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Structural Optimisation of an Aircraft Wing using rapid Analytical Methods

Masterarbeit

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Deutsches Zentrum für Luft- und Raumfahrt

Institut für Faserverbundleichtbau und Adaptronik

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Structural Optimisation of an Aircraft Wing using rapid Analytical Methods

Masterthesis
Technische Universität Braunschweig

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Aerospace Engineering
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Technische Universität Braunschweig

Masterthesis

For

Hendrik Traub

Matr.-No.: 4096878

Topic: Structural Optimisation of an Aircraft Wing using rapid analytical methods

Introduction

The design of modern lightweight aircraft structures is an iterative process and includes many disciplines. In conceptual and preliminary design, the objective is a rapid evaluation of many different designs in order to achieve a lightweight aircraft structure which can be further developed. Statistical methods are used in conceptual design to explore the design space based on existing designs. Such methods are only valid in the design space of existing designs. In order to find new designs the evaluation needs to be based on physical rather than empirical models. Standard structural optimisation processes use Finite Element Methods and need a great amount of time to find a lightweight and therefore cost-efficient structure. Such a time-consuming process limits the number of designs which can be evaluated in a feasible period of time. A much faster method for structural optimisation is therefore needed in preliminary and conceptual design.

At the DLR Institute of Composites and Adaptive Structures the tool PreDoCS is developed to evaluate wing stresses based on analytical cross-section theory. PreDoCS imports a provided aircraft model consistent with CPACS, DLR's Common Parametric Aircraft Configuration Schema. Besides structural design parameters, a CPACS model can also provide data from other disciplines, for example nodal loads. The PreDoCS calculation, based on nodal loads applied to a structural model, takes into account effects such as transversal shear and bending torsion coupling. Stresses within the cross-sections can be derived from the beam deflections. PreDoCS therefore offers a possibility of rapid structural evaluation of composite and metal structures.

A gradient-based optimisation environment for composite structures exists at the Institute of Composites and Adaptive Structures. The environment uses Finite Element Analysis (FEA) for structural evaluation, offering a vast amount of evaluation methods available for the optimisation including gradient calculation. Its aim is to find the minimum mass of a proposed design considering boundary conditions. In order to use a gradient-based optimisation, the composite material is defined in terms of lamination parameters and thickness. Failure criteria are evaluated with rapid analytical methods and consider buckling, strength and deformation.

The evaluation of a detailed FEM model is very time consuming. In structural optimisation several hundreds to thousands of evaluations are necessary to find a mass minimum. For this reason, an optimisation framework for wing structures based on rapid analytical methods is desirable. In the scope of this work such a generic structural optimisation framework has to be designed. Analytical failure criteria evaluation and the formulation of the optimisation problem in terms of lamination parameters and thickness can be adopted from the existing optimisation environment. A stress interface included in the framework allows the use of different structural analysis tools. In combination with PreDoCS the optimisation framework is a rapid structural sizing framework based on a physical model allowing the exploration of unknown design space in conceptual and preliminary design.

Tasks

The global objective of this project is the design, implementation and test of a rapid analytical sizing framework for wing-like structures. The following challenges need to be addressed in order to achieve such a framework:

1. The current state of research on wing structural optimisation should be given in a short introduction.
2. Set up a generic optimisation framework built in Python including a structural evaluation interface. The necessary steps are given as follows:
   a. Implementation of a coordinate system module for PreDoCS in order to choose the beam axis freely and therefore enable the use of aircraft models with swept wings.
   b. Loads from aerodynamic or aeroelastic calculations are stored in a CPACS model. Implementation of a load interface for PreDoCS to access the nodal loads of a CPACS model and to use them for structural evaluation.
   c. Design and implementation of a generic structural optimisation framework using gradient-based optimisation.
   d. Design of a generic interface to structural evaluation tools and use it to connect PreDoCS to the optimisation framework.
   e. Integration of existing analytical methods for failure criteria evaluation within a failure module into the optimisation framework.
3. Perform a case study based on the new sizing framework and compare its capabilities with existing results.
4. The development of the optimisation framework and its evaluation shall be well documented in a written report.

The editing period for this paper is six months as specified below. The paper has to be submitted electronically via the WISA-portal in time. Two samples of the paper have to be delivered to the Institute of Adaptronics and Function Integration. Exceptions are documented in the examinations regulations (Prüfungsordnung).

The Supervisor on behalf of TU Braunschweig will be Dr.-Ing. Naser Al Natsheh.

Changes to the above assignment are only permitted with the approval of IAF. The paper has to be provided to the IAF business office in duplicate, hardcover, including an electronic version.

Hereby I declare the receipt of the assignment:

Hendrik Traub

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Hendrik Traub

Processing time: 6 months

Date of issue: 01.03.19

Date of Dispatch:

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Prof. Dr.-Ing. Christian Hühne

Strukturoptimierung eines Flugzeug-flügels mit schnellen, analytischen Methoden

Einleitung

Das Auslegen moderner, leichter Flugzeugstrukturen ist ein iterativer Prozess, der viele Fachdisziplinen einschließt. In der Konzept- und Entwurfsphase ist es notwendig viele verschiedene Entwürfe auszuwerten um eine leichte Bauweise zu erzielen, die im Folgenden weiter ausgearbeitet werden kann. In der Konzeptphase wird der Designraum mit statistischen Methoden erforscht. Diese Methoden beruhen jedoch auf bereits existierenden Flugzeugdesigns und sind nur im Raum ihrer Datengrundlage gültig. Um neue Bauweisen zu erforschen muss die Bewertung einer Struktur auf Basis von physikalischen Modellen stattfinden. Übliche Auslegungstools für Flugzeugstrukturen benutzen Finite Elemente Methoden für die Strukturbewertung und brauchen daher viel Zeit um eine leichte und kosteneffiziente Struktur zu finden. Dadurch sinkt die Anzahl der Bauweisen erheblich, die innerhalb einer praktikablen Zeitspanne untersucht werden können. Für die Konzept- und Entwurfsphase wird deshalb eine sehr viel schnellere Möglichkeit der Strukturbewertung benötigt.

Am DLR Institut für Faserverbundleichtbau und Adaptronik wird das Tool PreDoCS entwickelt um Flügelstrukturen mit analytischen Querschnittstheorien zu bewerten. Für die Strukturberechnung nutzt PreDoCS ein CPACS Modell (Common Parametric Aircraft Configuration Schema), das neben den Strukturparametern auch Daten anderer Disziplinen speichern kann, beispielsweise Punktlasten. PreDoCS berechnet die Spannungen in den Querschnitten auf Basis eines Balkenmodells und an den Balkenknoten angreifenden Punktlasten. Dabei berücksichtigt PreDoCS auch transversalen Schub und Biegetorsionskopplung. PreDoCS bietet dadurch die Möglichkeit einer schnellen Bewertung von Faserverbund- und Metallbauweisen.

Eine gradientenbasierte Optimierungsumgebung für Faserverbundstrukturen wird ebenfalls am DLR Institut für Faserverbundleichtbau und Adaptronik entwickelt. Diese Umgebung benutzt eine Finite Elemente Analyse für die Strukturbewertung und ermöglicht so die Verwendung verschiedenster Berechnungsmethoden, unter anderem die Berechnung von Gradienten. Das Ziel der Optimierungsumgebung ist das Design mit der geringsten Masse zu finden, das gerade noch die gesetzten Randbedingungen erfüllt. Um gradientenbasierte Optimierungsverfahren benutzen zu können werden Faserverbundstrukturen durch kontinuierliche Lamination Parameter und durch die Materialdicke dargestellt. Versagenskriterien werden mit analytischen Handbuchmethoden ausgewertet und umfassen Zugfestigkeit, Beulen und maximale Verformungen.

Die Auswertung großer Finite Elemente Modelle dauert sehr lange. In der Strukturoptimierung sind oft einige hundert bis tausend Auswertungen nötig um ein Massenminimum zu finden. Aus diesem Grund scheint es sinnvoll eine Optimierungsumgebung für Flügelstrukturen basierend auf analytischen Methoden zu entwickeln. Im Rahmen dieser Arbeit soll eine generische Optimierungsumgebung entwickelt werden. Analytische Versagenskriterien und die Formulierung des Optimierungsproblems mittels Lamination Parameter und Dicke kann dabei aus der bestehenden Optimierungsumgebung übernommen werden. Ein Lasteninterface soll genutzt werden um Strukturlasten verschiedener Analysetools verwenden zu können. In Kombination mit PreDoCS als Analysetool wird dadurch eine schnelle Optimierungsumgebung auf Basis physikalischer Modelle für die Konzept- und Entwurfsphase geschaffen.

Aufgaben

Das Gesamtziel dieses Projekts ist der Entwurf, die Implementierung und der Test einer schnellen, analytischen Optimierungsumgebung für Flügelstrukturen. Folgende Punkte sollen zu diesem Zweck bearbeitet werden:

1. Eine Literaturrecherche soll den derzeitigen Stand der Strukturoptimierung zeigen.
2. Eine generische Optimierungsumgebung mit einem Strukturanalyseinterface soll entworfen und in Python implementiert werden:
   (a) PreDoCS muss um ein Koordinatensystem-Modul erweitert werden damit die Balkenachse frei gewählt werden kann, was die Verwendung von Flugzeugmodellen mit Pfeilflügeln ermöglicht.
   (b) Lasten aus aerodynamischen oder aeroelastischen Berechnungen können in CPACS Modellen gespeichert werden. PreDoCS soll mit einem Lasteninterface erweitert werden um diese Lasten aus dem CPACS Modell zu importieren und zu verarbeiten.
   (c) Eine generische Optimierungsumgebung für die Strukturoptimierung von Flügelmodellen soll entworfen und implementiert werden.
   (d) Die Umgebung soll ein Interface zu Strukturanalysetools bereitstellen und das Analysetool PreDoCS soll über diese Schnittstelle verbunden werden.
   (e) Ein existierendes Versagensmodell soll in einem Versagensmodul in die Umgebung integriert werden.
3. Die Optimierungsumgebung soll mit einer Fallstudie getestet und mit einer anderen Optimierungsumgebung verglichen werden.
4. Die Entwicklung der Optimierungsumgebung und die Fallstudie sollen in einer schriftlichen Ausarbeitung dokumentiert werden.

Statutory Declaration

I assure that this thesis is a result of my personal work and that no other than the indicated aids have been used for its completion. Furthermore I assure that all quotations and statements that have inferred literally or in a general manner from published or unpublished writings are marked as such. Beyond this I assure that the work has not been used, neither completely nor in parts, to pass any previous examination.

Braunschweig, 23.08.2019

Confidentiality Clause

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Abstract

The investigation of new aircraft designs requires robust and rapid evaluation methods on a physical basis allowing to explore the unknown design space. The structural optimisation of aircraft wings is particularly challenging due to aeroelastic coupling effects. The minimum structural mass of an aircraft wing with a given outer shape can only be found in a multi disciplinary optimisation including structural and load calculations. In this thesis a structural optimisation framework for wing-like structures is developed. Material formulations based on lamination parameters allow gradient based optimisation algorithms. The modular optimisation framework provides a general interface to structural solvers enabling a multi fidelity optimisation process. In this thesis the structural solver PreDoCS calculates internal structural loads with an analytical cross-section theory in combination with one dimensional finite beam elements. Outer loads are provided by external tools and imported through the standardised CPACS interface, allowing multi disciplinary coupling. A comparison of the optimisation results of PreDoCS and a finite element based structural solver establishes confidence in the framework. The optimisation of a mid range aircraft wing shows the potential of lamination parameter optimisation with a gradient based, analytical optimisation framework.

Contents

Nomenclature

Acronyms

API	Application Programming Interface
ATLAs	Advanced Technology Long-range Aircraft concepts
CPACS	Common Parametric Aircraft Configuration Schema
MDO	Multi Dicisplinary Optimisation
PreDoCS	Preliminary Design of Composite Structures
VErSO	Virtual Environment for Structural Optimization

Chapter 1

Introduction

Statistical methods are used in aircraft preliminary design for the mass estimation of new aircraft concepts. Such methods are only valid if the new concept is situated in the same design space as the data basis of the statistic. The mass estimation and the structural evaluation of new, promising aircraft concepts outside the existing design space demand preliminary design methods on a physical basis. Some key features of these methods are robustness, flexibility and a rapid evaluation. Analytical methods fulfil these requirements and have additional advantages such as exact solutions and a physical basis.

One important part of the aircraft preliminary design is the structural layout of the wings. The design process of aircraft wings and wind turbine blades is from a structural point of view identical and allows universal tools for both processes. The coupling of its elastic structure and its aerodynamics makes the wing structural optimisation particularly challenging. The dimensioning of the inner wing structure is based on the outer loads. The outer loads on the other hand change with the wings elastic properties. It is state of the art to update the loads in an iterative structural optimisation process. An actual multi disciplinary optimisation (MDO), in contrast to load updates, requires the aerodynamic parameters to be part of the optimisation parameter space. Only then an aeroelastic mass minimum can be found as a result of the wing optimisation process schematically shown in *figure 1.1*. The focus of this thesis is on the structural optimisation where load updates are covered to the extend of an universal load interface of the structural optimisation process.

The outer forces and moments applied at the wing have to be translated to internal loads by a structural solver. In this thesis the structural solver PreDoCS is used and extended with an universal load interface and an application programming interface (API). The load interface allows to import loads from a standardised CPACS model. PreDoCS uses an analytical cross section theory in combination with one dimensional finite beam elements to calculate the wing stiffness properties and its internal loads. The API provides all functionalities of the structural solver in a single interface. Export functionalities for a structural optimisation model and internal loads are included in the API and provide an interface to an optimisation process.

The objective of structural optimisation is to find a structure with a minimum mass withstanding all outer loads. In contrast to the structural optimisation, structural sizing is not aiming to find a global mass minimum. In the scope of this thesis a modular optimisation framework is developed which optimises the wing mass by varying skin thicknesses and composite layups. Instead of layer angles and stacking sequences lamination parameters, representing the main stiffness directions of composite materials, are optimised, having the advantage of continuously defined parameters and a small optimisation parameter space. Further, lamination parameter are the theoretical boundary for optimum laminate stacking sequence and layer angels. Constraints considering the feasible lamination parameter design space, analytical strength and stability criteria and manufacturing constraints are evaluated in a constraint module of the optimisation framework. The internal loads and the structural model which build the foundation for the constraint evaluation are provided by PreDoCS. The optimisation model collects optimisation parameters, objective function and constraint functions from the structural model and the constraint processor and hands them over to the optimisation algorithm. The material definition in lamination parameters allows to use a gradient based optimisation algorithm, in this thesis the method of moving asymptotes, which are faster and more robust than evolutionary algorithms.

Another tool with an interface to the optimisation framework is VErSO, in this thesis used as structural solver. VErSO offers interfaces to different finite element tools and thereby allows a structural optimisation based on internal loads calculated with finite element methods. In contrast to PreDoCS, VErSO is a high fidelity tool for the detailed design of an aircraft. The comparison of optimisations with PreDoCS and VErSO allows a validation of both tools and a first investigation of the optimisation frameworks abilities. A full thickness and layup optimisation of a mid-range jet (MiRaJet) shows the full potential of the developed optimisation framework. All optimisation results from the comparison of PreDoCS and VErSO and from the lamination parameter optimisation are presented graphically and are investigated for their physical meaning. The final optimisation results are thickness and lamination parameter distributions. A reconversion of the lamination parameters into stacking sequences and layer angles and an update of the CPACS model are not conducted in the scope of this thesis.

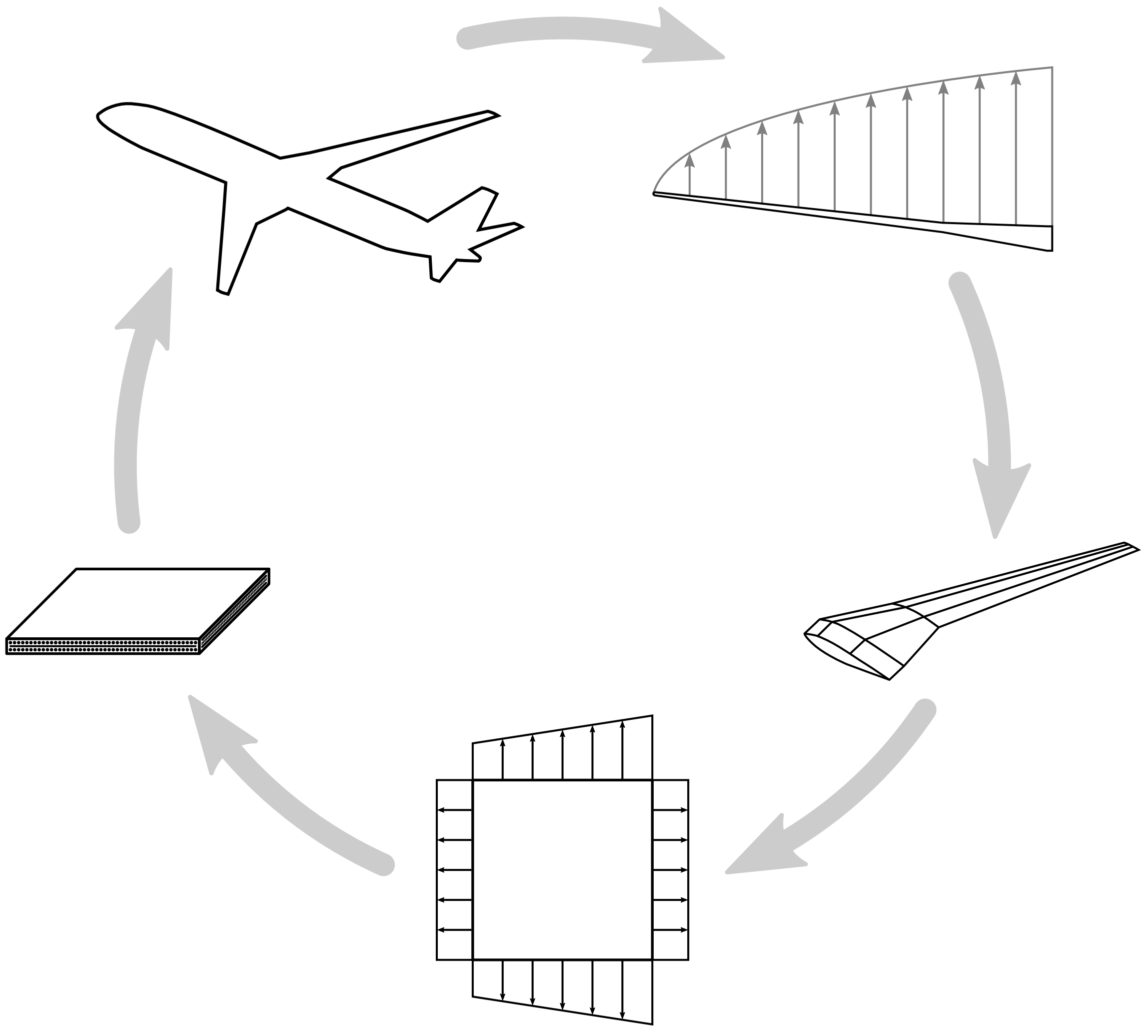


Figure 1.1: Multi disciplinary optimisation process

Chapter 2

Basic Tools and Engineering Models for the Structural Optimisation

The structural optimisation framework developed in the present work relies on other projects and tools developed by the DLR. The structural solver PreDoCS is based on analytical cross-section theory. This offers the opportunity of rapid structural evaluation and is therefore an appropriate tool for a rapid optimisation framework for conceptual and preliminary design. PreDoCS provides an universal interface to CPACS models, a standard for parametrised aircraft models developed at the DLR. Failure criteria used to calculate optimisation constraints can be adopted from VErSO, a finite element based structural optimisation environment, developed at the DLR Institute of Composite Structures and Adaptive Systems. PreDoCS and VErSO will be provided with an interface to the structural optimisation framework developed in this thesis, allowing a comparison and validation of both tools. The comparison of both structural optimisations is carried out at the example of a mid range jet wing as a part of the ATLAs project.

2.1 CPACS

CPACS is a Common Parametric Aircraft Configuration Schema developed by the DLR and used in national and international aircraft design projects [7]. The schema defines how aircraft related data is stored in an Extensible Markup Language (XML) file and therefore provides a multi disciplinary interface in aircraft development. The schema can be used to define parametrised models of rotors and aircraft of various designs. Besides structural parameters as geometry and materials, other aircraft related information such as load analysis and dynamic models are defined in the CPACS schema.

A variety of different software programs using CPACS models is developed in and outside the DLR. Libraries as TiGL [17] and TiXi developed at the DLR are tools made to access data in CPACS models. Other Tools as VAMPZero or even PreDoCS use CPACS models in discipline specific calculations. Results of such calculations can be added to the CPACS model in order to make them available to other disciplines and tools. Adding data to a CPACS model only requires the definition of such data in the common CPACS schema. For example panel stresses or cross section line loads are not defined in CPACS[1].

The advantage of a general model, as it is provided by the CPACS schema, is the simplified connection of different disciplines and the reduction of interfaces between them. *Figure 2.1* shows the reduction of interfaces of a general model with reference to tool specific models. The disadvantage of an universal model as provided by the CPACS schema is the model size. Large high-fidelity models, containing information from many disciplines, result into huge files and to access these files can be very slow.

[1]The possibility exists to save tool-specific data to CPACS models, but to do so is accompanied by loosing the defined interfaces of the CPACS schema together with the interdisciplinary advantages.

XML files store data in a text-based, human-readable format which is advantageous for the user but is only slowly processed by computers. In optimisation the performance disadvantages of a CPACS model defined in XML should be out-weighed by the advantage of the multi-disciplinary interfaces.

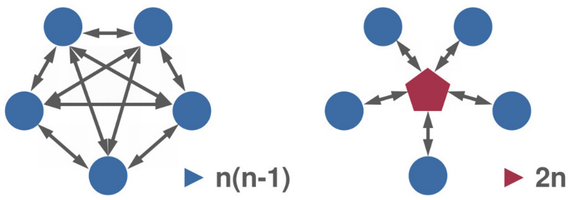


Figure 2.1: The CPACS scheme [7]

2.2 PreDoCS

PreDoCS is a structural solver developed for the Preliminary Design of Composite Structures at the DLR Institute of Composite Structures and Adaptive Systems. PreDoCS uses the wing of a CPACS model and evaluates its deflection stresses and strains based on an analytical cross section approach in combination with a 1D FEM beam model. The main objective of PreDoCS is to calculate a representative stiffness matrix of the wing. The tool is build up in three levels: the wing level, the cross section level and the material level [10]. On the wing level, PreDoCS uses finite beam elements to discretise the wing. Each beam element is described by a stiffness matrix defined on the cross section level. The wing cross sections are placed orthogonal to the beam axis and can be analysed with several analytical cross-sectional theories. The cross-sectional stiffness matrix is derived from material laws, defined on material level, in combination with the cross section geometry.

A CPACS interface module integrated in PreDoCS offers the possibility to generate a PreDoCS wing structure from a CPACS model using TiXi and TiGL. TiGL works with the Open Cascade (OCC) CAD kernel to compute aircraft surfaces as NURBS (non-uniform rational B-Splines). Cross section geometries are generated in PreDoCS from the three dimensional wing by means of defined intersecting planes. Material data is provided in the CPACS model in the form of stiffness matrix values and imported by using TiXi. Additional information about stacking sequences and layer angles allows the construction of laminae. Thus the import module allows the complete generation of a wing structural model for PreDoCS including composite structures. The CPACS model can also provide design load cases for the wing such as nodal loads applied at dynamic model reference points. Design loads are needed in PreDoCS as input for the stress evaluation. It is part of this thesis to add a load module to PreDoCS to import such loads and integrate them into the PreDoCS process.

2.2.1 Cross-Sectional Model

The cross sections of the PreDoCS model have to be always perpendicular to the beam elements in order to provide a representative stiffness for the beam element. PreDoCS beam elements are one dimensional elements which need to be all placed on one axis. For this reason a meaningful beam axis has to be placed approximately within the elastic axis of the wing. Because the elastic axis of the wing is usually not straight, the PreDoCS beam axis can only be an estimation of the actual elastic axis. To allow the user to freely set the PreDoCS beam axis, a beam coordinate system module is developed in *section 3.1*.

The number of cross sections used to discretise a beam model has a significant impact on the calculation time of the internal loads. For this reason a minimum number of cross sections has to be selected for each model. For each cross section the analytically correct stiffness matrix at the position of the cross section is calculated. A beam model with a constant cross section geometry along the wing axis, as shown in *figure 2.2*, only needs one cross section to describe the stiffness of the whole beam. Usually the shape of a wing changes along its beam axis. For this reason the stiffness along the wing axis is interpolated in PreDoCS with cubic splines. The minimum number of samples needed for a cubic spline results into a minimum of four necessary cross sections for each PreDoCS beam model.

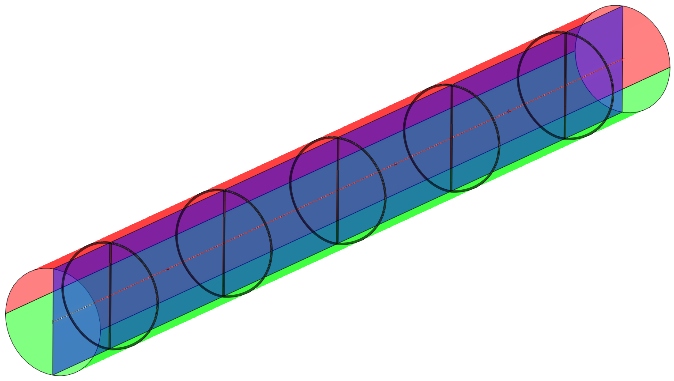


Figure 2.2: A CPACS tube beam model including PreDoCS cross-sections

Starting from the minimum of four, the number of cross-sections can be chosen freely. Apart from the change of the cross section geometry, also the change of materials has to be considered when the number of cross-sections is chosen. Thickness and lamination parameters are material properties of a laminate. In structural optimisation where the material properties are optimisation parameters, an optimisation region is defined as material region. Material changes along the beam axis therefore require a cross sectional discretisation. Enough cross sections for an accurate geometry discretisation along the beam axis and at least one cross section for each desired optimisation region are necessary.

