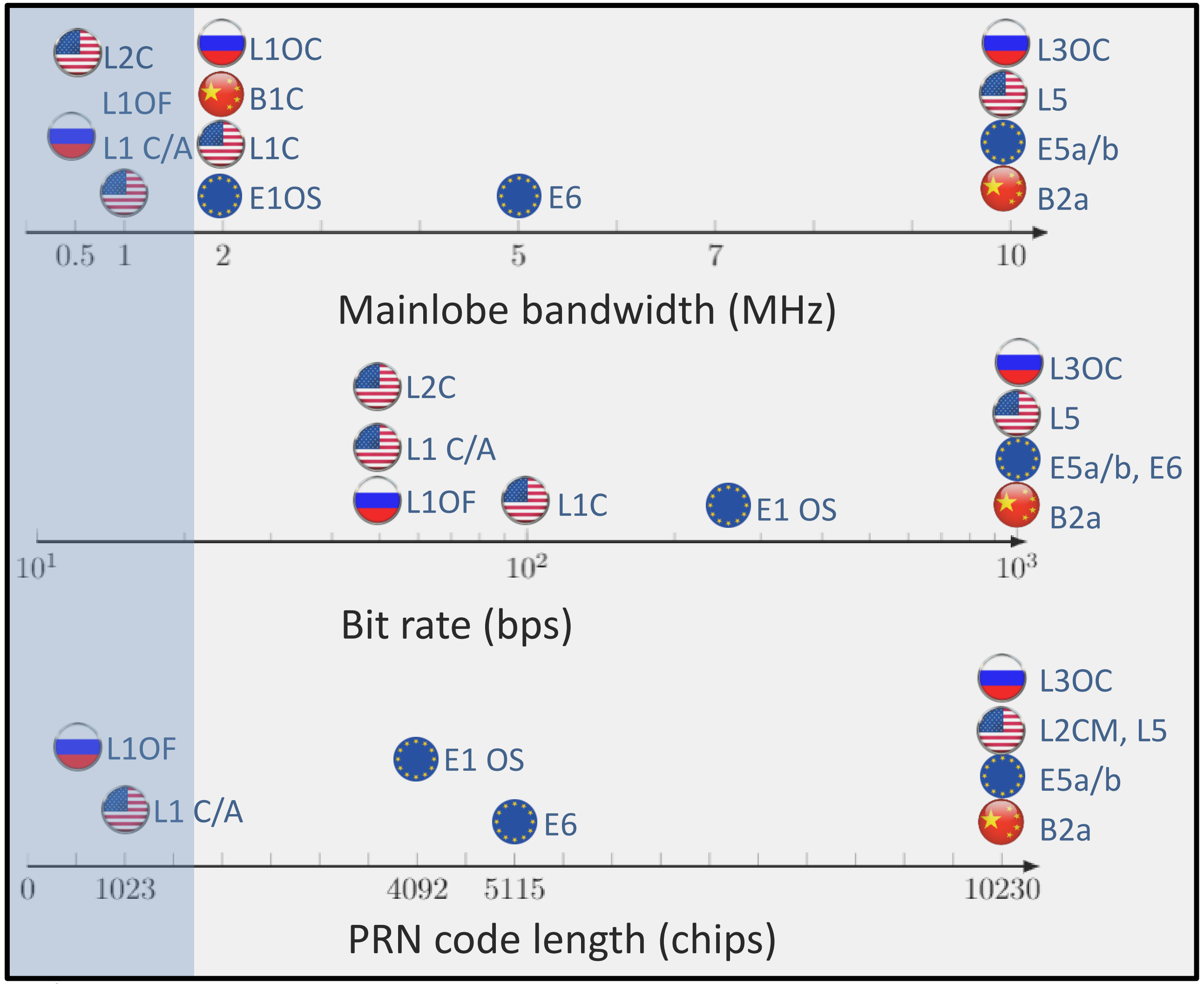




Bit transition in the middle of coherent integration interval

Civil GNSS signals ten years ago … and today!



- Trend in signal design 2000-2010:
  "Race for accuracy"
  - high bandwidth
  - high bit rate (or overlay code, symbols,…)
  - long PRN codes

- Trend in signal design 2015-ongoing:
  "Fast fix/low cost"
  - Time/energy per fix
  - Snapshot receivers, IoT devices, SpaceNav

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A possible C/A Signal for Galileo: "E1-D"

*) assuming 40 correlations per ms **) non-assisted, -158.5 dBW / 7x -153 dBW IF

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Galileo E1-D	4 ms	**32**	**341** or less	x1 BPSK(1)	**= 10912** or less	0.27 s or less	50 Hz or less	???

- Code length of 341 would reduce the acquisition complexity by a factor of 3
- Is such an acquisition signal still reliable?

