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Aeroelastic Analysis of Highly Flexible Aircraft Structures under Large Deformations

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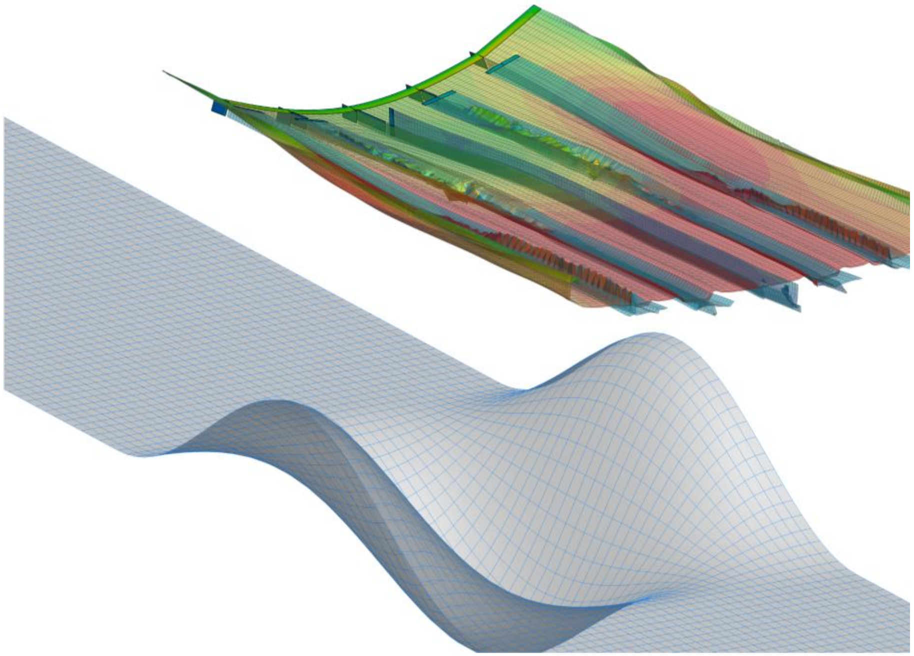
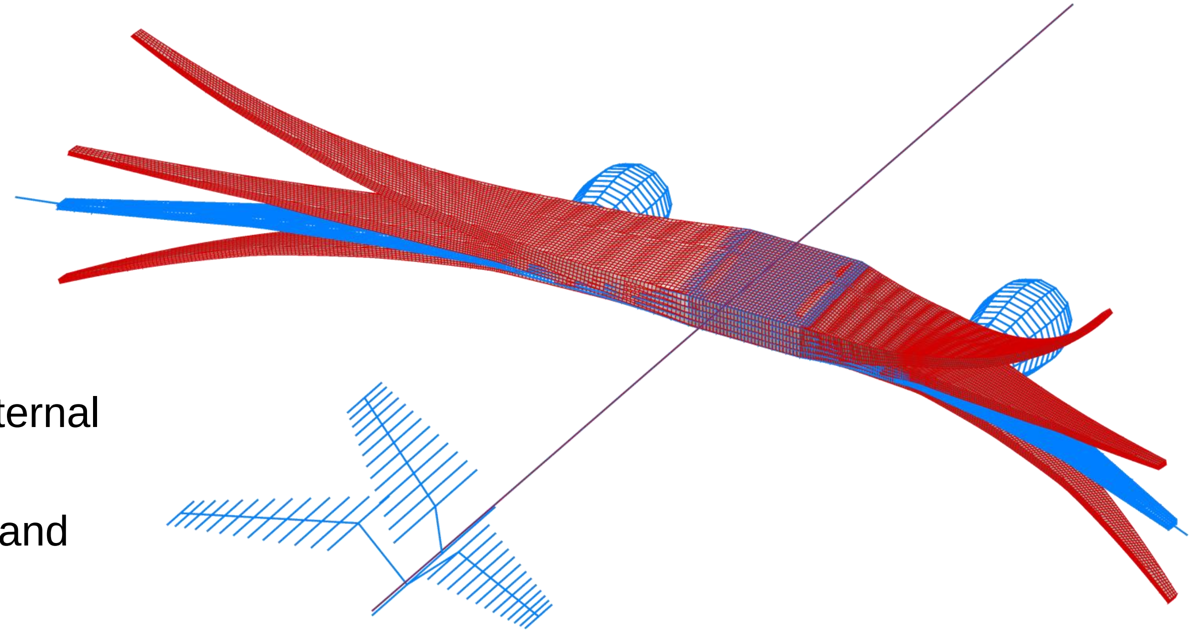


Motivation

- Large elastic deflections are inherent in particular types of A/Cs: HALEs, UAVs, Sailplanes
- Large deformations as the result of aerodynamic optimization
- e.g. Open Class sailplanes with very high aspect ratios
- HALEs and UAVs
  - extreme lightweight and span-loaded design
  - low overall wing deflection at design point but prone to atmospheric disturbances

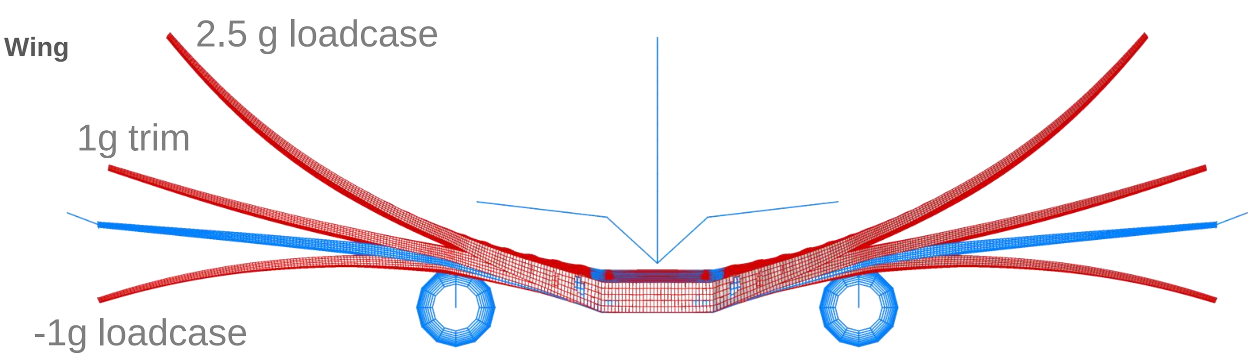
Motivation

- Jet transport aircraft are entering the regime of large deformations
- Bringing more flexibility into jet transport wings is a future design goal of aircraft industry



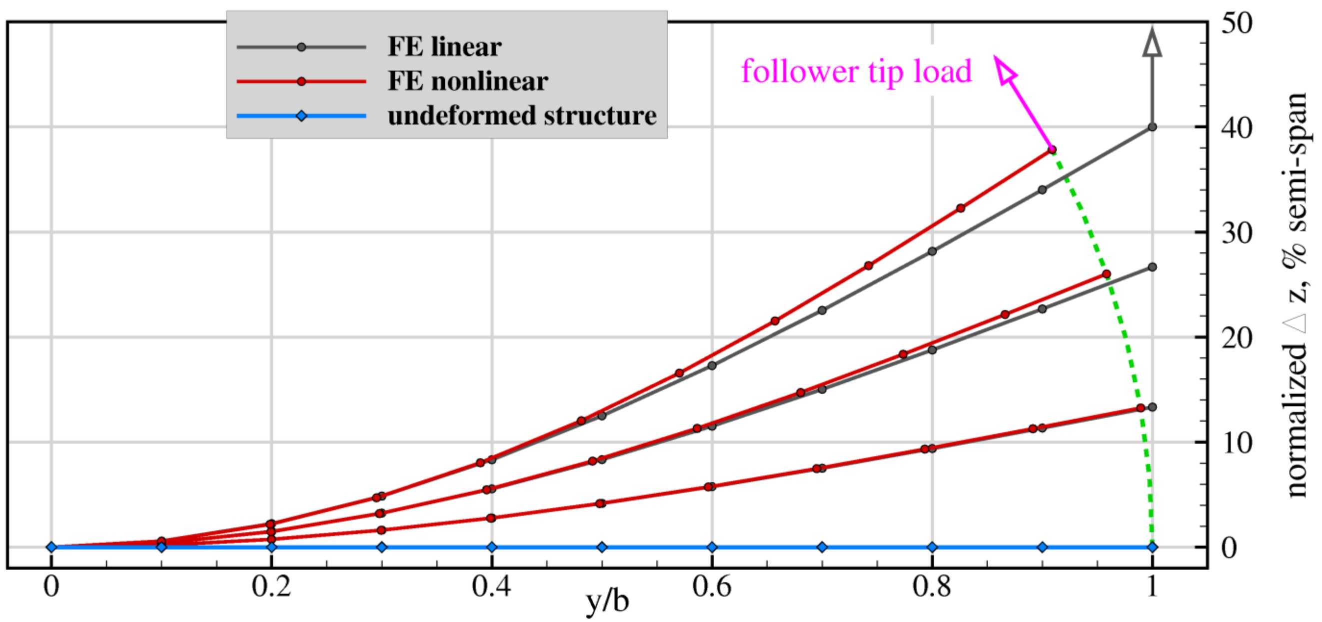
- Thinner airfoil → higher Ma_{crit}
- Absorption of energy from external disturbances
- Reduction of structural loads and increase of comfort

Markus Zimmer, **Design of a Highly Flexible Wing Structure**, ATLAS² Hybrid Project



Problem Statement

- Future aeroelastic analysis and design programs must account for nonlinearities
  - geometrically nonlinear aerodynamics (rotation of forces)
  - geometrically nonlinear structural dynamics (changes in mass and stiffness) ?
  - large rigid-body motions coupled with elastic deflections (stability, disturbances) ?
- Standard analysis and design approaches are not suitable for highly flexible A/C
- Aerodynamic methods are already advanced, but what about structural dynamics?



