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On the conductivity of moderately non-ideal completely ionized plasma

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ABSTRACT

Interrelations between the problems of electrical conductivity of a completely ionized plasma and the ion drag force in a dusty plasma are discussed. It is shown that a physically motivated modification of the Coulomb logarithm proposed in the context of ion-particle scattering in dusty plasma allows us to improve the theoretical description of the conductivity in the moderately non-ideal regime. A simple theoretical expression obtained is in reasonable agreement with available results from experiments and numerical simulations.

The electrophysical properties of matter are defined mainly by the properties of the electron component. Electrical conductivity of a completely ionized plasma is governed by electron–ion collisions. Elementary formula for the ideal plasma electrical conductivity is [1]

$$\sigma \simeq \frac{e^2 n_e}{m_e \nu_{\rm eff}}, \tag{1}$$

where e is the elementary charge, m_e and n_e are the electron mass and density, and $\nu_{\rm eff}$ is the effective electron–ion collision (momentum transfer) frequency. It is accurate up to a numerical coefficient, provided by the kinetic theory.

The simplest way to estimate the conductivity is to assume that the ions are immobile uncorrelated scatterers and the electrons do not interact with each other. This corresponds to the simple Lorentz gas picture. The calculation proceeds as follows [2]. The kinetic equation for the electron component reads

$$\frac{e\mathbf{E}}{m_e}\frac{\partial f_0}{\partial \mathbf{v}} = \nu(v) f_1, \tag{2}$$

where $\mathbf{E}$ is the electric field, f_0 is the unperturbed (Maxwellian) velocity distribution function, $\nu(v)$ is the effective velocity-dependent collision frequency, and f_1 is a small perturbation of the distribution function. The effective collision frequency is $\nu(v) = nv\sigma_s(v)$, where the classical momentum transfer (Coulomb scattering) cross section in the binary collision approximation is

$$\sigma_s(v) = 4\pi\int_0^{\rho_{\rm max}} \frac{\rho d\rho}{1+(\rho/\rho_0)^2} = 4\pi\rho_0^2\Lambda, \tag{3}$$

and Λ is the conventional Coulomb logarithm [3–6]

$$\Lambda = \frac{1}{2}\ln\left(1+\frac{\rho_{\rm max}^2}{\rho_0^2}\right). \tag{4}$$

Here $\rho_0 = e^2/m_e v^2$ is the Coulomb (Landau) radius and $\rho_{\rm max}$ is the maximum (cutoff) impact parameter. It has also been implicitly assumed that the ions are singly charged and the quasineutrality condition holds, $n_e \simeq n_i \simeq n$. The appearance of the Coulomb logarithm, Λ, is a special feature of scattering in the ultra-soft long-ranged Coulomb potential. The characteristic cross section of large-angle scattering, $\simeq 4\pi(e^2/m_e v^2)^2$ should be multiplied by a large number Λ to account properly for the (dominant) contribution from small-angle scattering. Note that we deal with the fully classical picture here, quantum mechanical effects [7–9] will not be considered.

Traditionally, the velocity dependence under the Coulomb logarithm is removed by assuming $m_e v^2 \simeq 3T_e$ [10], where T_e is the electron temperature in energy units. In addition, in the ideal (weakly coupled) regime, the maximum impact parameter is chosen as the electron Debye radius $\lambda_{\rm D} = \sqrt{T_e/4\pi e^2 n}$. This reflects the fact that electrons with larger impact parameters practically do not contribute to the momentum transfer due to exponential screening of the electrical potential; for smaller impact parameters approximation of the unscreened Coulomb potential is appropriate. The Coulomb logarithm can then be expressed

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in terms of the coupling parameter $\Gamma = e^2/aT_e$, as $\Lambda = \frac{1}{2}\ln(1 + 3/\Gamma^3)$, where $a = (4\pi n/3)^{-1/3}$ is the Wigner–Seitz radius. The condition of weak coupling $\Gamma \ll 1$ ensures that $\lambda_D \gg \rho_0$ and the Coulomb logarithm is large. In this case we get approximately $\Lambda \simeq \ln(\sqrt{3}/\Gamma^{3/2})$.