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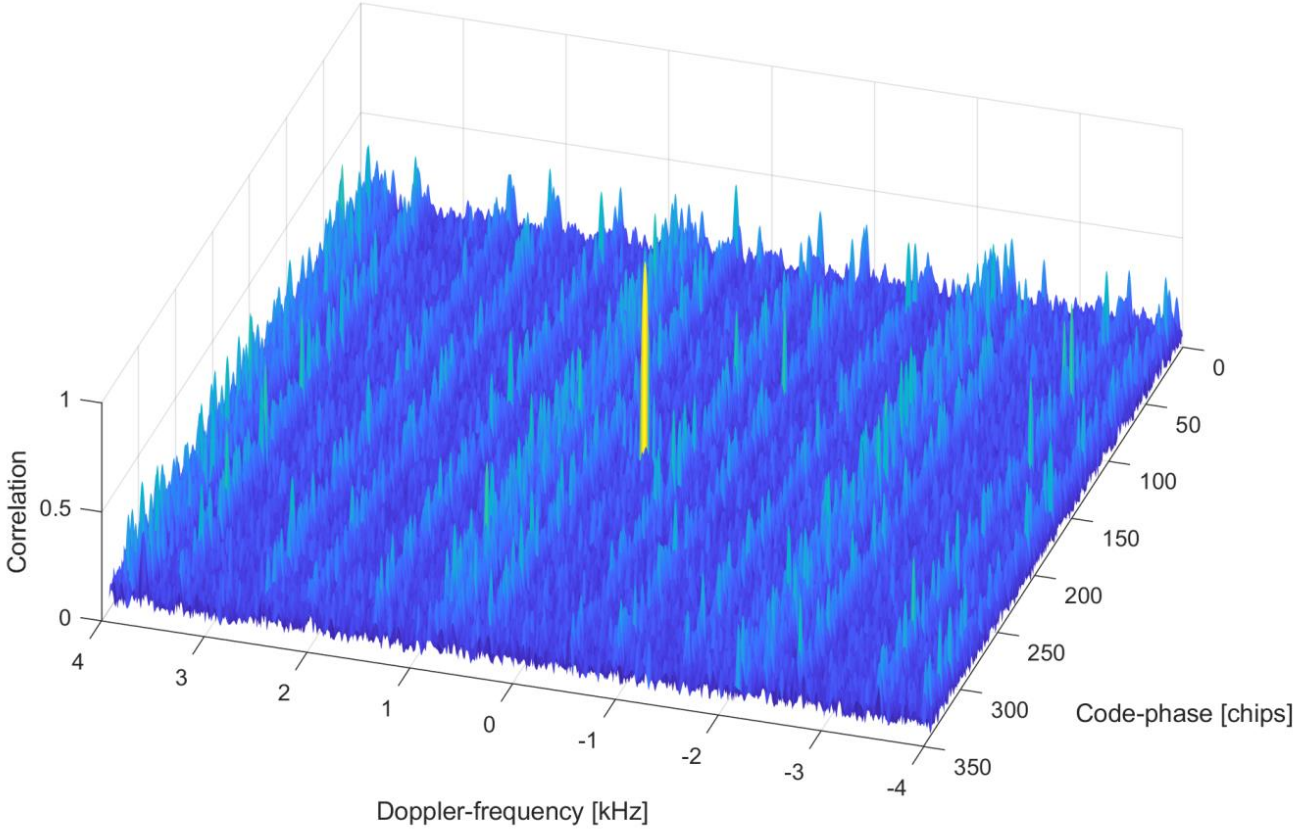
A possible C/A Signal for Galileo: "E1-D"



- 8 in-view satellites transmitting E1-D (as in Slide 7)
  - $k = 1$ to be acquired (-158.5 dBW)
  - $k = 2, .., 8$ interferers (-153 dBW)
- Interference affects some Doppler bins more than others
- Effect becomes more pronounced for
  - near-far scenarios
  - shorter codes
  - lower databit rate
- This effect is known from L1 C/A, but less pronounced