2.2.2 Element Discretisation

The stiffness calculation of a cross-section in PreDoCS is exact within the limitation of the selected analytical theory for straight elements. Curvatures of a cross-section are currently discretised with straight elements. This leads to a discrepancy compared with the exact solution. The influence of element discretisation on internal load distribution and magnitude is investigated at the example of the tube beam shown in *figure 2.2*. The tube beam has a radius of $0.5\,\mathrm{m}$ and and can be tested for a range of element lengths. The qualitative tension and compression distribution resulting from a bending moment at a cross section of the tube beam is shown in *figure 2.3* and *figure 2.4*. *Figure 2.3* shows the internal loads for the minimum element discretisation of one element per segment whereas in *figure 2.4* the internal loads for a more appropriate element discretisation are shown.

The tension and compression in the cross section due to a applied moment increase linear with the distance to the point of load application. Because in both discretisation cases the applied moment is the same, the distributed loads integrated over the cross section geometry of both discretisation levels have to be identical. Even though the total amount of summed up loads is identical for both element discretisation levels, the peak load is much higher for the low discretisation. The load distribution is exact in both figures for the shown geometries. Both geometries are an approximation of the circular cross section of the tube beam. Because of its closer geometry approximation, the load distribution shown in *figure 2.4* is a more accurate estimations of the analytically exact load distribution.

The maximum allowed element length and its impact on the resulting maximum load at one element is shown in *table 2.1*. The calculated difference in maximum loads from the two chosen discretisation levels is approximately $50\,\%$. A strong increase of the maximum internal loads causes high masses in a structural optimisation process. For this reason the element discretisation has to be chosen carefully depending on the models curvature.

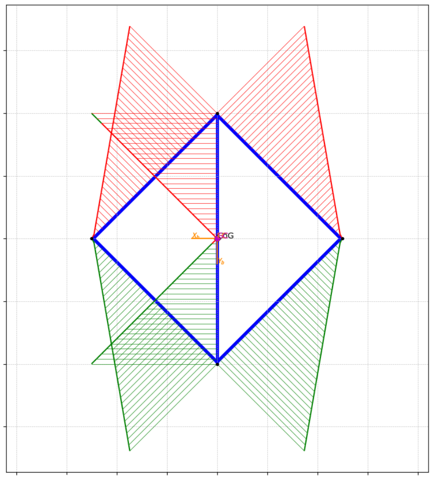


Figure 2.3: Low element discretisation

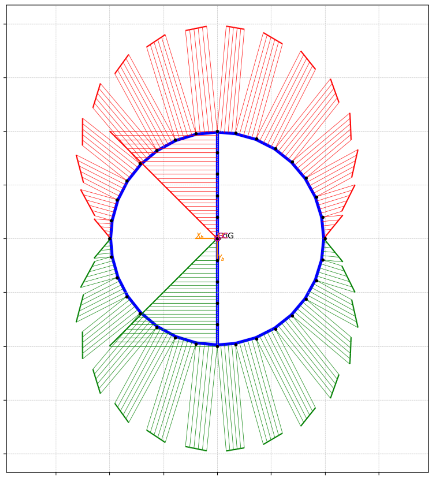


Figure 2.4: High element discretisation

	low discretisation	high discretisation	variation
maximum length	$0.71\,\mathrm{m}$	$0.1\,\mathrm{m}$	$607\,\%$
maximum load	$1391\,\mathrm{N\,mm^{-1}}$	$937\,\mathrm{N\,mm^{-1}}$	$48\,\%$

Table 2.1: The impact of maximum element length on maximum load.

The relation of element length and maximum load for the tube beam is shown in *figure 2.5*. Additionally the lower and the upper boundary of the maximum load and the $10\,\%$ line are given in the figure. By selecting a maximum element length of $0.25\,\mathrm{m}$ a load increase of less then $10\,\%$ with reference to the minimum load can be achieved. Normalised by the radius of the cross-section curvature of $0.5\,\mathrm{m}$ a factor is determined to estimate appropriate maximum element lengths of arbitrary profiles. The maximum element length approximation with a given minimum curvature radius of a cross-section is presented in *equation 2.1*. For aerodynamic profiles, the nose radius is usually known, allowing a rough estimation of an appropriate maximum element length for PreDoCS.

$$l_{element,10\,\%} \leq \frac{1}{2} * r_{min} \tag{2.1}$$

2.3 VErSO

The Virtual Environment for Structural Optimization is a FEM based structural optimisation tool developed at the DLR Institute of Composite Structures and Adaptive Systems. VErSO uses an interface to various finite element tools for internal load calculation and analytical failure criteria for the structural evaluation. By this combination VErSO is completely flexible in its structural description and not limited to beam like structures as for example PreDoCS. The opportunity to solve models with strong three-dimensional effects allows the optimisation of concepts such as blended wing body aircraft. Using high order elements and a high model discretisation, VErSO is a high fidelity structural optimisation tool.

In the scope of this thesis the analytical failure criteria used in VErSO are adapted to the newly developed modular optimisation framework. In a second step VErSO is equipped with an interface to the

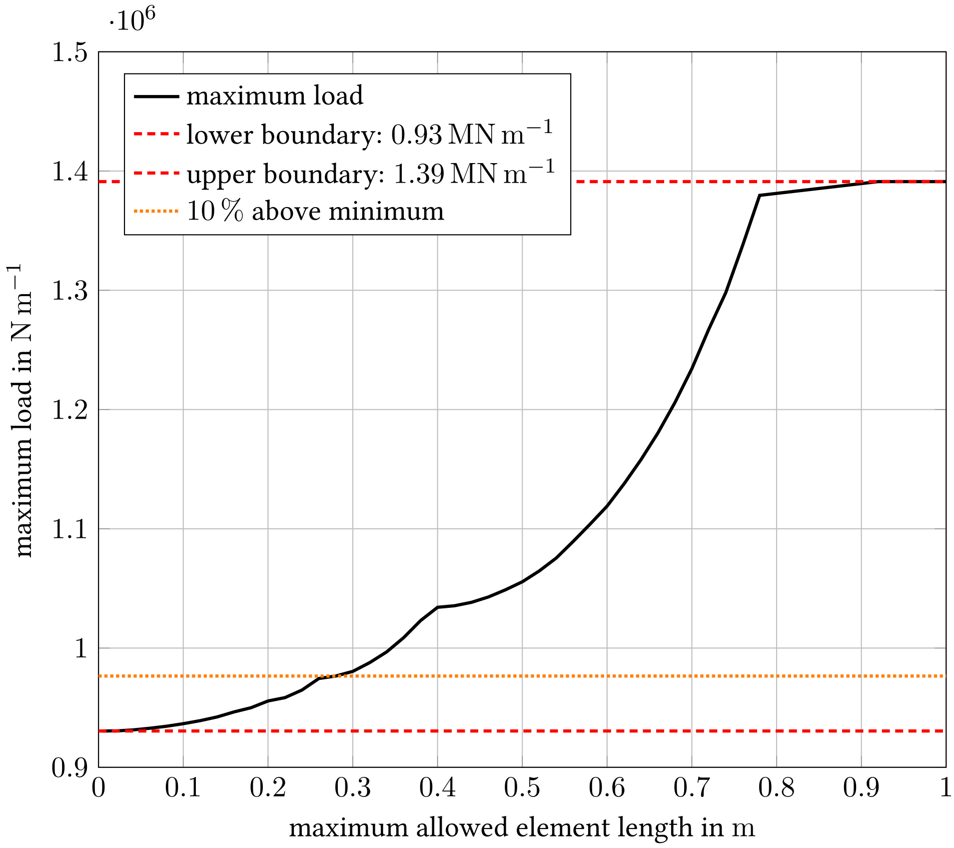


Figure 2.5: Influence of the element discretisation on maximum loads

optimisation framework. This allows a direct comparison of PreDoCS and VErSO in an full optimisation process. For future projects one optimisation framework with multiple structural solvers offers the possibility of multi-fidelity optimisation.

2.4 The ATLAs Project

In the ATLAs project, the DLR investigates Advanced Technology Long-range Aircraft-concepts. The development of multi-disciplinary design processes is evolved by linking the analysis tools of each discipline to a global design process. The CPACS schema is used as a central data model providing a general, parametrised interface to all tools. Two aircraft configurations are designed in the ATLAs project, a long range aircraft ("D250x") and a Mid-Range-Jet ("MiRaJet") which is a DLR-Airbus cooperation shown in *figure 2.6*. Next to the development of the aircraft concepts and configurations itself, the development of analysis tools and chains for the assessment of new technologies is the central target of the ATLAs project.

PreDoCS as a structural optimisation tool for wing like structures on a physical basis in preliminary design allows the assessment of new wing structures and technologies because its analysis does not depend on an empirical database of existing wings. Supporting transverse shear evaluation, PreDoCS can take into account effects as bending torsion coupling. Therefore using PreDoCS in a multidisciplinary optimisation includes all benefits from aeroelastic tailoring. Even without the aeroelastic coupling PreDoCS can be used for structural tailoring. Including such physical effects in a preliminary design tool makes PreDoCS a perfect match for the ATLAs project.

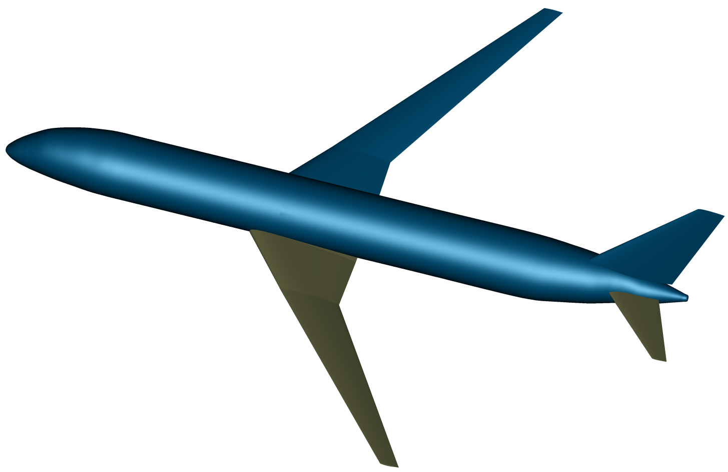


Figure 2.6: Mid Range Jet (MiRaJet)

2.5 Optimisation

The task of a mathematical optimisation is to find a minimum fulfilling all constraints. A schematic optimisation problem is shown in *figure 2.7*. Here f(x) is the objective function, g(x) is the inequality constraint, h(x) is the equality constraint and x is the optimisation parameter. The shown objective function has two minima, a local minimum and an absolute, global minimum. The inequality constraint is fulfilled when g(x) is lower or equal to zero. The equality constraint is only fulfilled when h(x) is exactly zero. Therefore the third minimum of the example is the restricted minimum where all constraints are fulfilled. The region where all constraints are fulfilled is the feasible region. It should be mentioned equality constraints usually can be replaced by one ore multiple inequality constraints and are therefore not considered in many optimisation algorithms. Equality and inequality constraints are implicit restrictions, only available after a function evaluation. The upper and lower boundaries of the optimisation parameter x are called explicit constraints.

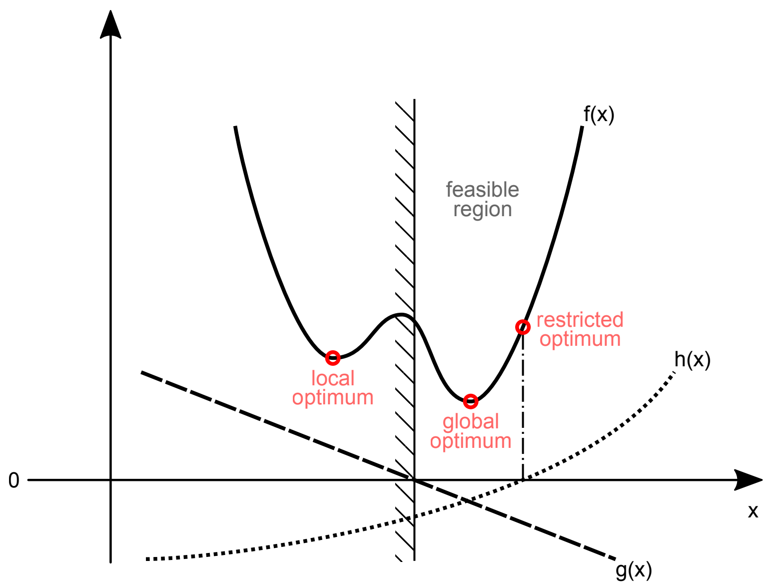


Figure 2.7: Schematic optimisation problem

The challenge of structural optimisation is to identify all optimisation parameters, constraints and the objective function. State of the art in the structural optimisation of wings is an optimisation of the wing mass by varying skin thicknesses, considering strength, strain and stability criteria. Starting from such a basic setting, many more optimisation parameters and objectives are possible, but require new constraints. Common additional parameters are for example ply angles or stacking sequence. A list of parameters and additionally required constraints is given in *table 2.2*.

	parameter	additional constraints
common	thickness	strength, stability
advanced	ply angles	composite design rules
	stacking sequence	manufacturability
research	lamination parameter	feasible design space

Table 2.2: Parameters in structural optimisation

In this thesis the focus is on the minimisation of structural mass, using thickness and lamination parameter as optimisation parameters. The only constraints which do not directly depend on the lamination parameter are the stability constraints. Strength constraints are usually evaluated layer-wise for composites and need a special formulation for lamination parameters. Additional necessary constraints such as composite design rules and manufacturability, e.g. ply continuity, are included in feasible design spaces for lamination parameter.

2.6 Lamination Parameter

Lamination parameter, first introduced by Tsai and Pagano [21] and Tsai and Hahn [20], emerged from the classical lamination theory (CLT)[25] [13] as an attempt to identify the influence of laminae rotation on the stiffness matrix of a laminate, the ABD matrix. Lamination parameter complemented by material invariants and the laminate thickness fully define the ABD matrix. In the CLT the ABD matrix of a laminate is build up from the material laws of rotated orthotropic layers. The ABD matrix therefore contains information on the stacking sequence and fibre orientation of a composite material. Stacking sequence and, for the purpose of manufacturing, also layer orientations are discrete parameters. Lamination parameters are continuously defined and hence more suitable for gradient based optimisation. The number of parameters needed in the CLT to set up the ABD matrix depends on the number of layers while the number of lamination parameters is independent of the number of layers. Thus lamination parameter are also advantageous in an optimisation process because using them reduces the number of parameters needed for the material description.

In the classical lamination theory the in plane and out of plane stiffness of any laminate is described in the ABD matrix, defining the relationship of line load and strain. To set up the ABD matrix, all layer thicknesses and angles as well as the stacking sequence and the material law of each layer have to be known. The number of parameters to determine the ABD matrix is proportional to to the number of layers, were for each layer the layer angle, thickness and material has to be known. Assuming all layers have the same thickness and material, only the two parameters stack-position and layer angle have to be considered for each layer. The stiffness properties of the laminate can be changed altering $2 * n$ parameters, where n is the number of layers and the material of the laminate is not changed.

Lamination parameter are a representation of the ABD matrix of an arbitrary laminate. Assuming all layers in the laminate have the same thickness and material, it is possible to describe the ABD matrix of any laminate with 12 lamination parameter, 5 material invariants and its total thickness resulting into 18 parameters. For symmetric balanced laminates the number of lamination parameters reduces to 5, resulting into a total of 11 parameters describing the stiffness properties of a laminate.

The advantage of lamination parameter in optimisation is their continuous definition and the small number of parameters needed to describe the material properties of a laminate with an arbitrary number of layers. However, the usage of lamination parameters in optimisation demands a closed look on their limitations as well. The most obvious limitation is the lost of information on stacking sequence and layer angles. To generate a laminate, i.e. stacking sequence and layer angles, with the information provided by the lamination parameter is a task subsequent to the gradient based optimisation. Liu, Featherston and Kennedy present a way to obtain a laminate from lamination parameters in [15]. Lamination parameters are trigonometric functions depending on the same angle and thus are not independent from each other. Different approaches have been made to define the feasible domain of physical meaningful lamination parameter under different conditions. Hammer et. al. [9] investigates the feasible region of lamination parameter with an analytical approach showing the feasible domain is always convex while Setoodeh Mostafa and Gürdal [16] offer a numerical approach based on convex hull functions (Hyperplanes). Continuity constraints can be evaluated based on lamination parameters using the same numerical approach as for the feasible region [23][24]. Also the evaluation of strength criteria with material definitions based on lamination parameters is possible [8] [11] [14]. This possibility will be further investigated in *section 4.4*.

Chapter 3

Adaption of PreDoCS for the Optimisation Process

For aircraft wings, the global CPACS coordinate system does not coincide with a potential wing beam axis. A new, PreDoCS specific coordinate system is needed in order to define the beam axis of an aircraft wing. Therefore PreDoCS is extended with a coordinate system module which allows the selection of a meaningful beam axis and handles the positioning of beam nodes and cross sections. The definition of the beam axis and the coordinate system module is described in *section 3.1*.

In the established optimisation process, the structural layout is based on a set of load cases. These so called design load cases are evaluated preliminary to the actual optimisation process and should represent the maximum loads a part has to endure during it's lifetime. The CPACS schema supports the storage of nodal loads applied at a set of given reference points. The design load cases in a CPACS model can be accessed with aeroelastic tools, which also provides the possibility of load updates in the optimisation process. Outer loads are the main driver of a structural optimisation and therefore essential in this thesis. For this reason PreDoCS is equipped with a load module handling the import CPACS loads and the load preparation for the structural evaluation. The load module is presented in *section 3.2*

The set up of a robust, modular optimisation framework working with the analytical structural solver PreDoCS is the main objective of this thesis. To connect PreDoCS to the optimisation framework, an interface has to be designed defining the structural model and the internal loads transfer. An application programming interface (API) is a top level module providing full control over the PreDoCS functionality without the need of extended knowledge or access to its low level modules. This allows the implementation of interface methods required by the optimisation framework. Other structural solvers providing the same interface methods can be easily connected to the framework. The PreDoCS API is introduced in *section 3.3*.

3.1 Coordinate System Module

PreDoCS structural representation is currently based on one dimensional beam elements requiring all elements to be placed on the same beam axis. The beam elements stiffness is defined by a cross-section placed in the middle of each beam element. Depending on the position of the beam axis, the cross section geometry and if applicable, also the material definition, changes. For a swept beam axis, cross-sections cannot be generated at the wing root and the wing tip because no full wing cut is possible here. This restriction limits the allowed placement of beam elements. Beam axis options as centre-of-gravity, shear-centre or elastic-centre are not possible because they don't line up in a straight axis along the wing and such a definition is not constant in optimisation. Another wing axis definition of the PreDoCS beam axis is therefore necessary, including the feasible region for cross-section placement.

The coordinate system module allows the definition of the first and the last node of the PreDoCS beam model. The axis through both points is the selected beam-axis, the z-axis in the PreDoCS coordinate system. The x-axis fulfils the requirement to be orthogonal to the z-axis and to be within the global x-y-plane. The PreDoCS y axis points upwards on the wing and is orthogonal to both other axis. Three options are available to select the beam start and end point:

- In order to be downward compatible to existing PreDoCS calculations, the global CPACS x-axis can be selected as beam axis. In this case the first beam node is the origin and the last beam node on x-axis is only defined by the wing span.
- As a geometrical approach, a point at 50 % camber-line of the wing-root aerofoil and the wing-tip aerofoil is chosen as beam start and end node respectively. This option is the preferred option if no dynamic model reference points are given in the CPACS model.
- The CPACS schema offers the possibility to define a dynamic model. Such a model is usually built from straight beam structures and defined with dynamic model reference points. In the dynamic model the fuselage and each lifting surface is modelled as a single beam element. Dynamic models are mainly used for aeroelastic calculations and share the challenge to simplify a complex beam model to a single axis. Because the outer loads are calculated with the same model, it seems promising to allow PreDoCS to use the dynamic model definition for its own calculation. In this case the first and the last dynamic model reference point is selected as beam start and end node. The definition of a PreDoCS coordinate system based on dynamic model reference points is shown in *figure 3.1*.

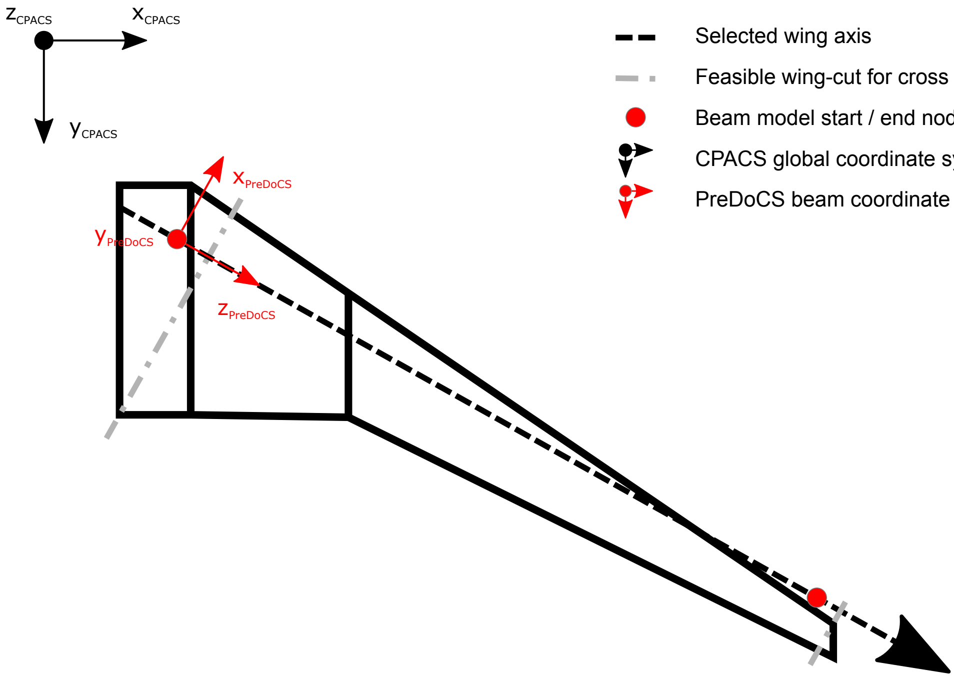


Figure 3.1: PreDoCS coordinate system definition

Based on the defined PreDoCS coordinate system, the beam start and end point and the chosen number of beam elements, the beam node positions and the cross sections positions are calculated. All beam nodes are uniformly distributed resulting in equal beam element lengths. For each beam element one cross section is placed in the middle of the beam element. This implies the cross section stiffness in the middle of a beam element is representative for it. The strong coupling of beam elements and cross sections ensures the quality of the solution is improving with the number of beam elements. *Figure 3.2* shows the beam nodes, beam element and cross section for an exemplary, swept back wing.

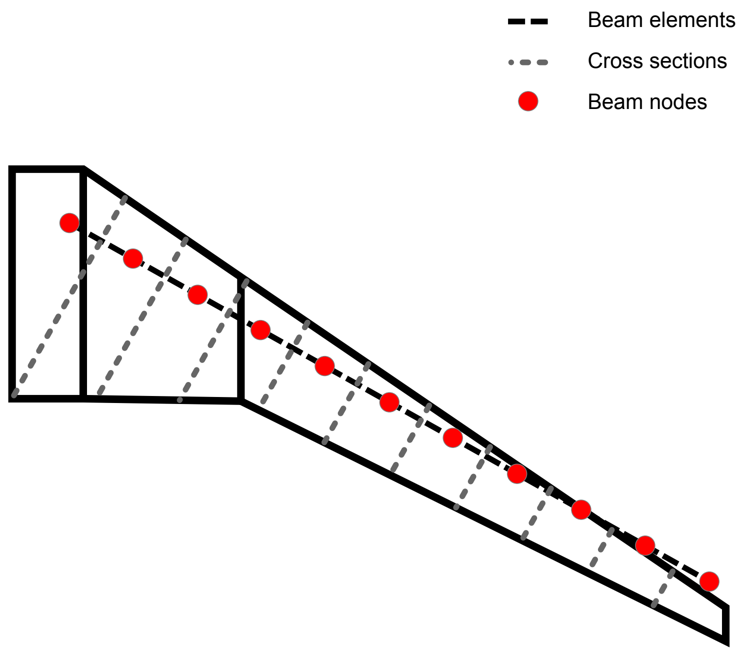


Figure 3.2: PreDoCS cross sections and beam elements

3.2 Load Interface

The previous development of PreDoCS focused on the analytical cross-section module, the beam model and the assembly of the beam stiffness matrix. Outer loads were assumed for tests and comparison of the calculation model whereas for structural evaluation maximum loads applied to the structure have to be known. In the case of wind turbine blades and aircraft wings such loads are the result of numerous aerodynamic and aeroelastic calculations. In the scope of this thesis, it is neither feasible nor necessary to include extensive aeroelastic calculations in PreDoCS. Instead loads from well established aeroelastic calculations can be added to the CPACS model and imported by PreDoCS. Using CPACS loads has the advantage of a defined interface and allows PreDoCS to work with different load calculation tools. In the context of structural optimisation, load updates in the optimisation process can be realized through such an interface.

