A wall-law for the mean velocity in the inner part of turbulent boundary layers at adverse pressure gradient and application for RANS turbulence modelling

Tobias Knopp *[1], Nico Reuther[3], Matteo Novara[1], Erich Schülein[1], Daniel Schanz[1], Christian Willert[2], Andreas Schröder[1], Andreas Krumbein[1], Cornelia Grabe[1], and Christian J. Kähler[3]

[1] *Institute of Aerodynamics and Flow Technology, DLR, Göttingen, Germany*

[2] *Institute of Propulsion Technology, DLR, Köln, Germany*

[3] *Institute for Fluid Mechanics and Aerodynamics, Universität der Bundeswehr München, Germany*

Summary We present a strategy to modify turbulence models based on the Reynolds averaged Navier-Stokes (RANS) equations for flow separation on smooth surfaces due to an adverse pressure gradient. We describe two new turbulent boundary layer experiments to study the behaviour of the mean velocity profile at adverse pressure gradients and at high Re. We observe that the log-law in the mean velocity profile is a robust feature, but its region is much thinner than in zero-pressure gradient flows, and its slope is altered. A square-root law emerges above the log-law extending to the wall-ward distance the log-law typically occupies. Then we modify a differential Reynolds stress turbulence model. The modified model accounts for a change of the log-law slope and the formation of a square-root law at adverse pressure gradient, improving the agreement with the experimental data for both experiments.

GOALS, EXPERIMENTAL SETUP, AND MEASUREMENT TECHNIQUE

Turbulent boundary layers subjected to an adverse pressure gradient at high Reynolds numbers still pose many open questions regarding their statistically averaged mean flow, see [1]-[5]. At the same time, they have a high relevance in technical applications, e.g., in aerospace research. At high Re, turbulence resolving simulation methods like large-eddy-simulation (LES) and hybrid RANS/LES methods still need a near-wall model based on the RANS equations and statistical turbulence models for the foreseeable future. Therefore the prediction of flow separation on smooth surfaces due to an adverse pressure gradient is still not reliable. We pursue a strategy (i) to set-up a high-quality data base by new experiments, (ii) to find an empirical wall-law for the mean velocity in the inner layer, and (iii) to improve the turbulence models based on the empirical wall-law.

The first goal is to find a new wall-law for the mean velocity based on the following questions: Q1: Does a log-law region still exist at APG [1]-[5]? Q2: Does the slope of the log-law change with the streamwise pressure gradient in inner scaling $\Delta p_x^+ = (\partial P/\partial x)^+$, as proposed by [1]? Q3: Is there a sqrt-law region above the log-law, see e.g. [2], [3]? For this purpose we designed two new wind-tunnel experiments. The second goal is to use this knowledge of a wall law for the mean velocity to improve RANS turbulence models.

In the wind-tunnel experiments, the turbulent boundary layer flow develops on the side wall of the wind-tunnel and then follows a ramp and a long flat plate at almost zero pressure gradient. Then the flow enters into the adverse pressure gradient section, first along a curved surface and then on a flat plate, which is the focus region. The flat plate segment has an opening angle of 14.4° and 18.6° resp. relatively to the zero-pressure gradient section. The flow remains attached in the first experiment and separates in the second.

The experiments were performed at different Re by variation of the velocity up to $U_\infty = 36\,\mathrm{m/s}$. The focus region is at $8\,\mathrm{m} \le x \le 10.5\,\mathrm{m}$, where the test-section starts at $x = 0\,\mathrm{m}$. We obtain a significant adverse pressure gradient with $\Delta p_x^+ > 0.02$ and Rotta-Clauser pressure gradient parameter $\beta_{\mathrm{RC}} > 30$ for the first experiment and even larger values for the second, and Re_θ up to 50000 in the adverse pressure gradient region.

The measurement technique uses 2D2C large-scale particle image velocimetry (PIV) for characterising the streamwise evolution of the flow over a streamwise distance of 2.1 m or 15 boundary layer thicknesses. In the adverse pressure gradient focus region, we use 2D2C microscopic (2D μPTV) and 3D3C Lagrangian particle tracking (3D LPT) to measure the mean velocity and the Reynolds stresses down to the viscous sublayer. Oil-film interferometry is used for the complementary direct measurement of the wall shear stress.

RESULTS FOR THE MEAN VELOCITY AND DISCUSSION

For the mean velocity profile in the adverse pressure gradient region, we observe that the log-law in the mean velocity profile $u^+ = \log(y^+)/K_i + B_i$ is a robust feature, but its region is much thinner than in zero-pressure gradient flows, and its slope is altered. The value of the log-law slope coefficient K_{i} is found to be decreasing with increasing values of the pressure gradient parameter Δp_x^+. We find a value of $K_{\mathrm{i}} = 0.370 \pm 0.017$ for $\Delta p_x^+ = 0.018$, lower than for the von Kármán constant for zero pressure gradient boundary layers $\kappa = 0.4 \pm 0.02$, see Fig. 1 (a). A square-root law emerges above the log-law extending to the wall-ward distance the log-law typically occupies, see Fig. 1 (b), and the slope diagnostic function for the square-root law shows a plateau.

*Corresponding author. E-mail: Tobias.Knopp@dlr.de.