State of the art: fine SSC

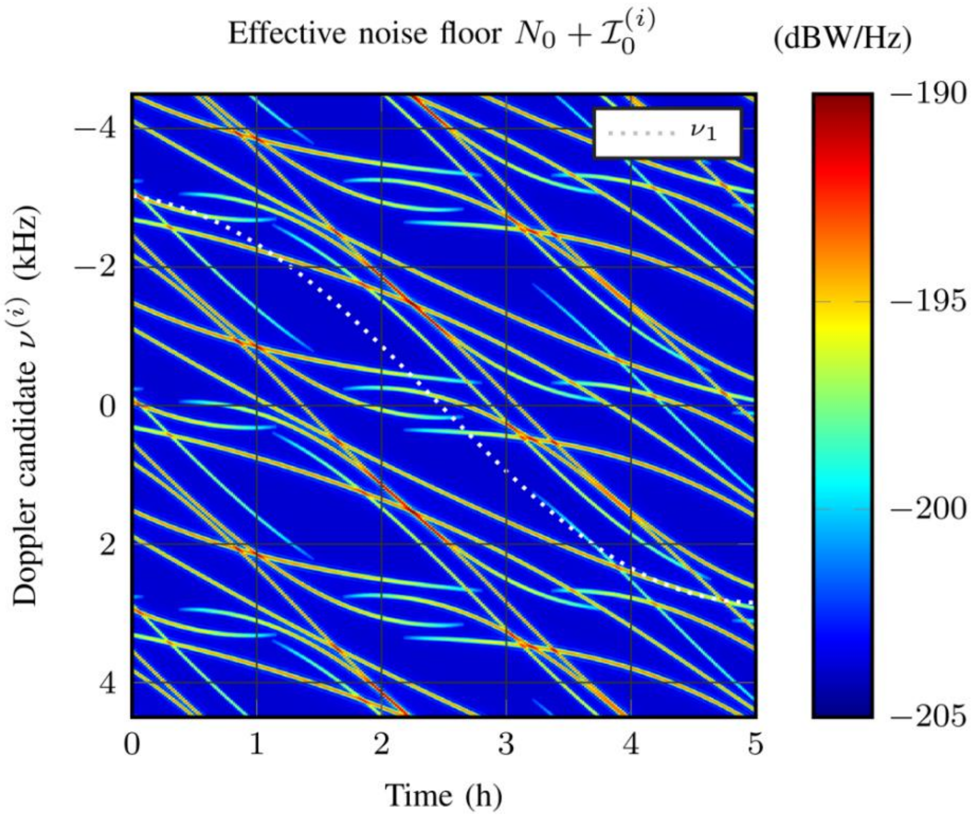


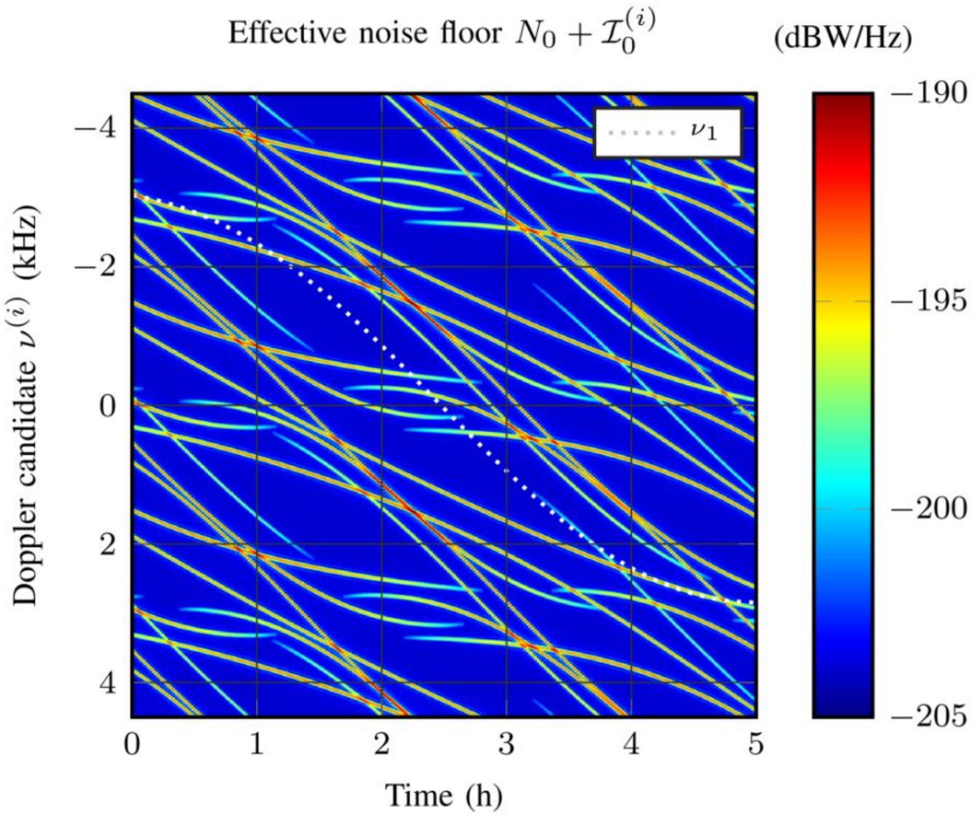
Figure: effective noise floor vs. Doppler bin vs. time for a Walker (24/3/1) constellation transmitting E1-D (as in Slide 7)

- Spectral separation coefficient (SSC): $\beta_{1,k}^{(i)} = \int \phi_1^{(i)}(f)\phi_k(f)df$
- The interference floor $I_0^{(i)}$ is a weighted sum of SSCs

$$I_0^{(i)} = \sum_{k=2}^{K} P_k \, \beta_{1,k}^{(i)} \,, \qquad P_k\text{: power of sat } k$$

- Interference can be modeled as Gaussian noise, using an *effective noise floor* $N_0 + I_0^{(i)}$
- Two SSC-versions
  1. Coarse SSC (low-res. spectrum features: order of MHz) = const.
  2. Fine SSC (high-res. spectrum features: order of sub-kHz)
- The results on the left are based on the fine SSC [Heg2019], [Dri2012]
  ➔ SSCs vary from bin to bin!

State of the art: fine SSC (cont'd)



- Fine SSC is large if the relative Doppler $\nu_k - \nu^{(i)}$ between the interferer k and bin i is a multiple of the PRN repetition rate 3 kHz (L1 C/A: 1 kHz)
- Sometimes, several such "Doppler crossings" occur in one bin at the same time (effective noise floor goes up by 15 dB)
- Straightforward (exact) solution: calculate bin probabilities $p_{\mathrm{fa}}^{(i)}, p_{\mathrm{det}}^{(i)}$ in Gaussian noise for
  - each fine SSC between every bin i and every interferer k
  - each possible constellation
  - each possible detection threshold

  then calculate global probabilities, e.g. $P_{\mathrm{FA}} = 1 - \prod_{i=1}^{N_{bins}} \left(1 - p_{\mathrm{fa}}^{(i)}\right)$.

  This is too complex for the evaluation of one signal design candidate!