In a CPACS model nodal loads are provided at dynamic model reference points in global CPACS coordinates. A coordinate transformation of the loads into the local PreDoCS coordinate system is therefore the first step in loads processing. The transformation of the force and moment vectors in PreDoCS is realised with a rotary matrix. Only in very special cases all dynamic model reference points coincide with the PreDoCS beam nodes. An interpolation of the loads along the dynamic model reference points axis makes sure one load is given at each beam node z-position. From here the bending forces are shifted in cross section wise direction to the shear centre position and the normal force to the beam axis, requiring a moment adjustment. As input for the one dimensional finite element analysis the loads are sorted by the beam node degrees of freedom.

3.2.1 Load Interpolation

Loads given at dynamic model reference points are usually condensed loads from a wing reference area. A direct interpolation of the nodal loads at the z-positions of the PreDoCS beam nodes does not consider a changing number of load output stations and therefore causes a significant change in load magnitude. For this reason the nodal loads are distributed on the dynamic model reference axis to a reference length equal to the node interval length. The load distribution is shown in the upper two plots of *figure 3.3* at the example of normalised loads for a reference wing. The distributed loads are redistributed to the intervals of the PreDoCS beam elements adding loads outside the beam elements to the first or the last beam element in order to ensure the total sum of loads is preserved. The redistributed loads are condensed to new nodal loads at the z-position of the PreDoCS beam nodes shown in the lowermost plot of *figure 3.3*. The same process, shown at the example of vertical forces, is used for all three forces and moments defined in the CPACS schema for nodal loads.

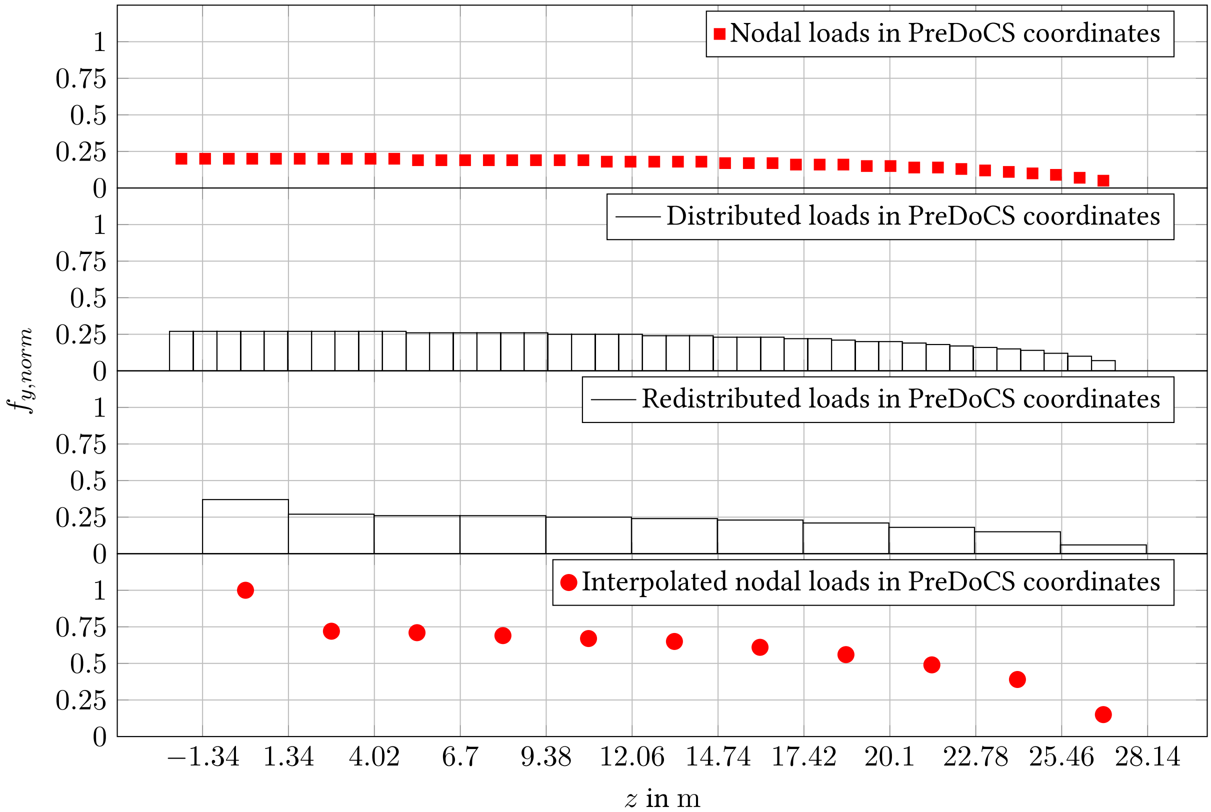


Figure 3.3: Load interpolation of normalised vertical forces

3.2.2 Load Shift to Beam Nodes

A coordinate transformation and a load interpolation are used to determine the outer loads in the PreDoCS coordinate system at the beam node z-positions. The analytical cross section theories available in PreDoCS expect transversal loads to be applied at the shear centre and longitudinal loads in the origin of each cross section. For each beam element, one cross section is defined. The shear centre position of all cross sections is calculated in PreDoCS and interpolated along the wing axis with cubic splines. As a result shear centre positions can be calculated at the beam node z-positions, where the loads have to be applied at the one dimensional beam finite element model. Within the z-plane of each beam node, the forces have to be shifted to the shear centre, adapting the moments accordingly. A schematic force shift and the resulting additional moment is shown in *figure 3.4*.

3.3 Application Programming Interface (API)

PreDoCS as a tool is designed as a python library providing functionalities for the structural evaluation of beam like structures. Its modular set up allows an easy exchange of singular functionalities. A full structural calculation on the other hand demands the combination of all modules resulting into a

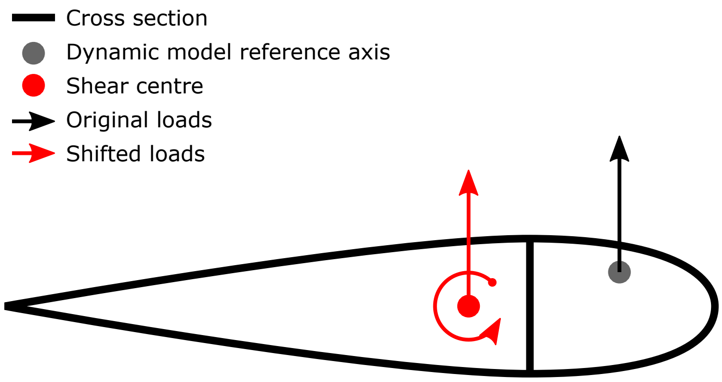


Figure 3.4: Load shifting to cross section shear centre

large control script which demands detailed knowledge of the low level modules. For an automated optimisation process it is beneficial to have an application programming interface (API), which also is useful to make PreDoCS available to non developing users. Such an interface combines the low level modules to top level functionalities which can be controlled by few, meaningful parameters. All parameters are well explained in the PreDoCS documentation and initialised with standard values for easy selection. Some of these parameters are:

- CPACS model
- PreDoCS coordinate system placement
- Cross section discretisation
- Element discretisation
- Analytical cross section theory

Based on these fundamental parameters, PreDoCS can built the cross sections and the beam model and solve the beam deformations. A post processing method allows also the calculation of stresses and strains. PreDoCS offers many plot methods to visualise the results. The application programming interface also allows the implementation of interface methods for the optimisation framework. Next to the generation of the structural model, methods returning panel stresses and deformations are available, complemented by a method to update the PreDoCS model based on the optimisation model.

Chapter 4

A Structural Optimisation Framework for Preliminary Design

The main objective of the present work is to set up a generalised structural optimisation framework for conceptual and preliminary design of wing like structures. In order to use this optimisation framework in the early design phase, its major characteristics are robustness and optimisation speed. Further it is desirable to set up the framework as general as possible so it can be used for multi-disciplinary and multi-fidelity optimisation. Two different approaches designing such a framework seem generally feasible, shown in *figure 4.1* and *figure 4.2*.

The most general approach is an optimisation framework directly based on a CPACS model. A CPACS based optimisation framework, shown in *figure 4.1*, has several advantages and disadvantages:

+ The optimiser has access to all information provided in a CPACS model.
+ Result data is universally available for other tools.
+ Interfaces are properly defined and documented through CPACS.
+ PreDoCS can be replaced by high fidelity tools in later design phases.
− The result of PreDoCS are internal structural loads which are provided in the form of element wise internal line loads. These loads are necessary for failure criteria evaluation and thus an essential input to the optimisation framework. CPACS schema on the other hand provides no data type to store such loads.
− In order to optimise the layup of composite materials with gradient based optimisation methods, the layup has to be represented by continuous design variables. A common approach is to use lamination parameter to define the material properties. Lamination parameter however are not supported by the CPACS schema material definition.
− CPACS models are stored in Extensible Markup Language (XML) format. To access such plain text files is very slow compared to binary formats.
− PreDoCS uses TiGL for the cross section generation which is slow. Thus the CPACS model import into PreDoCS is very slow compared to the actual PreDoCS calculation.

As discussed before an essential characteristic of the optimisation framework in the present paper is a rapid optimisation. The PreDoCS beam model generation with the current TiGL version 3.0.0, takes several minutes depending on the model size and cross section discretisation. Compared to a stress calculation time of less than one second the PreDoCS model generation is very time consuming and it is therefore not reasonable to generate a new PreDoCS beam model from a CPACS file in every iteration of the optimisation process. Instead a solution is needed where the PreDoCS beam model is only created once before the optimisation process. Only stiffness and thickness properties of this model are adjusted in the loop.

An approach for an optimisation framework which optimises a PreDoCS beam model is shown in *figure 4.2*. The CPACS model is imported into PreDoCS and not changed during the optimisation process. Subsequent to the optimisation process, the CPACS model is updated only with the final optimisation parameters. This approach results into a much faster process but also limits the optimisation framework to a purely structural optimiser. Data provided from other disciplines, as for example loads, can be updated after the optimisation process or have to be integrated explicitly into the optimisation process.[1].

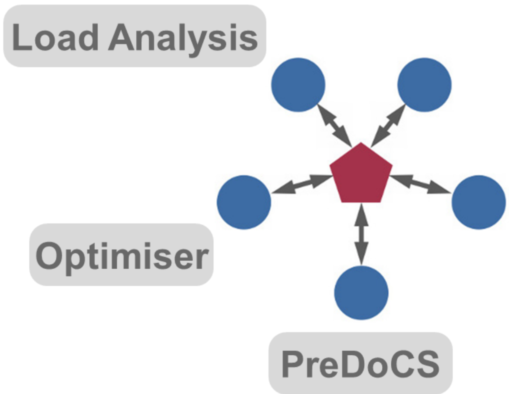


Figure 4.1: CPACS based framework

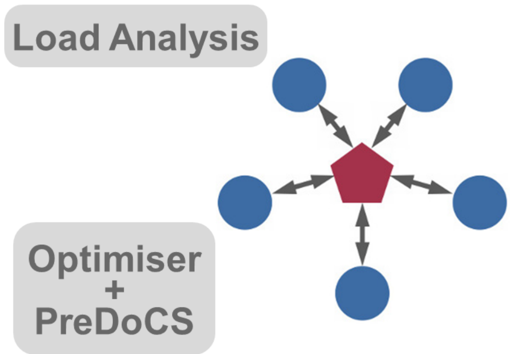


Figure 4.2: PreDoCS based framework

4.1 Modular Design of the PreDoCS based Optimisation Framework

The optimisation framework established in this thesis is shown in *figure 4.3*. The framework is build in a modular manner with clearly defined interfaces between the modules. This allows to exchange or enhance modules easily in the later development. In order to keep the optimisation framework independent from other tools it is equipped with its own structural and loads module. The constraint module and the optimisation module are solely based on information provided in the structural and the loads module.

4.2 Structural Model

The structural model provides the geometric and material information needed in the optimisation process. The concept of the structural model is shown in *figure 4.4*. On the lowest level a structure is described as a skin, which is either a composite material or a metal sheet. All skins need to provide the attributes thickness, density and stiffness which is given in the form of an ABD matrix. The skin defines a single material region and therefore represents the material optimisation region. A failure region is represented by a panel and usually limited by surrounding supports, such as stringers and ribs. The dimensions of a panel are therefore determined by the rib and stringer spacing. A simple panel needs two dimensions, a and b, and a reference to a skin. With these information analytical buckling and strength criteria can be evaluated and a calculation of the panel mass is possible. Multiple panels can reference to the same skin and therefore reference to the same optimisation region. If multiple panels point to the same skin, one optimisation region is divided into multiple failure regions which is equivalent to one optimisation variable and multiple constraints. Panels can be grouped into assemblies. Each assembly therefore contains a list of at least one panel. Further an assembly holds information of the panel connectivity and allows to derive the skin connectivity from it. Since the skins also represent

[1]A real Multi-Disciplinary-Optimisation (MDO) can be established based on this structural optimisation process by adapting a aerodynamic solver to work with a cross-section model as provided by PreDoCS.

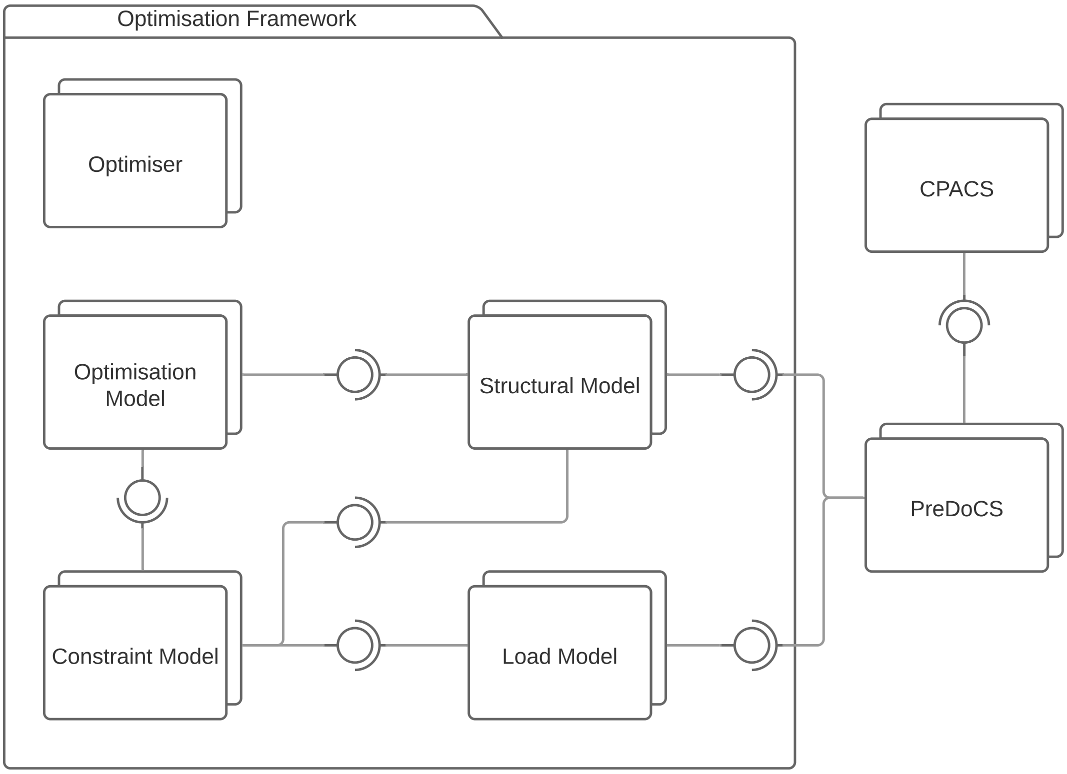


Figure 4.3: Optimisation framework with PreDoCS

optimisation regions the skin connectivity is needed as manufacturing constraints in the optimisation process.

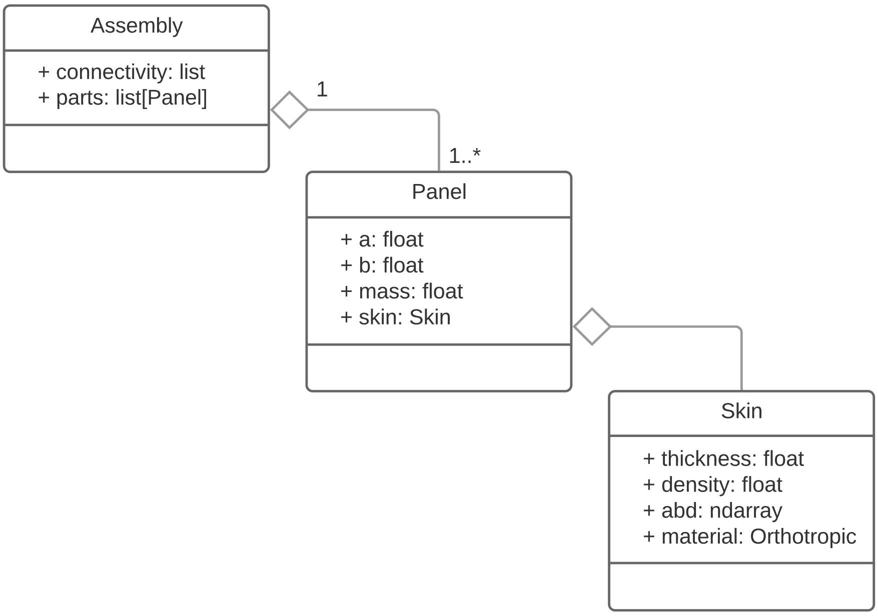


Figure 4.4: UML diagram of the structural model

A structural model as shown in *figure 4.4* can be derived from any kind of structural representation, for example from a CPACS schema conform model or from a model made for commercial FE software. In this thesis, the structural model is derived from a PreDoCS model in order to be consistent with the structural load analysis. The PreDoCS model is divided into assemblies for the upper cover, the lower cover and webs. The assembly configuration of the PreDoCS model is also used for the assembly configuration of the structural model in the optimisation framework. PreDoCS segments are defined as material regions of one cross sections with no branches and can be considered the equivalent of panels in the optimisation model. A minimum segment discretisation of a PreDoCS model is shown in *figure 4.5*. On material level PreDoCS components can be translated to skins in the structural model. The PreDoCS components are derived from the CPACS cells shown in *figure 4.6*. All equivalents are summarised in *table 4.1*.

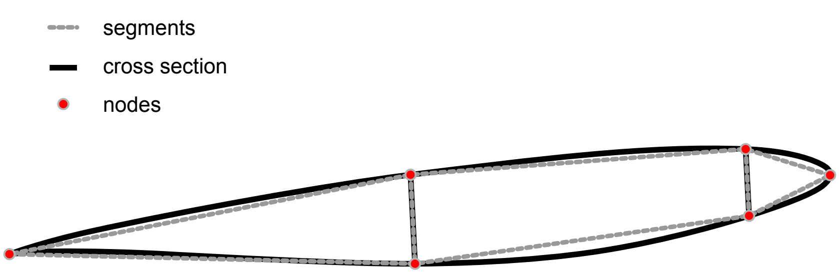


Figure 4.5: Minimum segment discretisation of a cross section in PreDoCS

Because PreDoCS does not support stringers and ribs, a conservative estimation of the panel dimensions has to be made. Segments in contrast to panels are one dimensional and therefore only have a length. The second dimension needed for a panel is defined as the distance of the PreDoCS cross sections

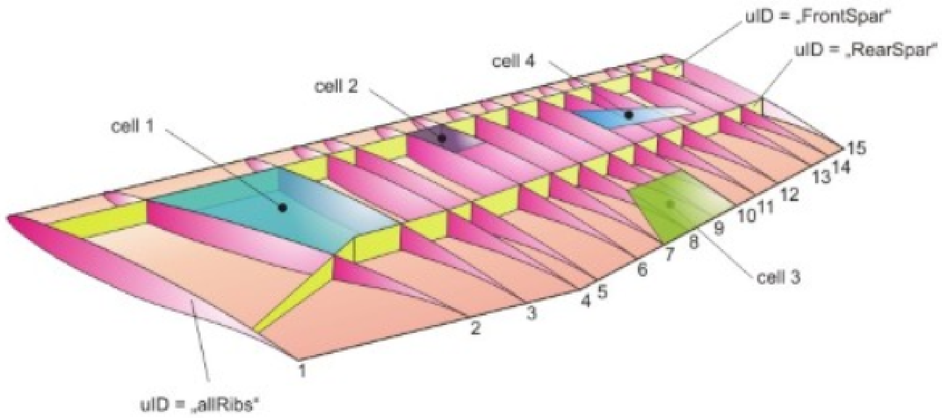


Figure 4.6: CPACS cells defined in the CPACS documentation [7]

CPACS	PreDoCS	Optimisation
component segment	assembly	assembly
	segment	panel
cell	component	skin

Table 4.1: Structural elements in CPACS, PreDoCS and the optimisation

which is equivalent to the PreDoCS beam element length. In analytical buckling analysis the critical buckling force decreases with panel size (the panel gets softer). As long as the PreDoCS beam elements are longer than the actual rib spacing and the segment length is longer than the stringer spacing both assumptions are conservative. The requirement of segments longer than the stringer spacing is for manufacturing reasons almost certainly fulfilled. The PreDoCS cross section discretisation on the other hand can be chosen freely. If a high cross section discretisation is chosen in PreDoCS, the assumption of conservative panel dimensions is violated and a conservative rib spacing has to be estimated by the user. The derivation of panels from a PreDoCS cross sectional model is shown in *figure 4.7*. The mass calculation of the structural model is based on the panel geometry resulting in a rough mass estimation for a rough cross section discretisation. The accuracy of mass calculation however increases with the cross section discretisation in PreDoCS.

In the assemblies of the optimisation framework structural model, panel connectivities in cord-wise and span-wise directions should be considered as manufacturing constraints and are assumed to improve the quality of the optimisation result. The connectivity of the structural model needs to be derived from the PreDoCS model. Therefore the connections between all segments within each assembly need to be identified. PreDoCS assemblies are divided into separate cross-sections. While within the cross sections neighbour segments share one node and can therefore be identified as such, no connection exists between the segments of two neighbouring cross sections. For this reason an algorithm is established, to find the cross-sectional and inter-cross sectional connections of the panels, as shown in *figure 4.8*, which compares the segments in each assembly. The cross-sectional connections of the segments can be found by searching shared nodes. The inter-cross-sectional connections are determined by comparing the number of segments in each cross section, distributing the segments equally and then connecting all neighbours. No actual positions or dimensions of the segments are taken into account.

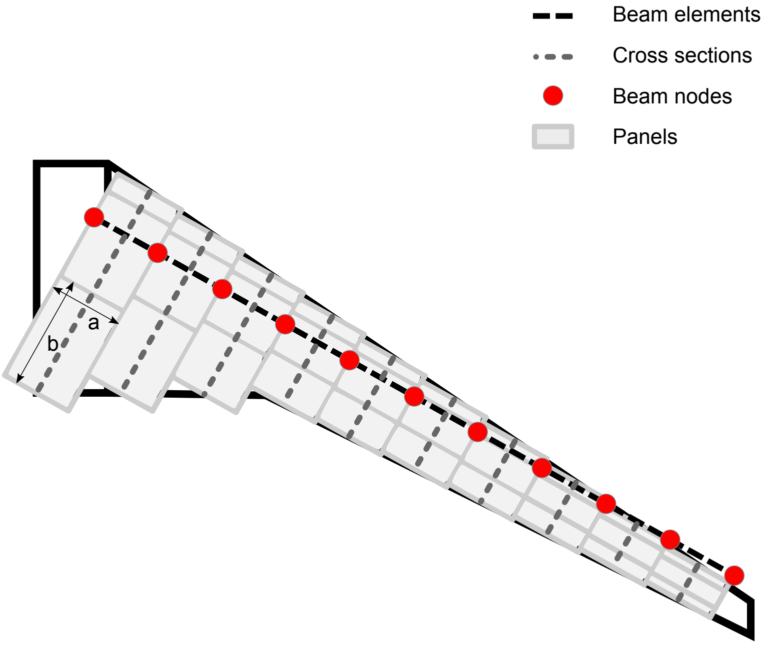


Figure 4.7: Panel derivation of a PreDoCS cross sectional model

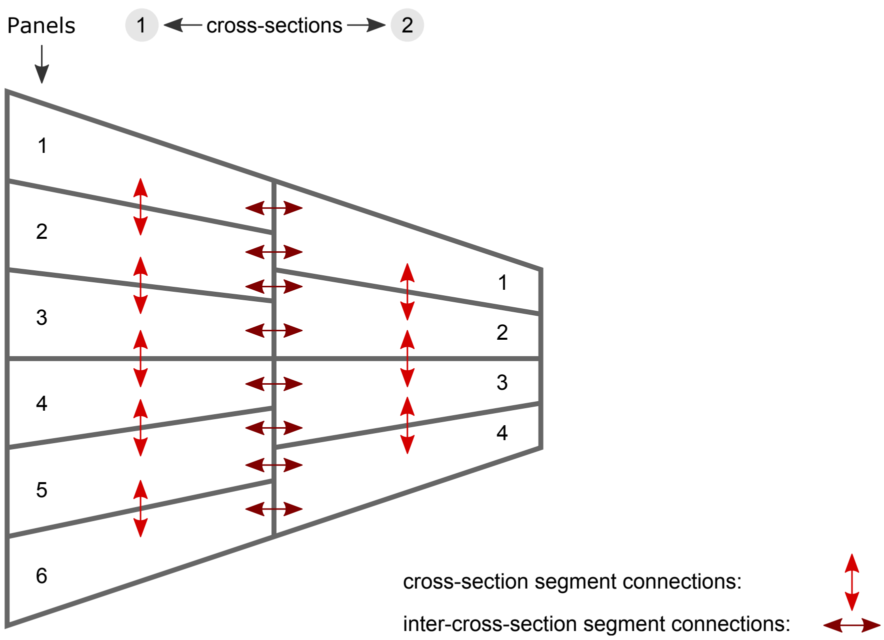


Figure 4.8: Illustration of the panel connectivity of an assembly

4.3 Load Model and Loads Processing

The strength and stability analysis of skins and panels in the constraint processor is based on the internal wing loads. PreDoCS calculates the internal loads of the wing for each cross section in the form of force and moments fluxes. In PreDoCS the curvature of a cross section is discretised by elements. For each element the internal loads are calculated as constant, linear or quadratic function, depending on the basic theory and load type. The longitudinal, internal line loads calculated with PreDoCS for the wing root of the MiRaJet baseline configuration is shown in *figure 4.9*. For each load case and each element a set of six load functions is available in PreDoCS. The strength and stability criteria implemented in the constraint processor require constant loads. For this reason the load functions of each element are evaluated at three points as shown in *figure 4.10*, then the maximum absolute load is selected. All internal loads of one element are collected in one panel load case. Each element is part of one distinct PreDoCS segment, which is the counterpart to a panel of the structural model. This allows to assign a list of panel load cases to each panel.