State of the Art – Industrial Practice

- Lagrangian methods make the state of the art in nonlinear FE analysis
  - total and updated Lagrangian (TL, UL, e.g. in Nastran *SOL400*)
  - loads are applied stepwise, kinematics and stiffness matrices are updated each step
- High computational costs due to iterative solution
- Commercial FE codes with these methods do not allow for rigid-body motions

Updating sequence of Lagrangian formulation

$$^{(n+1)}\boldsymbol{f} - {}^{(n+1)}\boldsymbol{r} = 0$$

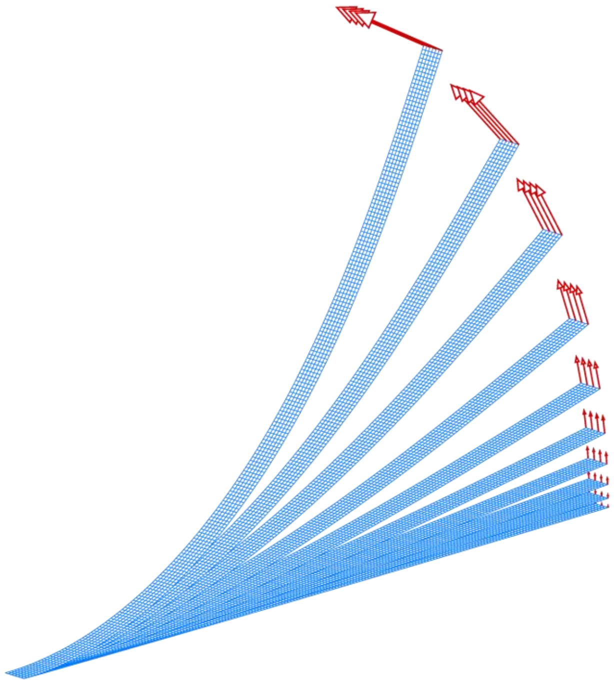
$$^{(n+1)}\boldsymbol{r} = {}^{(n)}\boldsymbol{r} + \Delta\boldsymbol{r}$$

$$\Delta\boldsymbol{r} = {}^{(n)}\boldsymbol{K}^t \Delta\boldsymbol{u}$$

$$^{(n)}\boldsymbol{K}^t = \frac{\partial\, {}^{(n)}\boldsymbol{r}}{\partial\, {}^{(n)}\boldsymbol{u}}$$

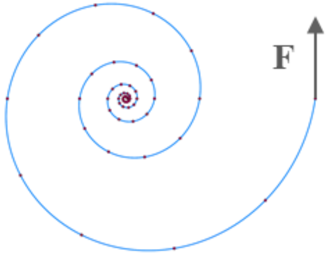
$$^{(n)}\boldsymbol{K}^t\, \Delta\boldsymbol{u} = {}^{(n+1)}\boldsymbol{f} - {}^{(n)}\boldsymbol{r}$$

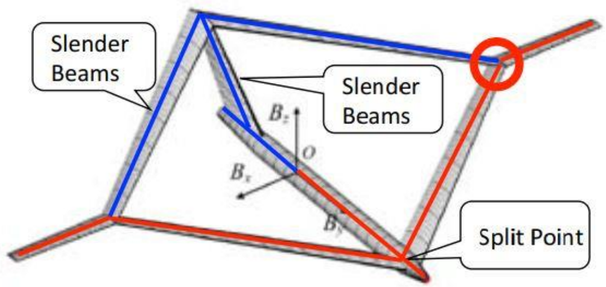
$$^{(n+1)}\boldsymbol{u} = {}^{(n)}\boldsymbol{u} + \Delta\boldsymbol{u}$$

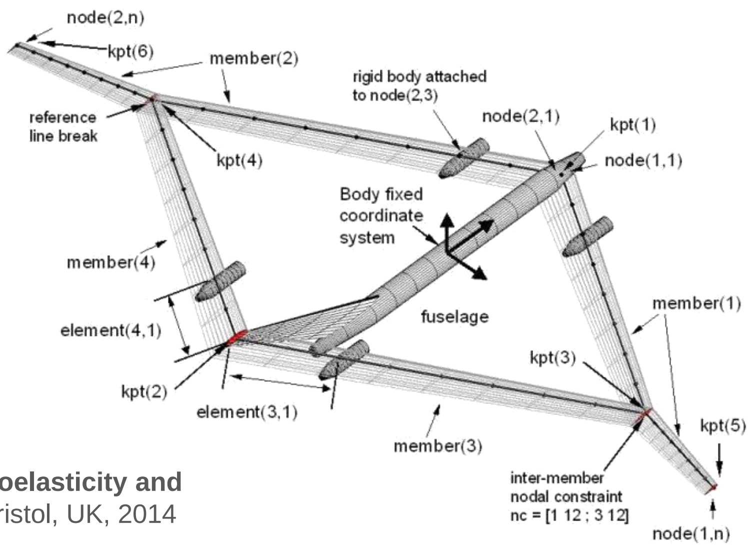


State of the Art – Research

- For flight dynamics of highly flexible free-flying aircraft, mostly *beam* type models are applied
  - *geometrically exact* beam theories (intrinsic, strain-based) are well-established
  - condensation of complex 3D FEM into "equivalent" beam model requires assumptions







C. E. S. Cesnik, **Workshop on Nonlinear Aeroelasticity and Flight Dynamics of Very Flexible Aircraft**, Bristol, UK, 2014

- ROMs for nonlinear structural dynamics account for nonlinear load-displacement behavior by *quadratic* and *cubic stiffness terms* and linear and dual modes for displacements*
- Modal expansions in terms of *quadratic modes* are used for rotating and aerospace applications to improve kinematic relations‡

* M. P. Mignolet et al., **A review of indirect/non-intrusive reduced order modeling of nonlinear geometric structures**, JSV, 2013

‡ L. H. van Zyl, E. H. Mathews, **Quadratic Mode Shape Components From Linear Finite Element Analysis**, ASME, 2011

Research Objective

- An improved structural method for geometrically nonlinear aeroelastic analyses is desired
- Main ideas of the new method:
  - Consider moderately large deflections, $\mathcal{O}(< 30\%)$
  - Assume nonlinear nodal displacement field is still composed of "modes"
  - Identify differences to linear modal approach and find extensions

Linear modal approach

$$\boldsymbol{u} = \boldsymbol{\Phi}\,\boldsymbol{q}$$

$$\boldsymbol{M}\ddot{\boldsymbol{q}} + \boldsymbol{D}\dot{\boldsymbol{q}} + \boldsymbol{\Omega}^2\boldsymbol{q} = \boldsymbol{Q}$$

→

Nonlinear extensions

$$\boldsymbol{u} = \boldsymbol{\Phi}\,\boldsymbol{q} + \widetilde{\boldsymbol{\Phi}}(\boldsymbol{q}^2, \boldsymbol{q}^3, \boldsymbol{q}^4) \qquad (1)$$

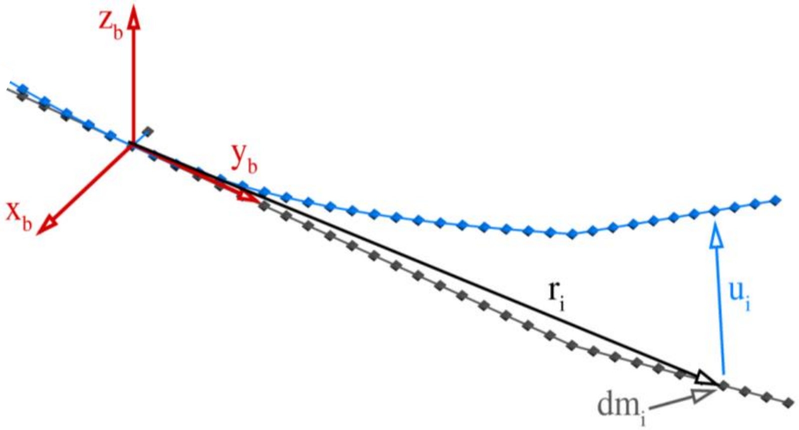
$$\boldsymbol{M}\ddot{\boldsymbol{q}} + \boldsymbol{D}\dot{\boldsymbol{q}} + \boldsymbol{G}(\boldsymbol{q}, \boldsymbol{q}^2, \boldsymbol{q}^3, \boldsymbol{f}) = \boldsymbol{Q} \qquad (2)$$

- Subject to:
  - low computational costs (few DOFs: modal space, no iterative solution)
  - applicable to complex/arbitrary FE models
  - easy extension for rigid-body motions

1. Higher-order modal components yield geometrical nonlinearities
2. Nonlinear stiffness terms for nonlinear force-displacement relations

Theoretical Derivation: Static Structural Equations

- Geometrically nonlinear displacements are represented by *higher-order mode components*



$$\boldsymbol{u}(\boldsymbol{q}) = \underbrace{{}^{p}\boldsymbol{\Phi}_0}_{\text{linear}}\, q_p + \underbrace{{}^{p}\boldsymbol{\Phi}_1^{i}}_{\text{quadratic}}\, q_p q_i + \underbrace{{}^{p}\boldsymbol{\Phi}_2^{ij}}_{\text{cubic}}\, q_p q_i q_j + \underbrace{{}^{p}\boldsymbol{\Phi}_3^{ijk}}_{\text{quartic/fourth order}}\, q_p q_i q_j q_k$$

mode components: linear quadratic cubic quartic/fourth order

The mode itself becomes a <u>function of the amplitude</u> $(\boldsymbol{q})$:

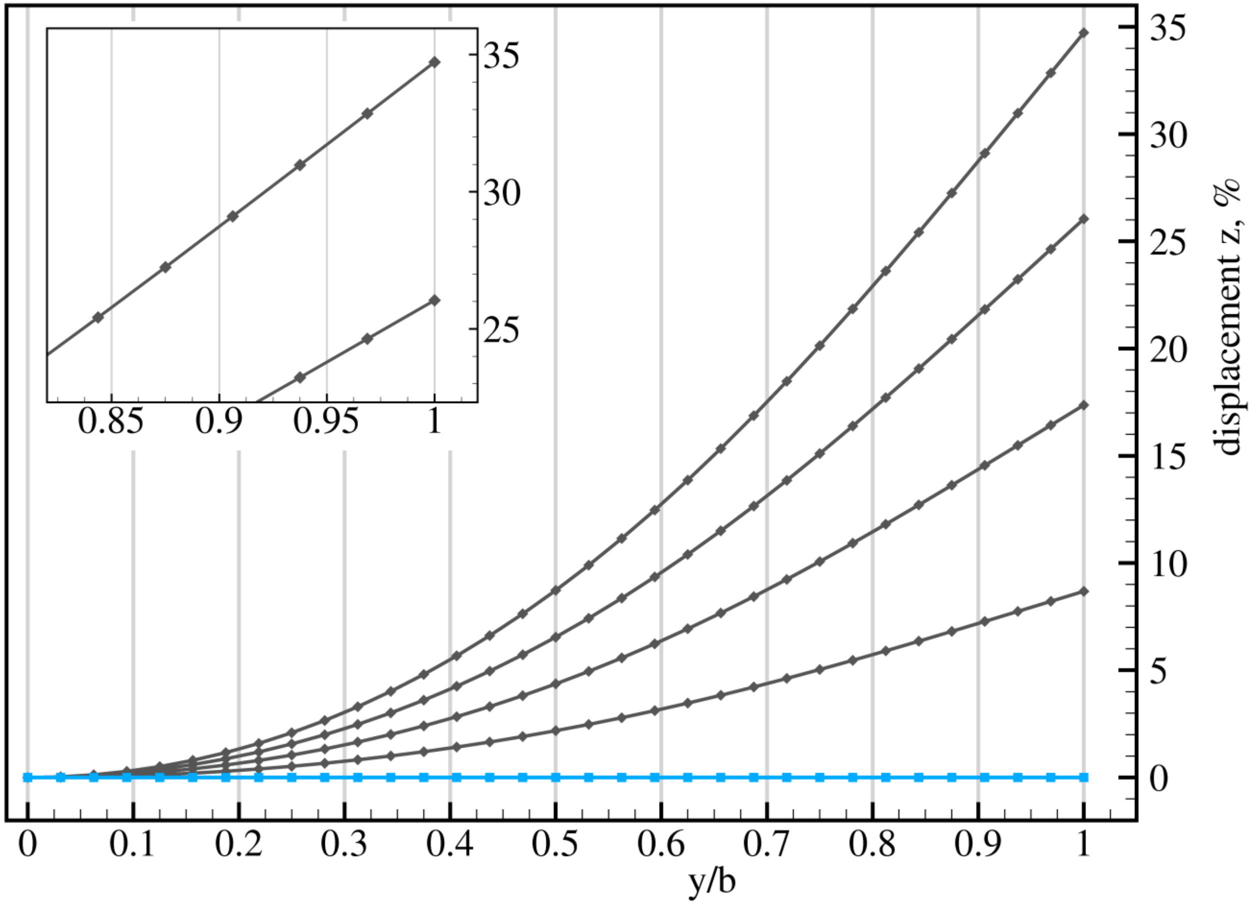
$$ {}^{p}\boldsymbol{\Phi}(\boldsymbol{q}) = {}^{p}\boldsymbol{\Phi}_0 + 2\,{}^{p}\boldsymbol{\Phi}_1^{i}\, q_i + 3\,{}^{p}\boldsymbol{\Phi}_2^{ij}\, q_i q_j + 4\,{}^{p}\boldsymbol{\Phi}_3^{ijk}\, q_i q_j q_k$$

${}^{p}\boldsymbol{\Phi}_1^{i}$, ${}^{p}\boldsymbol{\Phi}_2^{ij}$, ${}^{p}\boldsymbol{\Phi}_3^{ijk}$ tensors enable a "geometric coupling" of several modes

Theoretical Derivation: Static Structural Equations

- For illustration: Mode components of the first bending mode of a cantilever beam
- Displacements from linear mode component ("normal mode")

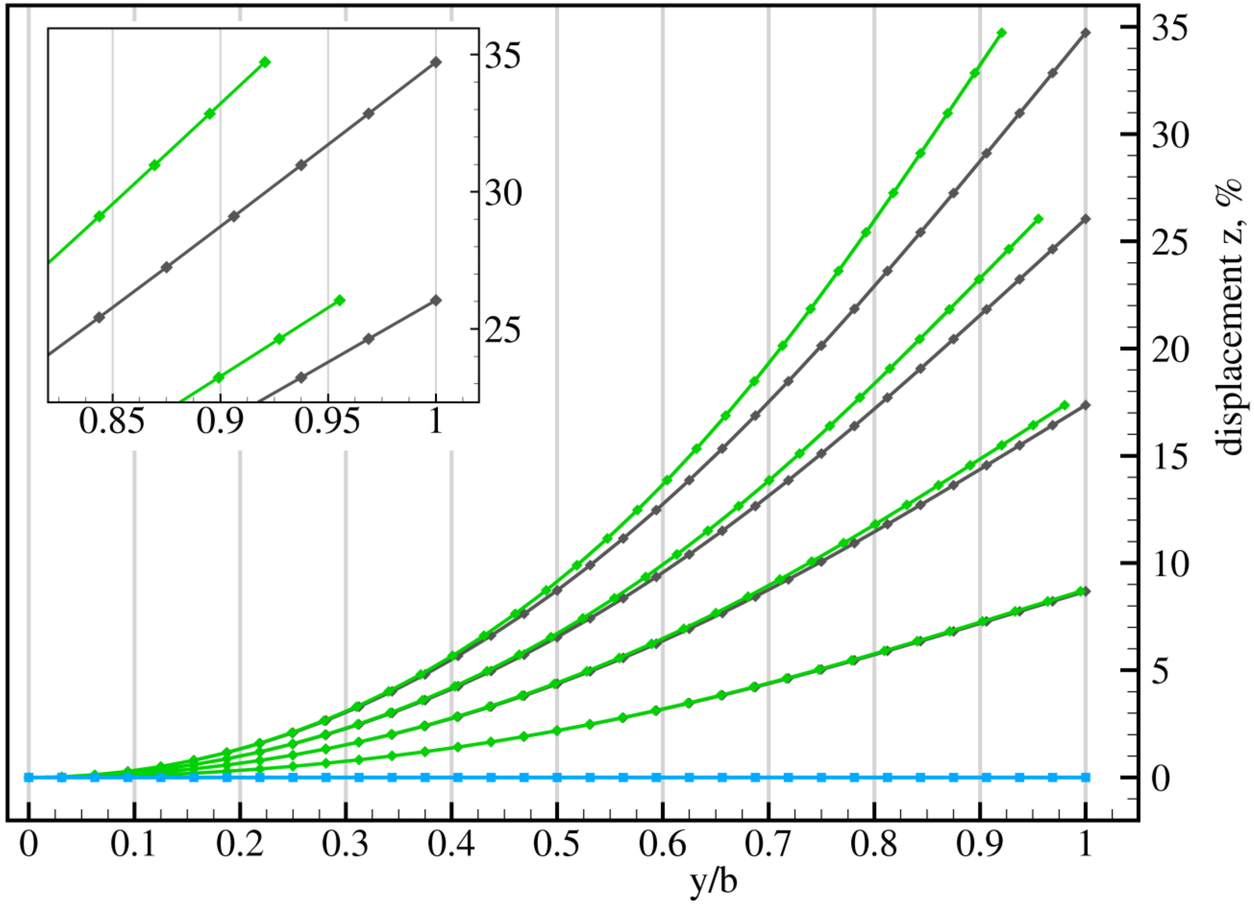
$$\boldsymbol{u}(\boldsymbol{q}) = {}^{1}\boldsymbol{\Phi}_0\, q_1$$



Theoretical Derivation: Static Structural Equations

- For illustration: Mode components of the first bending mode of a cantilever beam
- Displacements from linear and quadratic mode component

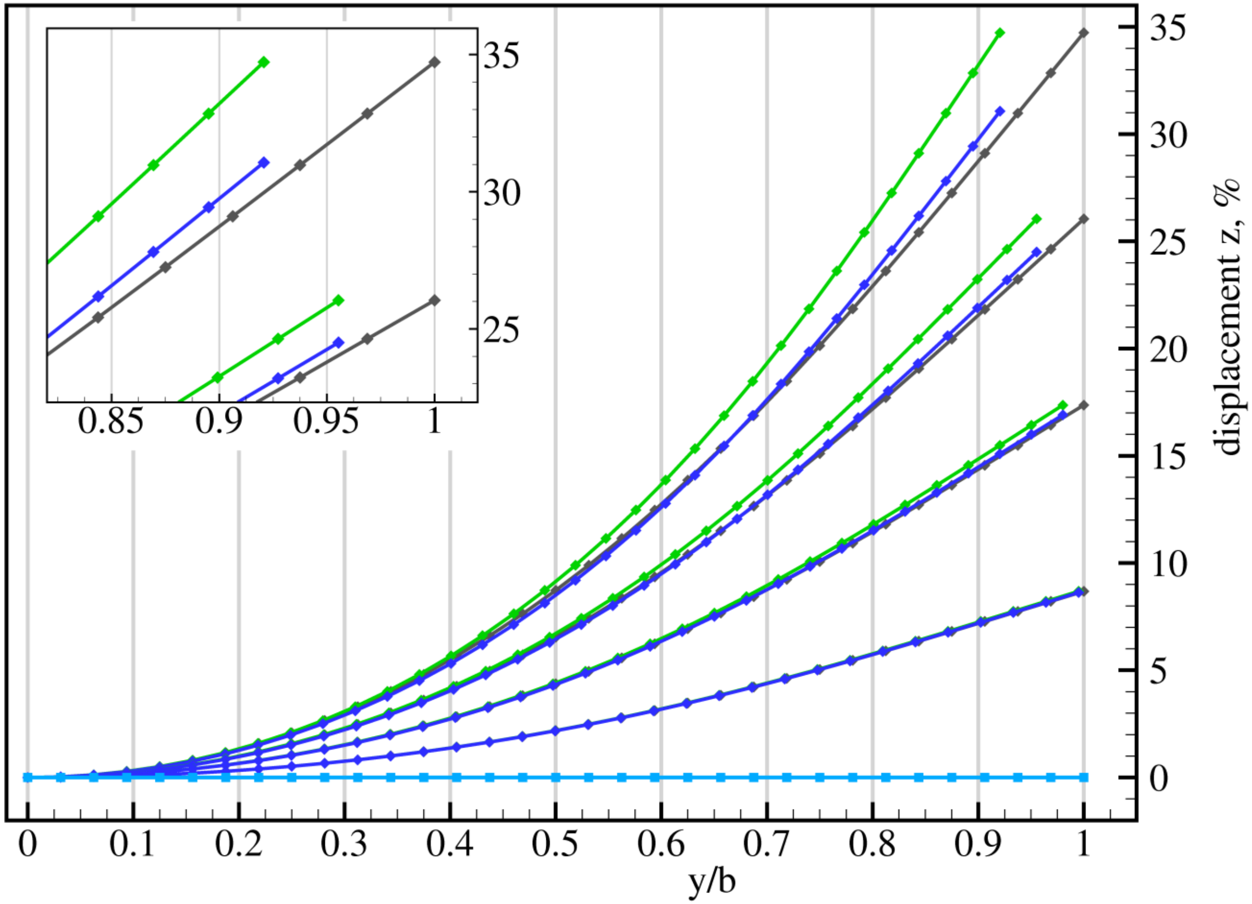
$$\boldsymbol{u}(\boldsymbol{q}) = {}^{1}\boldsymbol{\Phi}_0 \; q_1 + {}^{1}\boldsymbol{\Phi}_1^1 \, q_1 q_1$$