Simplified model: random SSC

- Given the instantaneous fine SSCs between all signals and bins, the bin probability of false alarm would be

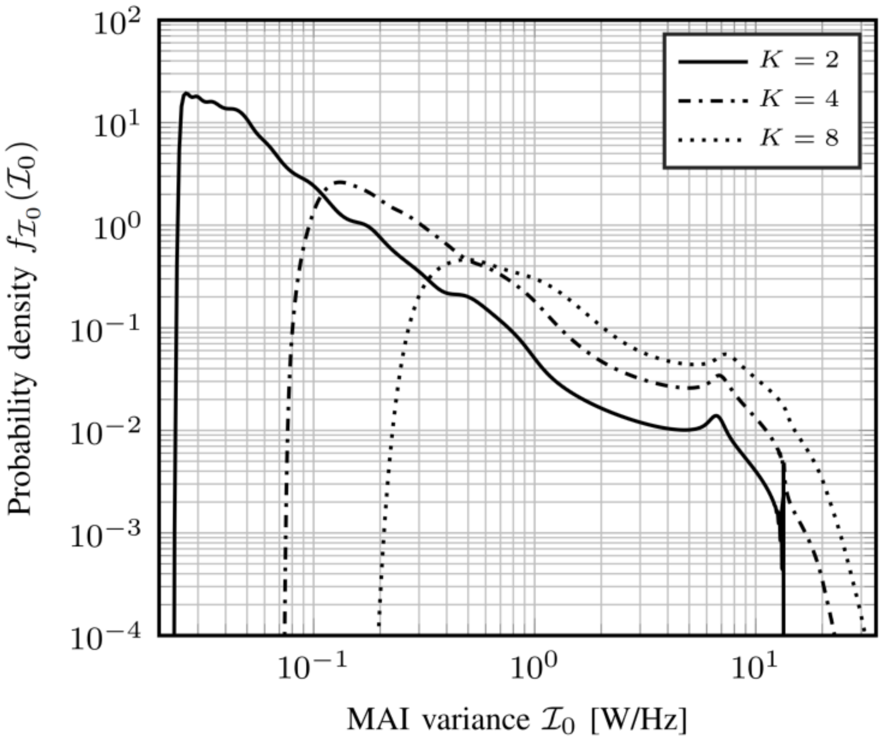
$$p_{\mathrm{fa}}^{(i)} = e^{-\frac{\lambda}{N_0 + I_0^{(i)}}}$$

λ: detection threshold

- Idea: do NOT calculate $I_0^{(i)}$ for each bin, but treat it as random variable i.i.d. for all bins with distribution $f_{\mathcal{I}_0}(\mathcal{I}_0)$

- Calculate the *compound* bin probability of false alarm, for random $\mathcal{I}_0$

$$p_{\mathrm{fa}} = e^{-\frac{\lambda}{N_0 + \mathcal{I}_0}}$$



K: number of in-view satellites

How to obtain this PDF: see model usage slides

Compound bin probabilities

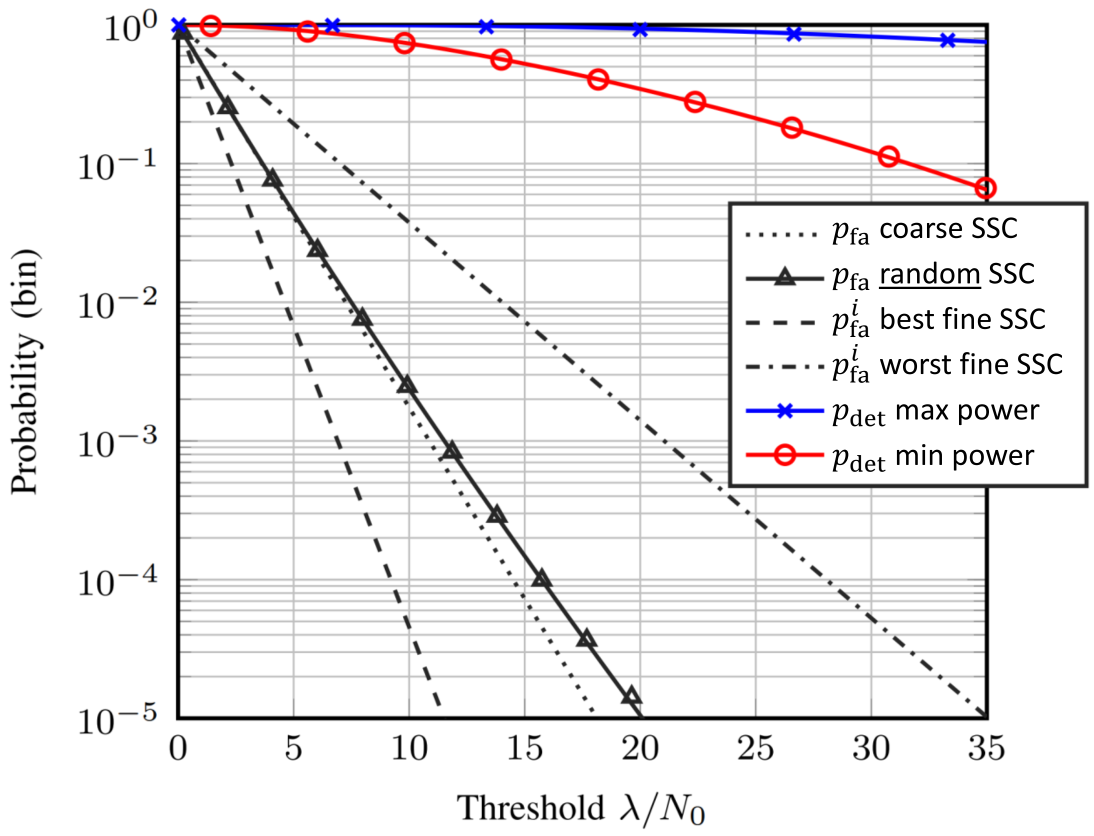
- The compound bin probability of false alarm
  - is independent of the bin index i
  - is representative for all search bins, but not for any particular search bin
  - is a mixture-Gaussian model (not a line on semilog axis!)

