IAC-19,C1,1,9,x51976

TRAJECTORY OPTIMIZATION IN THE THREE-BODY PROBLEM FOR A LUNAR TRANSPORTATION SYSTEM

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Abstract

Lunar missions and their design are in the agenda of most main space agencies around the world. With the goal of regular lunar missions, many of them manned, both time and fuel efficiency are currently being considered as key aspects of space mission design. With this in mind, new and more sophisticated methods of transportation to the Moon are to be designed as alternatives for the ones used by NASA during the Apollo era to improve sustainability. One of the most logical steps to take towards this goal is ISRU (In-Situ Resource Utilization), which would allow for a mission in which the ship is refueled with propellant produced on the Moon. This capability would be beneficial to a wide range of missions even beyond the Moon, potentially spanning other celestial bodies like Mars.

This project from DLR in collaboration with ISAE-SUPAERO consists of two ships which have to carry cargo from a Low Earth Orbit (LEO) to the lunar surface, while obtaining propellant from the regions near the lunar poles, in order to have an efficient transportation system that can benefit from lunar resources. The two ships exchange both fuel and cargo while in a Libration Point Orbit (LPO).

The aim of this work is to optimize the trajectories to and from the LPO in terms of ΔV and time of flight, taking into consideration the various constraints that this type of mission requires. These constraints include, for instance, an in-orbit time limit due to propellant boil-off (cryogenic LOx/LH2) or a minimum time in orbit to perform rendezvous. This optimization is computed over the entire duration of a mission, which extends from the departure of the payload from a LEO until its arrival in Low Lunar Orbit (LLO). Flexibility is allowed in terms of the inclination of the LLO, which is subject to change depending on the focus of the mission or the nature of the payload. While current estimations are available for both time and ΔV for single branches of the mission, this optimization is aimed at narrowing down those quantities to more precise, reliable results for realistic mission operation.

List of abbreviations (see Annex A)

1. Introduction

A reliable system for lunar travel is a key piece for the close future of space exploration. Even though Apollo era missions effectively succeeded in transporting people and research equipment to the Moon, current times and space agencies' agendas call for more sustainable, efficient and reliable methods. The Earth-Moon environment is also being used as the testing bed for future interplanetary missions to Mars and beyond. With this in mind, achieving close to permanent presence in the Moon is a logical intermediary step towards the interplanetary goal. With the possibility to have an outpost or station on the Moon, missions to Mars could be much more flexibly designed, allowing for refueling in orbit and carrying extra payload. Apart from the obvious difficulties of building spacecraft capable of reliably carrying humans safely beyond Low Earth Orbit, a rather challenging obstacle is presented when considering the dynamics dictating the trajectories leading to these objectives. From among the most promising and viable options, the usage of Lagrangian Points (equilibrium points from the 3-Body-Problem). As is widely discussed in [1], positioning vehicles in orbits around these points as intermediate steps in the mission brings many advantages over other classical methods which approximate Hohmann transfers. These advantages include potential ΔV savings over those traditional methods and the ability to have a stable intermediate orbit to dock spacecraft composing one mission, which greatly improves the flexibility of the whole system compared to directly using a Low Lunar Orbit.

Furthermore, this strategy can benefit from In-Situ Resource Utilization (ISRU), which is another research trending topic. Being able to harness and utilize space resources that would have been sent from the Earth otherwise can significantly reduce the cost of future missions. Even if the ISRU concept covers many different applications, the production of fuel on the surface of the Moon has a special interest in the scope of the work that is presented in this paper.

By taking advantage of these concepts, the DLR is studying the feasibility of a reusable lunar transportation system, in collaboration with ISAE-SUPAERO. The idea consists in sending payload from the Earth to the Moon by sending it first to a Low Earth Orbit (LEO) with a conventional or reusable launcher (not part of this study), recuperating it by using two additional spacecrafts charged with propellant produced in the Moon surface and transporting the docked payload to its destination, by using an intermediate LPO. These additional spacecrafts are the Reusable Trans-Lunar Vehicle (RTLV), which will cover the trajectories between the LEO and an LPO, and the Reusable Lunar Resupply Vehicle (RLRV), covering the trajectories between the LPO and the moon surface.

The objective of this paper is to show the results of the trajectory optimization of the proposed scenario, used to determine both the feasibility of the mission and the most appropriate LPO choice. This is an extension to the work presented in [2].

There is extensive state of the art related to the optimization of Earth-Moon single transfers by using LPOs. However, these results cannot be directly applied in the presented mission. The reason being the existence of several mission-specific constraints, mainly related to the total mission time and the rendezvous between spacecraft. Therefore, the approach that is presented here considers the end-to-end mission optimization. This fact yields to more realistic calculation of the total time and fuel consumption, adapted to the particular specifications of the scenario.

The document is divided in three main parts. Firstly, the sequence of the mission and the main assumptions are detailed in Section 2. Then, the methodology that has been used is presented in Section 3. Finally, the obtained results are shown and commented in sections 4 and 5. Further estimations related to the rendezvous and Moon ascent and descent are also detailed.

2. Mission description

2.1 Mission sequence

As introduced before, the objective of the proposed mission architecture is to transport payload from the Earth to the Moon surface, by using two additional spacecraft and an intermediate LPO. Furthermore, the launcher used in the Earth will transport the payload only to a LEO, and the RLRV and RTLV will be responsible for transferring it to the Moon surface. This reduces the requirements and thus the cost of the launchers, as they will only need to reach the given LEO instead of the Moon.

At the beginning of the mission, the RTLV is orbiting the LPO and the RLRV is on the Moon surface. Then, the RLRV takes the fuel produced on the Moon surface and transports it to the LPO, to transfer it to the RTLV after performing the rendezvous operations with it. When having transferred all the propellant, the RLRV returns to the Moon surface and the RTLV flies to the LEO. During this transfer, the payload is sent by a conventional launcher to the same LEO, where it is docked by the RTLV, which transports it to the LPO. Again, the RLRV goes from the Moon surface to the LPO to perform a rendezvous with the RTLV, this time to get the payload and transport it to the Moon surface. The RTLV stays in the LPO until the next mission cycle.