Theoretical Derivation: Static Structural Equations

- For illustration: Mode components of the first bending mode of a cantilever beam
- Displacements from linear, quadratic, and third order mode component

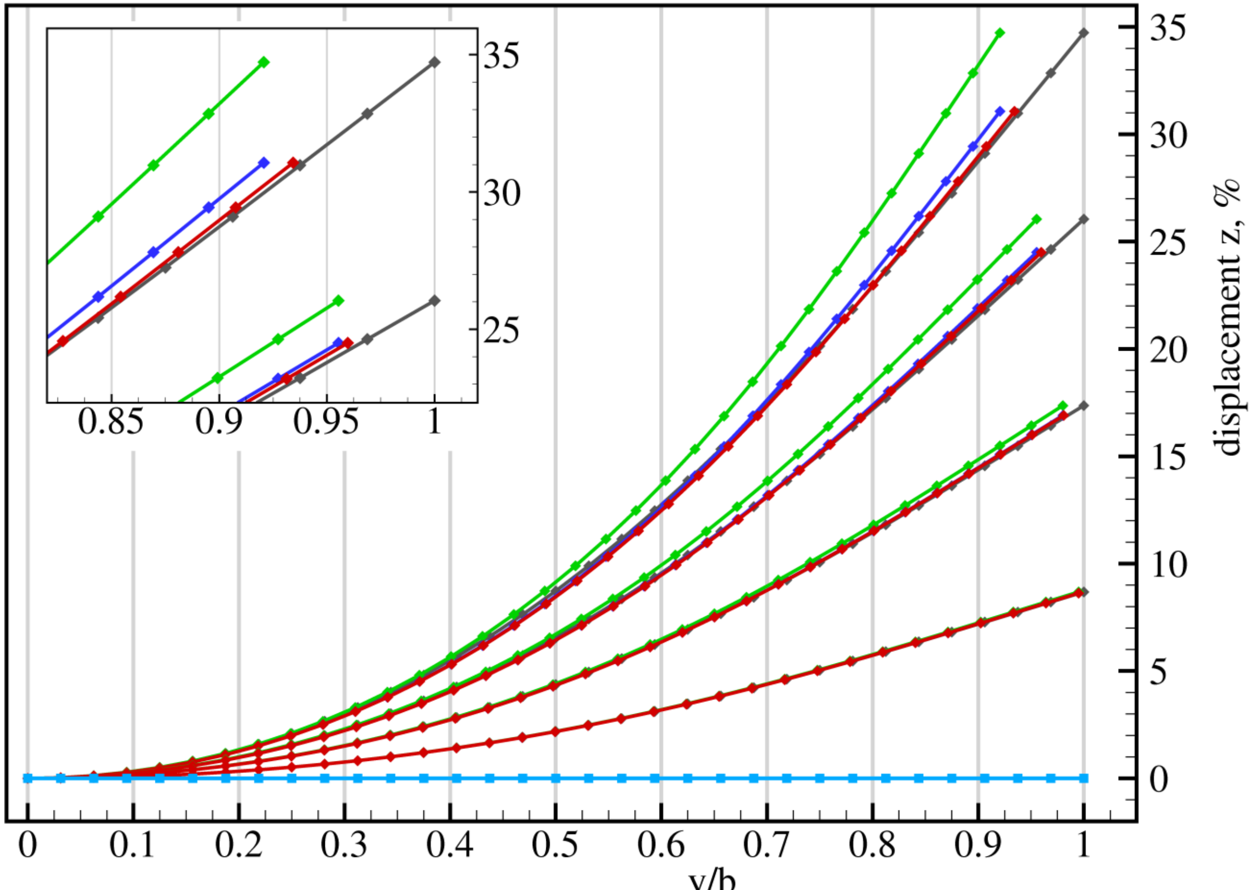
$$\boldsymbol{u}(\boldsymbol{q}) = {}^{1}\boldsymbol{\Phi}_{0}\ q_1 + {}^{1}\boldsymbol{\Phi}_{1}^{1}\ q_1 q_1 + {}^{1}\boldsymbol{\Phi}_{2}^{11}\ q_1 q_1 q_1$$



Theoretical Derivation: Static Structural Equations

- For illustration: Mode components of the first bending mode of a cantilever beam
- Displacements from linear, quadratic, third, and fourth order mode component

$$\boldsymbol{u}(\boldsymbol{q}) = {}^{1}\boldsymbol{\Phi}_{0}\; q_1 + {}^{1}\boldsymbol{\Phi}_{1}^{1}\; q_1 q_1 + {}^{1}\boldsymbol{\Phi}_{2}^{11}\; q_1 q_1 q_1 + {}^{p}\boldsymbol{\Phi}_{3}^{111}\; q_1 q_1 q_1 q_1$$



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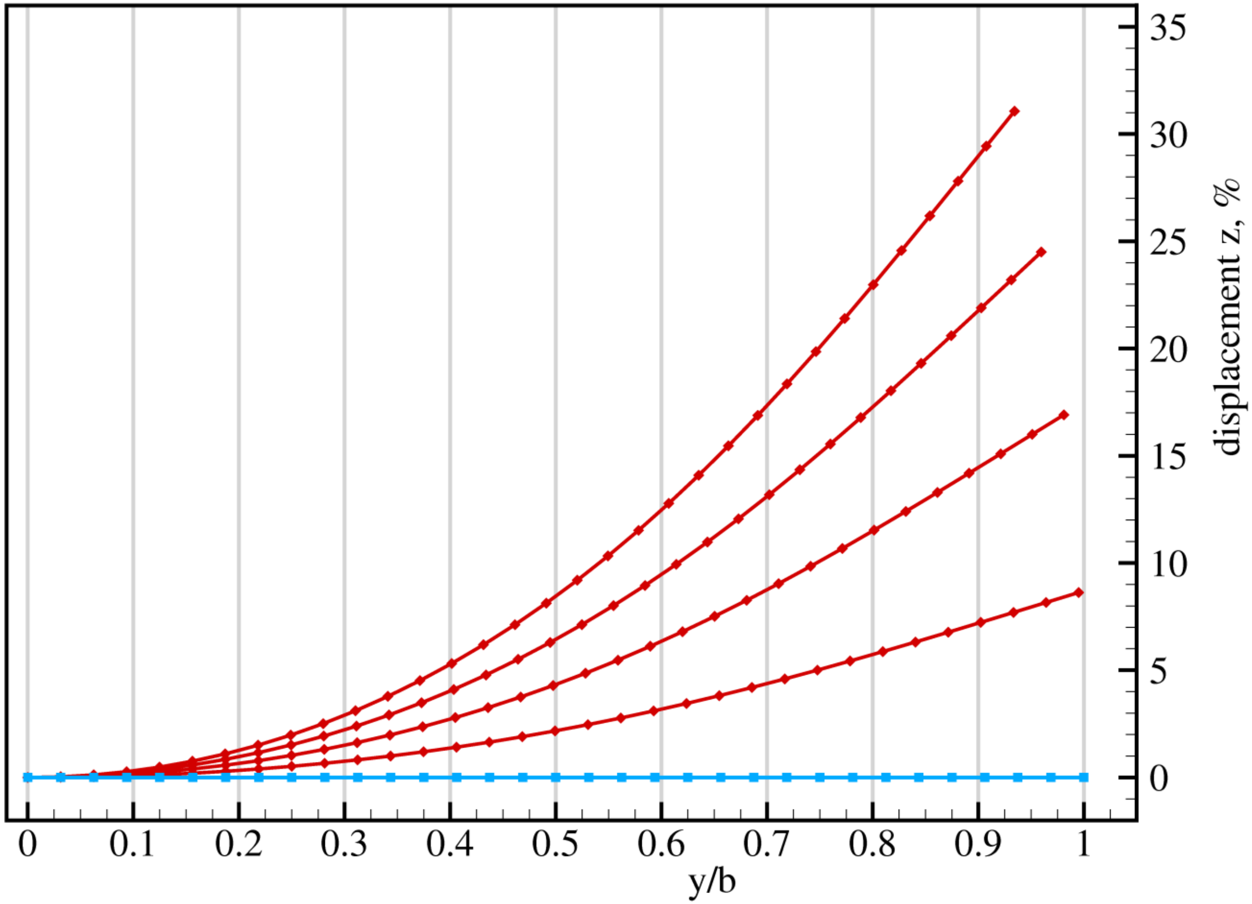