- The global probability of false alarm simplifies to

$$P_{\mathrm{FA}} = 1 - \prod_{i=1}^{N_{bins}} \left(1 - p_{\mathrm{fa}}^{(i)}\right) \approx 1 - (1 - p_{\mathrm{fa}})^{N_{bins}}$$

➔This facilitates acquisition signal design considerably!

- The bin probability of detection is hardly affected by interference ➔ use an accurate model, e.g. [Dri2007]



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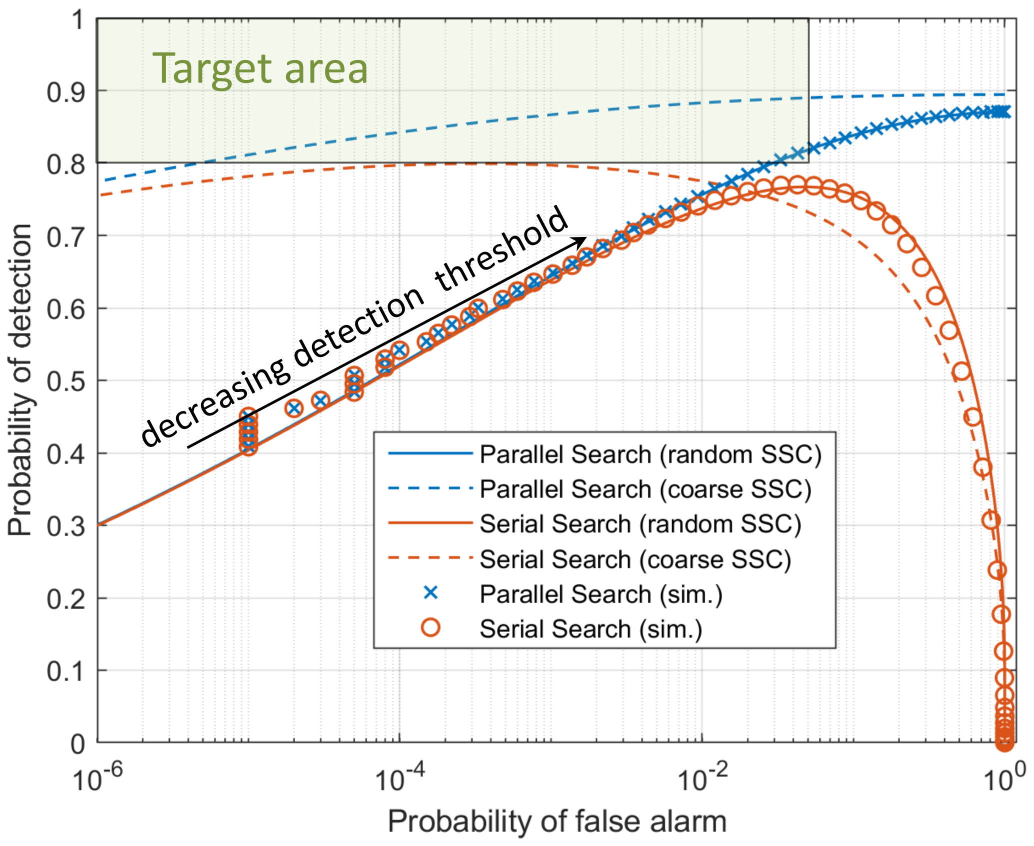
Receiver operating characteristic (ROC) curve



Code length	341
Bit rate	0 Hz (pure pilot)
Modulation	BPSK(1)
Coh. Integration	4 ms
Search bins	$32 \times 341 = 10\,912$
Signal of interest	-158.5 dBW (minimum)
Interferers	7×-153.0 dBW (maximum)
Noise floor	-204.0 dBW/Hz
Doppler spread	-4 kHz … 4 kHz
Target P_{DET}	$> 80\%$
Target P_{FA}	$< 5\%$