The sequence of the mission is also depicted in Figure 1 (Annex B). The sequence of the mission is also depicted in Figure 1. It is described from left to right, in terms of temporal evolution.

2.2 Mission assumptions

One of the specifications of the propulsion system is the use of LOx/LH2 type propellant for both the RLRV and the RTLV. It has the advantage of having a high specific impulse (~ 400 s in vacuum rocket engines), and a good availability from the Moon surface ice water. However, it has an important drawback related to its poor storability. Being a cryogenic propellant, its storage in the spacecraft tanks is more complex due to the boil-off effect. Heat coming from outside (such as solar radiation) will may lead to venting large amount of formed vapors. In order to minimize its impact on the mission, a maximum TOF of 20 days is set.

The production of the propellant is assumed to take place in the polar regions of the Moon, where massive quantities of water ice can be processed. This fact drives the specifications on the landing and launching location of the RLRV. For the sake of simplicity only the South Pole is considered. Furthermore, an LLO (100 km x 100 km) will be used as an intermediate orbit between the Moon surface and the LPO. It will be defined then as a polar orbit, having an inclination of 90 degrees. However, scientific stations are also likely to be situated in the far side of the Moon, in the equatorial region. So, all the LLO used in the mission will be defined as polar, except for that of the final landing with the payload, where both polar and equatorial LLO will be considered.

As explained before, the payload is firstly sent by a conventional launcher to a LEO. This orbit is assumed to be circular, with an amplitude of 200 km and an inclination higher than 4 degrees. The last constraint is a consequence of not having available launching spaceports with latitudes closer to the equator.

3. Methodology

The process is separated in two different parts. Firstly, the trajectories are created and optimized by assuming CR3BP dynamics in MATLAB R2018a, by using the SEMAT (Sun-Earth-Moon system in MATlab) tool developed at ISAE-SUPAERO. Then, the mission is reconstructed using GMAT [9] in order to introduce additional perturbation sources, having a more realistic solution.

3.1 Hypothesis

In this early stage of the study presented in this paper, only nominal open loop trajectories are considered, without taking into account any physical parameters (mass, geometry, etc.) of the payload or spacecraft. They are considered as punctual masses, propagated according to the given dynamics, without any maneuver execution error or state uncertainties. Furthermore, the burns are considered to be instantaneous and impulsive ΔV, and they occur only twice for each transfer (departure and arrival), as represented by the colored dots of Figure 1 (Annex B). Then, during the propagation of each transfer or orbit, no corrective maneuvers are needed.

Furthermore, there are no implemented rendezvous strategies in the full. It is considered that both spacecraft arrive at the same time at LPO insertion point, reaching zero relative distance but at an arbitrary relative speed. Then, the insertion burns are applied so that the RLRV and RTLV start orbiting the LPO at zero relative distance and velocity. Their position is propagated in these conditions during an amount of time that is an approximation of the required time for the actual rendezvous plus the time that is needed to transfer the propellant from the RLRV to the RTLV (first rendezvous) or the payload from the RTLV to the RLRV (second rendezvous). Similarly, the rendezvous between the RTLV and the payload is not detailed. The RTLV orbits the LEO during an amount of time that corresponds to the time that would be needed to perform this rendezvous.

3.2 Parametrization of the system

In the CR3BP, any arbitrary LPO is perfectly periodic with period T . This fact permits the parametrization of a position in a given LPO with a single variable θ. It is defined between 0 and 1, and maps to the position that is obtained by propagating a point in the LPO during $\theta \cdot T$ and starting at a reference position, which is defined as the position on the LPO that has the highest z value in the Earth-Moon synodic frame.

Once the position on the LPO is parametrized, the orbit still needs to be defined. Four different LPO types are assessed in this work, including the northern and southern variants of the Halo and NRHO (Near Rectilinear Halo Orbit) families. In the CR3BP, the LPO is fully defined by a single parameter, which is the amplitude along the synodic z-
axis. Also, a combination of two different LPO can be used in a single mission. The first one corresponding to the first rendezvous, where the fuel transfer takes place between the RTLV and the RLRV, and the second one for the rendezvous where the payload is transferred from the RTLV to the RLRV. This fact increases the flexibility on the design of the mission.

As all the different transfers of the mission have the LPO as either the origin or the destination, the full mission can be parametrized solely by defining the two LPO and the θ variable for the arrival or departure position of each transfer. From these variables, the complete mission can be constructed as explained in the next section. All the optimization variables that are used by the algorithm are summarized in Table 1. The type of each LPO is defined by the user before starting the optimization.

Table 1. Optimization variables.

Variable	Description
$A_{z,1}$	First LPO amplitude
$A_{z,2}$	Second LPO amplitude
θ_2	Arrival position in the LPO for the first LLO-LPO transfer
θ_5	Departure position in the LPO for the LPO-LEO transfer
θ_8	Arrival position in the LPO for the LEO-LPO transfer
θ_{11}	Departure position in the LPO for the LPO-LLO transfer

The process starts by finding the arrival position on the LPO of the first transfer LLO-LPO1 by the RLRV, which is defined by the optimization variable θ_1. A Hohmann transfer is computed to reach backwards the initial LLO altitude (which is 100 km), without taking into account the influence of the Earth. This is used as a first guess on a differential correction procedure, as done in [7], this time with the CR3BP dynamics, by fixing the position on the LPO and correcting the velocity until converging to a 100 km polar orbit and integrating backward in time. This will give the initial state of the RLRV at the departure point of the LLO

after the departure maneuver, which is computed by circularizing the orbit, and the trajectory to the LPO. The injection maneuver is computed by comparing the velocity computed by the differential correction and the necessary velocity to orbit the LPO at that position. It is assumed that the RTLV was orbiting the LPO, such that at the instant of the time when the RLRV arrives to the LPO and performs the injection burn, the position and velocity of both spacecraft match.