Now taking into account that $\partial f_0/\partial \mathbf{v} = -(\mathbf{v}m_e/T_e)f_0$, the perturbation of the electron velocity distribution function becomes

$$f_1 = -\frac{e\mathbf{E}\mathbf{v}}{\nu(v)T_e}f_0. \tag{5}$$

The electrical conductivity is then obtained from

$$\mathbf{j} = -en\int \mathbf{v}f_1 d^3v \equiv \sigma\mathbf{E}. \tag{6}$$

This results in the integral of the form

$$\sigma = \frac{4\sqrt{2}}{3\pi^{3/2}}\frac{T_e^{3/2}}{m_e^{1/2}e^2}\int_0^\infty \frac{e^{-x^2}x^7dx}{\Lambda}. \tag{7}$$

Neglecting the velocity dependence under the Coulomb logarithm, as discussed above, we end up with the conductivity in Lorentzian approximation:

$$\sigma = \frac{4\sqrt{2}}{\pi^{3/2}}\frac{T_e^{3/2}}{m_e^{1/2}e^2\Lambda}. \tag{8}$$

To further improve the accuracy of the derived expression it is necessary to account properly for the effect of electron-electron collisions. This has been done by Spitzer and Härm [10], who derived a correction factor $\gamma_E \simeq 0.5816$ for singly charged ions.

It is customary to define the effective electron collision frequency by [11,12]

$$\nu_{\rm eff} = \frac{4\sqrt{2\pi}ne^4\Lambda}{3\sqrt{m_e}T_e^{3/2}}. \tag{9}$$

Combining expressions (8) and (9) with the Spitzer and Härm correction factor we get

$$\sigma_{\rm SH} = \frac{32\gamma_E}{3\pi}\frac{ne^2}{m_e\nu_{\rm eff}} \simeq 1.97\frac{ne^2}{m_e\nu_{\rm eff}}. \tag{10}$$

A very similar numerical coefficient (1.96) appears in Braginskii' theory of transport processes in plasma [11] and is further quoted in NRL Plasma Formulary. Since the electrical conductivity is expressed in inverse seconds, it is more or less natural to normalize it by the electron plasma frequency $\omega_p = \sqrt{4\pi e^2 n/m_e}$. The reduced conductivity then reads

$$\sigma_* = \frac{\sigma_{\rm SH}}{\omega_p} \simeq \frac{0.34}{\Gamma^{3/2}\Lambda}. \tag{11}$$

As the coupling parameter increases and the non-ideal regime is approached, the problem of electron-ion scattering becomes to some extent reminiscent of the problem of ion scattering on charged grains and the ion drag force in a complex (dusty) plasma. The ion drag force is associated with the momentum transfer from drifting ions to massive highly negatively charged grains immersed in conventional weakly ionized plasma [3,13–16]. Ions lose their momentum in collisions with the dust grains and push them in the direction of the ion flow; this force is equal and opposite to the frictional force experienced by the ion component.

The important similarities between the two processes is that in both cases we deal with scattering of light particles on massive motionless centers, whose positions are uncorrelated in space. The scattering occurs in the screened Coulomb potential. The problems are not fully equivalent, because in the conventional completely ionized plasmas the asymmetric component of the electron velocity distribution function is formed by electron–ion collisions (and is affected by electron-electron collisions), whereas in complex plasmas ion-neutral collisions are normally responsible for that (another clear difference is the finite size of the grains, but we can easily avoid it by taking the point-like grain limit). However, as long as the ion mean free path between collisions with neutrals exceeds considerably the plasma screening length, binary collision formalism applies and the two scattering problems are essentially equivalent. For some examples of the ion drag force calculation in the opposite highly collisional regime for the ions see e.g. Refs. [17–20].