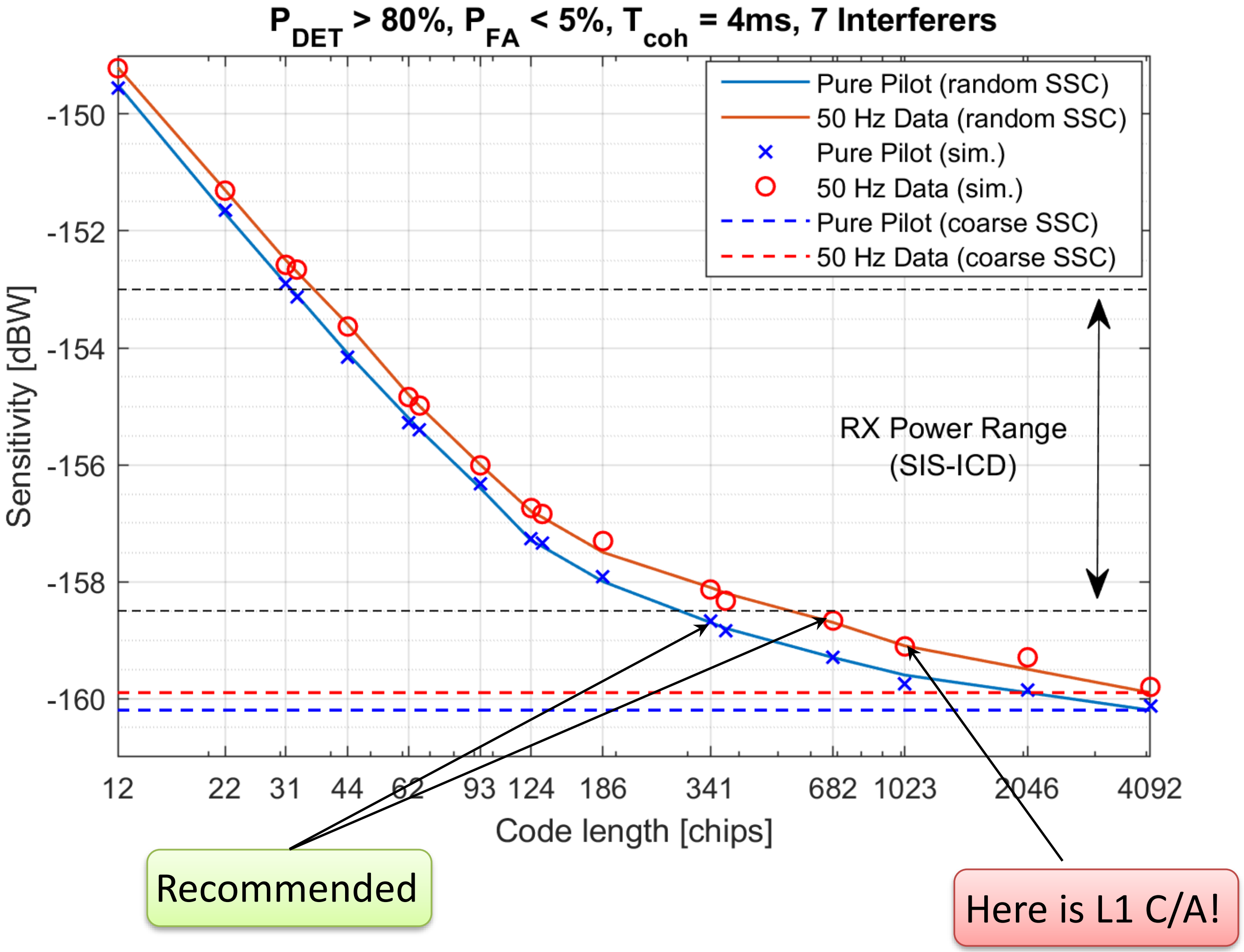
Sensitivity vs. code length

Given a scenario…
- tentative code length
- coherent integration time
- number and power of interferers $k = 2, \ldots, K$

and target global probability …
- of detection
- of false alarm,

what is the required received power for the satellite signal to be acquired, $k = 1$?

Integration time	4 ms
In-view satellites	8
Power per interferer	-153 dBW (max)
Probability of detection	> 80%
Probability of false alarm	< 5%



Conclusion

- New C/A-signals with codes shorter than 1023 (e.g. 341) chips are an option for low-cost acquisition, especially for Galileo
- Self-interference needs to be assessed
- New model (random SSC & compound bin probabilities) has been developed for accurate global probability of false alarm ⇔ state of the art:
  - Coarse SSC: very inaccurate for C/A-signals
  - Fine SSC: more accurate, but too complex for acquisition signal design
- 50 Hz bit sequence leads to acceptable sensitivity loss (0.3-0.5 dB as compared with pure pilot)
- Final design options:

Signal	Coh. Int.	Doppler bins	Code bins	Overall bins	Required time	Bit rate	P_{DET} (P_{FA})
L1 C/A	4 ms	**32**	**1023**	**= 32736**	0.82 s	50 Hz	82% (5%)
E1-D Pure Pilot	4 ms	**32**	**341**	**= 10912**	0.27 s	0 Hz	82% (5%)
E1-D Quasi Pilot	4 ms	**32**	**682**	**= 21824**	0.54 s	50 Hz	81% (5%)

References

[Heg2020] C. Hegarty, “A simple model for GPS C/A-code self-interference”, *ION Navigation*, Jan. 2020.

[Dri2012] C. O’Driscoll, J. Fortuny-Guasch, “On the determination of C/A code self-interference with application to RFC analysis and pseudolite systems”, *Proc. Int. Tech. Meeting Inst. Nav. ION/GNSS,* Nashville, TN, Sep. 2012.