Theoretical Derivation: Static Structural Equations

- Expansion of strain energy of a discretized structure in a Taylor series centered at zero (without linear term here):

$$U_s(q) = \frac{1}{2!}\sum_{i=1}^{m}\sum_{j=1}^{m}\frac{\partial^2 U_s}{\partial q_i \partial q_j} q_i q_j + \frac{1}{3!}\sum_{i=1}^{m}\sum_{j=1}^{m}\sum_{k=1}^{m}\frac{\partial^3 U_s}{\partial q_i \partial q_j \partial q_k} q_i q_j q_k + \frac{1}{4!}\sum_{i=1}^{m}\sum_{j=1}^{m}\sum_{k=1}^{m}\sum_{l=1}^{m}\frac{\partial^4 U_s}{\partial q_i \partial q_j \partial q_k \partial q_l} q_i q_j q_k q_l + \text{h.o.t.}$$

$U_s(q)$ = strain energy; q_i = generalized coordinates

- Application of Castigliano's first theorem yields steady governing equation in generalized coordinates:

$${}^{p}G_1^{i} q_i + {}^{p}G_2^{ij} q_i q_j + {}^{p}G_3^{ijk} q_i q_j q_k = Q^p \ ; \ (p = 1, \ldots, m)$$

- Quadratic and cubic stiffness terms account for nonlinear force-displacement relation

${}^{p}G_1^{i}$ is comparable to the linear stiffness matrix of the structure (eigenvalues)

${}^{p}G_2^{ij}$ and ${}^{p}G_3^{ijk}$ are tensors of 3rd and 4th order which enable coupling of several modes

Theoretical Derivation: Static Structural Equations

- Generalized forces, Q^p, are calculated by application of the PVW

$$\delta V = \delta \boldsymbol{u}^T \boldsymbol{f}$$

- Expanding $\delta \boldsymbol{u}$ using the linear and the quadratic mode shape components:

$$\delta V = \delta q_p^T \left({}^p\boldsymbol{\Phi}_0^T + {}^p\boldsymbol{\Phi}_1^{i\,T} q_i \right) \boldsymbol{f}$$

- Generalized forces are then given as:

$$Q^p = {}^p\boldsymbol{\Phi}_0^T \boldsymbol{f} + {}^p\boldsymbol{\Phi}_1^{i\,T} \boldsymbol{f}\, q_i$$

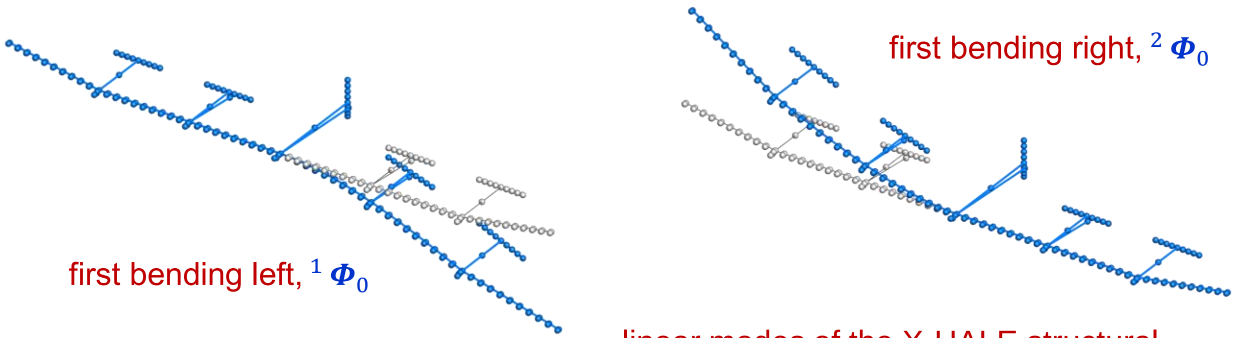
- Combining yields the final steady governing equation of the proposed method:

$$\left({}^p G_1^i \underline{- {}^p\boldsymbol{\Phi}_1^{i\,T} \boldsymbol{f}} \right) q_i + {}^p G_2^{ij} q_i q_j + {}^p G_3^{ijk} q_i q_j q_k = {}^p\boldsymbol{\Phi}_0^T \boldsymbol{f}$$

- Linear stiffness term is a function of the applied loads (comparable to a *tangent stiffness*)

Theoretical Derivation: EOMs Free-Flying Elastic Aircraft

- The equations derived so far enable geometrically nonlinear structural analysis
- The higher-order stiffness and mode tensors are calculated in pre-processing
- However, they can be calculated for <u>clamped structures</u> only



linear modes of the X-HALE structural model <u>clamped at the center</u>

- Clamped mode shapes require a formulation of flight dynamics EOMs with inertial coupling
- Lagrange's equations of the second kind are used for the derivation
- No mean axes assumptions

$$\begin{bmatrix} M_{tt} & M_{tr} & M_{te} \\ M_{rt} & M_{rr} & M_{re} \\ M_{et} & M_{er} & M_{ee} \end{bmatrix} \begin{bmatrix} \dot{V}_b \\ \dot{\omega}_b \\ \ddot{q} \end{bmatrix} + \begin{bmatrix} 0 & D_{tr} & 0 \\ D_{rt} & D_{rr} & D_{re} \\ D_{et} & D_{er} & D_{ee} \end{bmatrix} \begin{bmatrix} V_b \\ \omega_b \\ \dot{q} \end{bmatrix} + \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & \boldsymbol{G} \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ q \end{bmatrix} = \begin{bmatrix} \tilde{Q}_t \\ \tilde{Q}_r \\ \tilde{Q}_e \end{bmatrix}$$

t = translational
r = rotational
e = elastic

$$M, D, G = f(\dot{V}_b, V_b, \dot{\omega}_b, \omega_b, \ddot{q}, q, q)$$

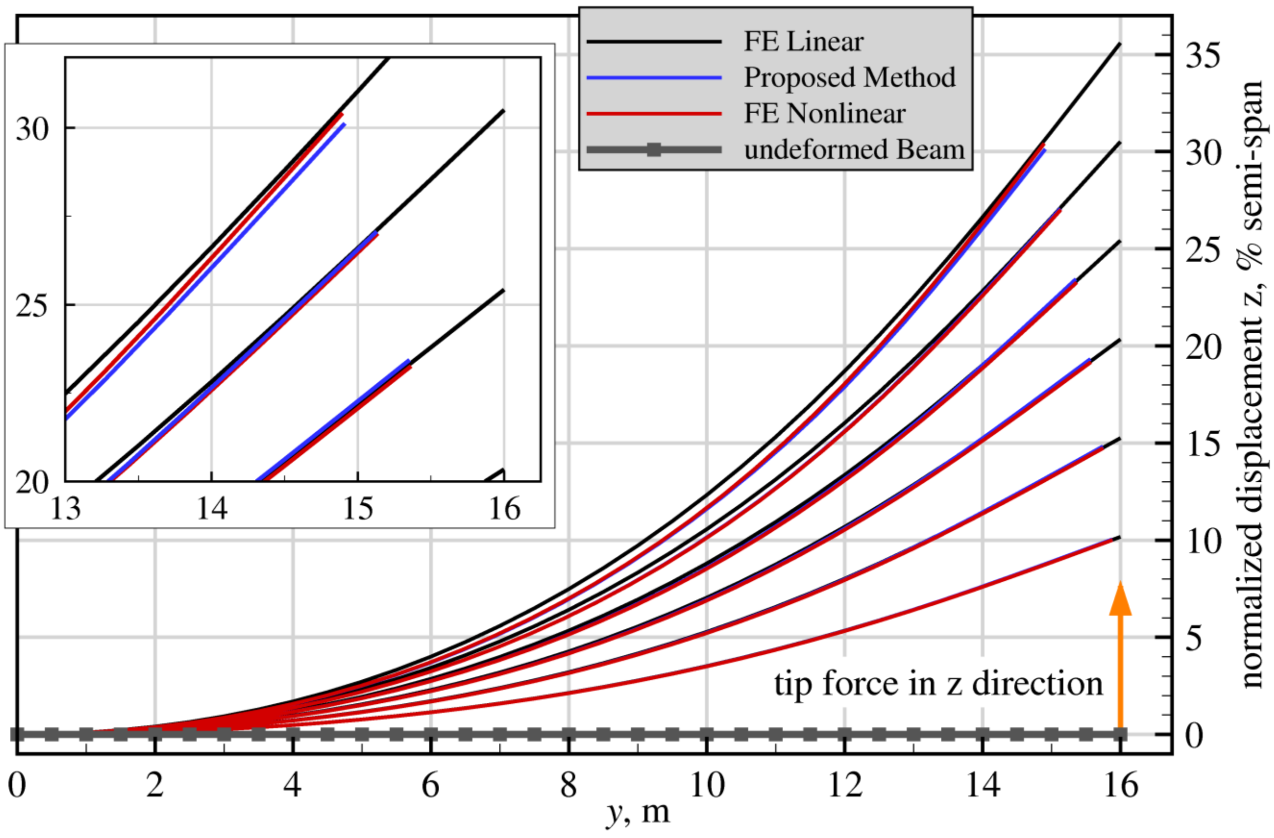
M. Ritter, J. Jones, C. E. S. Cesnik, **Free-flight Nonlinear Aeroelastic Simulations of the X-HALE UAV by an Extended Modal Approach**, IFASD, 2017