Then, the states are propagated until arriving to the position defined by the variable θ_2. When this occurs, both spacecraft perform the departure maneuver for their next transfers simultaneously. These transfers are computed in a similar manner as the first LLO-LPO1 (RLRV transfer to the first LPO where the fuel transfer takes place), but this time integrating forwards in time. The RLRV targets a polar LLO again, while the RTLV transfers towards a 200 km LEO with arbitrary inclination (higher than 4 degrees), to dock the payload. The next transfer, from the LEO to the second LPO, is computed by starting at the arrival position at the LPO, defined by θ_3, and performing a differential correction to the previous LEO by integrating backward in time. Note that in this case, the differential correction is more restrictive because the inclination and longitude of the ascending node must match those that were found for the LEO in the previous transfer. Furthermore, the departure from the LEO must occur after having spent enough time orbiting to be able to perform the rendezvous and dock the payload ($T_{payload}$). During this transfer, in parallel, the RLRV flies from the LLO to the LPO in such a way that both spacecraft arrive at the same position (given by θ_3) at the same time. Then, they perform the injection burn and are propagated, while transferring the payload to the RLRV, until arriving to the position given by θ_4, where the RLRV initiates its last transfer to either a polar or equatorial LLO. The RTLV stays orbiting the LPO for the next mission.

The sequence of the different transfers, as well as their integration direction and the instants of time where the differential corrections are started are summarized in Table 2 (Annex C).

3.3 Optimization process

The highly nonlinear nature of these dynamics makes gradient based optimization algorithms non-effective. Then, a method called pattern search [10] has been implemented in order to find the set of trajectories and orbits that minimize the total cost of the mission. This algorithm can be used from the MATLAB optimization toolbox, and further information can be found in its official documentation.

The variable to be minimized is the sum of the absolute value of the ΔV (computed as in (1))

$$\Delta V_{total} = \sum_{n=1}^{6} \Delta V_{i,LEO/LLO} + \Delta V_{i,LPO} \quad (1)$$

of all the maneuvers of both spacecraft. This criterion aims to find the most optimal transfers and LPO in terms of fuel, which drives the propellant storage capability of the RTLV and RLRV.

Regarding the constraints of the optimization algorithm, the total TOF of the mission must be lower than 20 days as explained before. Furthermore, there must be enough margin for each rendezvous to be performed, even if they are not implemented, taking into account also the times needed to transfer the fuel and fuel between the spacecraft. For the first rendezvous, the minimum time that has been set as constraint is 18 hours, including the time needed to transfer the fuel. This time corresponds to the minimum value of $T_{LPO1}(\theta_2 - \theta_1)$, and has been determined based on constraints from the DLR to allow for the necessary fuel transfer. For the second rendezvous, the minimum time constraint is set to 6 hours (given that there is no fuel transfer), including the time that is needed to transfer the payload from the RTLV to the RLRV. It corresponds to $T_{LPO2}(\theta_4 - \theta_3)$.

3.4 GMAT implementation

The code used in MATLAB to build the trajectories of the mission considers the simplified model CR3BP. In order to extend the analysis of the mission to a more realistic case, the simulations should take into account a completer and more reliable model, including the ephemeris of all the bodies of the solar system, additional perturbations such as the spherical case, the simulations should take into account a completer and more reliable model, including the ephemeris of all the bodies of the solar system, additional perturbations such as the spherical harmonics of the Earth, its magnetic field or the solar radiation pressure.

The software that has been used in order to take into account these perturbations, validate the previous results and get the final reliable solution is NASA's General Mission Analysis Tool (GMAT). The maneuvers of each transfer have been tuned using GMAT's built-in tools, in order to reach the desired final states.

One of the major issues is to find a continuous trajectory for the RTLV at the arrival, orbiting and departure of the LEO, having computed separately both transfers. The procedure starts by propagating the forward solution to the LEO, then computing the maneuver to circularize it, and propagating it (orbiting around the Earth) an amount of time of a value that will be tuned. The idea is to do a maneuver right after this amount of time, to get the velocity vector that was computed by the backward propagation optimization when arriving at the LEO, and minimize the resulting flight path angle. Then, another shooting method is implemented by tuning the last maneuver so that the spacecraft will reach the required LPO position at the given time (which was the initial conditions for the separated backward propagation shooting method that was given before that gave the initial guess for the LEO departure). After this procedure, the trajectory is always continuous and reaches the target point at the required time.

4. *Results*

In this section, four representative cases are presented. This selection was made based on lowest ΔV cost and the rest of the variables within the constraints, while also representing a variety of LPO combinations. The chosen results are (listed by type of LPO and inclination of final lunar orbit):

- Halo orbit (Southern-Southern), final equatorial LLO.
- NRHO (Southern-Northern), final polar LLO.
- NRHO - Halo (Northern-Northern), final equatorial LLO.
- Halo orbit - NRHO (Southern - Northern), final polar LLO.

Each result comprises of a whole mission, with a different combination of the two LPO, and their numeric data can be found within the appended data sheets. The amplitudes of the LPOs are a result of the optimization process, and are listed for every result. The results show both the CR3BP dynamics optimization in MATLAB, and the comparison with the implemented solutions in GMAT.

4.1 Halo orbit (Southern-Southern), final equatorial LLO

First, a solution using only Halo orbits is proposed. In this solution, all transfers are performed to and from the same Halo orbit from the “Southern" Halo family. A particularity of this case is that the Halo orbit has a rather low amplitude compared to the other solutions (Az = 7257 km). This fact also causes a low “out-of-plane" velocity, which helps with convergence when trying to perform a transfer to an equatorial LLO.

Figure 2 (Annex B) shows a plot in the synodical frame of the whole mission, with all the transfers to and from the Halo orbit. The ΔV and time of flight (TOF) values for each transfer are plotted in figure 3 (Annex B). As will be seen in all examples, a clear conclusion is that the most demanding transfers are those performed to and from the LEO orbit, which are performed by the RTLV spacecraft.

Finally, some key data of the mission is gathered in table 3. This includes the times available for both rendezvous in the Halo orbit, the time available at the Moon surface and the total ΔV and TOF.

Table 3. Several relevant values of the mission. Halo orbit (Southern - Southern), 20 days constraint, final equatorial LLO.

1st RDV time (days)	1.27
Moon surface time (days)	3.33
2nd RDV time (days)	2.11
Total TOF (days)	17.31
Total ΔV (km/s)	11.01

4.2 Near Rectilinear Orbit (Southern-Northern), final polar LLO

For this case, a combination of northern and southern NRHO families is configured and its effects studied in terms of ΔV and time of flight. Essentially, northern and southern families of Halo or NRHO families are symmetric with respect to the Z = 0 plane, and the sense of rotation is opposite for both families. The considered NRHOs have an amplitude of Az = 74530 km.

