Comments on ”Influence of the Statistical Properties of Phase and Intensity on Closure Phase”

Francesco De Zan, Fabio Rocca, Alessandro Ferretti, Paco López-Dekker, Michael Eineder

***Abstract*—A recent publication claims that closure phases in SAR interferometry bear no relation to physical changes of the scatterer, only to statistical properties of the averaged pixels. We disprove this claim with a simple counterexample and remind the reader of cases in which closure phases indicate a clear physical content, including the exploitation of closure phases in other fields.**

***Index Terms*—SAR interferometry, closure phase**

I. Introduction

In the paper [1] the authors claim to demonstrate that closure phases in SAR interferometry do not carry any physical information but are only related to the dispersion of phase and amplitude. To our knowledge the first physical model predicting explicitly the presence of closure phases was in [2]. Implicitly, closure phases are in the standard models for volumetric scattering and decorrelation, as in [3].

In particular we want to disprove the following statements:

(1) ”We show that nonzero phase triplet is only related to the statistical properties of the pixels within the multilooked window.”

(2) ”We showed that closure phase [...] similar to InSAR coherence, [it] contains no information about the magnitude of physical changes.”

(3) ”We showed that phase closure [...] does not relate to the magnitude of physical, deforming and nondeforming changes.”

These are rather general statements, and we are going to disprove them with a counterexample.

II. A Counterexample

Let us take a scatterer made of two sub-populations, represented as the stochastic variables a and b, with $\mathrm{E}[a] = \mathrm{E}[b] = 0$ and $\mathrm{E}[ab^*] = 0$. If we consider the following three SAR data sets, comprising for instance laid over returns, with some relative motion:

$$y_1 = a + b \cdot e^{j\varphi_1} \tag{1}$$
$$y_2 = a + b \cdot e^{j\varphi_2} \tag{2}$$
$$y_3 = a + b \cdot e^{j\varphi_3}, \tag{3}$$

where φ_1, φ_2 and φ_3 are constants, then expected values of the resulting interferograms are:

$$i_{12} = \mathrm{E}[y_2 y_1^*] = \sigma_a^2 + \sigma_b^2 e^{+j(\varphi_2 - \varphi_1)} \tag{4}$$
$$i_{23} = \mathrm{E}[y_3 y_2^*] = \sigma_a^2 + \sigma_b^2 e^{+j(\varphi_3 - \varphi_2)} \tag{5}$$
$$i_{31} = \mathrm{E}[y_1 y_3^*] = \sigma_a^2 + \sigma_b^2 e^{+j(\varphi_1 - \varphi_3)}. \tag{6}$$

For $\sigma_a^2 \neq \sigma_b^2$ and $\varphi_n \neq \varphi_k$, it is immediate to verify that the closure phase is not zero[4]. We can take for instance $\sigma_a^2 = 1$, $\sigma_b^2 = 0.5$, $\varphi_1 = 0$ deg, $\varphi_2 = 90$ deg, and $\varphi_3 = 180$ deg. The three interferograms are

$$i_{12} = 1 + 0.5j \tag{7}$$
$$i_{23} = 1 + 0.5j \tag{8}$$
$$i_{31} = 0.5 + 0j, \tag{9}$$

and the closure phase is $\angle i_{12} i_{23} i_{31} = \arctan(0.5) + \arctan(0.5) + \arctan(0) \neq 0$.

Finally, if the φ's are proportional to moisture levels (or any physical quantity, for what it matters), then the magnitude of the closure phase will obviously reflect the magnitude of the moisture variations (or of any physical quantity). For example, a constant moisture will produce a zero closure phase, whereas if moisture levels and consequently the φ's are changing the closure phase will be typically different from zero. The relation might be complex, but it exists, since $\angle i_{12} i_{23} i_{31}$ is clearly a function of the φ's.

In order to be more precise as for the minimal number of looks to be used: closure phases are already evident with two looks; with three looks the 3x3 covariance matrix that yields the three interferograms needed for a closure phase can reach full rank; with increasing number of looks, the dispersion of the phase closure will get smaller (see [4], Eq. (23)). Incidentally, in [1], Eq. (15), the covariance terms are missing.

III. Discussion

The crucial point of the model presented in the previous section, ignored by the authors in [1], is that we are considering two different populations of scatterers, each with a distinct phase history. This is not contemplated in equation (20), which shows only one scatterer population with varying phase and intensity. With such a model, it is not surprising that the simulations give consistently zero-average closure phases (Fig. 2 in [1]) and the only visible effects are those of noise. The critic here is that the assumed model is not general enough.

In our example the two populations are statistically present in every single look pixel and one could wonder if this is

FDZ and ME are with the DLR (German Aerospace Center), Oberpfaffenhofen, D-82230 Wessling, Germany (e-mail:francesco.dezan@dlr.de); FR is with Politecnico di Milano, Milan, Italy; AF is with TRE-A, Milan, Italy; PLD is with TU Delft, Delft, The Netherlands.

necessary. Indeed the spatial segregation of the two populations to neighboring pixels will not change the conclusion, considering that the mixing at the multilooking stage has an equivalent impact.