Static Structural Validation: 16m Cantilever Beam

- Static deformation for tip forces along the z direction

$$\left({}^{p}G_{1}^{i} - {}^{p}\boldsymbol{\Phi}_{1}^{i\,T}\boldsymbol{f} \right) q_i + {}^{p}G_{2}^{ij} q_i q_j + {}^{p}G_{3}^{ijk} q_i q_j q_k = {}^{p}\boldsymbol{\Phi}_{0}^{T}\,\boldsymbol{f}$$

$$\boldsymbol{u}(\boldsymbol{q}) = {}^{p}\boldsymbol{\Phi}_{0}\; q_p + {}^{p}\boldsymbol{\Phi}_{1}^{i}\; q_p q_i + {}^{p}\boldsymbol{\Phi}_{2}^{ij}\; q_p\, q_i q_j + {}^{p}\boldsymbol{\Phi}_{3}^{ijk}\; q_p q_i q_j q_k$$



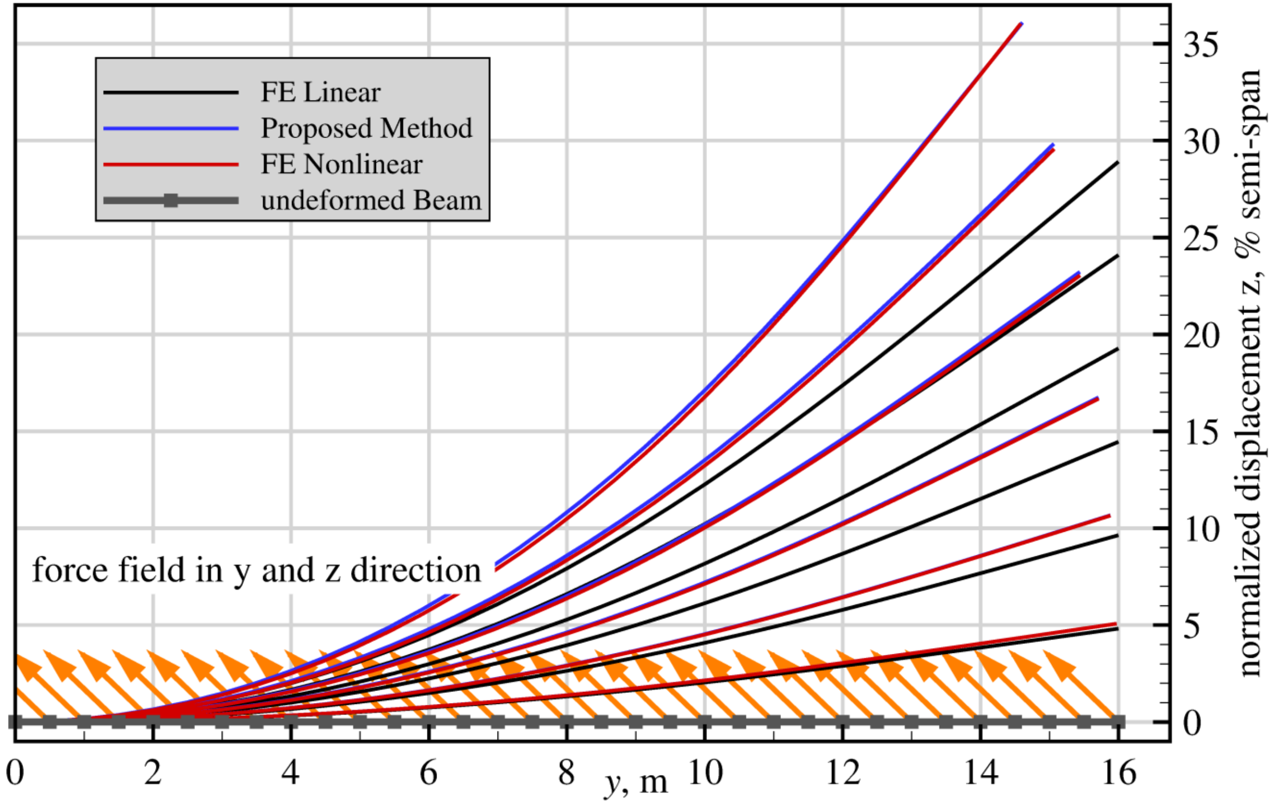
Static Structural Validation: 16m Cantilever Beam

- Static deformation for tip forces along the z and the y direction

$$\left({}^{p}G_1^i - {}^{p}\boldsymbol{\Phi}_1^{i\,T}\boldsymbol{f}\right) q_i + {}^{p}G_2^{ij} q_i q_j + {}^{p}G_3^{ijk} q_i q_j q_k = {}^{p}\boldsymbol{\Phi}_0^T \boldsymbol{f}$$

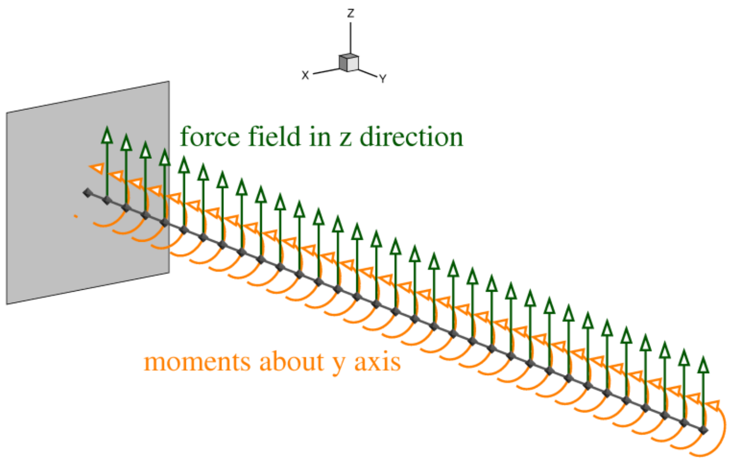
$$\boldsymbol{u}(\boldsymbol{q}) = {}^{p}\boldsymbol{\Phi}_0\, q_p + {}^{p}\boldsymbol{\Phi}_1^i\, q_p q_i + {}^{p}\boldsymbol{\Phi}_2^{ij}\, q_p q_i q_j + {}^{p}\boldsymbol{\Phi}_3^{ijk}\, q_p q_i q_j q_k$$

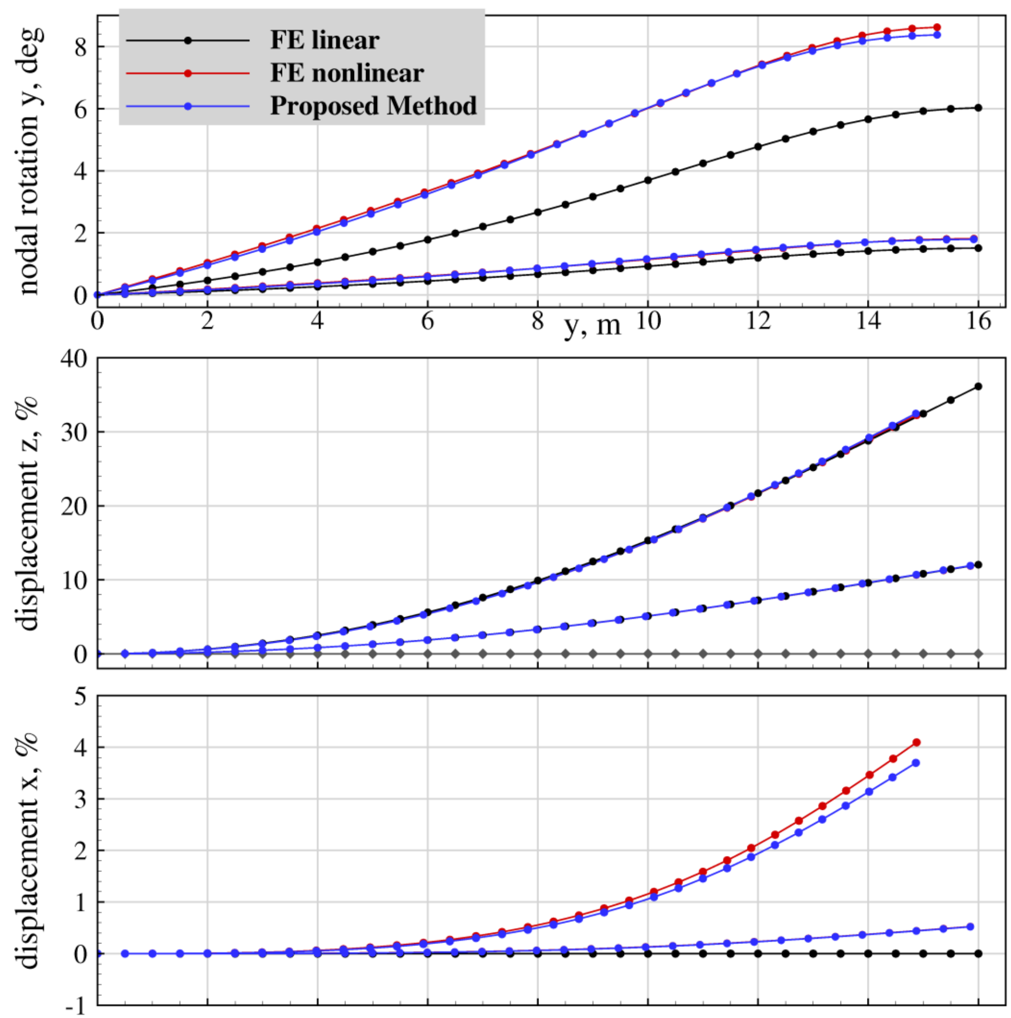
Linear stiffness term becomes a function of the force field



Static Structural Validation: 16m Cantilever Beam

- Static bending deformation for forces along the z direction and moments about the y axis