The value of ΔV achieved with this configuration is close to the previous Halo case, at around ΔV = 11.1 km/s, while the TOF stays within the limit (20 days) at TOF = 17.87 days. Plots of the mission in the synodical frame and a detail plot can be seen in figures 4 (Annex B) and 5, respectively.

Table 4 Several relevant values of the mission. NRHO orbit (Southern - Northern), 20 days constraint, final polar LLO.

1st RDV time (days)	1.50
Moon surface time (days)	5.19
2nd RDV time (days)	0.25
Total TOF (days)	17.87
Total ΔV (km/s)	11.1

As it could be expected due to the elongation of NRHOs, forcing a final polar LLO in the last trajectory makes the optimizer choose positions of the NRHO that are further towards the point of less relative velocity of the orbit to perform the transfer (see figure 4 in Annex B).

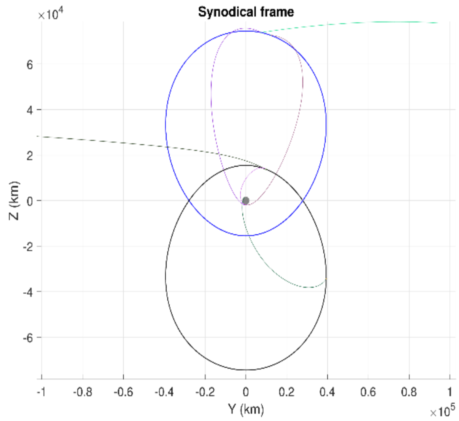

Figure 5. Lunar transfers detail view, synodical frame of reference. NRHO, TOF = 17.87 days, ΔV=11.1 km/s, final polar LLO.

In figure 6 (Annex B), a big difference in TOF is observed with respect to the previous Halo orbit, with lunar transfers taking longer times, due to the very elongated NRHO shape (transfers happen mostly at the highest peak, except for a shorter one happening close to the Moon). Finally, some numerical data points are gathered in table 4.

4.3 Near Rectilinear Halo Orbit - Halo (Northern-Northern), final equatorial LLO

A combination of different orbit types is also possible given the flexibility of the solver. An example using both northern NRHO and northern Halo has been computed, with a result of ΔV = 11.2 km/s and TOF = 16.82 days. These values are very close to previous results using exclusively one family of orbits. However, performing the last transfer from a Halo orbit improves convergence since the target is an equatorial LLO orbit (less out-of-plane velocity). The dimensions of both orbits are Az = 94208 km for the NRHO and Az = 28189 km for the Halo orbit.

To help with convergence, an extra manoeuvre has been implemented into the last transfer (in pink, marked as transfer number 6), which in turn makes it more expensive in terms of ΔV (this effect can be observed in figure 7, Annex B). This procedure is especially useful when performing a transfer from an LPO to a low-inclination LLO orbit (high out-of-plane velocity). Finally, a general and detail plot of the mission and lunar transfers are displayed, respectively, in figures 8 and 9 some key numerical figures of the mission are displayed in table 5.

Table 5. Several relevant values of the mission. NRHO – Halo orbit (Southern - Northern), 20 days constraint, final equatorial LLO.

1st RDV time (days)	0.75
Moon surface time (days)	1.23
2nd RDV time (days)	0.25
Total TOF (days)	16.82
Total ΔV (km/s)	11.2

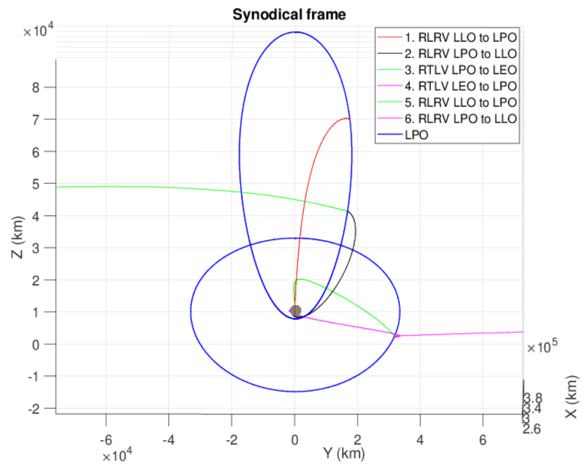

Figure 9. Lunar transfers detail, synodical frame of reference. Near Rectilinear orbit - Halo orbit, TOF = 16.82 days, ΔV=11.2 km/s, final equatorial LLO.

4.4 Halo orbit - Near Rectilinear orbit (Southern - Northern), final polar LLO

A last example combining NRHO and Halo orbits is presented, which also combines Northern and Southern orbit families. This combination achieves the lowest ΔV among the studied solutions (ΔV = 10.94 km/s), and does so mainly by performing relatively long transfers from the Moon to the NRHO and back (transfers 5 and 6). All this data can be observed in figure 10 (Annex B). Dimensions for both orbits are Az = 24680 km for the Halo orbit and Az = 83595 km for the NRHO.

Figure 11 (Annex B) shows a plot of the whole mission in the synodical frame. In this figure, the necessary out-of-plane transfer to the top of the NRHO can be easily observed (initial LLO is considered polar for all computed missions), as well as the transfers from that area to the LLO.

The data in table 6 shows a larger time in the first rendezvous and the Moon surface than previous solutions, while the time for the second rendezvous is the minimal allowed for the optimizer (0.25 days).

Table 6 Several relevant values of the mission. Halo - NRHO orbit (Southern - Northern), 20 days constraint, final polar LLO.

1st RDV time (days)	2.18
Moon surface time (days)	3.83
2nd RDV time (days)	0.25
ºTotal TOF (days)	19.75
Total ΔV (km/s)	10.94

5. *GMAT comparison and validation*

The considered two best solutions have been built in GMAT [9] and compared with the previous MATLAB results in terms of ΔV and TOF.