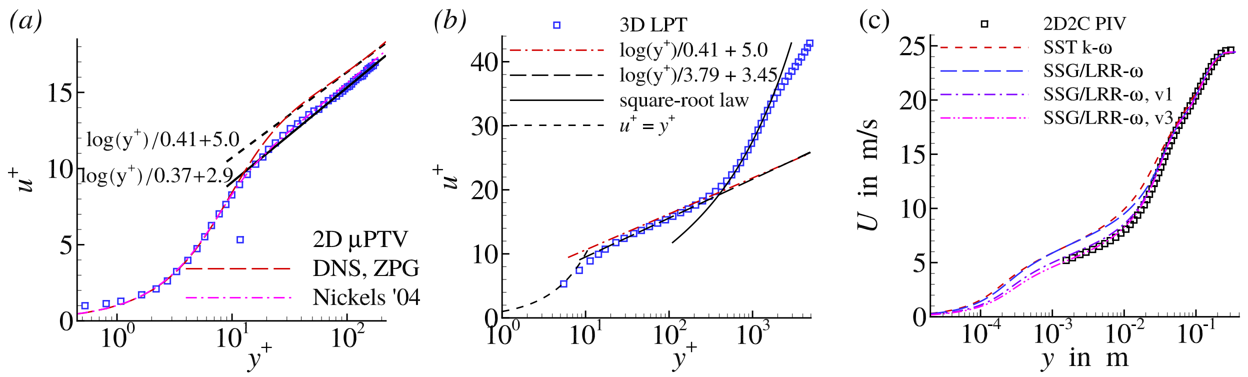

Figure 1: (a) Near wall profile of u^+ for $\Delta p_x^+ = 0.018$ and change of log-law slope. (b) Reduced extent of log-law region and square-root law for $\Delta p_x^+ = 0.011$. (c) RANS results for modified SSG/LRR-ω RSM using the square-root law modification (v1) and additional modification of the log-law slope (v3) at $\Delta p_x^+ = 0.027$.

The wall-distance, where the mean-velocity profile changes from the log-law to the square-root law, can be characterised by two similarity parameters, i.e., the pressure gradient parameter Δp_x^+ and the acceleration parameter $\Delta u_{\tau,x}^+ = (\partial u_\tau/\partial x)^+$, being the streamwise gradient of the wall shear stress velocity. Put in other words, we find mean-flow similarity in the inner layer for the present flow and other flows in the literature, provided that they are at the same values of Δp_x^+ and $\Delta u_{\tau,x}^+$ and provided that low-Re effects are small, i.e., $Re_\theta > 10000$. In the outer layer, the profiles can differ due to the different upstream history. Similarity in the inner layer and differences in the outer layer can be explained using the idea of the eddy turn over time.

RANS MODEL IMPROVEMENT FOR ADVERSE PRESSURE GRADIENTS

Then we present a RANS study for the two experiments using two popular models, i.e., the SST k-ω model and the differential Reynolds stress model (DRSM) SSG/LRR-ω. The SST model is based on the turbulent viscosity hypothesis. The SSG/LRR-ω model solves the transport equation for the Reynolds stress tensor using empirical models for redistribution and turbulent transport. For both, the dissipation ϵ is modelled using the specific dissipation rate ω, for which an additional empirical transport equation is solved. We observe that both models predict a too high velocity in the inner layer and cannot predict a square-root law, see Fig. 1 (c).

We apply the above findings for the mean velocity profile to modify the RANS models, extending the work in [5]. First, the coefficient which controls the slope of the log-law is made a function of the local value of Δp_x^+, instead of being a constant value in the standard model. Second, we add a model deficiency term so that the modified model gives a square-root law at adverse pressure gradient. The deficiency term is determined by analytical solution of an inverse boundary layer problem. Both modifications increase the dissipation in the inner part of the inner layer in agreement with the findings in [4]. This leads to a reduction of turbulent transport of momentum into the near-wall region, thus making the model more susceptible for flow separation. The modified model yields a much better agreement with the experimental data, see Fig. 1 (c).

CONCLUSIONS

In an analysis of two new turbulent boundary layer experiments, we find that the mean velocity profile at adverse pressure gradient in the inner layer shows a thin log-law region, whose slope is altered compared to the zero pressure gradient case, and a square-root law which emerges above the log-law extending to the wall-ward distance the log-law occupies at zero pressure gradient. These findings are used to modify a differential Reynolds stress RANS turbulence model to account for a change of the log-law slope and the formation of a square-root law at adverse pressure gradient, leading to improved agreement with the experimental data.

References

[1] Nickels, T. B. Inner scaling for wall-bounded flows subject to large pressure gradients. *J. Fluid Mech.* **521**: 217-239, 2004.

[2] Perry, A. E., Bell, J. B., and Joubert, P. N. Velocity and temperature profiles in adverse pressure gradient turbulent boundary layers. *J. Fluid Mech.* **25**: 299-320, 1966.

[3] Coleman, G.N., Rumsey, C. L., and Spalart, P. R. Numerical study of turbulent separation bubbles with varying pressure gradient and Reynolds number. *J. Fluid Mech.* **847**: 28-70, 2018.

[4] Kitsios, V., et al. Direct numerical simulation of a self-similar adverse pressure gradient turbulent boundary layer at the verge of separation. *J. Fluid Mech.* **829**: 392-419, 2017.

[5] Knopp, T., et al. Modification of the SSG/LRR-omega RSM for turbulent boundary layers at adverse pressure gradient with separation using the new DLR VicToria experiment. In: Proceedings of 21th STAB/DGLR Symposium 2018. Springer, 2019.