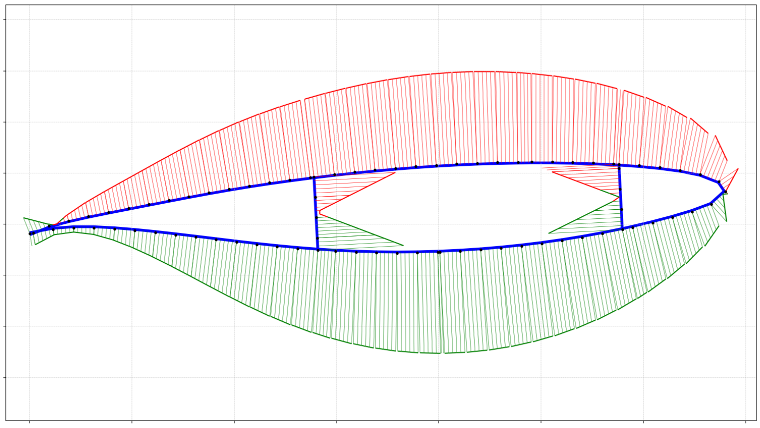


Figure 4.9: Distributed internal line loads with element-wise linear load functions

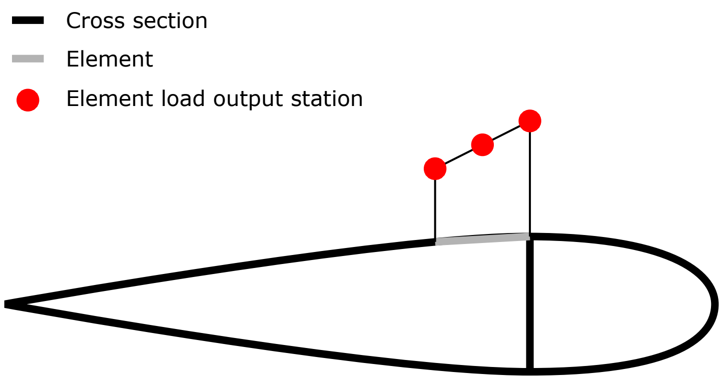


Figure 4.10: Reference load output stations for the generation of load cases

Before the evaluation of strength and stability criteria can take place, the panel load cases have to be assigned to an appropriate criteria, e.g. compression load cases to buckling criteria, shear load cases to buckling and strength criteria etc. Furthermore, if many load cases exist for each panel, it can be advantageous for optimisation process speed to reduce the number of panel load cases in order to reduce the number of inequality constraints. However such a selection of the "sizing panel load cases" as it can be found in commercial optimisation tools as HyperSizer [2] has two major disadvantages in an optimisation process. The first problem is a change of the sizing panel load case can result into a jump of the constraint value accompanied with physically meaningless gradients of the constraint function. The even severe problem is the change in the number of constraints functions if the number of selected panel load cases changes. To avoid both issues in this work all used strength and stability criteria are evaluated with all available panel load cases independently of the containing loads. For this reason it has to be ensured in the constraint processor, that all criteria are valid in the complete load space, e.g. buckling criteria return a meaningful value in the tensile regime etc.

4.4 Constraint Processor

The allowable space of the design variables in an optimisation process is defined by constraints, where the number of constraints is independent of the number of design variables. Most optimisation algorithms accept constrains as equality constraints, usually $h(x)$, or inequality constraints, usually $g(x)$, defined in *equation 4.1* and *equation 4.2* respectively. It is noteworthy that all equality constraints can usually be transformed into inequality constraints. These so called implicit constraints define, together with the explicit lower and upper boundaries of the parameters x_L and x_U, the allowable design space of the optimisation problem. In the constraint processor of the proposed optimisation framework only implicit constraints in the form of inequality constraints are used. Explicit constraints, the lower and upper boundaries of the parameter vector, are defined in the optimisation model. The inequality constraints are divided into four categories:

- Laminate design rules (skin level, e.g feasible domain of the lamination parameters)
- Strength (skin level)
- Stability (panel level)
- Manufacturing (Assembly level, e.g. ply continuity, materials)

$$h(x) = 0 \tag{4.1}$$

$$g(x) \leq 0 \tag{4.2}$$

$$x_L \leq x \leq x_U \tag{4.3}$$

All constraints can be evaluated with the information provided by the structural model and the loads model with the advantage of independence from the chosen solver. Each constraint is evaluated on a different level. On the skin level, defining the material properties of an optimisation region, the feasible region of the lamination parameter set is determined and the strength evaluation takes place. On panel level the stability analysis considers compression and shear buckling phenomena in a combined form. Manufacturing constrains ensure a percentaged continuity of the laminate across optimisation regions (material regions). The methodology of the feasible region of lamination parameters and the implementation of manufacturing constraints for lamination parameters is very similar and hence described together in *section 4.4.1*.

In *section 4.4.2* and *section 4.4.3*, different strength and stability criteria are compared. The objective of the comparison is to find few suitable strength and stability criteria covering the complete load space in order to reduce the number of constraints to a minimum and keep the computational effort minor. The main result of the strength and stability analysis is a safety factor λ. The safety factor is the ratio of critical load and applied load as defined in *equation 4.4* and can be transformed into a constraint as shown in *equation 4.5*. The safety factor is a function of the applied loads n and the critical loads $n_{critical}$, which depend on the laminate layup and material. The safety factor can therefore be described as a surface in the three-dimensional $[n, n_{critical}, s_f]$ space. For the sake of representation the safety factor is investigated firstly for a change in applied load keeping the critical load constant and secondly for a change in critical load, keeping the applied load constant. The variation of the critical load is achieved by rotating a given laminate in the range $\alpha = [-90^\circ \ldots 90^\circ]$. It is expected the safety factor is high when the fibre orientation is equal to the load orientation.

$$\lambda = \frac{n_{critical}}{n} \geq 1 \tag{4.4}$$

$$g(x) = \frac{n}{n_{critical}} - 1 \leq 0 \tag{4.5}$$

$$g(x) = \frac{1}{r_f} - 1 \leq 0 \tag{4.6}$$

The analysis of strength and stability criteria uses the laminates and materials provided in *table 4.2* and *table 4.3* in combination with the load cases provided in *table 4.4*. The first laminate is made of a single uni-directional layer in order to observe influences of layer angles on the strength and stability criteria. The second laminate is a quasi-isotropic laminate used to investigate differences between layer-wise criteria and such based on lamination parameter at a more realistic laminate. The third laminate is used for the same investigations but is designed with a preferred direction as it is probable for most design cases. All three laminates use a common unidirectional carbon fibre tape for general purpose commercial and military structural applications defined in *table 4.3*. All load cases are uni-directional load cases improving the physical understanding of the strength and stability criteria. The test panel for the investigations with the dimensions $a = 1.0\,\mathrm{m}$ and $b = 1.0\,\mathrm{m}$ is defined in *figure 4.11*.

Laminate	Material	Layup
Laminate 1	T-300 15k/976	[0]
Laminate 2	T-300 15k/976	[0, 45, 90, -45, -45, 90, 45, 0]
Laminate 3	T-300 15k/976	[0, 45, 90, -45, 0, 0, -45, 90, 45, 0]

Table 4.2: Laminates used for the strength and stability investigations

4.4.1 Feasible Region, Laminate Design Rules and Manufacturability

The stiffness properties of an optimisation region can be described in the structural model as lamination parameters. Lamination parameters of one laminate are by definition not independent of each other. All valid combinations of lamination parameters together form the feasible region of the lamination parameter set. Laminate design rules can be realised in the same way, further diminishing the feasible region[24]. Manufacturing constraints consider the difference in the layup of two adjacent laminates. In order to increase the manufacturability, this difference should be kept to a minimum. The difference in layup can also be expressed as a difference of the lamination parameter sets of each skin. The maximum valid differences of two adjacent skins therefore can be represented again by a feasible region, this time for the differences of two lamination parameter sets. Both feasible regions can be defined by a set of hyperplanes forming a convex hull[16][23]. This approach also assures a convex design space.

An optimisation region is represented in the structural model by a skin. A skin is defined by a basic material, a thickness and a lamination parameter set standing for the layup of the laminate. The feasible region of each skin is determined in the constraint processor as the distance of the lamination parameter set to the hyperplanes of the convex hull. In this thesis, the convex hull is discretised by 49 hyperplanes. The feasible region only allows 0°, $\pm 45^\circ$ and 90° layers and symmetric and balanced

Material	T-300 15k/976	
Stiffness		
E_1	$133.70\,\mathrm{GPa}$	
E_2	$9.24\,\mathrm{GPa}$	
G_{12}	$6.27\,\mathrm{GPa}$	
G_{23}	$3.50\,\mathrm{GPa}$	
ν	0.318	
Strength	**Tension**	**Compression**
σ_1	$1427.21\,\mathrm{MPa}$	$1503.06\,\mathrm{MPa}$
σ_2	$39.02\,\mathrm{MPa}$	$206.84\,\mathrm{MPa}$
τ_{12}	$76.53\,\mathrm{MPa}$	
τ_{23}	$76.53\,\mathrm{MPa}$	

Table 4.3: Material definition for the strength and stability investigations[6]

Loadcase	n_x	n_y	n_{xy}
Loadcase 1	$100\,000\,\mathrm{N\,m^{-1}}$	$0\,\mathrm{N\,m^{-1}}$	$0\,\mathrm{N\,m^{-1}}$
Loadcase 2	$1\,000\,000\,\mathrm{N\,m^{-1}}$	$0\,\mathrm{N\,m^{-1}}$	$0\,\mathrm{N\,m^{-1}}$
Loadcase 3	$0\,\mathrm{N\,m^{-1}}$	$0\,\mathrm{N\,m^{-1}}$	$1\,000\,000\,\mathrm{N\,m^{-1}}$
Loadcase 4	$-5\,000\,000\,\mathrm{N\,m^{-1}}$	$0\,\mathrm{N\,m^{-1}}$	$0\,\mathrm{N\,m^{-1}}$
Loadcase 5	$-100\,\mathrm{N\,m^{-1}}$	$0\,\mathrm{N\,m^{-1}}$	$100\,\mathrm{N\,m^{-1}}$
Loadcase 6	$-100\,000\,\mathrm{N\,m^{-1}}$	$0\,\mathrm{N\,m^{-1}}$	$100\,000\,\mathrm{N\,m^{-1}}$

Table 4.4: Load cases used for the strength and stability investigations

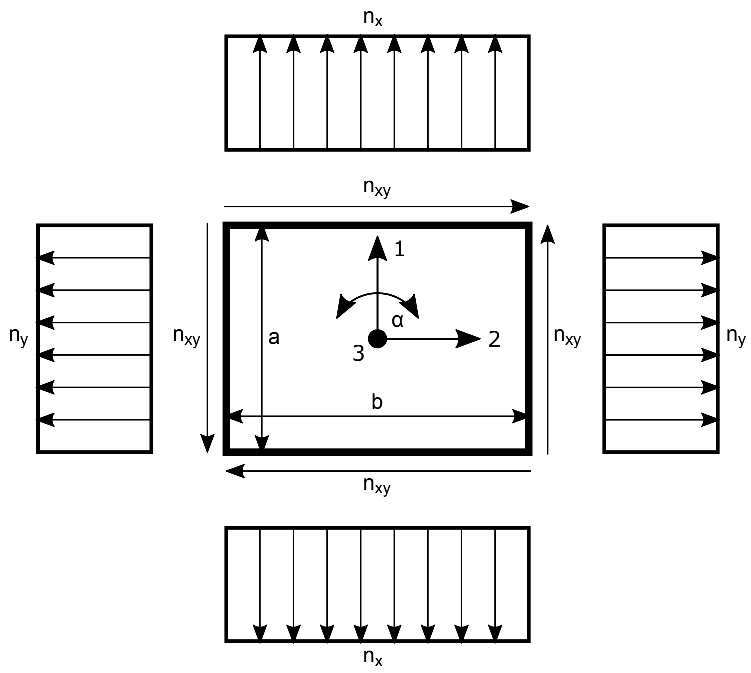


Figure 4.11: Panel definition for strength criteria and stability criteria comparison

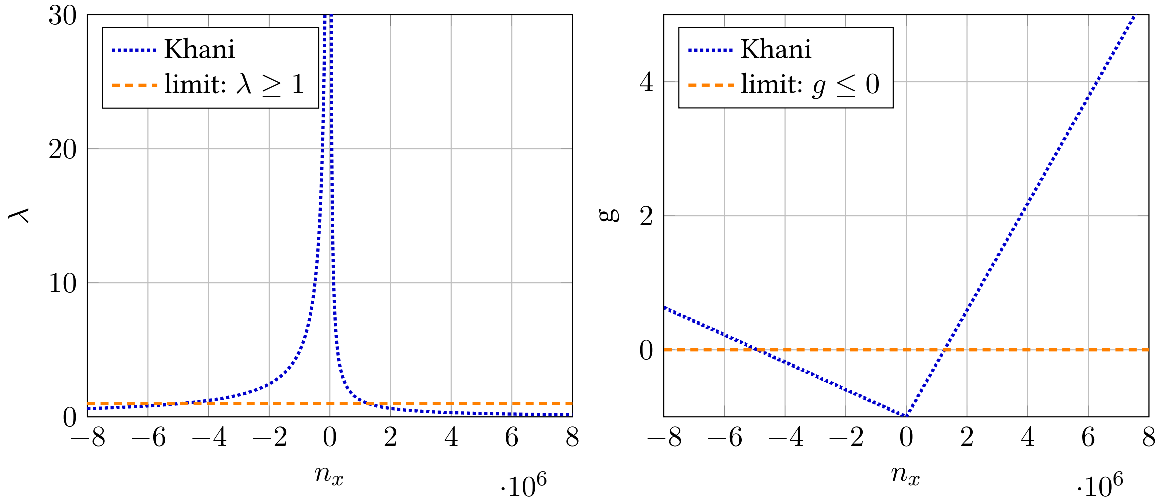
laminates. The distance of the lamination parameter set to each hyperplane generates one constraint resulting into 49 constraints per optimisation region. Because the feasible region is determined prior to the optimisation and a hyperplane is represented by a linear equation, the constraint evaluation needs only little computational power resulting in a rapid process.

Manufacturing constraints can be implemented as feasible region in the lamination parameter design space. Instead of one set of lamination parameters, this feasible region is valid for the difference of two sets of lamination parameter. In this thesis the feasible region for the manufacturing constraints is described as a convex hull of 7 hyperplanes resulting into seven constraints per connected optimisation regions. The volume enclosed by the convex hull for manufacturing constraints depends on the total thickness of the two connected lamina. The continuity of thicker lamina is limited by a smaller volume in the lamination parameter design space. The number of layer combinations providing the required continuity increases with the layer numbers. At the same time the number of allowed combinations with respect to the total number of combinations decreases. This explains the reduced volume of the feasible design space for thick laminates. The restriction of thin laminates to the design space of thick laminates is not likely result into actual laminates fulfilling this ply continuity criterion. On the other hand restricting thick laminates to the design space of thin laminates, to find actual laminates is far more likely. For this reason a minimum laminate thickness has to be estimated and the feasible region for manufacturing constraints for an equivalent number of plies has to be used. In this thesis the mid range aircraft MiRaJet is investigated. A minimum ply thickness of three millimetre is chosen, roughly equivalent to a laminate of 30 plies. In this way the feasible design region for lamination parameter with manufacturing constraints defined in[24] can be used in this thesis.

4.4.2 Strength Evaluation on Skin Level

Lightweight wing structures are usually exposed to considerable bending and torsion loads causing tension, compression and shear loads in the structure. Compression is the dominating load on the upper cover, tension on the lower cover and shear on the webs. For all three cases a strength analysis is necessary, usually performed for each layer of a composite material. The material definition in the structural model of the optimisation framework is based on lamination parameter. Therefore information of the stacking sequence and layer angles of the composite material are not available during the optimisation process. Due to this fact standard formulations of strength criteria evaluated on layer level cannot be used and a formulation of a strength criteria using lamination parameter is needed. Ijsselmuiden [11] proposes a strength criteria based on lamination parameter derived from the criteria of Tsai and Wu [22] using a transformation to a conservative strain space. This formulation was further developed by Khani [14] considering a rotation of the main stiffness axis of the composite.

The criteria by Khani, shown in *figure 4.12* at the example of "Laminate 2" under uni-directional load, is defined for tensile strength, shear and compression. While the safety factor λ shown in sub-figure (a) has a singularity for $n_x = 0$, the constraint g is a piecewise linear function providing C^0 continuity and is defined in the complete load space as demanded in *section 4.3*. Since the strength criterion by Khani is already defined in the complete load space, no further modelling of the constraint function is necessary.



(a) safety factor (b) constraint

Figure 4.12: Safety factor and constraint of "Laminate 2" (*table 4.2*)

In *figure 4.13* the safety factors are compared for the strength criteria by Ijsselmuiden et al.[11] and by Khani et al. [14] and for the layerwise criteria of Tsai and Wu[22] and Yamada and Sun[26]. For this comparison the uni-directional (UD) laminate "Laminate 1" (*table 4.2*) is rotated in the range $\alpha = [-90^{\circ} \ldots 90^{\circ}]$ while the tensile load of "Loadcase 1" (*table 4.4*) is applied. It can be observed the lamination parameter based criteria are conservative with respect to the layerwise criteria for all layer angles. It is noteworthy the difference in safety factor between Khani and Tsai/Wu in fibre direction (0°) is more than a factor of two while the difference vanishes perpendicular to the fibres ($\pm 90^{\circ}$). This observation confirms the conservatism of the approach already described by Ijsselmuiden and Khani [11][14]. In a strength driven optimisation a margin of over 100% in safety factor would increase the required material thickness by the same amount. For this reason the strength criteria by Ijsselmuiden and Khani show a limited usability for the optimisation of UD layers.

Figure 4.14 shows the same comparison of strength criteria for the more general layup "Laminate 3" defined in *table 4.2*. This laminate has a main stiffness direction at an angle of 0° in order to distinguish in fibre strength and inter-fibre strength. It can be observed the strength criteria by Ijsselmuiden and Khani are still conservative with respect to the layer based criteria. While the criteria by Ijsselmuiden is very conservative for all angles of the laminate, the criteria by Khani is a closer approximation of both layer-wise criteria. However the secondary stiffness direction of "Laminate 3" at approximately $\pm 70^{\circ}$ is not described by the criteria of Khani and Ijsselmuiden. Despite this limitation the strength criteria by Khani shows good resemblance to the layer-wise criteria of Tsai and Wu and Yamada and Sun for tensile

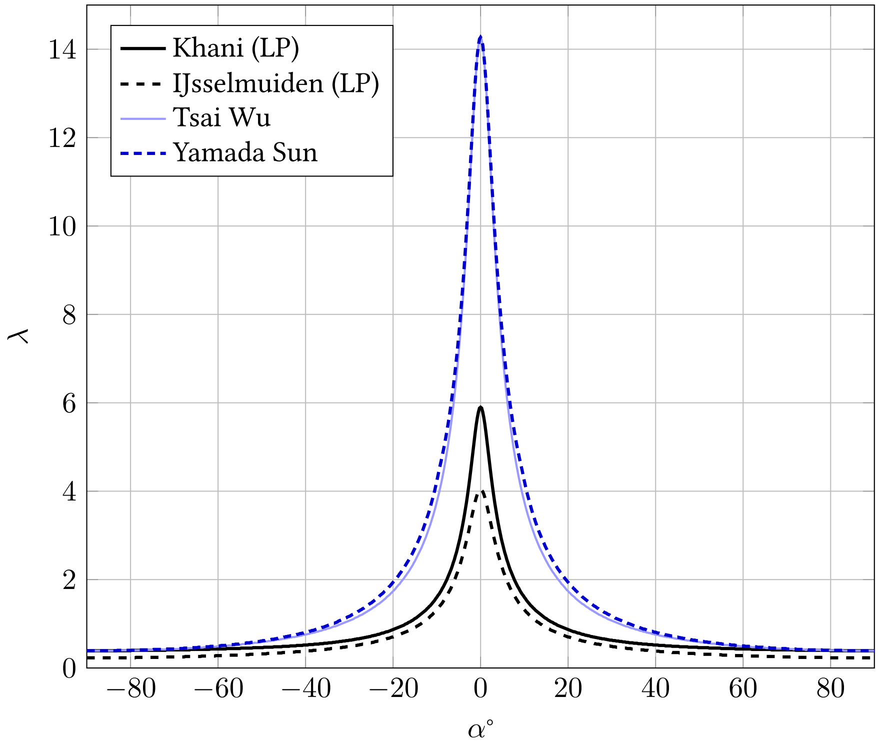


Figure 4.13: Strength criteria comparison for tension ("Laminate 1" and "Loadcase 1")

strength. *Figure 4.15* shows the safety factor for "Laminate 3" with the pure shear load case "Loadcase 3". The general shape of the safety factors with a peak at $\alpha = 45°$ where the $0°$ layers are rotated into the load axis is described by all criteria. For this loading the safety factors resulting from both lamination parameter based criteria are a good match. Compared to the layer-wise criteria of Tsai and Wu the criteria by Khani and Ijsselmuiden are very conservative with a more than factor 2 difference in safety factor. Therefore a strength driven optimisation where shear is the main load is expected to return very conservative results when using the lamination parameter based strength criteria. A comparison of strength criteria for the compressive loads is given in *appendix A*. This comparison is not further discussed here because for compressive loads stability is expected to be the sizing criteria.

The strength criteria by Khani shows promising results for laminates with general layups for tension and compression load cases. The criteria has limitations for uni-directional laminates and pure shear load cases where it results into very conservative safety factors. Uni-directional laminates such as "Laminate 1" make only sense on rare occasions but should be avoided in general due to laminate design rules[2]. Tension and compression loads in thin-walled, beam-like structures such as wings are usually one order of magnitude higher compared to shear loads and are therefore the driving loads in a structural optimisation. Lamination parameter based strength criteria only have to be evaluated once per laminate while the layer-based criteria have to be evaluated for each layer of a laminate. Considering the emphasis on rapid constraint evaluation in preliminary design this is an advantage of the layer-based criteria. Taking into account the conservatism of the strength criteria by Khani, its limitations are outweighed by the possibility to include strength evaluation in a purely lamination parameter based optimisation process. It is therefore used for the strength evaluation on skin level in the constraint processor of the optimisation framework developed in the scope of this thesis.

[2]Pure unidirectional laminates are only appropriate for real unidirectional loads. Therefore such laminates should only be used in bars or spar cabs for example.