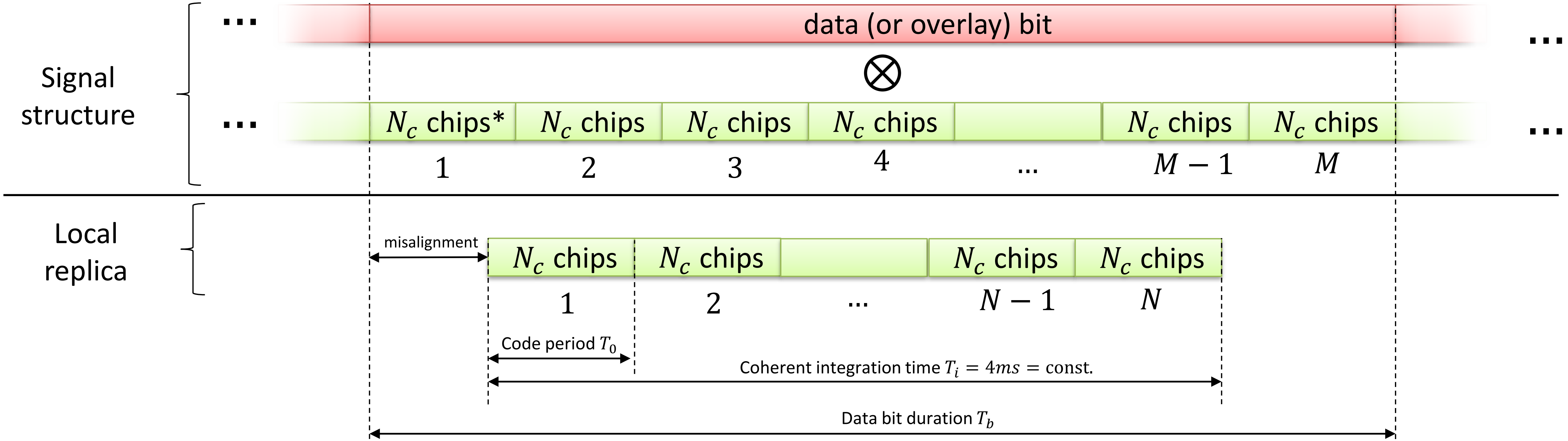
[Dri2007] C. O’Driscoll, “Performance analysis of the parallel acquisition of weak GPS signals”, PhD Thesis, National University of Ireland, Cork, 2007.

[Enn2018] C. Enneking, F. Antreich, André L. F. de Almeida, “Gaussian Approximations for Intra- and Intersystem Interference in RNSS”, *IEEE Comm. Letters*, Jul. 2018.

[Enn2019] –, “Pure Pilot Signals for GNSS: How Short Can We Choose Spreading Codes?”, *ION ITM 2019*, Reston, Virginia, Jan. 2019.

Thank you for your attention!

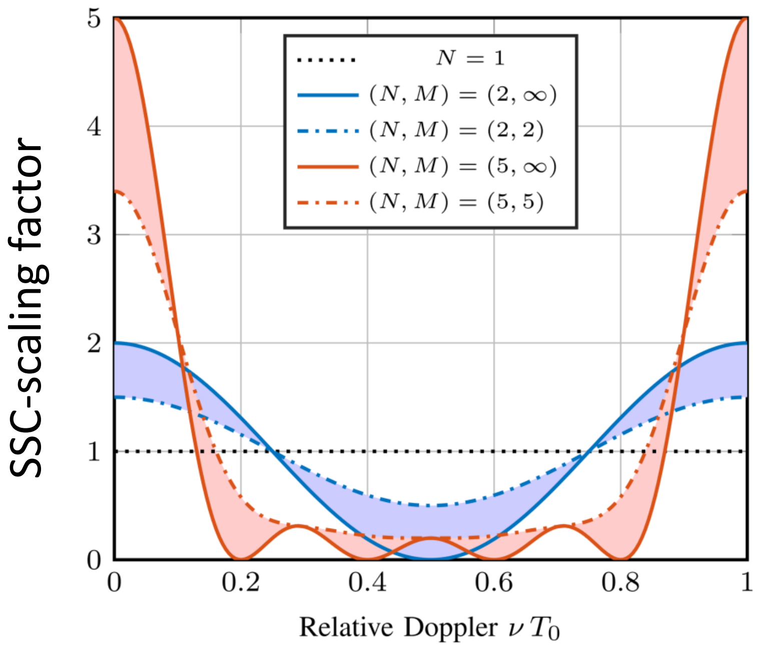
Model usage – Step 1: Identify TX and RX parameters



	Signal	Galileo E1-C	GPS L1 C/A	Galileo E1-D
TX parameters	Code length: N_c	4092	1023	341
	PRNs per bit: M	1	20	60
RX parameter	PRNs per integration: N	1	4	12