5.1 Southern Halo – Northern NRHO

The first solution has been selected from the others because it has the lowest ΔV. It has been implemented in GMAT with very similar results. Table 7 (Annex C) shows the comparison between the results of the GMAT and the MATLAB solution. Each row corresponds to a transfer of the mission. Differences in ΔV are around some tens of meters per second. This is due to the fact that the additional perturbations that GMAT takes into account modify the mission requirements, both in terms of ΔV and time, which is difficult to foresee. The final GMAT solution is 80 m/s more expensive that the computed previously by MATLAB.

Regarding the time of flight, the differences are up to few hours for the different transfers. The final total time of flight is just 52 minutes above the time computed with MATLAB. Furthermore, the GMAT solution gives 24 hours of orbiting around the LEO, which gives enough margin to dock the payload. For the RLRV, the margin that is given for the landing, refuelling and launching is 5 days. The fact that the results are so close serves as a validation for the MATLAB code as a reliable tool for preliminary calculations.

5.2 Southern NRHO – Northern NRHO

The second solution that has been selected showed total ΔV values similar to the previous one, but with a total time of flight slightly lower. In this case, the first LPO is a southern NRHO, while the second is a northern NRHO. Regarding the results, table 8 (Annex C) shows that, again, the MATLAB and GMAT solutions are close in all the transfers. In this case, the GMAT solution shows a higher difference in terms of total time of flight, with a difference of around 14 hours with respect to the MATLAB solution.

The total time in LEO in this case is 23 hours, which again gives enough time for the payload docking. In the case of the RLRV, the total margin given for the landing, refuelling and launching is 5 days and 13 hours.

6. *Additional ΔV estimation*

With the goal of providing a more complete estimation of the potential cost of the mission regarding ΔV, the manoeuvres for the rendezvous between spacecraft and for the ascent or descent to the Moon's surface should be accounted for. These two procedures will be assessed separately with different numerical methods and should give an estimate for the mission requirements regarding those parts of the mission.

6.1 Rendezvous ΔV estimation

For this computation, a rendezvous strategy has been designed where the RTLV vehicle acts as a "target" in the LPO, while the RLRV acts as a "chaser". All physical manoeuvres in this scenario are performed by the chaser, which will be placed in a slightly lower orbit ($A_{z_{chaser}} = 0.999 \cdot A_{z_{target}}$) and a lower anomaly within the orbit (around 5º behind the target). The implemented solution is taken from [5] [6], and uses the "corridor" approach [3] for the trajectory design with angles for the cone of approach of $\alpha = \beta = 16°$, as shown in figure 11 (Annex B).

Following this method, a good estimate has been found around 500 m/s for HALO LPOs, and around 200 m/s for NRHOs. This value is heavily dependant on the TOF, which should be taken into account given that both rendezvous manoeuvres have different time limits.

6.2 Lunar ascent and descent ΔV estimation

For the very critical phase of lunar ascent and descent, that is the injection into LLO and the lunar landing, an estimate is necessary to the sizing of the mission requirements. These computations will be performed using the TOSCA [11] (*Trajectory Optimization and Simulation of Conventional and Advanced Spacecraft*) software, developed and used by the SART (*Space Launchers Systems Analysis*) department of the DLR. This software finds optimal solutions for ascent and descent from a given LLO, given certain parameters such as engine performance, dry mass, propellant mass, etc.

For this case, a circular LLO (100 km, I = 0º / 90º) is considered, and the software is set up to optimise ΔV consumption. For both ascent and descent, an intermediate orbit of 15 km x 100 km is used, which reduces the effect of gravity losses. The obtained values of ΔV are represented in table 9 (Annex C).

These results seem to indicate a somewhat similar ΔV requirement for both legs of the mission, around 1.8 km/s. These results can be checked against values from the bibliography, namely from the well-known Apollo missions. From [4] the projected nominal values of ΔV for ascent and descent from a similar LLO were respectively 6056 ft/s (1.85 km/s) and 6827 ft/s (2.08 km/s), which are close to the estimated values from the TOSCA software.

7. Conclusions

The main goal of this study is the analysis of the feasibility of a lunar transportation system based on two vehicles: the Reusable Lunar Resupply Vehicle (RLRV) and the Reusable Trans-Lunar Vehicle (RTLV). These vehicles perform a rendezvous manoeuvre in an LPO. The operational aspects of the mission (manoeuvres and transfers between different branches) are optimized with respect to ΔV and TOF.

Bibliography research pointed out that many previous studies and reports focus on single branches or transfers. These studies give estimations of ΔV and TOF that may no longer be valid when specific constraints are imposed through a complete mission. This work takes into account the numerous constraints of this specific DLR mission and applies several numerical tools (SEMAT, GMAT) in order to optimize it according the constraints.

Firstly, a pattern search optimization algorithm has been applied to the highly non-linear 3-body problem in order to find feasible solutions for the mission. The trajectories of two vehicles composing the mission have been computed with several time constraints for minimum ΔV requirements. All optimization attempts have been performed taking into account an entire mission, both symmetrically (using the same LPO orbit for both rendezvous) and asymmetrically (using different LPO orbits).

The optimizing method of pattern search has been found to be adequate due to its robustness to find optimized solutions for non-linear, highly constrained problems. While solutions cannot be guaranteed to be global optimums, the high number of constraints and the sensibility due to non-linearities make it very difficult for other methods (such as gradient-based optimizers) to succeed in achieving similar results.

Numerical results show a clear correlation between the time constraints and the presence of global minima, which follows the logical conclusion that for a longer time of flight, lower ΔV solutions are achievable. This is especially noticeable when considering highly elongated NRHOs, where the manoeuvre's burn would take place at the point which is furthest from the main body. At this point, the relative velocity of the spacecraft and the main body is the lowest and thus the applied thrust is more effective, however resulting in a longer trajectory. Moreover, due to the in-plane and out-of-plane velocity components of the considered LPOs, HALO orbits (which generally have a stronger in-plane velocity component) are preferred when transferring to an equatorial LLO, while NRHOs (with a stronger out-of-plane velocity component) are generally better for transfers to polar LLOs, in terms of ΔV cost. Total optimized solutions offer ΔV total costs (for transfers between LEO – LPO – LLO) of around 11 km/s.

The results have been first optimized in MATLAB, in the simplified Circular Restricted Three Body Problem, and then constructed and validated in GMAT with a more complete force model. Both results show values of ΔV and TOF that are close between them with a maximum relative difference of 3.1% (see relative difference percentages in tables 7 and 8).