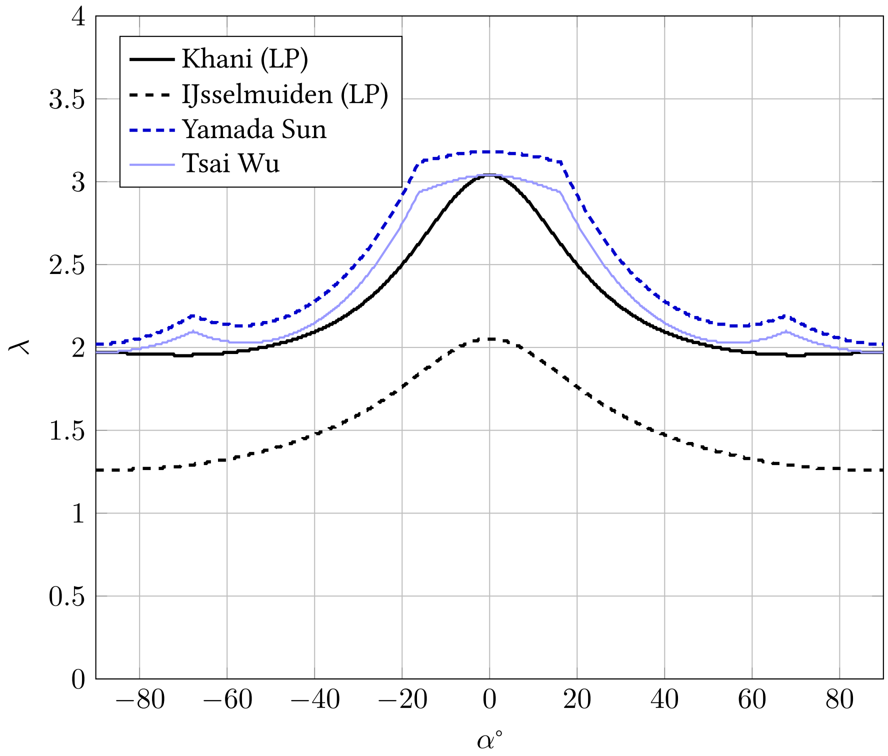


Figure 4.14: Strength criteria comparison for tension ("Laminate 3" and "Loadcase 2")

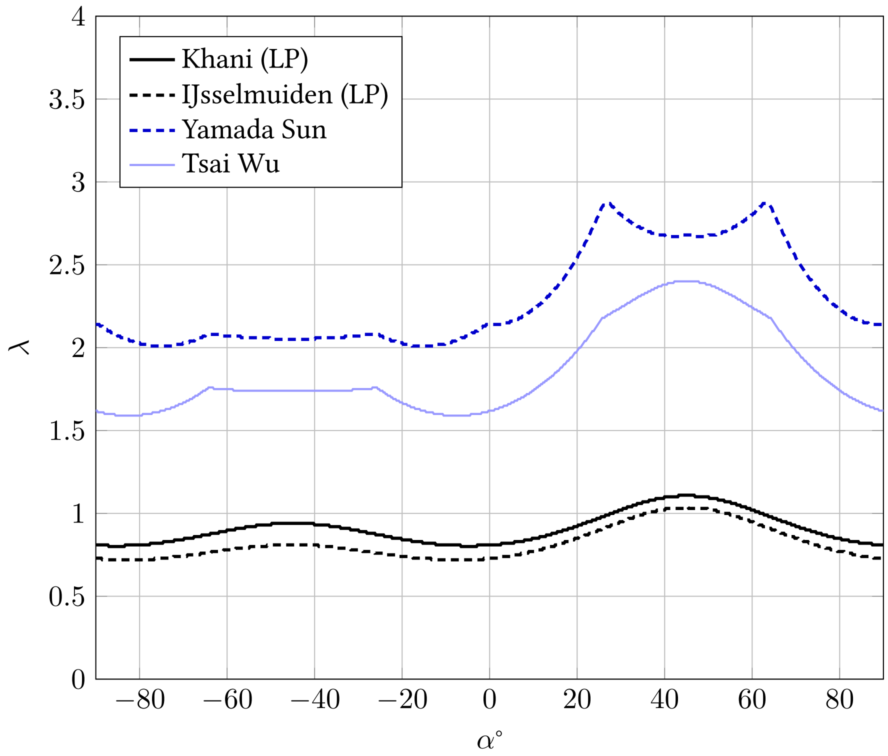


Figure 4.15: Strength criteria comparison for shear ("Laminate 3" and "Loadcase 3")

4.4.3 Stability Evaluation on Panel Level

Bending and torsion of thin walled beam like structures can cause considerable compression and shear loads at wing covers and spars. For such load conditions the structural stability is usually the sizing criteria compared to strength. For a structural optimisation in preliminary design, few, simple stability criteria are needed in order to keep the number of constraints and hence the computational effort minor. The structural analysis manual of the Luftfahrttechnisches Handbuch (HSB) [1] offers analytical buckling criteria for orthotropic plates with different load and support conditions. For simplification, only criteria with constant loading conditions as shown in *figure 4.11* are given consideration.

A further reduction of potential stability criteria is possible when only the governing loads expected in thin walled, beam like structures are taken into account. The main expected loads in the upper cover, lower cover and spar caps are stresses in span wise direction. In the spar webs shear loads are expected to be the sizing criteria. Transversal shear loads are neglected in the evaluation of strength and stability criteria, although they are evaluated in the structural solver. The critical buckling loads of orthotropic plates under compressive loads can be described by the analytic formula given in HSB 45111-08. The plate dimensions and loads of the criteria are shown in *figure 4.16*. Shear buckling of orthotropic plates can be described by HSB 45112-02 buckling criteria shown in *figure 4.17*. In the common case of a twisted and bended wing, shear loads are applied to the upper and lower cover additionally to the compressive stresses. A combination of both criteria provided by [25] is given in *equation 4.7* in terms of safety factor. The resulting buckling envelope of the combined shear and compression buckling criteria is shown in *figure 4.18*.

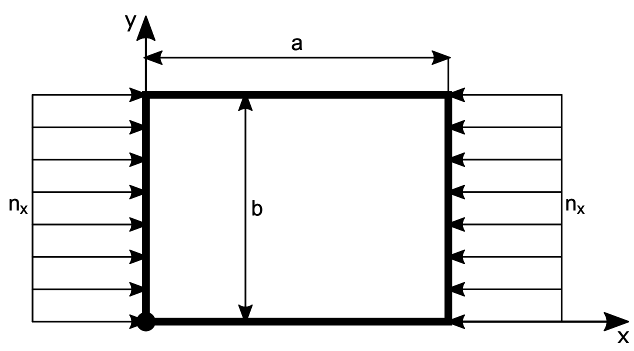


Figure 4.16: Definition of compression buckling criteria for orthotropic plates (HSB 45111-08)

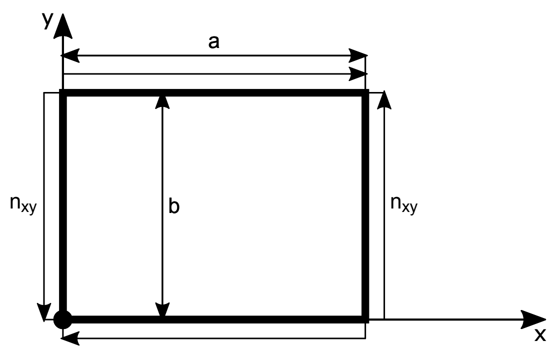


Figure 4.17: Definition of shear buckling criteria for orthotropic plates (HSB 45112-02)

$$\lambda_{combined} = \frac{1}{\lambda_{compression}} + \left(\frac{1}{\lambda_{shear}}\right)^2 \tag{4.7}$$

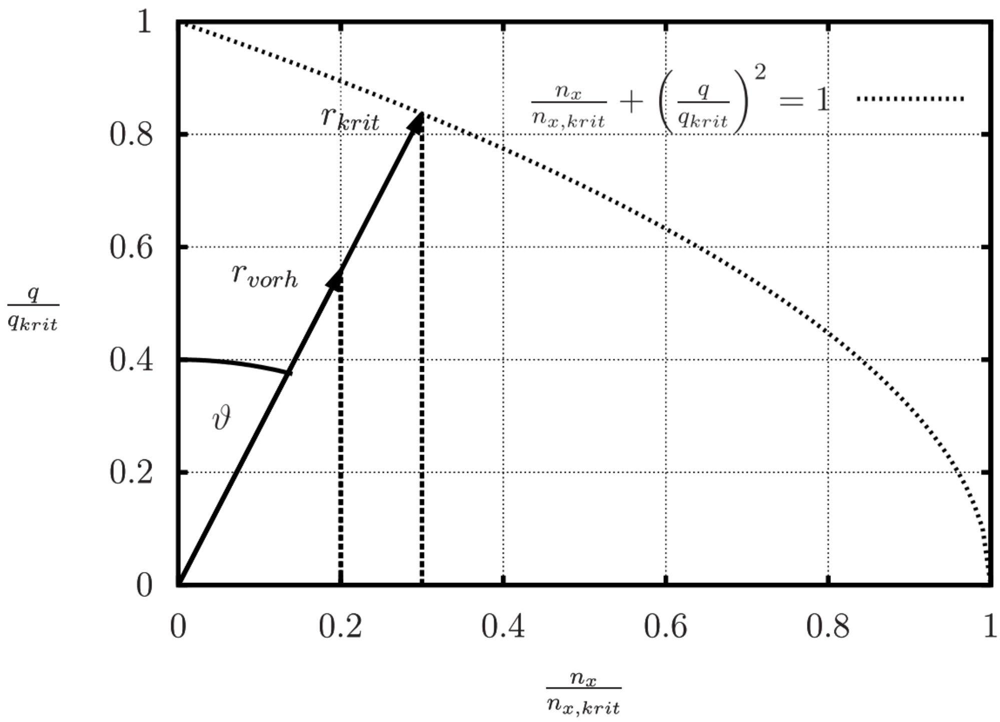
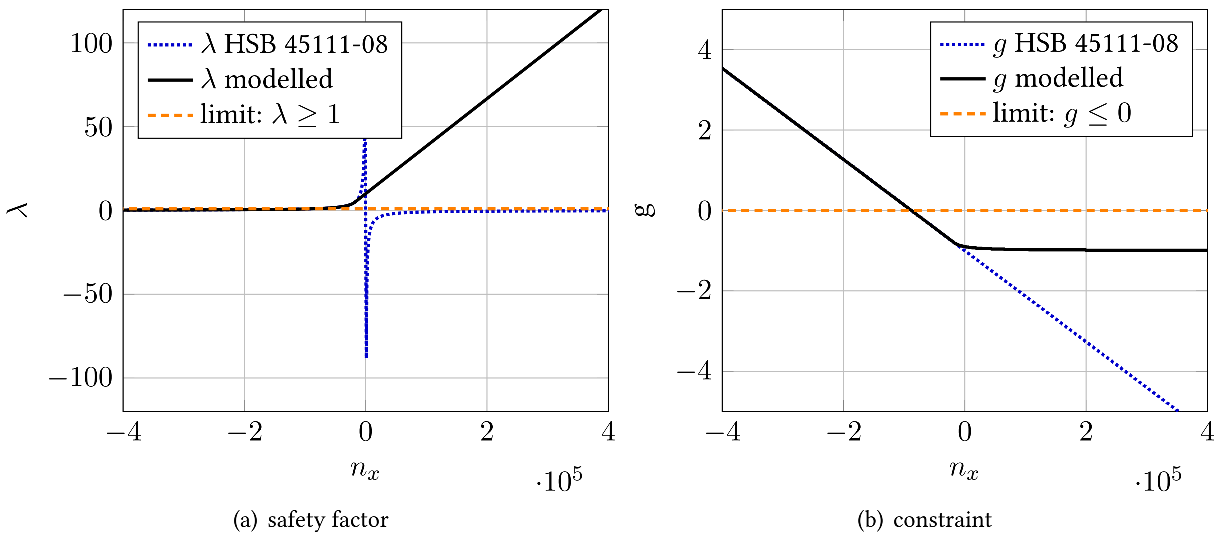


Figure 4.18: Safety factor of a combined shear-pressure-buckling criteria [3]

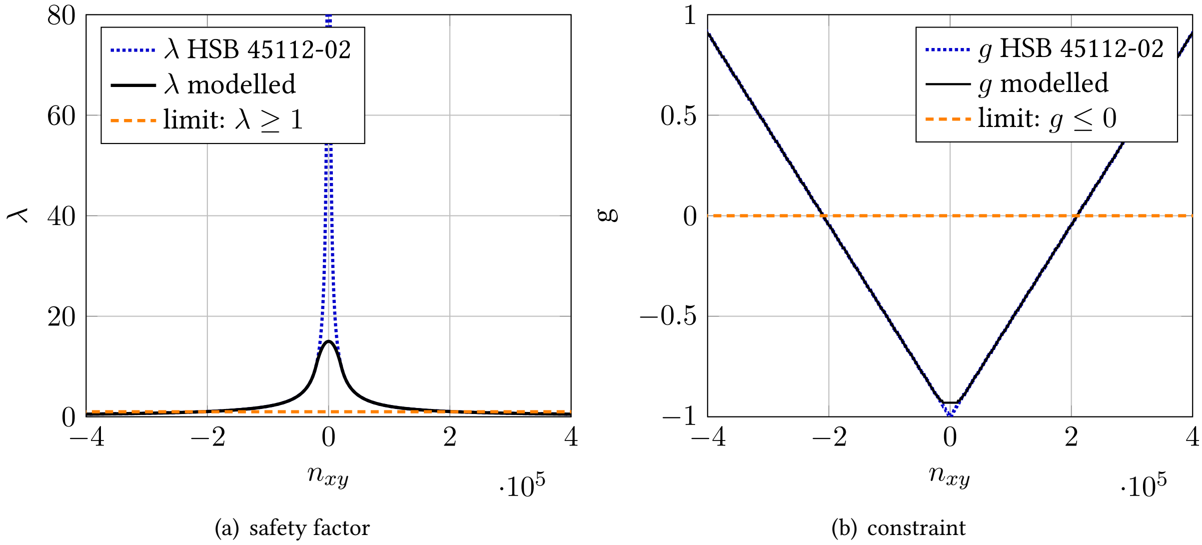
The result of each buckling criteria is a critical buckling load for given plate dimensions and material stiffness (ABD-matrix). The safety factor can be determined for both criteria by *equation 4.4* as ratio of applied load and critical load. The safety factor of the compression buckling criteria HSB 45111-08 is only defined in the the compression regime. In *section 4.3* it is demanded all safety factors used for the constraint calculation have to be defined in the complete load space. For this reason the safety factor is modelled as a linear function for tensile stresses as shown in *figure 4.19 (a)*. The linear function is designed to continue the graph as its tangent starting at a safety factor $\lambda_c = 5$ in the compression regime. *Figure 4.19 (b)* shows the according constraint for the original and the modelled safety factor. It can be observed the original constraint for compression is physically meaningful in the complete load space. The combination of compression buckling criteria and shear buckling criteria are based on safety factors. In order to calculate the combined compression shear buckling criteria, the modelling of the safety factor is necessary. A modelling of the single constraints on the other hand does not seem to bring any advantage for an optimisation process.

The shear buckling criteria HSB 45112-02 is defined for positive and negative shear loads. With no loading ($n_{xy} = 0\,\mathrm{N\,m^{-1}}$) the safety factor is infinite decreasing towards positive and negative shear loads. Even though a infinite safety factor for zero loading is physically meaningful, it results into a discontinuity in the constraint function. Such a discontinuity can cause issues with different optimisation algorithms because of the related jump in constraint gradients. In order for this optimisation framework to be as general as possible and to support different types of optimisation algorithms, the safety factor of the shear buckling criteria is modelled for $\lambda \geq 10$ as quadratic function shown in *figure 4.20 (a)*. Because of the symmetry of the shear buckling criteria around $n_{xy} = 0\,\mathrm{N\,m^{-1}}$ it is possible to model the safety factor with a quadratic approach as continuous function in the complete load space. As shown in *figure 4.20 (b)* this modelling results into a continuous constraint function.



(a) safety factor (b) constraint

Figure 4.19: Modelled safety factor and constraint of compression buckling criteria



(a) safety factor (b) constraint

Figure 4.20: Modelled safety factor and constraint of shear buckling criteria

The stability criteria for compression and shear loads can be combined into a single criteria accounting for shear and compression. Using the combined criteria has the advantage of reducing the number of constraints and avoiding the distinction of shear and compression cases. The combination of the criteria safety factors using *equation 4.7* is only possible where $\lambda \neq 0$ to avoid division by zero. The safety factor of the original compression buckling criteria can become zero for $n_x = 0\,\mathrm{N\,m^{-1}}$ and is therefore not suitable for combination. Instead the modelled safety factors of both criteria are used for the compression shear buckling criteria shown in *figure 4.21*. This new stability criteria is defined continuously in the complete load space of shear, tension and compression and is further investigated for defined load cases in the following paragraphs.

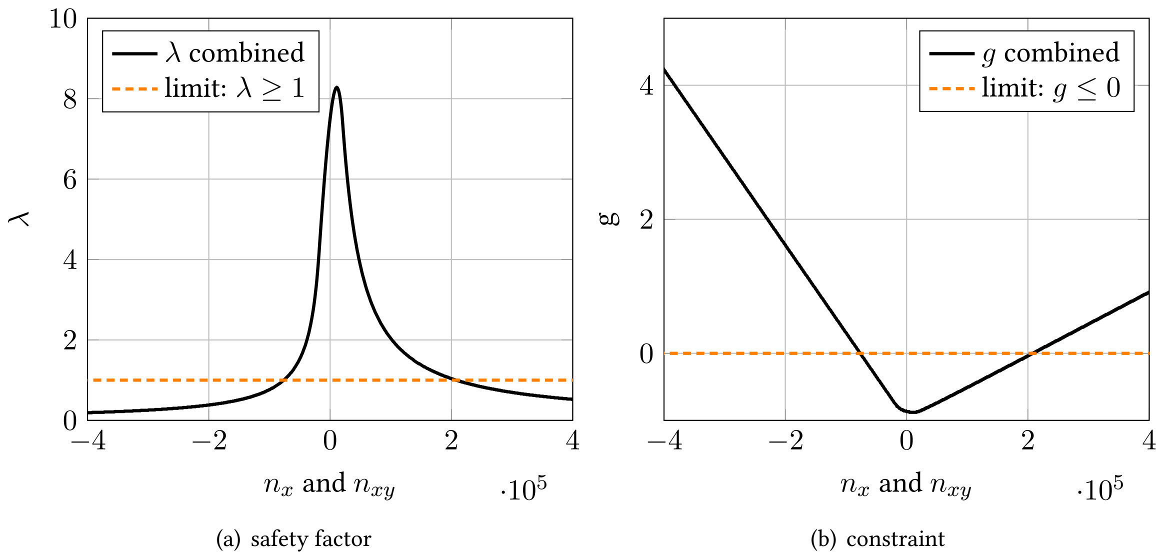


Figure 4.21: Safety factor and constraint of combined buckling criteria

In the previous paragraphs stability criteria for shear buckling, compression buckling and combined buckling were introduced and their behaviour for varying loads analysed for constant critical loads. During structural optimisation the wing stiffness properties are changed causing the critical loads of the stability criteria to change as well. The influence of the stiffness properties is hence investigated in the following for a uni-directional laminate ("Laminate 1"), and for a more likely layup ("Laminate 3"). The change in the stiffness properties is again caused by rotating the test panel defined in *figure 4.11* in the range $\alpha = [-90° \ldots 90°]$.

The safety factor calculated for a test panel made from the uni-directional laminate "Laminate 1" and loaded with "Loadcase 5" is shown *figure 4.22*. It can be observed the safety factor of the shear buckling criteria is almost a factor three higher with respect to the compression buckling criteria for the same shear and compressive load. The combination of both criteria results into a safety factor very close to the compression criteria. Even though the influence of the shear buckling criteria is minor in this example, it cannot be neglected since it still could be the sizing criteria in other load constellations, for example in the spar web. The second observation is the distinct resistance against buckling of fibres diagonal to the main load axis resulting into safety factor peaks at $\alpha = \pm 45°$. *Figure 4.23* shows similar results for the test panel with "Laminate 3" and "Loadcase 6". The variation in safety factor is decreased compared to uni-directional laminate as it could be expected for a more orthotropic laminate. Both investigations show physically meaningful results and no major limitations became obvious. For this reason solely the combined compression-shear buckling criteria is used for stability analysis in the constraint processor of this optimisation framework.

All constraints used in the constraint processor of the optimisation framework are summarised in *table 4.5*. All constraints together form a reasonable parameter space were a realistic wing design is possible. Some of these constraints can be eased for more sophisticated designs. It seems reasonable for example

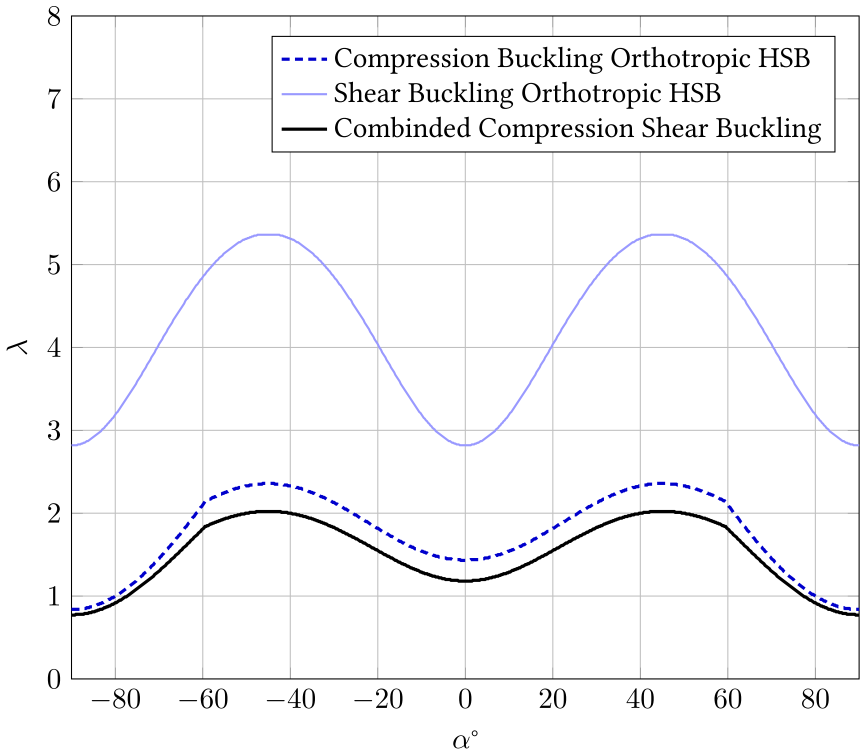


Figure 4.22: Stability criteria for compression ("Laminate 1" and "Loadcase 5")

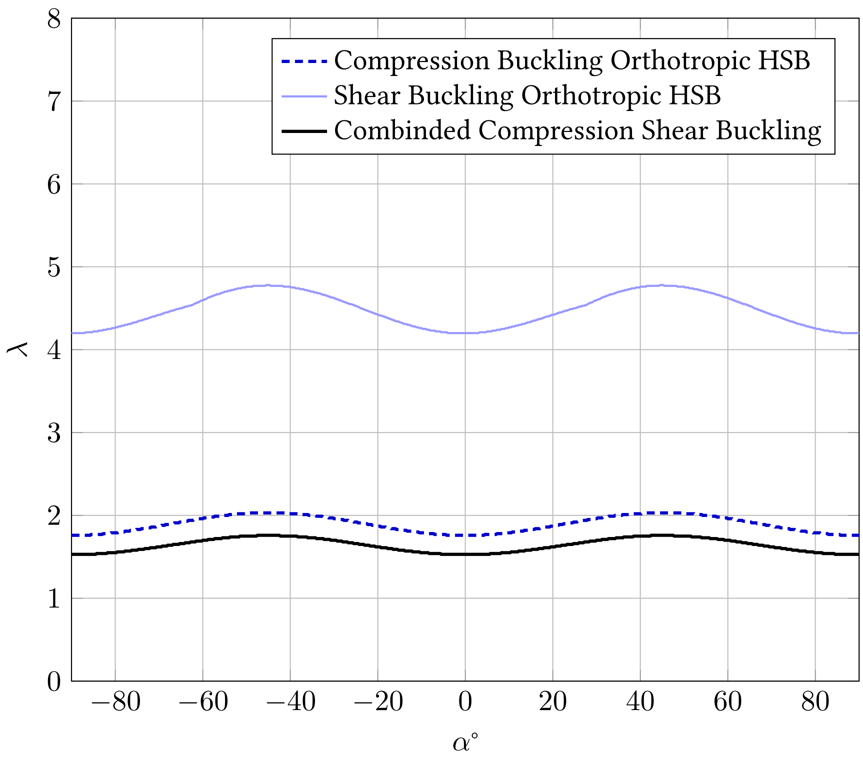


Figure 4.23: Stability criteria for compression ("Laminate 3" and "Loadcase 6")

to extend the range of allowed angles and open up the parameter space for unbalanced and unsymmetric laminates. Other constraints can be refined, for example buckling constraints are available not only for constant internal loads loads, but also for linear and higher order internal loads. In the scope of this thesis the main task is to build up a robust and rapid optimisation framework for preliminary design of wing structures. Improvements can be integrated into the framework later on when the basic working principle is proven.

constraint type	structural level	constraint	value
laminate design rules	skin	allowed angles	$0°, \pm45°, 90°$
		symmetric	yes
		balanced	yes
strength	skin	tension	
		compression	
		shear	
stability	panel	compression	
		shear	
manufacturing	assembly	taper ratio	0.2 (30 plies)
		ply continuity	0.6 (30 plies)

Table 4.5: Summary of all constraints

4.5 Optimisation Model

The optimisation model translates the physical optimisation problem represented by the structural model and the constraint processor into a mathematical model. In order to achieve such a transformation, in the optimisation model all information needed for the mathematical optimisation of a non-linear constraint problem are collected and arranged in an appropriate form. The total mass of the structural model is used as return value of the objective function. Thicknesses, lamination parameter or layer angles of the skins (material regions / optimisation regions) defined in the structural model can be used as parameters. The constraints are collected directly from the constraint processor. The optimisation model handles also the reverse translation of the parameter set into thicknesses, lamination parameters and layer angles.

Depending on the definition of the structural model, several parameter types can be selected for the optimisation process. For all selected parameter types explicit upper and lower boundaries are set in the optimisation model. All parameters available, their boundaries and the according definition of the structural model is shown in *table 4.6*. A pure thickness optimisation is the most common and basic option in structural optimisation and the only one needed for isotropic optimisation. Lamination parameters or layer angles can be considered as parameters for the optimisation of composite materials. In this thesis the emphasis is on gradient based optimisation with lamination parameters. The layer angles are only listed here for the sake of completeness[3]. Lamination parameters can either be selected in a general form, or in the form of symmetric and balanced lamination parameters only. In the latter case the number of lamination parameters reduces from 12 to 5 (*section 2.6*) where the remaining 7 parameters are set to zero.