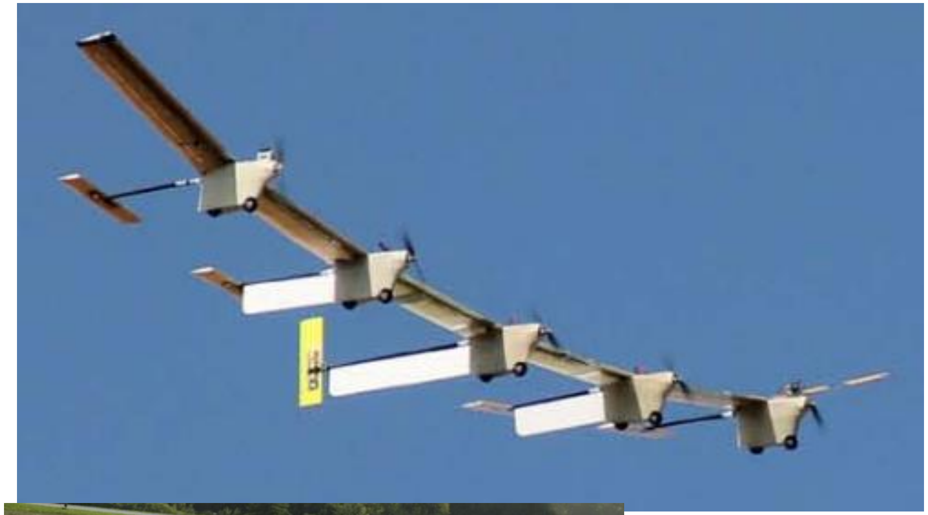


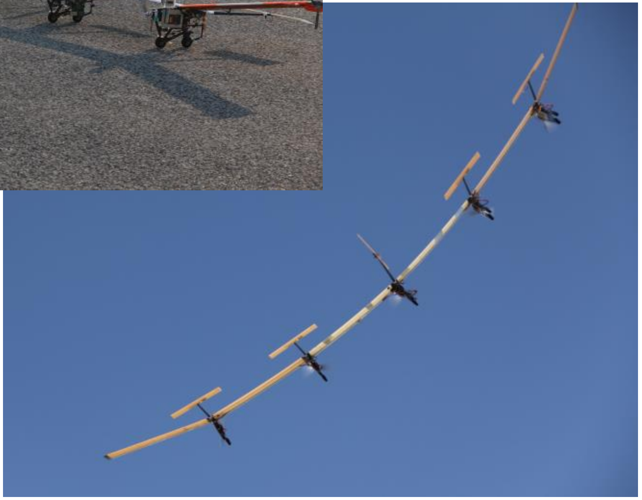


Flight Dynamics Simulation: X-HALE UAV

- Properties of the X-HALE UAV from the University of Michigan

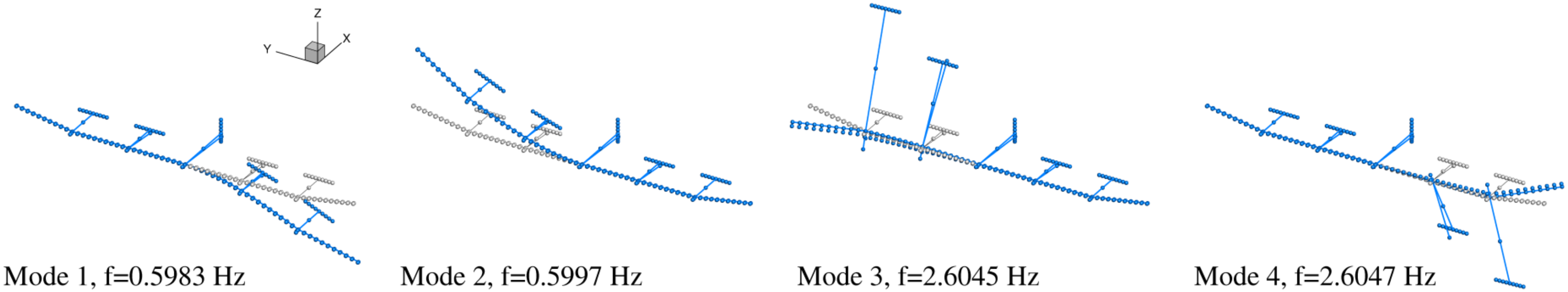
Property	***X-HALE 6m RRV***
Span (wing)	6 m
Mean chord (wing)	0.2 m
Area (wing)	1.2 m^2
Airfoil (wing)	EMX07 reflex
Aspect ratio (wing)	30
Flight velocity (trim)	16 m/s
Mass	10.86 kg
max. Engine thrust	5×8.5 N
Materials	Fiberglass/graphite/foam



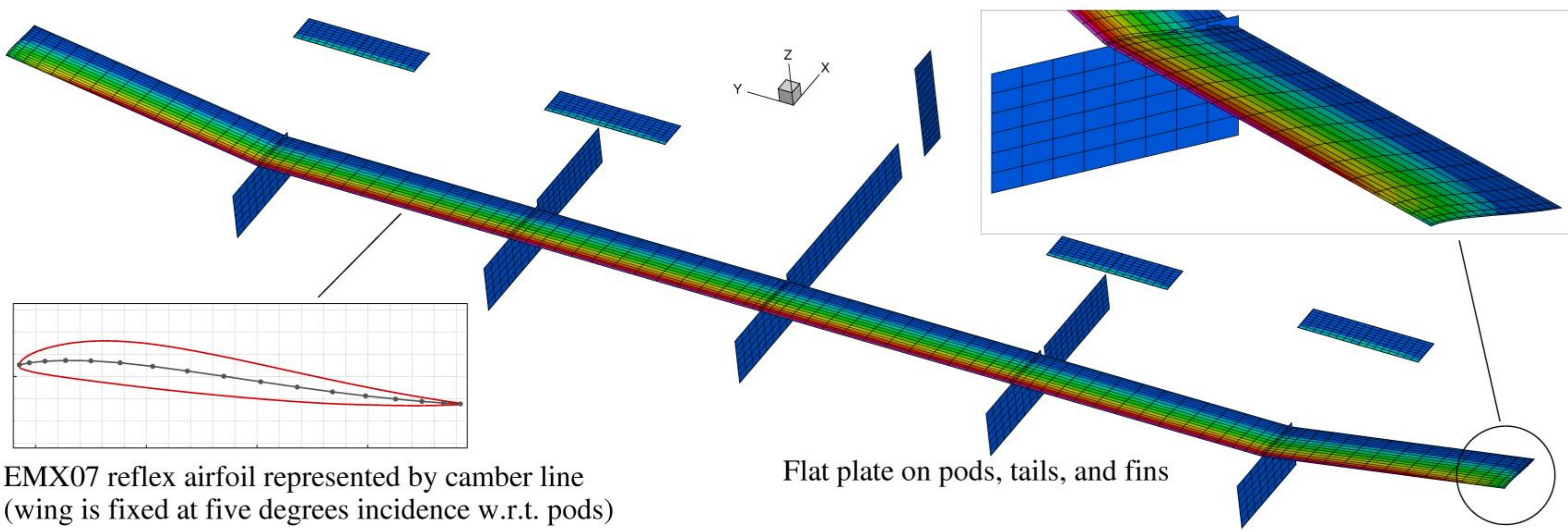


Flight Dynamics Simulation: X-HALE UAV

- Nastran FEM with beam elements and discrete masses based on UM/NAST input files
- Mode components calculated in pre-processing appear in pairs left/right



Mode 1, f=0.5983 Hz Mode 2, f=0.5997 Hz Mode 3, f=2.6045 Hz Mode 4, f=2.6047 Hz



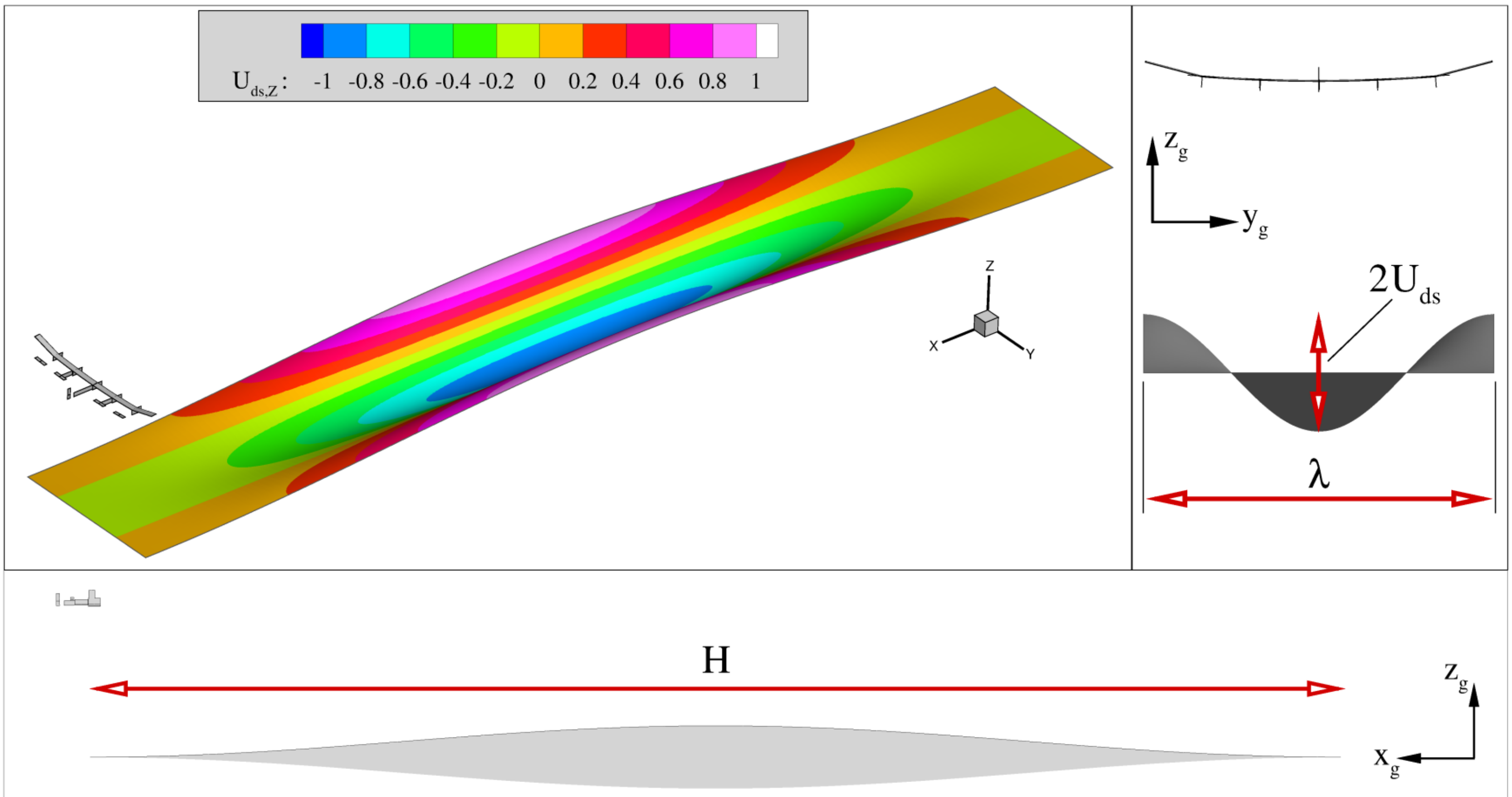
EMX07 reflex airfoil represented by camber line
(wing is fixed at five degrees incidence w.r.t. pods)