Furthermore, additional estimations have been performed for the rendezvous strategy ΔV and time, and also for the lunar ascent and descent, which would serve the purpose of dimensioning the technical requirements. The ΔV cost of lunar ascent and descent has been found to be particularly substantial, compared with the total cost of the mission, with combined values of around 3.5 km/s. Together with the rendezvous operation, a rough estimate of 4 km/s is to be expected for the RLRV vehicle for each cycle in-between landings. For the RTLV, the estimate for total ΔV capacity for one mission cycle is estimated at 8 km/s. Finally, extra ΔV costs should be expected for the necessary station-keeping manoeuvres when the spacecraft is in the LPO, which is out of the scope of this work.

Annex A: list of abbreviations

- ***CR3BP*** *– Circular Restricted Three-Body Problem*
- ***DLR*** *– German Aerospace Center*
- ***GMAT*** *– General Mission Analysis Tool*
- ***ISRU*** *– In-Situ Resource Utilization*
- ***LEO*** *– Low Earth Orbit*
- ***LLO*** *– Low Lunar Orbit*
- ***LPO*** *– Libration Point Orbit*
- ***LPO1*** *– Libration Point Orbit for fuel transfer rendezvous*
- ***LPO2*** *– Libration Point Orbit for payload transfer rendezvous*
- ***NASA*** *– National Aeronautics and Space Administration*
- ***NRHO*** *– Near-Rectilinear Halo Orbit*
- ***RLRV*** *– Reusable Lunar Resupply Vehicle*
- ***RTLV*** *– Reusable Trans-Lunar Vehicle*
- ***TOF*** *– Total Time of Flight*
- ***TOSCA*** *- Trajectory Optimization and Simulation of Conventional and Advanced Spacecraft*
- ***SART*** *– Space Launchers Systems Analysis*
- ***SEMAT*** *– Sun-Earth-Moon system in MATlab*

Annex B: Figures

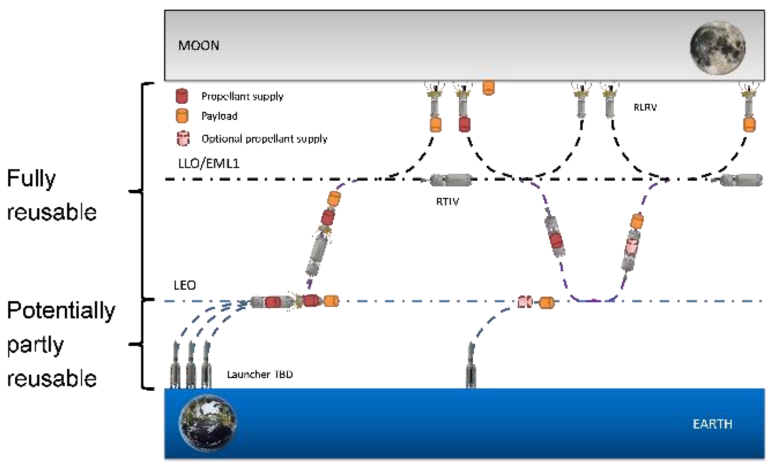

Figure 1. Possible operational scenario for the proposed Moon transportation system.

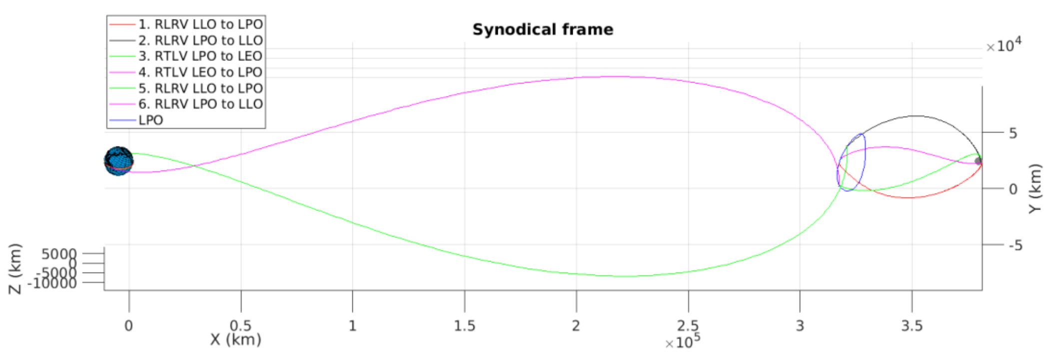

Figure 2. General mission overview, synodical frame of reference. Halo orbit, TOF = 17.31 days, ΔV=11.01 km/s, final equatorial LLO.

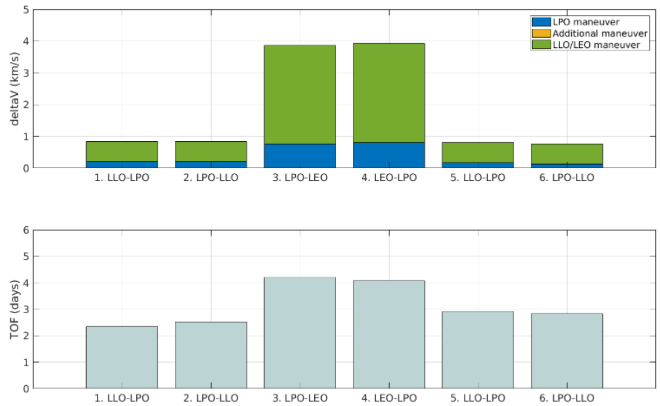

Figure 3. Breakdown of ΔV and TOF for each transfer of the mission. Halo orbit, TOF = 17.31 days, ΔV=11.01 km/s, final equatorial LLO.