[3]An optimisation of layer angles is generally possible with the presented optimisation framework. In this case the skins of the structural model have to be provided in composite formulation. Further, in the constraint processor layer based strength criteria as Tsai-Wu or Yamada-Sun have to be chosen.

During the collection of parameters in the optimisation model, the collection pattern is stored internally and allows the mapping of a modified parameter set onto the structural model. When parameters are collected in the optimisation model, the selected parameters of the structural model are set and the update methods of the solver and the constraint processor are triggered. No further actions are necessary and the new parameters, objective and constraints can be collected from the optimisation model again.

parameter type	skin definition	lower boundary	upper boundary
thickness	LP / composite	$0.1\,\text{mm}$	$500\,\text{mm}$
$LP_{sym,bal}$	LP	-1	1
LP_{all}	LP	-1	1
layer angles	composite	$-90°$	$90°$

Table 4.6: Parameters available in the optimisation

4.6 Optimiser

Numerous commercial and open source optimisation algorithms are available in the form of libraries or stand alone tools. The investigation of different algorithms suitable for structural optimisation is not part of the thesis. Rather an easily available, reliable, open source optimisation library for non linear constraint problems is needed for the initial set up of a robust and rapid structural optimisation framework. A library, including a python interface and providing multiple gradient based and evolutionary optimisation algorithms is the Non Linear optimisation (NLopt) library [12]. The gradient based algorithms available in the NLopt library as for example truncated Newton, BFGS, SQP and MMA are well known and tested. In the optimisation model developed in this thesis, the MMA algorithm (Method of Moving Asymptotes) is chosen because it showed a stable and reliable behaviour and returned reasonable results.

The Method of Moving Asymptotes first introduced by Svanberg[18] uses convex approximation of objective and constraint functions to generate a convex design space where a minimum, not necessarily a global one, is guaranteed. The function approximation used in the Method of Moving Asymptotes is very flexible and especially appropriate for all graphs similar to linear and exponential functions. Such a flexible approximation makes the method suitable for many different types of objective and constraint functions as long as function gradients can be provided. A good approximation also improves convergence speed. Further information on MMA and the precise formulation used in NLopt can be found in Svanberg[19].

The NLopt library does not provide any functionality to calculate gradients of the objective and constraint functions. For this reason gradients have to be calculated in the optimiser and handed over to the selected optimisation algorithm in each iteration of the optimisation. An easy way to calculate gradients is to use finite differences. In order to obtain a linear approximation of the actual gradient vector a forward difference quotient can be used. The objective and constraint functions have to be evaluated once with the given parameter vector (centre of expansion) and once for a small step into each dimension of the parameter space. Finite differences are very sensitive to the chosen step width. The step width h, dependent on the parameter value x, used in the optimiser for gradient calculation is given in *equation 4.8*[5], where ϵ is the machine epsilon of the numeric format.

$$h = \sqrt{\epsilon} * (1.0 + |x|) \tag{4.8}$$

The end of an optimisation can be defined with several termination conditions available in the NLopt library. In this framework the combination of objective tolerance, tolerance of the parameter sum and the maximum wall clock time are used. The maximum wall clock time is the back up termination condition ensuring the optimisation will come to an end after a maximum time. The tolerance of the parameter sum is triggered, when the relative change in the sum of all parameters is less than the defined tolerance. The tolerance of the parameter sum includes information of the change of lamination parameter which have no influence on the objective itself. Because it is possible to reduce the constraints by altering the lamination parameters and thereby expand the feasible space where a further reduction of the objective is possible, the tolerance of the parameter sum is the most relevant termination condition when using lamination parameter. Finally the objective tolerance terminates the optimisation process if the relative change in the objective is less than the defined tolerance. Since a minimum of the objective function is the actual goal of the optimisation, this condition can be used where no significant improvement can be expected anymore.

While the number of iterations needed in one optimisation is in the order of magnitude $n_{iterations} = [50, \dots, 100]$ the number of parameters can easily exceed 100 and is for aircraft wings usually in the range $n_{parameters} = [100, \dots, 500]$. The number of parameters determines the number of function evaluations needed in each iteration to calculate the gradient vector of the objective and the constraint functions when using finite differences. Therefore the major computational effort in the optimisation is caused by the gradient calculation. In the determination of the gradients, the single calculations of each variation of the parameter vector are independent of each other and can therefore be parallelised using pythons dask distributed library[4]. The parallelisation of the gradient calculation reduces the optimisation time by a factor nearly the size of the parameter numbers. A major improvement of this optimisation framework could be achieved by the implementation of analytical gradients into the solver PreDoCS and into the constraint processor.

4.6.1 Optimisation of a Test Panel

In the previous sections, the optimisation framework is described and explained in detail. In this section the optimisation of a composite test panel is used as proof of concept. The test panel defined in *figure 4.11*, with the dimensions $a = 1.0\,\mathrm{m}$ and $b = 1.0\,\mathrm{m}$, the laminate "Laminate 3" defined in *table 4.2* and the load cases "Loadcase 2", "Loadcase 3" and "Loadcase 6" defined in *table 4.4* are chosen to be consistent with previous investigations. All settings of the test panel optimisation are summarized in *table 4.7*.

parameters	thickness, $LP_{sym,bal}$, LP_{all}
objective	mass
constraints	laminate design rules, strength, stability
algorithm	MMA
termination condition	parameter sum tolerance: 0.0001
	objective tolerance: 0.00001

Table 4.7: Setting for the optimisation of the test panel

The optimisation of the test panel is compared for three different parameter sets. The most basic optimisation approach uses the panel thickness as the only parameter. In a second and a third optimisation first symmetric lamination parameters and then all lamination parameters are added to the optimisation parameter vector. In the first optimisation run, laminate design rules are neglected since the layup is not optimised. The laminate design rules used in the second and the third run restrict the lamination parameters to the space of symmetric, balanced laminates with $0°$, $\pm 45°$, $90°$ layers (*table 4.5*).

Figure 4.24 shows the development of the mass over the iterations for all three test optimisations. In the first ten iterations the mass development seems to be independent of the chosen parameter set. *Tabel 4.8* shows all constraints for the thickness optimisation of the test panel. In iteration 10 the stability constraint (combined compression shear buckling) for load case 3 (LC3) is zero whereas all other constraints are below zero. Therefore the load case three is the sizing load case and the first failure to be expected is stability failure. The thickness optimisation is stopped after iteration 10 because all constraints are fulfilled and the feasible relative change in thickness without violating a constraint is smaller than the set tolerance of the parameter sum.

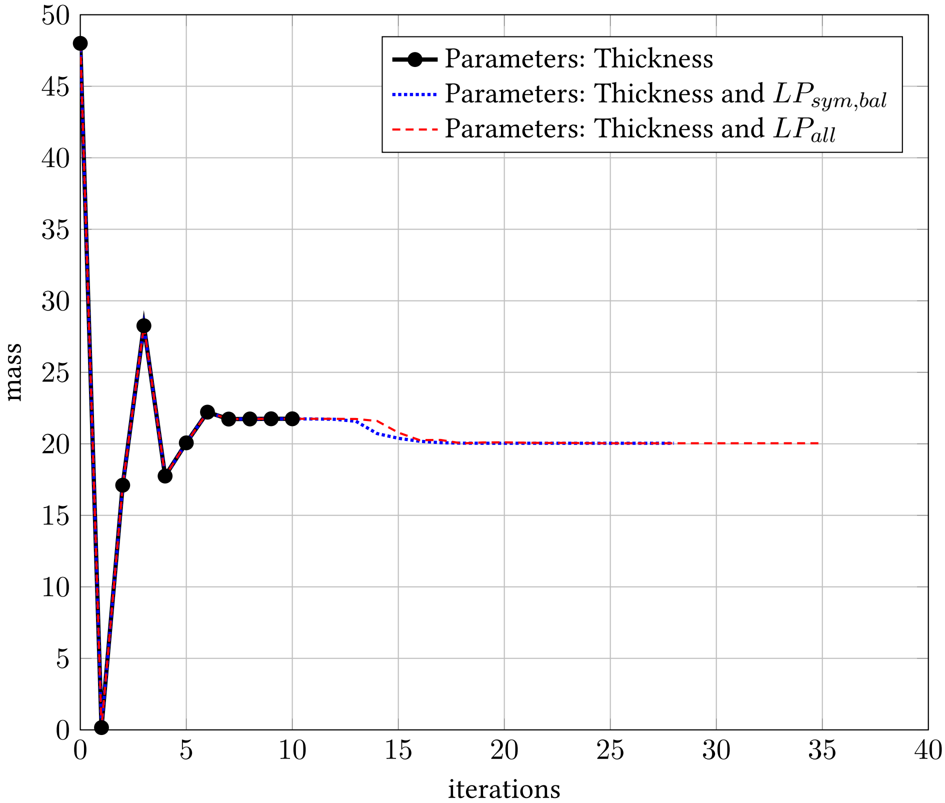


Figure 4.24: Mass development of the test panel

The optimisation of both cases including lamination parameters does not stop after ten iterations, even though it seems the change in mass seems converged. In this case the parameter sum tolerance is not reached because the lamination parameters still change. This change is only visible in the constraint values and not in the mass development. The optimiser changes the lamination parameters in order to reduce the stability constraint of "Loadcase 3". The constraint table for this optimisation run can be found in *appendix B.1*. By reducing the active constraint with a change in lamination parameters, the optimiser can further reduce the panels thickness without a constraint violation. The reduction of the stability constraint of "Loadcase 3" is accompanied by a reduction of the strength constraint of "Loadcase 3", showing both criteria are affected by the changes in the main stiffness direction. After the optimisation, the test panel is well adjusted to the load cases given in the beginning, which is the real advantage of composite design.

The difference in final mass of the optimisation with symmetric balanced lamination parameters and all lamination parameters in *figure 4.24* is negligible. Also the comparison of the constraints in *appendix B.1* and *appendix B.2* shows no major differences. The reason for the identical results is that the laminate design rules are set to symmetric and balanced laminates with $0°, \pm 45°, 90°$ layers. In both cases the feasible design space of the lamination parameters limits the further reduction of stability constraints. This may suggest, unsymmetric unbalanced laminates with unlimited layer angles can be used to further reduce the panel mass. Such a discussion however exceeds the scope of this thesis. The

	Stability			Strength		
i	LC2	LC3	LC6	LC2	LC3	LC6
0	−0.89296	−0.84701	−0.86206	−0.89032	−0.58771	−0.96168
1	−0.93333	10.00000	10.00000	10.00000	10.00000	10.00000
2	−0.92926	1.00014	−0.46396	−0.69225	0.15682	−0.89249
3	−0.91792	−0.51449	−0.83464	−0.81368	−0.29963	−0.93491
4	−0.92881	0.79707	−0.51981	−0.70333	0.11515	−0.89636
5	−0.92696	0.25862	−0.66796	−0.73766	−0.01388	−0.90835
6	−0.92494	−0.05825	−0.75417	−0.76300	−0.10914	−0.91720
7	−0.92542	0.00247	−0.73825	−0.75773	−0.08930	−0.91536
8	−0.92541	0.00148	−0.73851	−0.75781	−0.08962	−0.91539
9	−0.92540	−0.00023	−0.73897	−0.75796	−0.09017	−0.91544
10	−0.92540	0.00000	−0.73891	−0.75794	−0.09009	−0.91543

Table 4.8: Constraints of the test panel thickness optimisation

final reduced parameter set is given for the three test optimisations in *table 4.9*.

parameters	thickness	$V_{A,1}$	$V_{A,2}$	$V_{D,1}$	$V_{D,2}$	$V_{D,3}$
t	13.59 mm					
t, $LP_{sym,bal}$	12.52 mm	0.178	−0.723	0.023	−1.000	0.000
t, LP_{all}	12.52 mm	0.185	−0.708	0.023	−1.000	0.240

Table 4.9: Final parameters of the test panel optimisation

Chapter 5

Optimisation of the MiRaJet Wing

In *chapter 4* an optimisation framework for PreDoCS with multi fidelity capability is introduced and tested for a simple panel. In this chapter the framework is used to optimise the wing structure of the MiRaJet which is investigated in the ATLAs project (*section* 2.4). A comparison of the optimisation results generated with PreDoCS and VErSO for a thickness only optimisation of the MiRaJet wing is used to investigate the reliability of the optimisation framework and to validate PreDoCS with a FEM based solver. The influence of the PreDoCS element discretisation on internal load size and distribution was shown in *section 2.2.2*. The number of cross sections and elements used in PreDoCS are expected to have an impact upon the optimisation speed and results. A sensitivity study is performed for the cross-section and element discretisation used in PreDoCS in order to identify a reasonable discretisation as a trade-off between optimisation speed and accuracy. When the PreDoCS optimisation results are validated by VErSO and reasonable discretisation settings for PreDoCS are identified, an optimisation of thickness and symmetric balanced lamination parameters is conducted. The full potential of the optimisation framework is investigated using PreDoCS in a laminate optimisation of the MiRaJet wing, also showing the weight saving potential of composites in aircraft design.

5.1 Comparison of PreDoCS and VErSO

PreDoCS and VErSO are two structural solvers with an interface to the optimisation framework presented in *chapter 4*. While PreDoCS uses an analytical cross-section based theory to build up one dimensional beam elements, VErSO uses a finite element approach to build up a wing model from two dimensional shell elements. Both tools calculate the internal element loads from outer nodal loads. The FEM solver controlled by VErSO achieves this transformation using shape functions and numerical integration methods. PreDoCS uses analytical functions and integration methods within the cross sections and only relies on shape functions and numerical integration methods on beam level.

PreDoCS and VErSO use very different approaches for the calculation of internal loads. Therefore the comparison of both tools not only poses an opportunity to compare their fitness for optimisation, but can also validate the results from both tools. In order to obtain comparable results, it is essential both optimisations start from on the same baseline model. In the scope of this thesis the MiRaJet CPACS model which is part of the ATLAs project is used for both optimisations. VErSO needs a FEM model as input which can be generated from the CPACS model. The discretisation of the FEM model generated from the CPACS model depends on geometrical information, such as rib-spacing, and is therefore not adjustable by the user. PreDoCS directly uses the ATLAs CPACS model to generate its own calculation and structural model. The PreDoCS model discretisation can be chosen arbitrary by the user. For the comparison of both tools, the attempt is made to approximately match the discretisation of the VErSO FEM model with the PreDoCS model.

The FE-models generated for structural optimisation usually consider only the box beam wing. The

PreDoCS model considers the complete cross section of the wing for its analysis. In order to make both models comparable, PreDoCS offers the possibility to label the wing box panels in the structural model of the optimisation framework as "box-wing", thus allowing to select the box beam wing as optimisation region. Because the parts not included in the box wing are still part of the PreDoCS model and the load calculation, their thickness is set to $0.2\,\mathrm{mm}$ in order to reduce stiffness and load carriage to a negligible minimum. The structural model generated for the optimisation framework by VErSO and the structural model generated by PreDoCS only including the selected optimisation region is shown in *figure 5.1.*

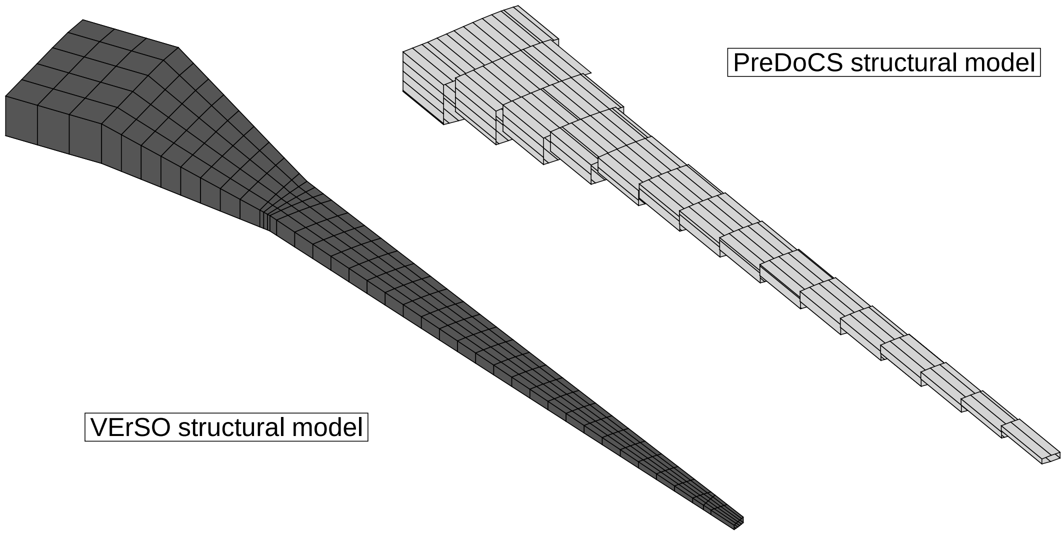


Figure 5.1: Comparison of VErSO and PreDoCS structural models

The comparison of PreDoCS and VErSOs structural optimisation capabilities are investigated using the optimisation framework introduced in *chapter 4.* It is assured both tools are used with identical settings given in *table 5.1.* Additionally the discretisation settings used in PreDoCS for cross-sections and elements are given. The span wise discretisation of both structural models does not match. In order to avoid discrepancies caused by the span-wise discretisation, a virtual rib-spacing which determines the span-wise buckling length of the constraint regions, is set to $0.8\,\mathrm{m}$. Both structural models use lamination parameters for the material stiffness description in combination with the material invariants of T-300 15k/976 carbon fibre tape defined in *table 4.3.* All lamination parameters are set to zero when the structural model is created in order to simulate a quasi isotropic material. Only the thicknesses of the optimisation regions are used as parameters while the lamination parameters are kept constant during the optimisation. The main purpose of manufacturing constraints is to ensure ply continuity in composite materials. Because all optimisation regions are defined with identical lamination parameters, i.e. with identical layups, no continuity constraints are needed for the comparison. The Method of Moving Asymptotes (MMA) is used as optimisation algorithm for the comparison. It is essential for this algorithm to start the optimisation in the feasible region which can be achieved by setting the initial thickness of all optimisation regions to a high, estimated value. In the case of the structural model from PreDoCS $0.06\,\mathrm{m}$ is chosen and validated resulting into an initial mass of roughly $8.5\,\mathrm{t}$. The initial thicknesses chosen for the VErSO structural model are even higher resulting into a initial mass of roughly $17.5\,\mathrm{t}$[1]. The external loads used for the optimisation are the limit load cases of the CPACS model and do not change during the optimisation.

parameters	thickness
objective	mass
constraints	strength, stability
algorithm	MMA
termination condition	parameter sum tolerance: 0.0001
	objective tolerance: 0.001
material	T-300 15k/976
rib-spacing	$0.8\,\mathrm{m}$
initial thickness (PreDoCS)	$0.06\,\mathrm{m}$
Nr. of cross-sections (PreDoCS)	15
Max. element length (PreDoCS)	$0.25\,\mathrm{m}$
Optimisation region (PreDoCS)	"box-wing"
Solver (PreDoCS)	Hybrid-Song

Table 5.1: Optimisation settings for the Comparison of PreDoCS and VErSO

5.1.1 Mass Development and Distributed Mass

The objective of the compared optimisations is the reduction of the total wing mass. *Figure 5.2* shows the development of total wing mass over the iterations for the PreDoCS and the VErSO optimisation of the MiRaJet wing. Generally the development of both graphs is very similar. Starting from a high mass the curves develop exponentially decreasing towards the asymptote of the final mass. Despite of their similar development, both graphs show a distinct offset of about $1.5\,\mathrm{t}$ at iteration 40 shrinking towards an $0.5\,\mathrm{t}$ offset of the final masses. The high offset at the beginning of the optimisation is caused by the set initial thickness and the resulting mass. The offset of the final mass however needs another explanation.

The total wing mass calculated from the structural model depends on its area, thickness and density. The density is set with the material which is the same in both optimisations. The thicknesses, as the actual optimisation parameters, are changed by the optimiser and should evolve towards similar final values in both optimisations. The total wing surface area of the VErSO structural model is a good approximation of the actual box-beam wing area. The PreDoCS structural model only offers a rough approximation of the wing surface area due to its cross-section wise composition. Especially the centre wing box at the wing root is only partially represented by the PreDoCS structural model. For this reason the final mass of the PreDoCS optimisation needs to be corrected by the ratio of the actual box beam wing area and the area approximation of the PreDoCS model. The masses, areas and corrected areas are given in *table 5.2*[2]. The final mass calculated with the VErSO optimisation and the area-corrected final mass from the PreDoCS optimisation have a discrepancy of less than $5\,\%$. Many explanations are possible for the remaining discrepancy, starting from the calculation of the internal loads in the solver up to differences in the discretisation of the structural model. To further investigate these mass differences

[1]A high initial thickness was chosen in VErSO in order to avoid a parameter study providing a minimum initial mass.

[2]Although here the wing area calculated by VErSO is used for mass correction of PreDoCS, to get the actual wing surface area from the CPACS model in order to use PreDoCS as a stand alone tool is possible

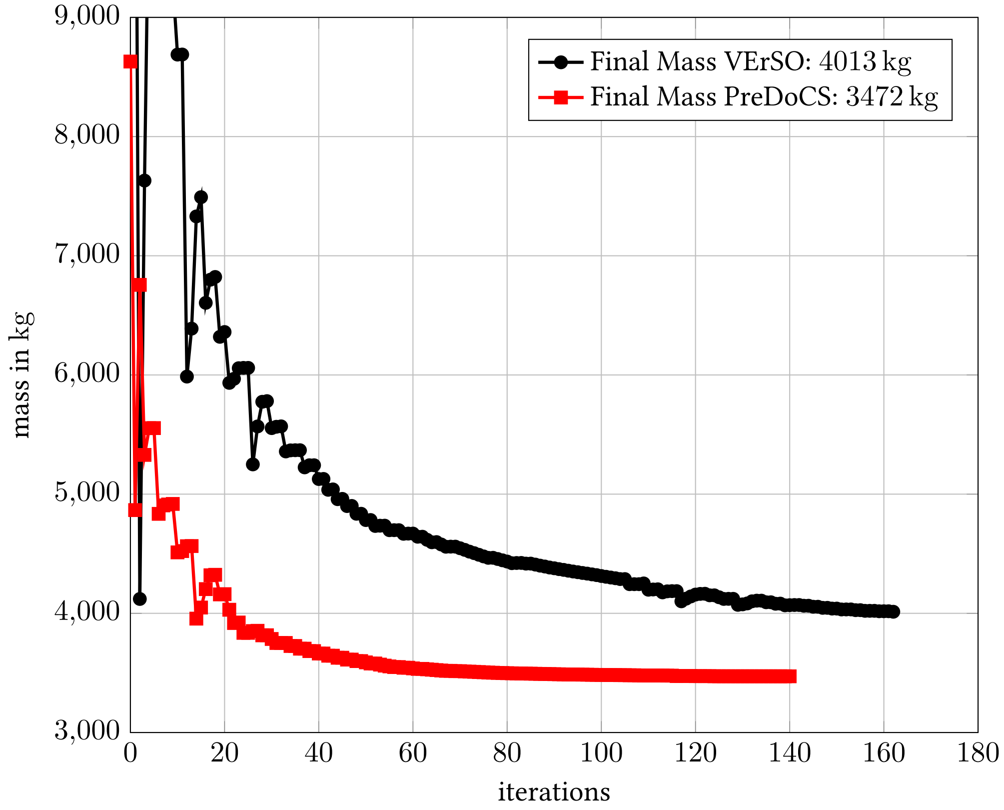


Figure 5.2: Mass development of the MiRaJet wing with PreDoCS and VErSO

might result into precise knowledge of PreDoCS and VErSOs optimisation capabilities, which is the primary objective of this section. However, only more detailed optimisation results as mass, thickness and constraint distributions are expected to deliver the required insight.

	PreDoCS	VErSO	deviation
mass	3472 kg	4013 kg	−13.49 %
area	89.89 m^2	109.49 m^2	−17.90 %
corrected mass	4093 kg	4013 kg	1.99 %

Table 5.2: Model area and mass

The total masses resulting from both compared optimisations of the MiRaJet wing show good agreement after correction with the wing area. However the total wing mass does not provide any insight on mass or thickness distribution over the wing. In order to avoid further mass corrections by total area and to gain some knowledge of the mass distribution, the average mass per area is plotted over the wing span in *figure 5.3*. Since the material and therefore the density does not change along the wing span, the distributed mass can be treated as a synonym of the average thickness. The mass distributions of both optimisations show a very similar development along the wing span. While the mass distribution of PreDoCS and VErSO is a close match at the the outer wing, the wing optimised with PreDoCS has a higher mass per area in the middle wing and a slightly lower mass per area at the wing root[3].

The wing parts where the distributed mass optimised with PreDoCS exceeds the distributed mass of VErSO outweighs the wing parts where the mass distribution is lower. This result is in accordance with the higher corrected total mass of PreDoCS. It is noteworthy that the wing optimised with VErSO

[3]The kink at the wing tip in VErSOs mass distribution is discussed in *section 5.1.2*.