The specifics of dusty plasmas is that the grain charge is normally rather high and the condition $\lambda_D \gg \rho_0$ is usually *not satisfied* for near-thermal ions. In this case, it is not sufficient to consider the ions with impact parameters below λ_D, because there exist considerable fraction of ions which can approach close to the particle even if the impact parameter is larger than λ_D. In terms of electron–ion collisions this situation corresponds to the regime $\Gamma \gtrsim 1$. A modification to the conventional Coulomb scattering theory in this regime has been put forward in Ref. [3]. Here it was proposed to take into account all the ions that *approach* the grain closer than the Debye radius. This resulted in a simple analytical expression for the momentum transfer cross section. The modification mainly affects the Coulomb logarithm, which becomes

$$\Lambda = \ln\left(1 + \frac{\lambda_D}{\rho_0}\right). \tag{12}$$

This modified Coulomb logarithm reduces to the conventional one in the weak coupling limit $\lambda_D \gg \rho_0$, but provides much better estimate of the momentum transfer cross section at $\lambda_D{\sim}\rho_0$. The form of Eq. (12) is relevant to the attractive interaction between the charges of different signs, the difference from repulsive collisions between the particles of the same sign becomes significant at moderate and strong coupling [21,22]. At even higher coupling Eq. (12) is inappropriate; specifics of scattering in strongly attractive potentials has to be invoked [13,23,24]. The discussed modification of the Coulomb logarithm is the only modification of the scattering process description that we make use of; all other special features of collisions in dusty plasmas are not relevant in the present context.

The suggested modification is expected to provide better accuracy at $e^2/T_e\lambda_D{\sim}\Gamma^{3/2}{\sim}\mathcal{O}(1)$. Since in this regime the argument of the logarithm is not large, velocity dependence under the logarithm should not be omitted. Performing the same steps of the derivation [that is combining Eqs. (5) and (6) with the scattering cross Section (3) and the Coulomb logarithm (12)] we arrive again at Eq. (8), but with the modified Coulomb logarithm. For the problem considered it reads

$$\Lambda_{\rm mod} = 3\left[\int_0^\infty \frac{x^7e^{-x^2}dx}{\ln\left(1 + \frac{2x^2}{\sqrt{3}\Gamma^3}\right)}\right]^{-1}. \tag{13}$$

We also chose to keep the Spitzer-Härm correction factor unaffected, although there have been predictions that its magnitude can somewhat increase towards unity when Γ increases [25].

In Fig. 1 (a) we plot the ratio of the modified-to-conventional Coulomb logarithms. It is observed that, as expected, they nearly coincide at weak coupling, $\Gamma \ll 1$, whereas the modified version becomes considerably larger at $\Gamma \gtrsim 1$. In Fig. 1 (b) a comparison with available experimental and numerical simulation results on electrical conductivity is presented. The data displayed correspond to the Coulomb part of the conductivity of partially ionized plasma of different elements extracted from the total conductivity measurements. Because of uncertainties of the extraction procedure the accuracy of these data is limited and relative deviations of about 30% are possible. The used experimental data set has been tabulated in Ref. [26], the original experimental results are from Refs. [27–31]. An extended collection of experimental data can be found for instance in Ref. [32]. Also shown in the Figure are Molecular Dynamics (MD) simulation results [26] from a series of recent publications [26,33,34].

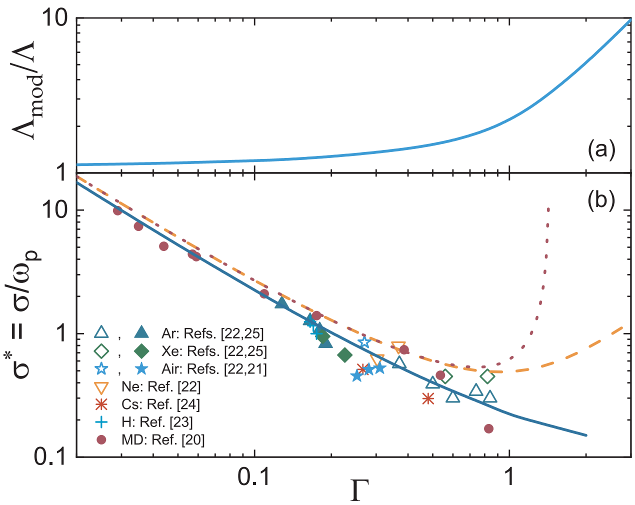

Fig. 1. Ratio of the modified and conventional Coulomb logarithms (a). Reduced conductivity of a moderately non-ideal ($\Gamma \sim 0.02 - 2$) fully ionized plasma as a function of the coupling parameter Γ (b). Symbols correspond to experimental results and MD simulations (see the legend). Dotted line corresponds to the weakly coupled limit of the conventional Coulomb logarithm, dashed line to the unsimplified conventional Coulomb logarithm, and the blue solid line to the modified Coulomb logarithm of Eq. (13).

