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Verification and Validation of a 2D energy based peridynamic state-based failure criterion

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The paper presents a validation approach for an enhanced energy-based failure model for state-based peridynamics. Model robustness and validity of predicted crack initiation and propagation is demonstrated for a double cantilever beam. Tensile tests using the same material and energy release rate from the prior simulations are used to predict specimen failure. The model is implemented in the software Peridigm and will be published open-source.

1 Introduction

Motivated by ideas of molecular dynamics and to overcome the deficits of the classical continuum and fracture mechanics models, Stewart Silling developed the fundamental Peridynamics theory in the early 2000's as an alternative approach. In this theory the fundamental partial differential equations of the momentum conservation used in classical continuum mechanics is replaced by an integral equation. Singularities at discontinuities are avoided. Following the assumptions and notations from Silling [4], within the neighborhood $\mathcal{H}$ with the volume V_q, defined by a spherical domain the horizon δ, the force volume density state $\underline{\mathbf{T}}$ for the bond interaction between the positions $\mathbf{x}$ and $\mathbf{x}'$ is defined as the integral balance of momentum

$$\int_{\mathcal{H}} (\underline{\mathbf{T}}(\mathbf{x},t)\langle \mathbf{x}'-\mathbf{x}\rangle - \underline{\mathbf{T}}(\mathbf{x}',t)\langle \mathbf{x}-\mathbf{x}'\rangle) dV_{\mathbf{q}} + \mathbf{b} = \rho\ddot{\mathbf{u}}. \tag{1}$$

Thereby, $\mathbf{b}$ is the external force density vector, ρ the mass density and $\ddot{\mathbf{u}}$ the acceleration. To model damages within a material, specific criteria for the initiation are needed, which utilize measurable parameters. Foster et al. [2] described an energy-based failure criterion which is valid for state-based peridynamic analysis. The assumption is that each bond stores a maximum elastic potential. The model was implemented in the numerical framework Peridigm and verified in [5]. This paper will present the first validation attempts. For this purpose a peridynamic plane stress formulation is used to reduce the computational effort and compared with KIC experiments utilizing a double cantilever beam.

2 Theory

The maximum elastic bond potential value w_c was derived by Foster et al. [2] based on the energy release rate G_0 and the horizon δ. The elastic bond potential is given in equation 2 (a) for three-dimensional and equation 2 (b) for the two-dimensional case with thickness h. If w_c exceeds a critical valued damage initiates due to a bond failure. This criterion has been implemented in the open-source peridynamic code Peridigm [3,5].

$$w_c = \frac{4G_0}{\pi\delta^4} \quad \text{(a)} \qquad\qquad w_c = \frac{3G_0}{\pi\delta^3 h} \quad \text{(b)} \tag{2}$$

Following Bobaru et al. [1] the force density scalar state $\underline{t}$ for the plane stress state can be determined as

$$\underline{t}(\mathbf{q}) = \chi(\underline{e},t)\omega\left(\frac{4K(\mathbf{q})G(\mathbf{q})\theta}{(3K(\mathbf{q})+4G(\mathbf{q}))m_v(\mathbf{q})}\underline{x} + \frac{8G\theta(\mathbf{q})}{m_v(\mathbf{q})}\underline{e}^d(\mathbf{q})\right) \tag{3}$$

with

$$\theta(\mathbf{q}) = \frac{3}{m_v(\mathbf{q})}\int_{\mathcal{H}(\mathbf{q})} \omega\underline{x}\underline{e}dV \quad \text{and} \quad \underline{e}^d(\mathbf{q}) = \underline{e} - \frac{4G(\mathbf{q})\theta\underline{x}}{9K(\mathbf{q})+12G(\mathbf{q})} \tag{4}$$

By replacing $\mathbf{q}$ with $\mathbf{x}$ and $\mathbf{x}'$ the force density scalar state for the corresponding opposite point can be obtained. Therein, K is the compression modulus, G the shear modulus, $\underline{e}^d$ the deviatoric part of the bond extension scalar state, m_v the weighted volume, θ the dilatation, t the time and $\underline{x}$ the undeformed scalar state.

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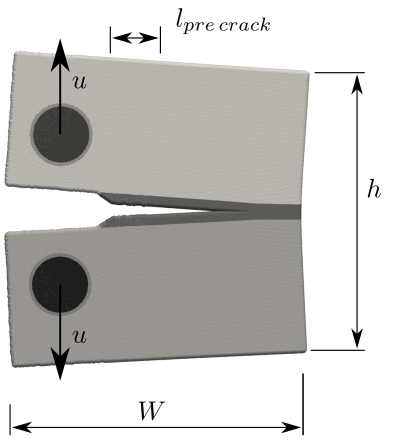
3 Results

The experimental setup is shown in the Fig. 1a. Bolts (dark circle-surfaces) can rotate free within experiment. An external displacement u is applied to the bolts center. The model parameters are varied (see Fig. 1b) in order to reflect the scatter in the experimental test results.



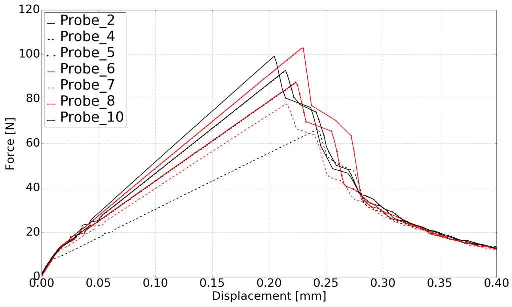
(a) Virtual test setup – ISO 13586:2000(E) ($W = 35mm$, $h = 41\,mm$, $\nu = 0.33$, and $\rho = 1000kgm^{-3}$).

Parameter	mean val.	lower bound	upper bound
$G_0\ [N/m]$	0.274	0.243	0.311
$l_{pre\,crack}\ [mm]$	19.33	16.72	21.56
$B\ [mm]$	5.27	5.48	5.48
$E\ N/mm^2$	1962	1828	2069

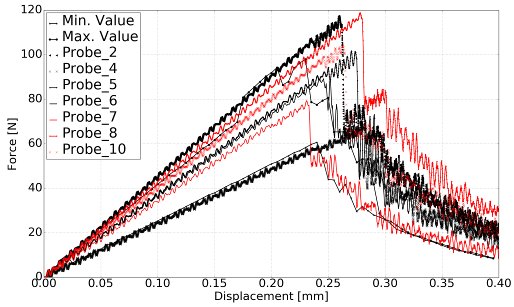
(b) Input data measured from the experimental specimens.

Fig. 1: KIC model of the virtual experiment.

Fig. 2a shows the experimental results. In terms of stiffness the numerical results (cf. Fig. 2b) fit well to the experimental results. The decrease of the load response due to crack initiation is also well represented. However, the variation of the input parameters significantly influences the calculated stiffness as well as the predicted damage initiation. For all simulated specimens the damage initiates too late. This means that the maximum load carrying capacity of the material is overestimated by approximately 20%.
Comparisons with tensile tests also showed that the material shows strong plastic properties, which probably have to be taken into account. Therefore, an extension of the material and damage model is currently taking place.



(a) Experimental results.



(b) Comparison of the numerical and experimental results.

Fig. 2: Experimental and numerical results.

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