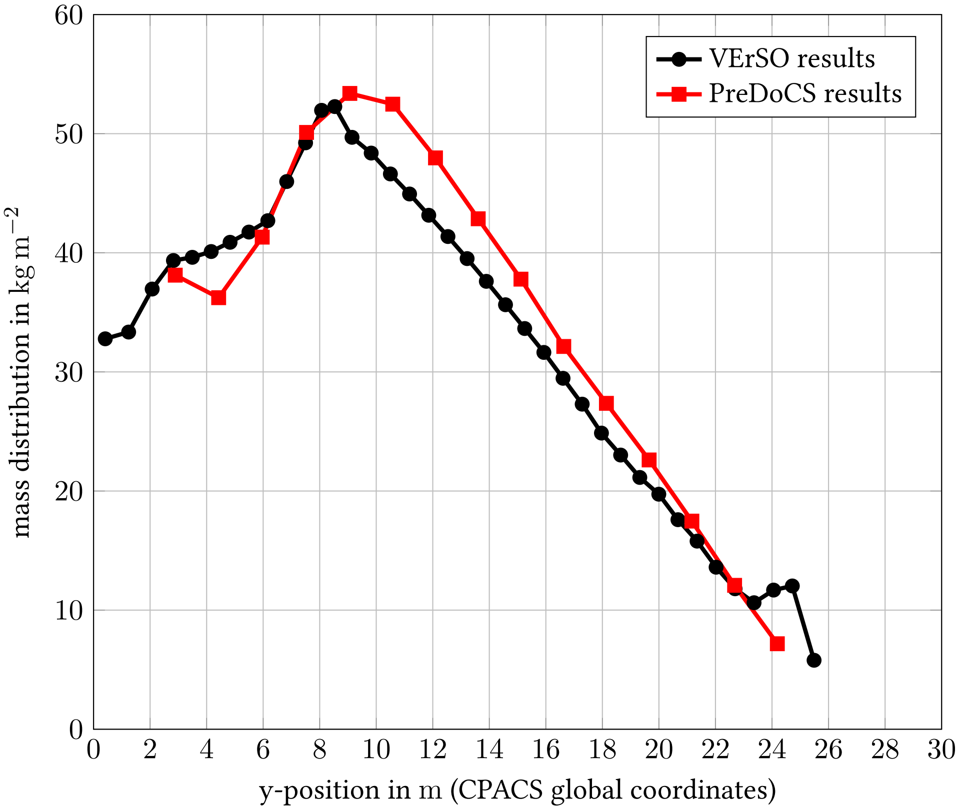


Figure 5.3: Mass distribution of the ATLAs wing

shows kinks in the distributed mass at y-positions of approximately 3 m and 8 m which correlate with kinks in the wing geometry. The wing model of PreDoCS is smoother due to its cross-section wise composition and also shows a smoother course of the mass distribution curve. The lower distributed mass of PreDoCS at the wing root can be explained by a higher cord-wise discretisation of the optimisation regions (material regions) compared to VErSOs model explained in the next paragraph.

The mass distribution over the wing span is a quantitative comparison of both optimisation results where similarities and discrepancies can be easily observed. *Figure 5.4* shows the wing thickness for each optimisation region of PreDoCS and VErSOs structural model. In VErSOs structural model, the optimisation regions of each assembly, e.g. upper cover, lower, cover, spars, divide the wing span-wise into one material region for each rib-bay. No discretisation of optimisation regions in cord-wise direction is realised, resulting into a stripe-model. In the structural model of PreDoCS some cross sections at the wing root are also divided into separate optimisation regions in cord-wise direction[4].

The final thickness distribution after the optimisation is very similar for both models. Two main differences between both thickness distributions can be observed however. As mentioned before, a more differentiated thickness distribution at PreDoCSs wing root due to the higher discretisation of the optimisation regions results into slightly lower average masses at the wing root. The second discrepancy between both optimisation results is the spar thicknesses. While PreDoCS has an average spar thickness at the wing root between 0.012 m − 0.020 m, VErSO can reduce the thickness to 0.004 m − 0.008 m. Except for the wing tip, this phenomenon can be observed on the whole wing explaining the increased distributed mass resulting from the PreDoCS optimisation and also explaining the higher area-corrected total mass. Before a conclusion can be drawn concerning the differences in spar thickness, the constraints have to be reviewed in order to reassure the spar thicknesses are limited by active constraints and no further reduction is possible.

[4]The cord-wise discretisation in the PreDoCS structural model is not set intentionally, but a result of the cross-section generation across multiple CPACS component segments.

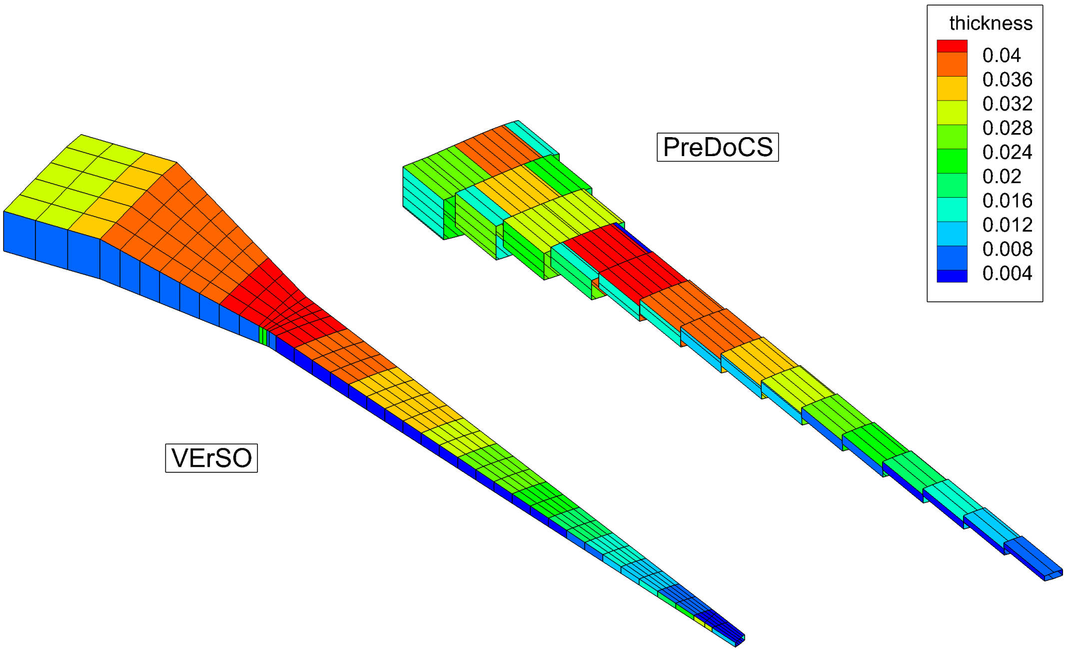


Figure 5.4: Thickness distribution and optimisation regions of VErSO and PreDoCS

5.1.2 Constraints

Strength and stability criteria represent two types of constraint functions used in the comparison of PreDoCS and VErSO. Constraint functions can be evaluated at all elements and for all load cases of each optimisation region, i.e. material region. Only the maximum constraint of an optimisation region is actively sizing the structure. In advance to the optimisation it is impossible to determine which element will deliver the sizing load and constraint. For this reason the constraint functions are evaluated for all load cases at each element. All constraints need to be lower or equal to zero in order to be fulfilled. Constraints which are exactly zero are called active constraints and prevent the optimiser from further reducing the objective at the corresponding optimisation region.

Figure 5.5 shows the maximum constraint values plotted on the wing structural models of PreDoCS and VErSO. All regions with a constraint value above -0.1, i.e. red regions, are considered active. A thickness reduction is only possible for optimisation regions with no active constraint inside. Optimisation regions are not represented by elements and therefore not shown in *figure 5.5*. They can be identified by comparison with *figure 5.4*, where each optimisation region has a single thickness value. In the PreDoCS optimisation, no material region without an active constraint is left. A further mass reduction is therefore not possible and the optimisation is considered as converged. At the VErSO optimisation, the model is not yet completely converged at the wing tip. Especially the spars, but also the upper and lower cover at the very end of the wing, allow further thickness reduction. The kink in the mass distribution at the wing tip in *figure 5.3* can be explained by the not fully converged optimisation.

The wing spars at the inner and middle wing of the VErSO optimisation are widely converged. In context of the much thinner spars of the VErSO model this leads to the conclusion that VErSO returns small internal loads within the spars compared to PreDoCS. While PreDoCS returns linear load distributions for each element and the maximum load of each element is picked for constraint evaluation, VErSO returns only constant load values per element. Additionally in PreDoCS the spar is discretised by several elements while VErSO uses only one element for spar discretisation. Usually the spar webs carry mainly shear loads which have a quadratic distribution over the spar height. In addition at the spar caps tension and compression are the governing loads. In PreDoCS it seems the combined compression shear buckling is the sizing criteria at the upper spar cap with a load combination of shear and superposed compression. For the VErSO optimisation a mean shear value and a mean stress value, containing tension and compression, has to be returned for a whole spar element. It seems unlikely that the mean values considers the actual maximum load combination of shear and compression at the spar. Therefore in this case the VErSO optimisation results are expected to be less conservative compared to the PreDoCS results. The calculation of the internal loads in the spars is therefore identified as a major reason for the difference of VErSOs final mass and PreDoCS area corrected final mass.

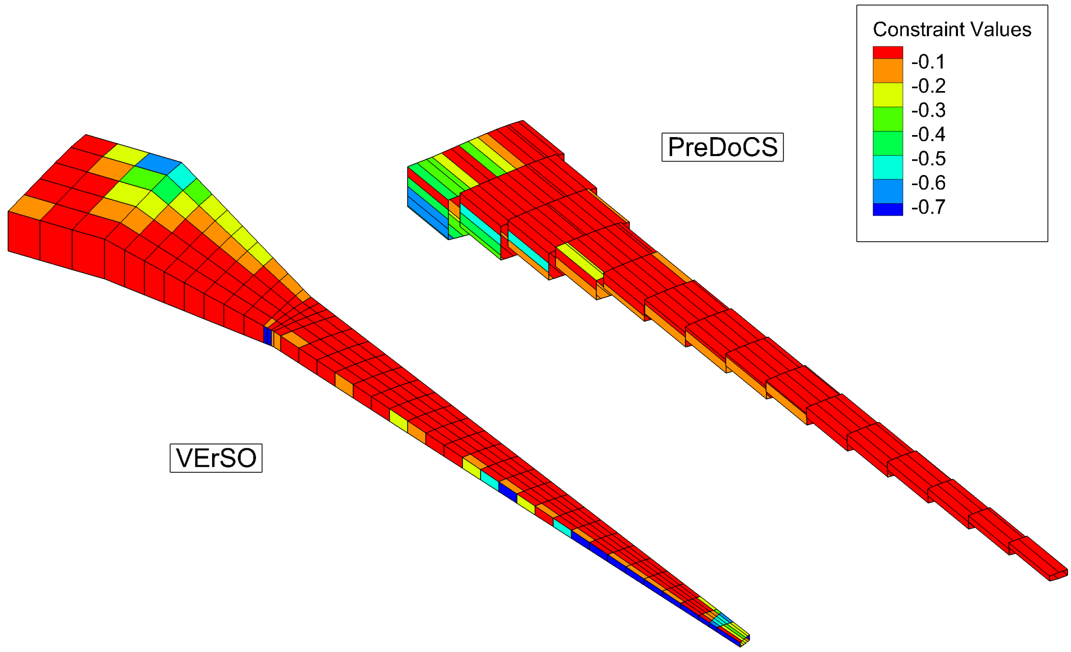


Figure 5.5: Constraint value comparison of VErSO and PreDoCS

Besides the maximum constraint value, also the constraint type at each evaluation region on the wing is of interest. It was already speculated in the above paragraph that the combined shear compression buckling criteria might be the sizing criteria of the inner wing spar. In *figure 5.6* it is shown where the two types of constraints, stability and strength, are the dominating criteria. For both optimisations the combined shear compression buckling stability criteria is the governing criteria on the upper shell and on the spars. In the PreDoCS model, spar elements at the inner and middle wing are dominated by strength criteria, but the elements with strength constraints are less critical compared with the stability constraints on the same optimisation region as shown in *figure 5.5*. Only in the middle wing and only on the lower shell both optimisation results are clearly sized by strength constraints.

The constraint type distribution, showing buckling constraints on the upper cover and strength constraints in the middle of the lower cover, is very typical for an aircraft wing. From such a constraint distribution it can be deduced that the main sizing load case is a bending load case. The bending load case leading to the maximum wing tip displacement is shown in *figure 5.7*. In this case the complete wing is shown for the PreDoCS optimisation results, not only the box beam wing. As explained before, the panels which are no part of the box beam, only have negligible impact on stiffness and deformation and are not part of the optimisation or the mass calculation. The additional parts are only included here for better visualisation of the wing deformation. Both wing deformations are based on the same

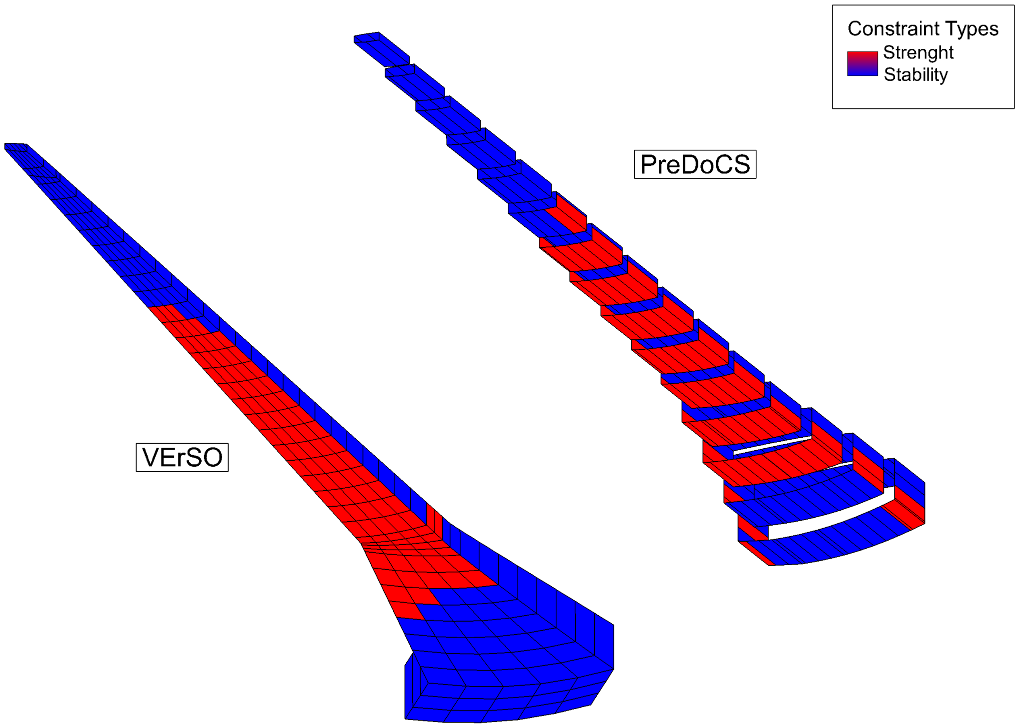


Figure 5.6: Constraint type comparison of VErSO and PreDoCS

outer loads and, except for the spar thicknesses, both structural models are very similar in terms of thickness and geometry. According with these conditions also the wing deformations are very similar. A close observation of the wing tips shows only a small difference of the maximum deformation. The deformation in z direction of both optimisation results is given in *table 5.3*. The reduced bending of the PreDoCS optimised wing is in accordance with its thicker spars. Both bending results are within the expected range of wing bending for mid range aircraft.

	PreDoCS	VErSO	deviation
deformation in z	$4.18\,\mathrm{m}$	$4.54\,\mathrm{m}$	$-7.90\,\%$

Table 5.3: Maximum wing bending and the difference between PreDoCS and VErSO

The comparison of the MiRaJet wing optimisation results from PreDoCS and VErSO show good agreement of mass distributions, thickness distributions, constraint type and value distributions and the wing deformations. Deviations of the thickness distributions can be explained with the higher cord-wise discretisation at the wing root and the thicker spars of the PreDoCS model. The mass and the area of the PreDoCS structural model are only a rough estimation. It seems promising to correct the wing mass by the ratio of the actual wing surface area and the estimated area. The results of the comparison suggest a detailed load comparison of PreDoCS and VErSO with the focus on the wing spars. An evaluation of the optimised wing structures with a well known, possibly commercial sizing tool, seems to be a promising approach for the final tool validation.

5.2 Influence of Cross-Section and Element Discretisation in PreDoCS

The influence of the model discretisation on the total wing mass and wing mass distribution is an important factor for the accuracy of the optimisation results. In order to achieve discretisation settings

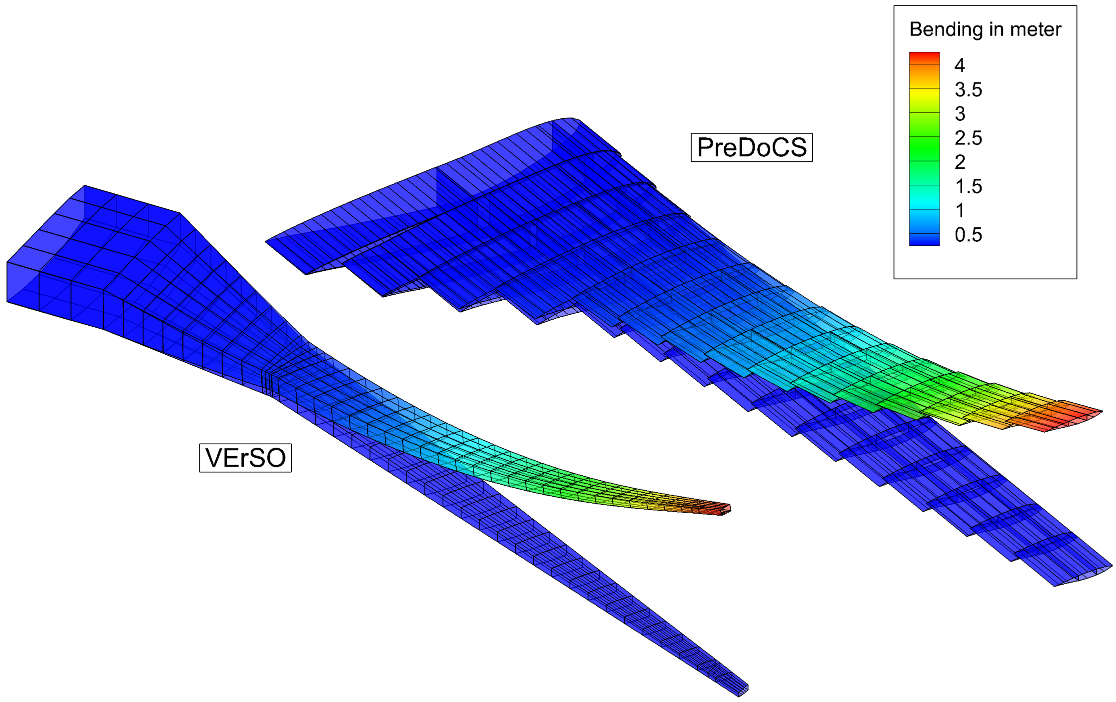
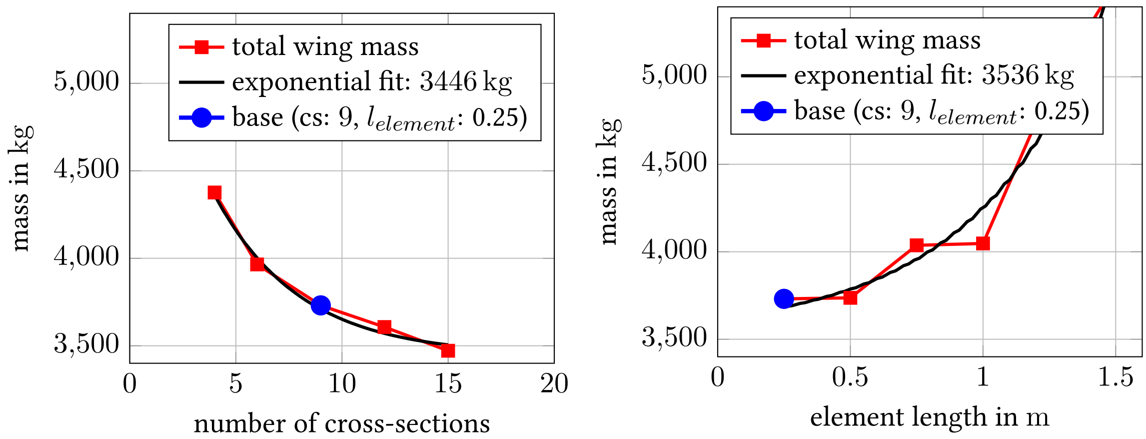


Figure 5.7: Wing bending of VErSO and PreDoCS

resulting into accurate masses, a parameter study is performed for the element discretisation and the cross section discretisation. In *section 2.2* the influence of cross-section and element discretisation on the load calculation is discussed. In *section 4.2* the correlation of the structural model mass calculation and the cross section discretisation is underlined. In this section the impact of the model discretisation on the optimisation results is investigated at the example of the MiRaJet wing. As a result of the study a discretisation correction factor for an assumed infinite discretisation is presented for the MiRaJet wing. A discretisation of the MiRaJet wing model for lamination parameter optimisation is chosen as a trade-off between accuracy and computational time.

In the parameter study, thickness optimisations of the MiRaJet wing are performed with varying discretisation settings. All other optimisation settings are the same as in the PreDoCS-VErSO comparison defined in *table 5.1*. The influence of cross-section and element discretisation on the total wing mass is shown in *figure 5.8*. As baseline for both parameter studies, a discretisation of nine cross sections and a maximum element length of $0.25\,\mathrm{m}$ is chosen, marked as base in *figure 5.8*. The cross-section discretisation study in subfigure (a) is calculated with a maximum element length of $0.25\,\mathrm{m}$. The element discretisation study in subfigure (b) is evaluated with 9 cross-sections. Both studies indicate a exponential development reaching a asymptote for an infinitely high discretisation. For this reason a least square exponential fit is calculated for both studies also shown in the figures. The asymptotic mass is the theoretical minimum mass which can be reached for a infinitely high discretisation.

The mass distributions for all optimisations of the cross-section discretisation study are shown in *figure 5.9*. The graphs of all mass distributions are very similar. Only for a cross-section discretisation below nine cross-sections slight deviations can be observed at the centre wing and for a discretisation of 4 cross sections also at the wing tip. It is noteworthy, that the deviation of the actual data points shown as markers, is negligible except for the 4 cross-section discretisation. The mass distribution over the wing span is determined by the inner structural loads. For this reason it seems justified so say that the impact of cross section discretisation on the loads is negligible as long as the basic model



(a) cross-section discretisation (b) element discretisation

Figure 5.8: Influence of discretisation on total wing mass

geometry is represented. The lowest cross-section discretisation showing the detailed mass distribution development is the one using nine cross sections.

The change of total mass with the cross section discretisation shown in *figure 5.8 (a)* cannot be explained with the minor changes in mass distribution observed in *figure 5.9*. The only additional parameter with an influence on the total mass, is the wing geometry. The decreasing cross section discretisation lowers the accuracy of the geometry representation and therefore causes an increase in total mass. The wing geometry does not change during the optimisation process. For a rapid optimisation, the possibility presents itself to optimise the wing with very few cross sections and interpolate the mass distribution, e.g. with cubic splines. The interpolated mass distribution can be used in combination with an accurate geometry representation, to precisely estimate the actual wing mass. Such a process however exceeds the limitations of this thesis.

The mass distributions for a variety of element lengths are shown in *figure 5.10*. A strong increase of the distributed mass from high element discretisations towards low element discretisaitons can be observed. Interesting is the stepwise change in the distributed mass. While the distributed masses for maximum element lengths of $0.25\,\mathrm{m}$ and $0.50\,\mathrm{m}$ are nearly identical, a mayor step is visible between maximum element lengths of $0.50\,\mathrm{m}$ and $0.75\,\mathrm{m}$. The maximum element length has less impact on the wing root, where a major change only can be observed for element lengths above $0.75\,\mathrm{m}$. The influence of the element length on the internal loads of curved profiles is shown in *section 2.2.2*. Here higher maximum loads were caused by a low element discretisation. In the optimisation the higher maximum loads result into increased distributed masses. At the wing root the curvature of the profile with respect to the element length is less distinct. Therefore the influence of the element length at the wing root is only visible for high element lengths.

A cross-section discretisation of nine cross sections shows a good span-wise representation of the wing geometry in the distributed mass. For this reason nine cross sections are chosen for the further studies. The best optimisation results can be achieved with maximum element lengths of either $0.25\,\mathrm{m}$ or $0.50\,\mathrm{m}$. Because the higher element length reduces the number of constraints in the optimisation and is therefore beneficial for the computational time, a maximum element length of $0.50\,\mathrm{m}$ is chosen. The asymptotes of the exponential fits in *figure 5.8* predict a minimum mass for each discretisation. The expected reduction in mass for an infinite small discretisation with respect to the chosen discretisation is given in *table 5.4*. This theoretical mass reduction of $-13\,\%$ is estimated to be reached for continuously varying optimisation parameters and can be understood as the lower boundary for the actual wing mass. The area factor determined in *section 5.1.1* has to be considered additionally.