*) fixed chip rate: 1.023 MHz

Model usage – Step 2: Calculate probability density function of fine SSC



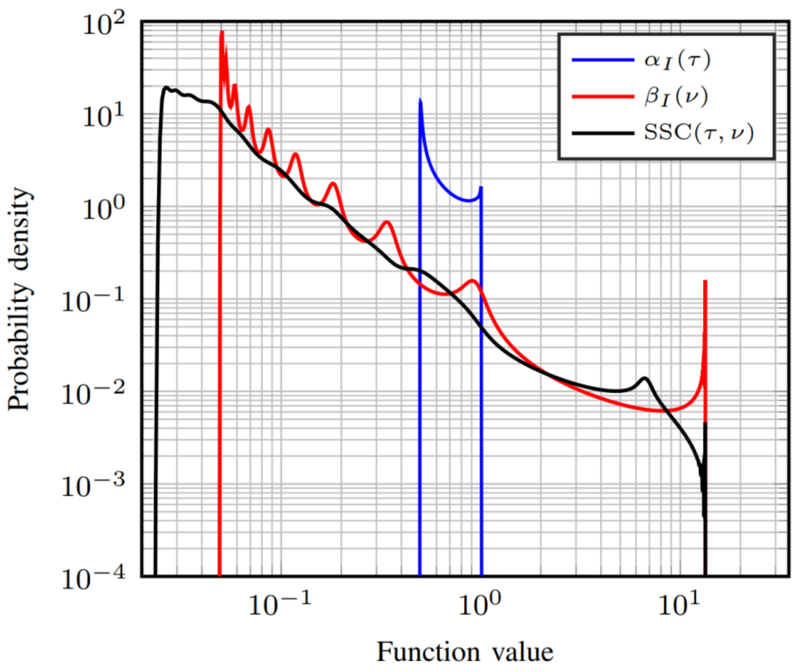
- Use fine SSC-models for Doppler ν [Heg2020] and (optionally) delay τ [Enn2018]
- It is sufficient to consider the intervals

$$\nu \in \left[0, \frac{1}{T_0}\right], \tau \in [0, T_c]$$

- Bin the resulting fine SSCs to obtain the PDF of the fine SSC

Figure top: Fine SSC vs. Doppler for TX parameter M and RX parameter N

Figure right: PDF of (dimensionless) fine SSC for uniform delay/Doppler



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Model usage – Step 3: Convolutions

- Weight with the received powers P_k, and perform $K-2$ convolutions (for $K-1$ interferers)

$$f_{I_0}(I_0) * \cdots * f_{I_0}(I_0)$$

- Now, the PDF of the interference floor is obtained

- Good alternative: sample the PDF directly from constellation simulations, using [Heg2020]

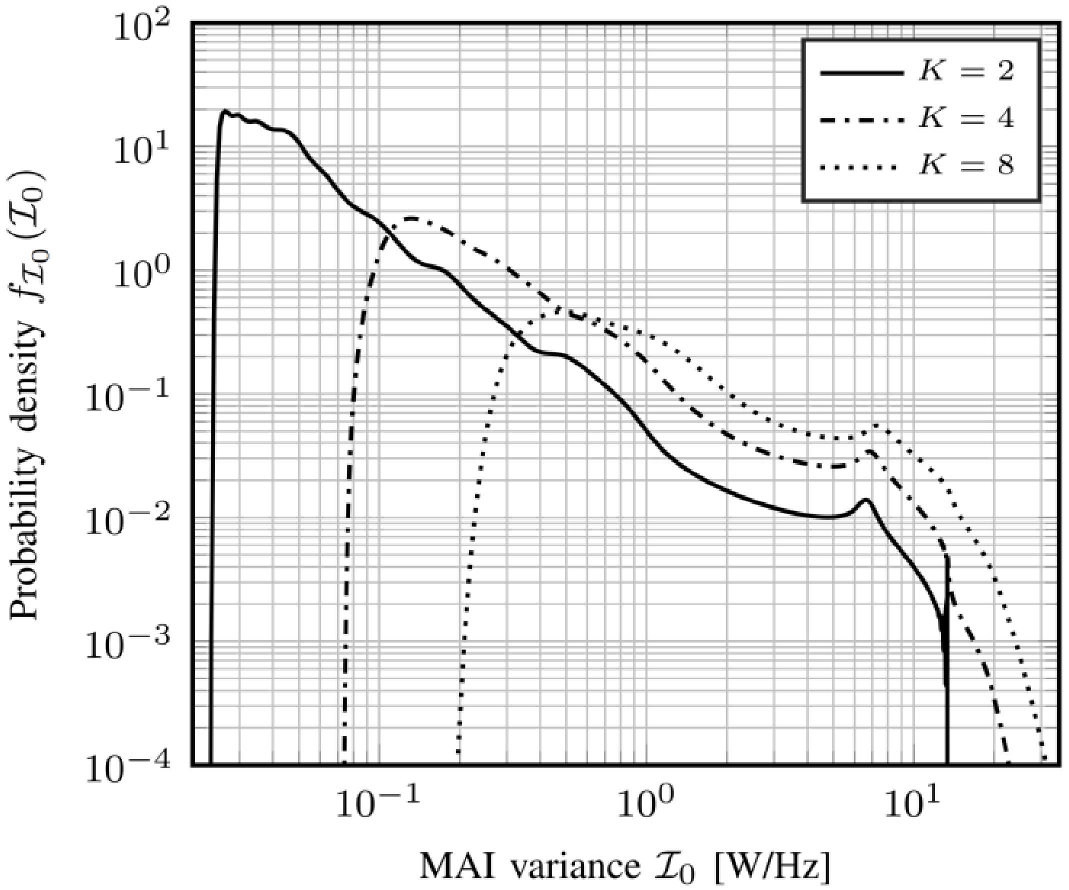

Figure: PDF of interference floor for $K-1$ interferers with unit power

Model usage

For more details on this model, stay tuned for our forthcoming journal paper:

C. Enneking, F. Antreich, André L. F. de Almeida
“Receiver Operating Characteristic of GNSS Coarse/Acquisition Signals With Short Codes”, approx. end of 2020.

Acknowledgment

This work has been carried out within the framework of the project “R&D for maritime safety and security and corresponding real time services” led by the Program Coordination Defence and Security Research within the German Aerospace Center (DLR).