Comparison demonstrates that in the ideal plasma limit ($\Gamma \ll 1$) the difference between the conventional and modified approaches vanishes. Here the Coulomb logarithm of the simple weakly coupled form $\Lambda = \frac{1}{2}\ln(3/\Gamma^3)$ does a very good job. On approaching the moderately non-ideal regime with $\Gamma \sim 1$, this simple form predicts the conductivity divergence and should not be used. The use of the non-simplified Coulomb logarithm $\Lambda = \frac{1}{2}\ln(1 + 3/\Gamma^3)$ allows one to avoid divergence, but this form still somewhat overestimates most of the experimental and MD data. The modified Coulomb logarithm defined by Eq. (13) provides better agreement with experiments and simulations. Taking into account scattering of available data, this simple model seems to provide a useful tool for semi-quantitative conductivity estimates in the moderately non-ideal plasma regime, which can be required in practical calculations [35,36].

There have been predictions from MD simulations [37] and theoretical consideration [38] that the reduced conductivity may exhibit increase with Γ in the nonideal regime with $\Gamma > 1$. To the best of our knowledge, this tendency has not yet been observed experimentally. Our present model is not consistent with this behavior. At the same time, it should be noted that in the strongly non-ideal regime, this model overestimates the momentum transfer cross section [3]. In addition, the assumption of uncorrelated scatterers becomes inappropriate in this regime. Moreover, in a recent paper it has been suggested that the "classical Coulomb logarithm" approach is itself insufficiently accurate for $\Lambda < 3$, and quantum calculations are required in this domain [5].

Let us summarize the applicability limits of the obtained results. Completely ionized plasma does not mean that neutrals are not allowed at all. The important requirement is that the electron-ion collisions dominate over the electron-neutral collisions, $\nu_{\rm eff} \gg \nu_{en}$. Note, that electron-neutral collisions can also affect $\nu_{\rm eff}$ itself [6,39], but this effect is not relevant in the considered highly ionized plasma regime. The classical analysis has been performed, which is appropriate when the electrons and ions are seldom found at distances comparable with the thermal electron wavelength $\lambda_e = \hbar/\sqrt{2mT_e}$ [32]. Since the characteristic radius of the ion-electron interaction is e^2/T_e, the condition of classicality can be written as $\lambda_e \ll e^2/T$. Therefore, a plasma is classical at relatively low temperatures, $T \ll {\rm Ry} = e^4 m/\hbar^2$. On the other hand, classical statistics can be employed when the electron Fermi energy is smaller than the kinetic energy. This is equivalent to require $\lambda_e \ll a$, which limits the temperatures from below by the condition $T \gg \hbar^2 n^{2/3}/m_e$ [40]. From the side of coupling parameters, the applicability of the modified Coulomb logarithm (12) is limited by Γ values about unity. The derived expression should not be used in the strongly coupled regime with $\Gamma \gg 1$. The result is based on the analysis of trajectories in the central screened Coulomb potential. This, for instance, limits the applicability by sufficiently weak electric fields, when the average relative electron drift velocity is subthermal. Only in this case an anisotropy in electron-ion interaction does not result in considerable corrections to the momentum transfer cross section and a shifted Maxwellian distribution for drifting electrons is appropriate.

To conclude, we have suggested a connection between the conductivity of a moderately non-ideal plasma and the ion drag force in dusty plasma. Physically motivated modification of the Coulomb logarithm proposed initially in the context of ion-grain collisions in dusty plasmas performs equally well when applied to electron-ion collisions in completely ionized plasma. Reasonable agreement (especially taking into account experimental uncertainties) between theory, experimental results, and MD simulations has been documented.

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Declaration of Competing Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

CRediT authorship contribution statement

Sergey A. Khrapak: Conceptualization, Investigation, Writing - original draft, Writing - review & editing. **Alexey G. Khrapak:** Investigation, Writing - review & editing.