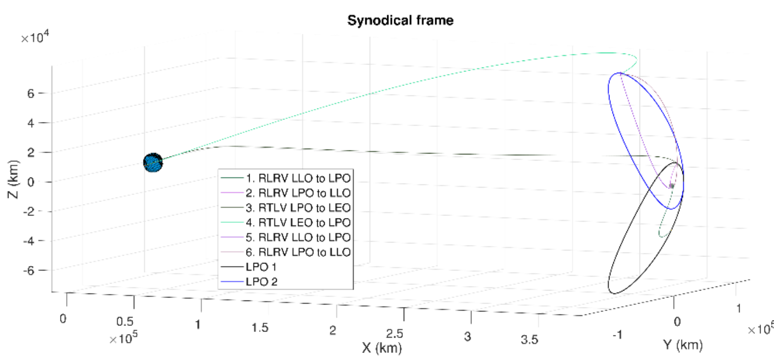

Figure 4. General mission overview, synodical frame of reference. NRHO (Southern – Northern), TOF = 17.87 days, ΔV=11.1 km/s, final polar LLO.

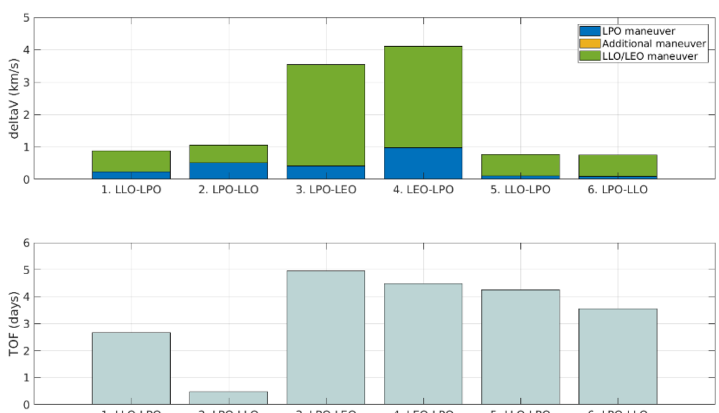

Figure 6. Breakdown of ΔV and TOF for each transfer of the mission. NRHO (Southern – Northern), TOF = 17.87 days, ΔV=11.1 km/s, final polar LLO.

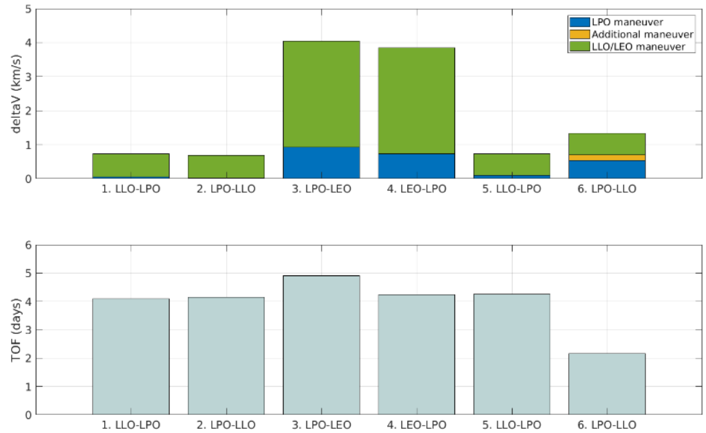

Figure 7. Breakdown of ΔV and TOF for each transfer of the mission. NRHO (Northern) - Halo orbit (Northern), TOF = 16.82 days, ΔV=11.2 km/s, final equatorial LLO.

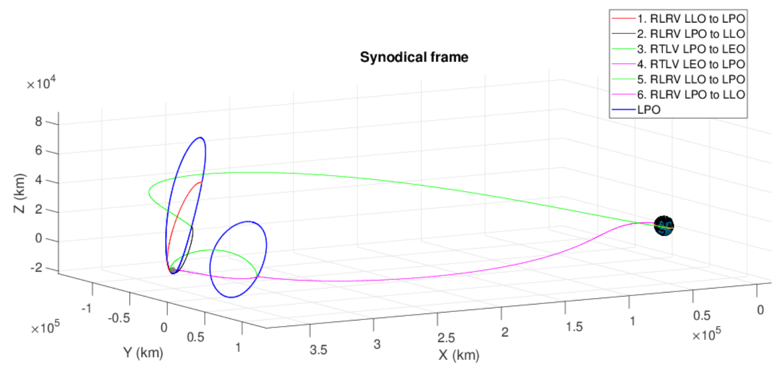

Figure 8. General mission overview, synodical frame of reference. NRHO (Northern) - Halo orbit (Northern), TOF = 16.82 days, ΔV=11.2 km/s, final equatorial LLO.

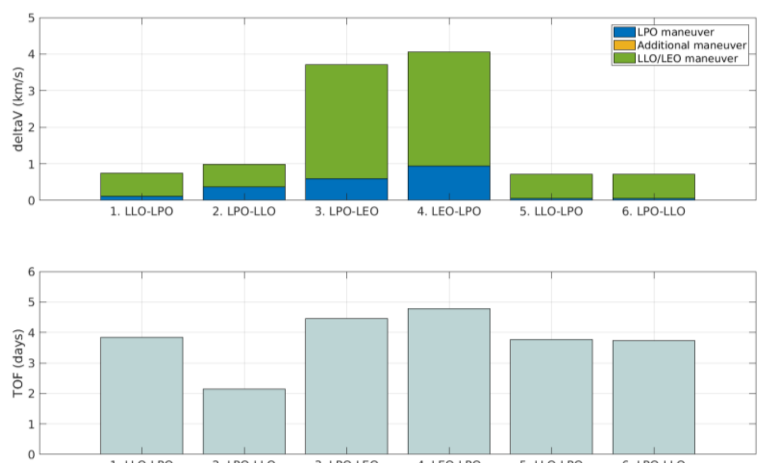

Figure 10. Breakdown of ΔV and TOF for each transfer of the mission. Halo orbit (Northern) – NRHO (Southern), TOF = 19.75 days, ΔV=10.94 km/s, final polar LLO.