Flat plate on pods, tails, and fins

Application: X-HALE DARPA Gust

- DARPA gust maneuver with $V_0 = 16$ m/s, $U_{ds} = -1.5$ m/s, $H = 40$ m, $\lambda = 6$ m, $\Phi = 0$

$$U_{ds,Z}(x_g, y_g) = \frac{1}{2} U_{ds} \left(1 - cos\left(\frac{2\pi x_g}{H}\right)\right) cos\left(\frac{\pi y_g}{\lambda} - \phi\right)$$



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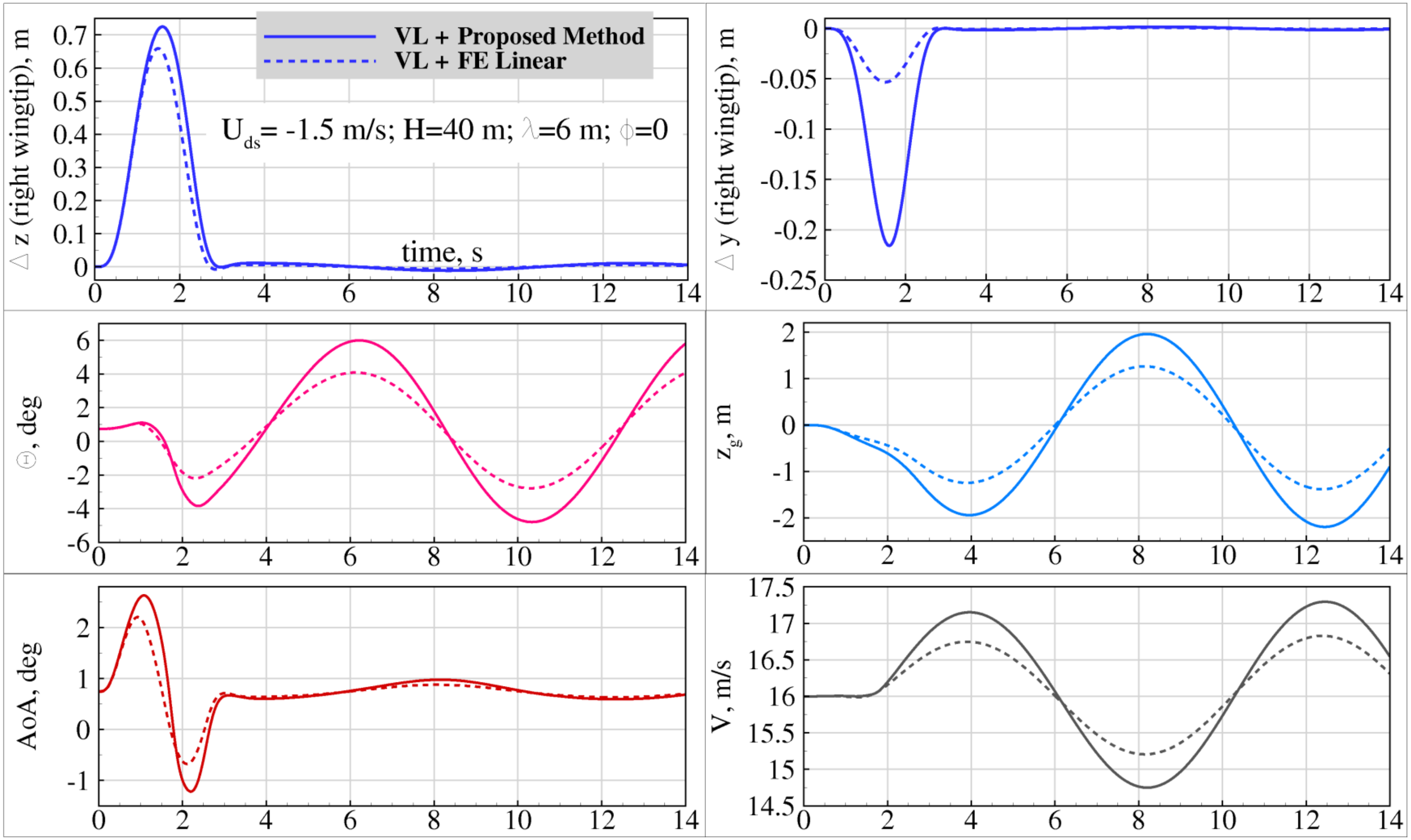
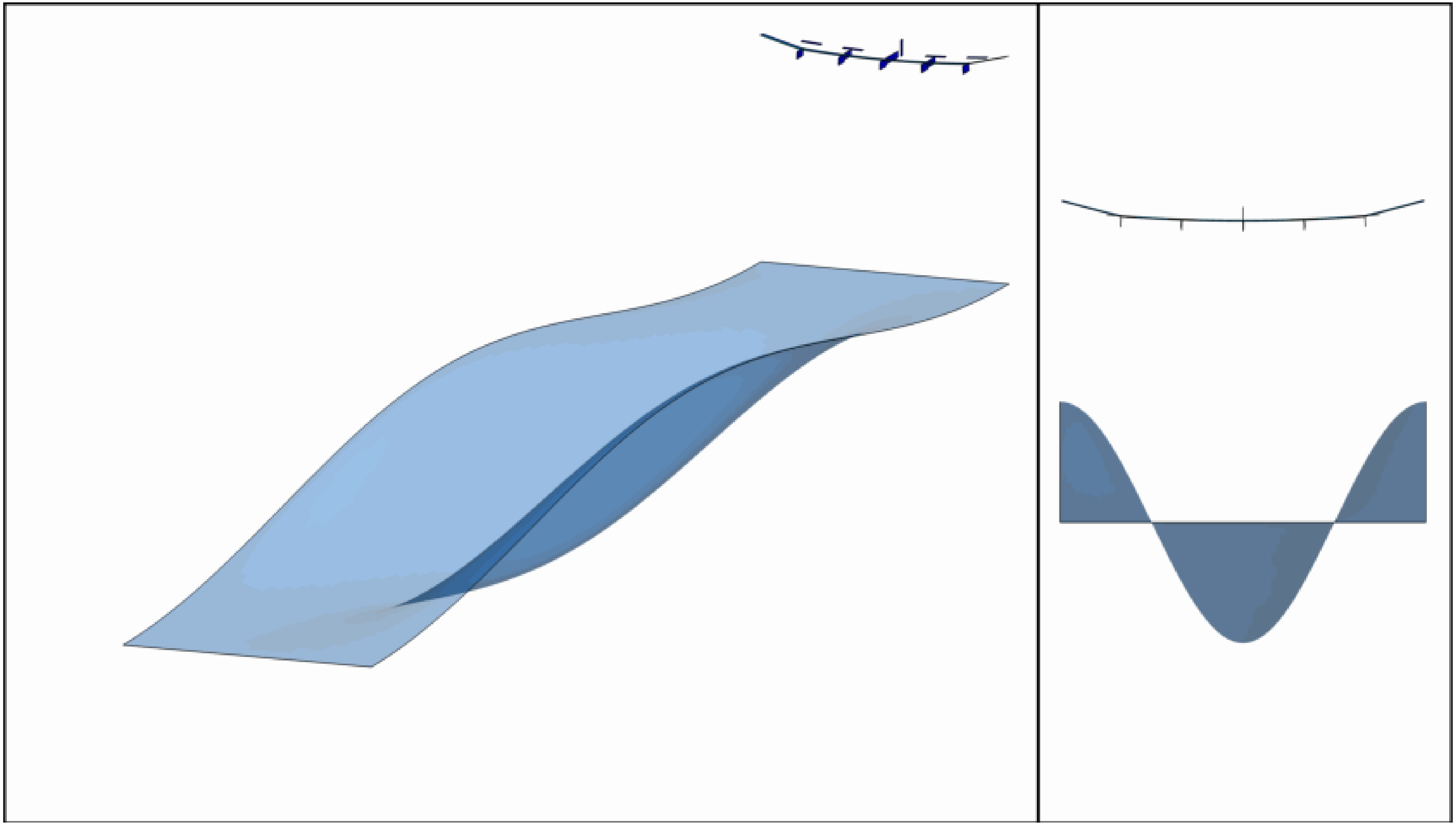


M. Ritter, P. C. Teixeira, C. E. S. Cesnik, **Comparison of Nonlinear Aeroelastic Methods for Maneuver Simulation of Very Flexible Aircraft**, AIAA SciTech, 2018

Application: X-HALE DARPA Gust



Summary and Outlook

- Development of an improved, modal based method which considers:
  - Geometrical nonlinearities in the nodal displacement field
  - Nonlinearities in load-displacement relation
- Validation with static and flight dynamic test cases
  - Good agreement with static nonlinear reference solutions up to ≈ 30 % displacement
  - Good agreement with reference solutions for X-HALE tail input maneuver
- Derivation and implementation of methods for loads recovery
- Modeling and validation of more complex 3D GFEM structures (e.g. jet transport)
- Application of the method to rotating structures such as wind turbines

DLR