References

[1] Spitzer L. Physics of Fully Ionized Gases. Dover Publications; 2013.
[2] Pitaevskii LP, Lifshitz EM. Physical Kinetics. Oxford: Butterworth-Heinemann; 2012.
[3] Khrapak SA, Ivlev AV, Morfill GE, Thomas HM. Phys Rev E 2002;66:046414.
[4] Stanton LG, Murillo MS. Phys Rev E 2016;93:043203.
[5] Starrett CE. Phys Plasmas 2018;25:092707.
[6] Hagelaar G, Donko Z, Dyatko N. Phys Rev Lett 2019;123:025004.
[7] Daligault J. Phys Plasmas 2016;23:032706.
[8] Daligault J. Phys Rev Lett 2017;119:045002.
[9] Krommes JA. J Plasma Phys 2019;85:925850101.
[10] Spitzer L, Härm R. Phys Rev 1953;89:977.
[11] Braginskii SI. Reviews of Plasma Physics vol. 1. New York: Consultants Bureau; 1965. p. 205.
[12] Chen W-T, Witte C, Roberts JL. Phys Rev E 2017;96:013203.
[13] Khrapak SA, Ivlev AV, Morfill GE, Zhdanov SK. Phys Rev Lett 2003;90:225002.
[14] Semenov IL, Khrapak SA, Thomas HM. Phys Plasmas 2017;24:033710.
[15] Khrapak S, Huber P, Thomas H, Naumkin V, Molotkov V, Lipaev A. Phys Rev E 2019;99:053210.
[16] Thomas HM, Schwabe M, Pustylnik MY, Knapek CA, Molotkov VI, Lipaev AM, Petrov OF, Fortov VE, Khrapak SA. Plasma Phys Control Fusion 2019;61:014004.
[17] Ivlev AV, Khrapak SA, Zhdanov SK, Morfill GE, Joyce G. Phys Rev Lett 2004;92:205007.
[18] Khrapak SA, Zhdanov SK, Ivlev AV, Morfill GE. J Appl Phys 2007;101:033307.
[19] Patacchini L, Hutchinson IH. Phys Rev Lett 2008;101:025001.
[20] Momot AI. Phys Plasmas 2017;24:103704.
[21] Khrapak SA, Ivlev AV, Morfill GE. Phys Rev E 2004;70:056405.
[22] Shaffer NR, Baalrud SD. Phys Plasmas 2019;26:032110.
[23] Khrapak SA. Phys Plasmas 2014;21:044506.
[24] Khrapak SA. Phys Rev E 2014;89:032145.
[25] Zaika EV, Mulenko IA, Khomkin AL. High Temp 2000;38:1.
[26] Bobrov AA, Bunkov AM, Bronin SY, Klyarfeld AB, Zelener BB, Zelener BV. Phys Plasmas 2019;26:082102.
[27] Andreev S, Gavrilova T. High Temp 1975;15:151.
[28] Ivanov YV, Mintsev VB, Fortov VE, Dremin AN. J Exp Theor Phys 1976;44:112.
[29] Gunther K, Popovic MM, Popovic SS, Radtke R. J Phys D: Appl Phys 1976;9:1139.
[30] Sechenov VA, Son EE, Shcekotov OE. High Temp 1977;15:346.
[31] Popovic MM, Vitel Y, Mihajlov AA. Strongly Coupled Plasma Physics. Elsevier;

1990. p. 561–9.
[32] Fortov VE, Iakubov IT, Khrapak AG. Physics of Strongly Coupled Plasmas. Oxford: Clarendon Press; 2006.
[33] Bobrov AA, Vorob'ev VS, Zelener BV. Phys Plasmas 2018;25:033513.
[34] Bobrov AA, Bronin SY, Klyarfeld AB, Zelener BB, Zelener BV. Phys Plasmas 2020;27:010701.
[35] Apfelbaum EM. Phys Plasmas 2017;24:052702.
[36] Apfelbaum EM. Contrib Plasma Phys 2019;59:e201800148.
[37] Morozov IV, Norman GE. J Exp Theor Phys 2005;100:370.
[38] Khomkin AL, Shumikhin AS. High Temp 2019;57:462.
[39] Khrapak SA. Reduction of the Coulomb logarithm due to electron-neutral collisions. Phys Rev E 2020. submitted.
[40] Alexandrov AF, Bogdankevich LS, Rukhadze AA. Principles of Plasma Electrodynamics. Berlin Heidelberg: Springer-Verlag; 1984.