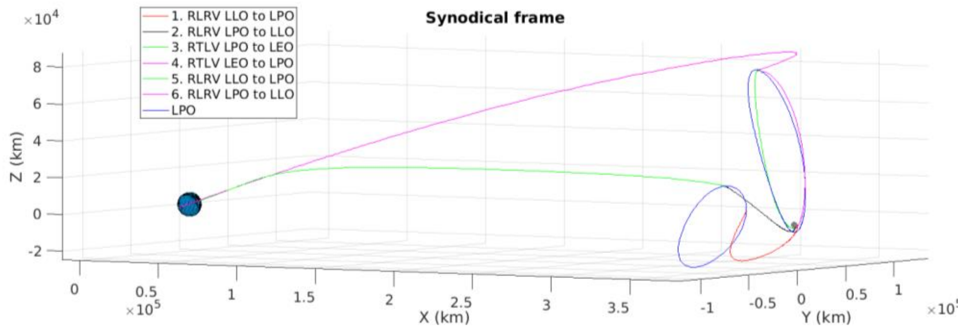

Figure 11. General mission overview, synodical frame of reference. Halo orbit (Southern) – NRHO (Northern), TOF = 19.75 days, ΔV=10.94 km/s, final polar LLO

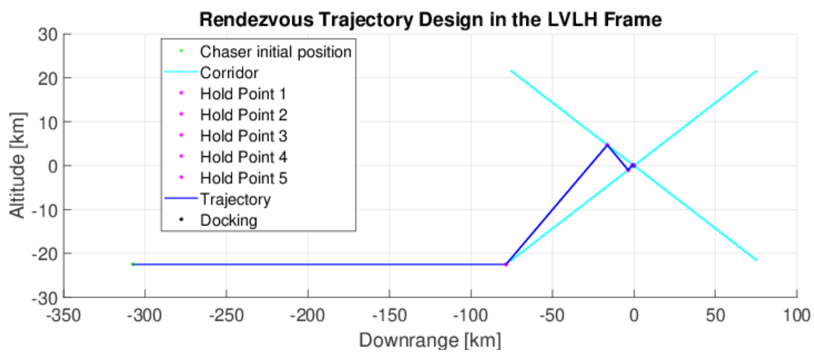

Figure 12. Trajectory design of the rendezvous operation. Hold points are plotted along with the trajectory joining them and the corridor itself.

Annex C: Tables

Table 7. Selected Southern Halo - Northern NRHO mission results comparison between GMAT and MATLAB. Notation for each branch is the same as in table 2.

Results	GMAT ΔV (km/s)	MATLAB ΔV (km/s)	GMAT TOF (days)	MATLAB TOF (days)
RLRV LLO-LPO1	0.734	0.745	3.970	3.833
RLRV LPO 1 - LLO	0.959	0.984	1.847	2.140
RTLV LPO 1 - LEO	3.748	3.709	4.073	4.462
RTLV LEO – LPO 2	4.088	4.063	4.440	4.782
RLRV LLO – LPO 2	0.727	0.719	3.657	3.771
RLRV LPO 2 - LLO	0.757	0.714	3.823	3.734
Total RLRV	3.177	3.162		
Total RTLV	7.837	7.772		
Total	**11.014**	**10.934**	**19.709**	**19.745**
% error (ref. GMAT)	**0.73**		**0.18**	

Table 8. Selected Southern NRHO - Northern NRHO mission results comparison between GMAT and MATLAB.

Results	GMAT ΔV (km/s)	MATLAB ΔV (km/s)	GMAT TOF (days)	MATLAB TOF (days)
RLRV LLO-LPO1	0.887	0.881	2.651	2.655
RLRV LPO 1 - LLO	1.060	1.061	0.653	0.472
RTLV LPO 1 - LEO	3.557	3.536	5.386	4.938
RTLV LEO – LPO 2	4.120	4.101	4.175	4.477
RLRV LLO – LPO 2	0.772	0.761	4.232	4.250
RLRV LPO 2 - LLO	0.784	0.747	3.411	3.553
Total RLRV	3.504	3.450		
Total RTLV	7.678	7.637		
Total	**11.182**	**11.089**	**18.445**	**17.871**
% error (ref. GMAT)	**0.8**		**3.1**	

Table 9. Values of ΔV for the several phases of lunar ascent and descent into LLO (100 km x 100 km).

	Ascent		**Descent**	
	Equator	**South Pole**	**Equator**	**South Pole**
ΔV (m/s)	1815	1820	1768	1772
Circularization ΔV (m/s)	19.7	19.7	0	0
Circularization prop. mass (kg)	88.4	88.1	0	0

References

[1] A., Charles, and J. E. Dennis Jr. “Analysis of Generalized Pattern Searches.” SIAM Journal on Optimization. Volume 13, Number 3, 2003, pp. 889–903

[2] W. S. Koon, M. W. Lo, J. E. Marsden, and D. R. Shane, Dynamical Systems, the Three-Body Problem and Space Mission Design. Marsden Books, 2008.

[3] E. Dumont, W. Goh, M. Delbourg, S. Lizy-Destrez and B. Le Bihan, Suitability of reusability and in-situ propellant production for a Lunar transportation system, IAC-17-D2.3.1, 68th International Astronautical Congress (IAC).

[4] S. Manglaviti, A. Campolo, B. L. Bihan, S. Lizy-Destrez, Safety analysis for near rectilinear orbit close approach rendezvous in the circular restricted three-body problem, IAC-17-D2.3.1, 68th International Astronautical Congress (IAC), Adelaide, Australia, 2017, 25-29 September.

[5] F. V. Bennet, Apollo Lunar Ascent and Descent Trajectories, AIAA 8th aerospace sci. meeting, New York, 1970, 21 January

[6] V. Quet, Le Rendez-vous Spatial sur des Trajectoires en Orbite au Voisinage des Points de Lagrange du Système Terre-Lune pour des Missions Habitées, ISAE-SUPAERO, 2018.

[7] A. Campolo, Safety analysis for near rectilinear orbit close approach rendezvous in the circular restricted three-body problem (final thesis), Politecnico di Milano, ISAE-SUPAERO, 2017.

[8] H. Zeng, J. Zhang and R. Qi, Study of Time-Free Transfers into Libration Point Orbits with Multiple Constraints, Journal of Guidance, Control and Dynamics, 2017

[9] General Mission Analysis Tool (GMAT) User Guide, version R2018a, NASA

[10] Audet, Charles, and J. E. Dennis Jr. “Analysis of Generalized Pattern Searches.” SIAM Journal on Optimization. Volume 13, Number 3, 2003, pp. 889–903.

[11] Petrović, Stacha and Dumont, Etienne. “Lunar Landing Simulations With Tosca” (2015) DLR-Forschungsbericht, DLR-Interner Bericht. SART TN-006/2015, 165S.