That closure phases carry physical information is often evident from the observations, when they display obvious areas of uniform values (Fig. 1 and Fig. 2). A meta-argument in favor of the potential for physical content of closure phases is the fact that they are routinely exploited in astronomical interferometric imaging [5]. There are applications also in seismic imaging [6]. We have shown the possibility to invert moisture series from closure phases in L-band in [7]. The inversion algorithm is complex and has some limitations but a forward modeling from ground truth is not difficult to realize. This is shown for example in [8]. A newly submitted manuscript ([9]) is also presenting observations related to deformation biases which can only be explained by non-zero physical closure phases.

One could eventually ask where is the logical pitfall of the demonstration in [1]. It looks like the thesis that *the closure phases is zero when using the expected values of the interferograms* is inadvertently introduced between Eq. (13) and (14) when one reads "By considering $\varphi_{0,1} + \varphi_{0,2} - \varphi_{0,3} = 0$...". The assumption might be true for single-pixels, but is not valid for average phases after a multilooking process.

IV. Conclusions

Physical closure phases are mathematically possible and exist beyond doubt in the real world too. They are quantitatively predictable with more realistic scattering models than have been used in the past, and they can indeed be used for the successful retrieval of those scattering mechanisms. The readers are encouraged to look through the telescope for themselves, besides considering the mathematical evidence.

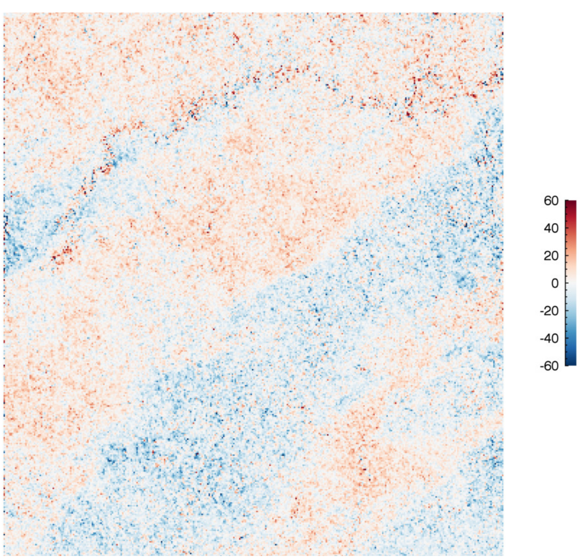

Fig. 1. Closure phase observed over North Carolina/Virginia, from L-band images acquired by PALSAR-2. The color scale is in degrees. Acquisition dates: 2017-06-19, 2017-07-03, 2017-11-06. The scene size is approximately 70 km × 70 km.

V. Acknowledgments

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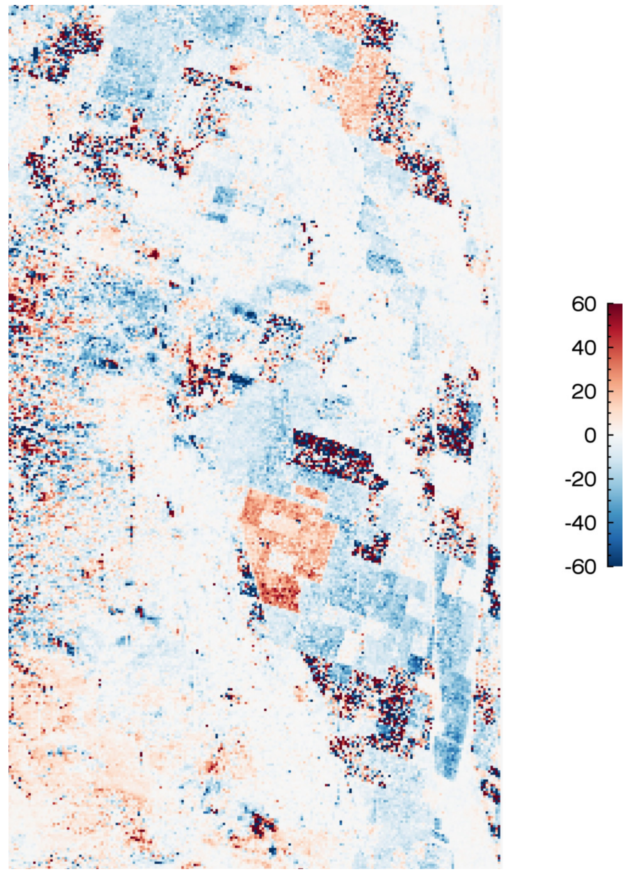

Fig. 2. Closure phase observed over San Joaquin County, California. The L-band SAR images were acquired by JPL with UAVSAR. The color scale is in degrees. Acquisition dates: 2017-04-04, 2017-10-04, 2017-11-07. The scene size is approximately 40 km × 20 km.

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