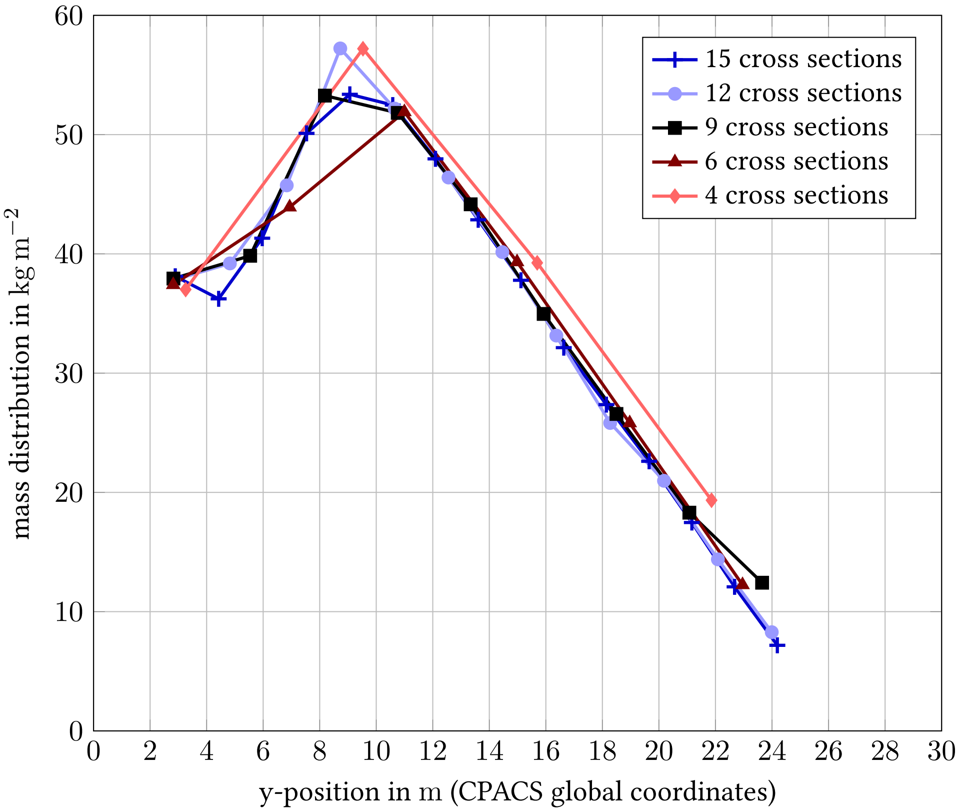


Figure 5.9: The influence of cross-section discretisation on mass distribution

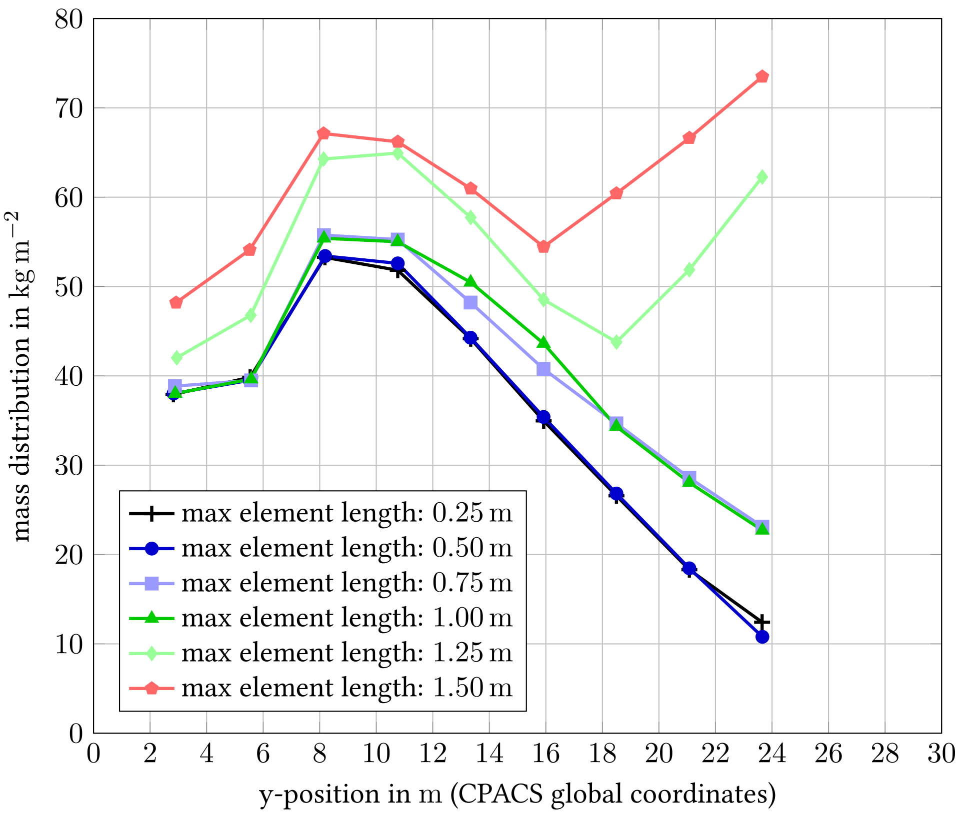


Figure 5.10: The influence of element discretisation on mass distribution

	9 cross-sections	infinite cross-sections	deviation
mass	3731 kg	3446 kg	−7.64 %
	max. element length 0.5	infinitely small elements	deviation
mass	3737 kg	3536 kg	−5.38 %
total deviation			−13.01 %

Table 5.4: The expected mass reduction for infinite discretisation

5.3 Optimisation of Thickness and Lamination Parameters

In *section 5.1* and *section 5.2* of this chapter skin and spar thicknesses are optimised to compare PreDoCS and VErSO and to investigate discretisation sensitivities. The optimisation framework presented in *chapter 4* is designed for a full composite layup optimisation using lamination parameters. The optimisation of lamination parameter is investigated in this section by adding lamination parameters to the optimisation parameter space and considering laminate design rules constraints. The discretisation settings of PreDoCS for the lamination parameter optimisation are based on the results of *section 5.2*. All optimisation settings are summarised in *table 5.5*.

parameters	thickness, lamination parameter
objective	mass
constraints	laminate design rules, strength, stability
allowed angles	0°, ±45°, 90°
algorithm	MMA
termination condition	parameter sum tolerance: 0.001
	objective tolerance: 0.01
material	T-300 15k/976
rib-spacing	0.8 m
initial thickness (PreDoCS)	0.06 m
initial lamination parameters	isotropic (all zero)
Nr. of cross-sections	9
Max. element length	0.5 m
Optimisation region	"box-wing"
Solver	Hybrid-Song

Table 5.5: Settings for the lamination parameter optimisation

Figure 5.11 shows the mass distributions of the MiRaJet wing with optimised lamination parameters and thicknesses, and the reference results were only thicknesses are optimised. A significant reduction of the distributed mass for the lamination parameter optimisation can be observed in the middle wing. Also at the inner wing a slight decrease of the distributed mass can be observed. Only at the very tip of the wing a small increase is visible, having negligible impact on the total mass because of the relatively small mass at the wing tip. The change in mass distribution is accompanied by a 5.16 % reduction of the total wing mass, shown in *table 5.6*.

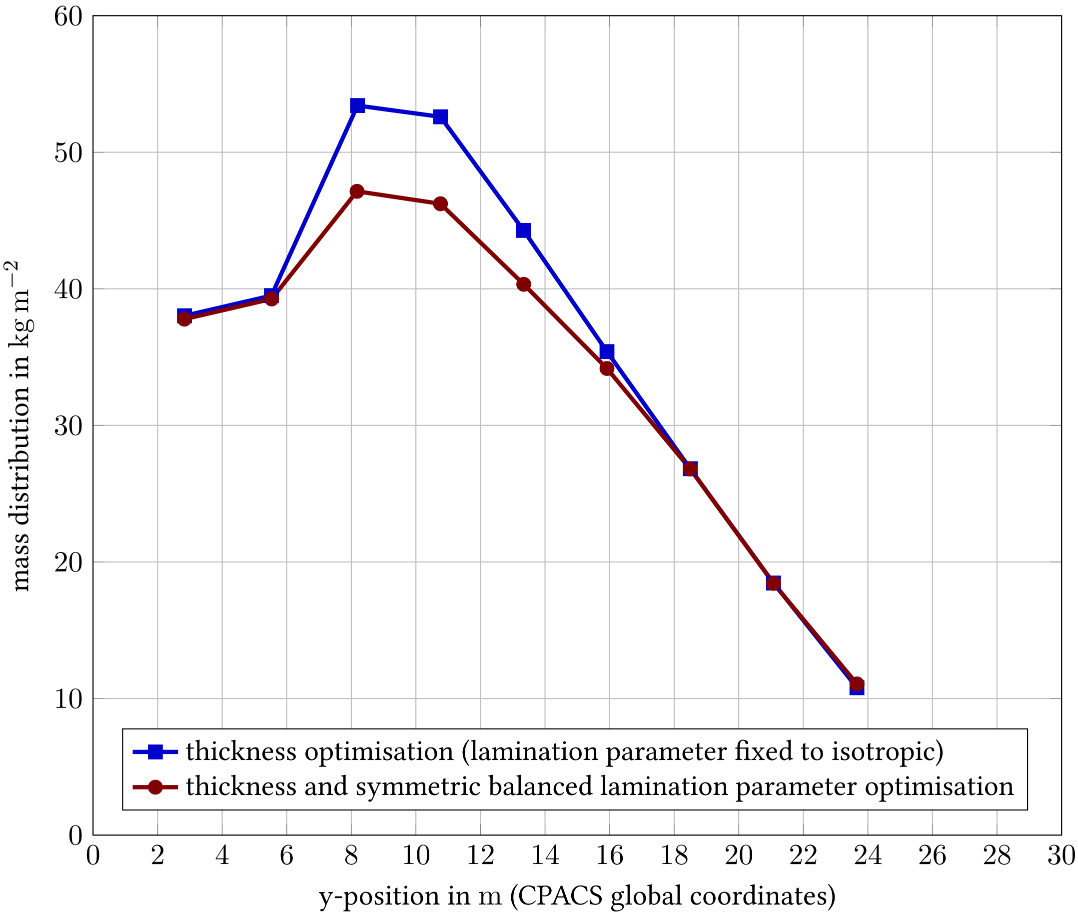


Figure 5.11: Wing mass distribution for lamination parameter optimisation

thickness	thickness + lamination parameter	deviation
3737 kg	3544 kg	−5.16 %

Table 5.6: Total wing mass reduction of the lamination parameter optimisation

All lamination parameters are set to zero prior to the optimisation in order to start with a quasi-isotropic material. The *figures 5.12 - 5.15* show the lamination parameter distributions for the symmetric balanced lamination parameters $V1_A$, $V2_A$, $V1_D$ and $V2_D$. In *figure 5.12* and *figure 5.13* the lamination parameters used for the construction of the A-matrix are shown with the same scale. Changes of the lamination parameters with respect to the initial, isotropic values can only be observed at the lower wing shell in the centre of the wing. At this location strength constraints are the sizing constraints, as shown in *section 5.1.2*. The lamination parameters determining the set up of the D-matrix are shown in *figures 5.14* and *figures 5.15* with the same scale. The most significant changes from the initial, isotropic configuration are at the upper cover in the wing middle, where the highest buckling loads were found. In contrast to the V_A lamination parameters, the V_D lamination parameters are modified on the complete wing.

The A-matrix represents the in-plane stiffness matrix in the classical lamination theory. A change of the A-matrix values can be interpreted as a change of the main fibre angles, not only increasing the stiffness in a selected direction but also the strength. The reduction of mass at the middle wing in combination with the modified lamination parameters $V1_A$ and $V2_A$ at the same location shows the positive impact of fibre orientation on the strength criteria allowing a significant mass reduction. The D-matrix describes the resistance of a composite against bending and is closely connected to the stacking sequence of the laminate. Using $\pm45°$ layers as the topmost and bottommost layers for example, is increasing the critical buckling load. Therefore the V_D lamination parameters can be used to increase the stability of composite panels. The mass reduction due to the change in V_D is small compared to the reduction caused by V_A.

The reference optimisation calculated with a discretisation of 9 cross sections and an element length of $0.5\,\mathrm{m}$ results into a mass of $3737\,\mathrm{kg}$. Considering the area correction defined in *section 5.1* of $17.90\,\%$, the area corrected reference mass is $4406\,\mathrm{kg}$. If also the reduction in mass due to a higher model discretisation is taken into account, the mass reduces $13.01\,\%$ to $3833\,\mathrm{kg}$. All total wing masses for one wing for the reference optimisation and for the lamination parameter optimisation are summarised in *table 5.7*. It has to be considered that a infinitely small discretisation implies a continuous material distribution of the wing and therefore poses the minimum boundary for the optimised total wing mass.

thickness optimisation (reference)	
calculated mass	$3737\,\mathrm{kg}$
area corrected mass	$4406\,\mathrm{kg}$
discretisation corrected mass	$3833\,\mathrm{kg}$
thickness and lamination parameter optimisation	
calculated mass	$3544\,\mathrm{kg}$
area corrected mass	$4178\,\mathrm{kg}$
discretisation corrected mass	$3635\,\mathrm{kg}$

Table 5.7: Theoretical wing mass for infinite discretisation and lamination parameter

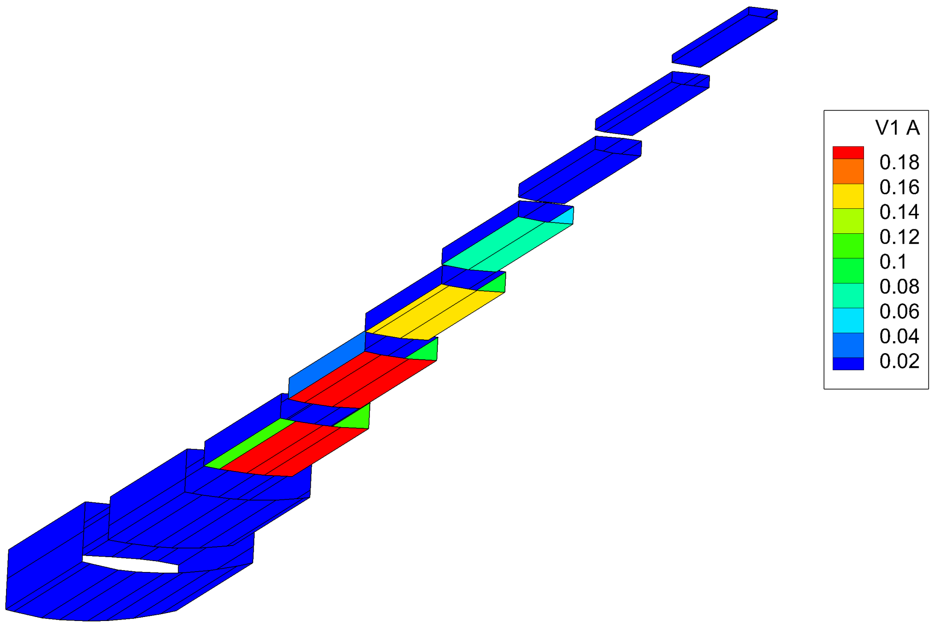


Figure 5.12: Optimised distribution of $V1_A$

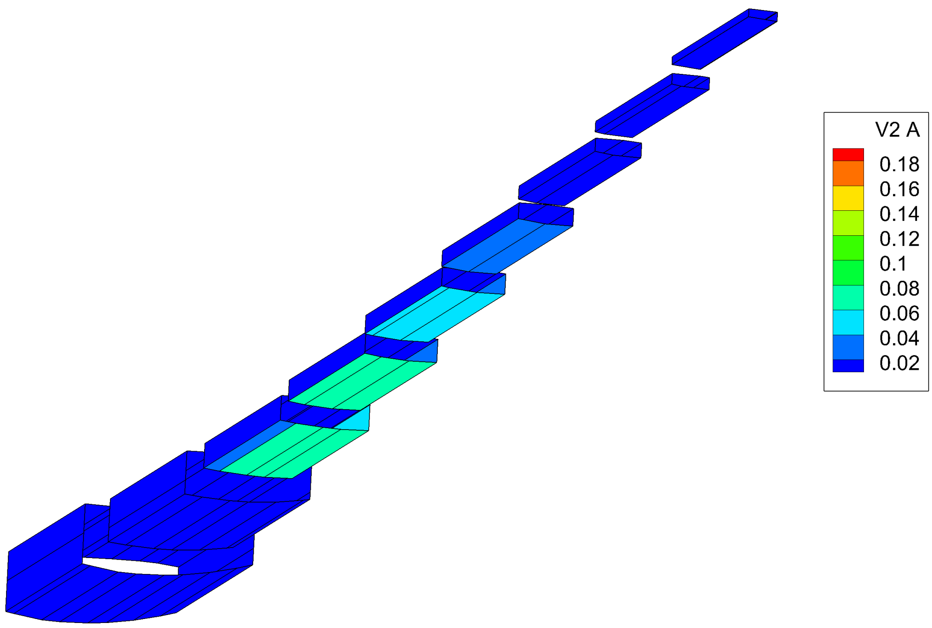


Figure 5.13: Optimised distribution of $V2_A$

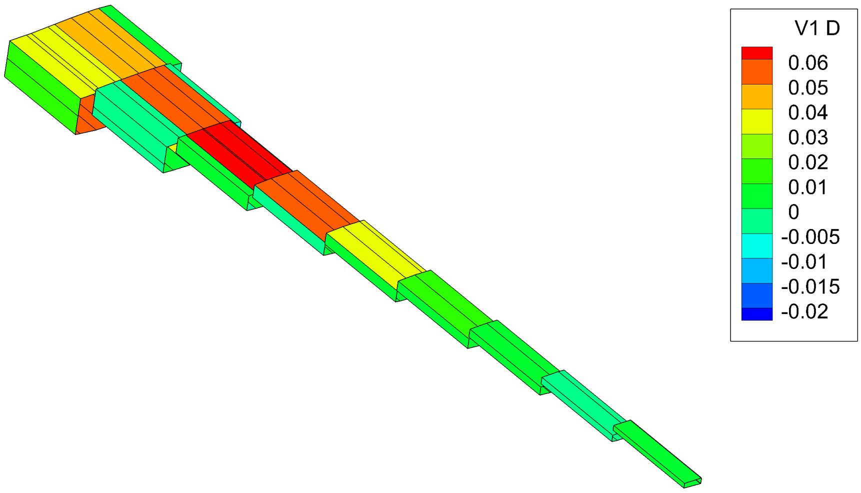


Figure 5.14: Optimised distribution of $V1_D$

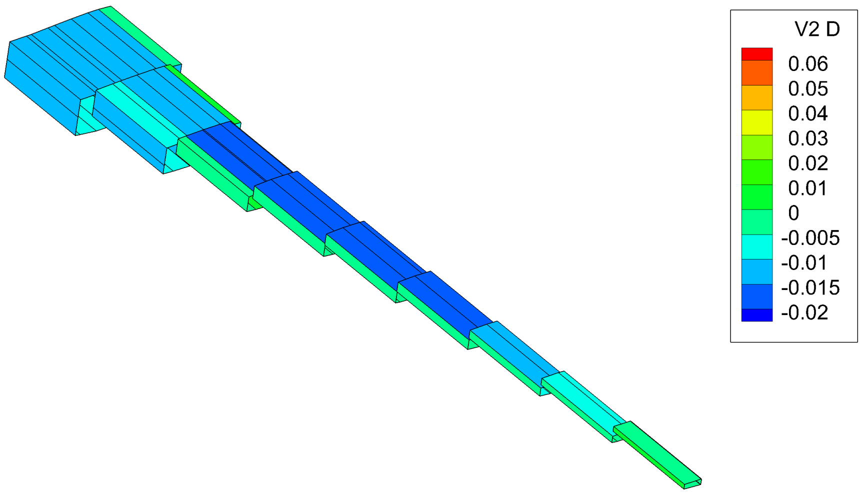


Figure 5.15: Optimised distribution of $V2_D$

Chapter 6

Summary

A robust, modular and multi-fidelity optimisation framework based on analytical methods is developed in this thesis. The structural solver PreDoCS is used to calculate internal loads of beam-like structures based on the geometry, materials and loads defined in a CPACS schema conform wing model. Using the CPACS standard provides PreDoCS with an universal interface to other disciplines supporting a multi disciplinary optimisation. A material definition based on lamination parameters allows the usage of gradient based algorithms for layup and thickness optimisations. The analytical set up of the optimisation framework allows a rapid optimisation of wing like structures.

The finite element based optimiser for high fidelity design developed at the DLR Institute of Composite Structures and Adaptive Systems is VErSO. The constraint module used in the optimisation framework in this thesis is extensively adapted from VErSOs own constraint module. An interface between VErSOs structural solver and the optimisation framework developed here allows the comparison of structural optimisations with PreDoCS and VErSO. A optimisation of skin and spar thicknesses with an isotropic material focuses on the different results of both tools. Because the solvers are independent of each other and use very different calculation methods, the comparison not only allows the evaluation of their optimisation fitness but can also be seen as a validation of both tools.

An investigation of the discretisation sensitivity of the PreDoCS model reveals promising approaches for the further improvement of PreDoCS with the focus on element formulation and computational effort. Based on the investigation a reasonable discretisation for the MiRaJet wing optimisation including thickness and lamination parameter is chosen and the potential of an infinitely small discretisation is presented. While in the comparison of PreDoCS and VErSO thicknesses are optimised for an isotropic material, the optimisation of thicknesses and lamination parameters shows the potential of composite structures for lightweight wing design. A relation of lamination parameters and strength and stability criteria indicates the physical meaning of lamination parameters in the optimisation process.

The optimisation results of PreDoCS and VErSO show good agreement and are within the range of expectations. The comparison of the analytical solver PreDoCS and the finite element solver provides confidence in both tools. A comparison of the optimisation results with a third party structural sizing tool is suggested as a further validation of the optimisation process. The optimisations of thicknesses and lamination parameters show a further mass reduction of the wing structure and therefore an improvement of the optimisation results. This thesis underlines the legitimation of a gradient based structural optimisation process using lamination parameters and analytical methods in preliminary design. The possibility of a high discretisation or increased order elements in PreDoCS opens up the question whether the same process can be used as a high fidelity structural optimisation, covering the full range from preliminary design to full scale design.

Outlook

In this thesis the concept of a structural optimisation process using gradient based optimisation and analytical methods is developed and proven. Many opportunities for further improvements of the framework opened up during the development, which are presented here:

- The calculation of the actual mass of the wing structure is not possible with a discretised structural model. For this reason it makes sense to calculate the wing mass of the CPACS model with updated materials.

- The final results of the structural optimisation in this thesis are thicknesses and lamination parameters for each optimisation region of the wing. A translation of the lamination parameters into stacking sequence and layer angles is inevitably the next step towards manufacturable structural design. An update of the CPACS model with the optimised composite material definitions including skin thicknesses is necessary for multi disciplinary design.

- PreDoCS uses linear elements for the curvature discretisation resulting into many elements for strongly curved profiles. The computational effort of PreDoCS increases with the number of elements. The element number can be significantly reduced using cubic elements in PreDoCS. This improvement is considered a major step towards a truly rapid optimisation.

- Internal line loads are returned on PreDoCS elements as analytical, parametrised functions. On the other hand constant loads are used for the evaluation of strength and stability criteria. For this reason the maximum load of the parametrised function is calculated for each element. Analytical strength and stability criteria are also available for higher order line loads, e.g. linear or quadratic line loads. Higher order constraints improve the failure estimation and allow a more lightweight construction.

- In this thesis no discrete or distributed stiffeners as ribs, stringers, sandwich materials etc. were used in the optimisation process. Such stiffeners increase the critical buckling loads and therefore further reduce the total structural mass. Stiffener concepts are already available in VErSO and can be ported to the optimisation framework structural model.

- Finite differences are used in the optimisation process for gradient calculation. A significant reduction of computational effort accompanied by a strong increase of accuracy can be achieved with analytical gradients. Because of its analytical basis, the optimisation framework using PreDoCS as structural solver allow the implementation of analytical gradients. Analytical gradients then have to be part of the structural model, the solver and all constraints.

- Analytical gradients and element discretisation are two important factors for the further improvement of the optimisation speed. Other factors may be found in a performance analysis of the framework using a profiler.

- The potential of cord-wise discretisation of the optimisation regions is shown in this thesis. A truly lightweight design can only be achieved with a reasonable cord-wise optimisation region discretisation allowing the optimiser to place material where it has the highest impact on strength and mass.

- The impact of the PreDoCS beam axis placement on load distribution and magnitude is not yet investigated. Also the deviation of the actual and the calculated wing stiffness for swept wings is unknown. Both effects may influence the quality of internal loads calculated with PreDoCS and are worth a closer investigation.

- The final step towards a multi-disciplinary optimisation framework is to include other disciplines, e.g. aerodynamics and aeroelastics, into the optimisation framework, possibly using the universal CPACS interface.

The most important points of all proposed improvements are an accurate calculation of the total wing mass and the validation of the optimisation process with a high confidence commercial or industrial tool. The accurate mass calculation requires an update of the composite materials in the CPACS model and therefore a transformation of lamination parameters into a laminate definition. A realisation of these points allows to use the established multi-fidelity optimisation process in aircraft preliminary design.

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List of Figures

List of Tables

Appendix A

Strength Analysis

Figure A.1 shows the comparison of safety factors for layer based and lamination parameter based strength criteria with compressive loads. For this analysis "Laminate 3" defined in *table 4.2* is used in combination with "Loadcase 4" defined in *table 4.4.* The lamination parameter based criteria of Khani and Ijsselmuiden are a close, widely conservative approximation of the layer based Tsai Wu criterion. However, compared to the criterion of Yamada and Sun, the criterion of Tsai and Wu returns a higher, not conservative safety factor. For thin walled structures as for example wing structures it can be assumed stability is the governing criteria in compression regime. Therefore the non-conservatism of the lamination parameter based criteria can be accepted here.

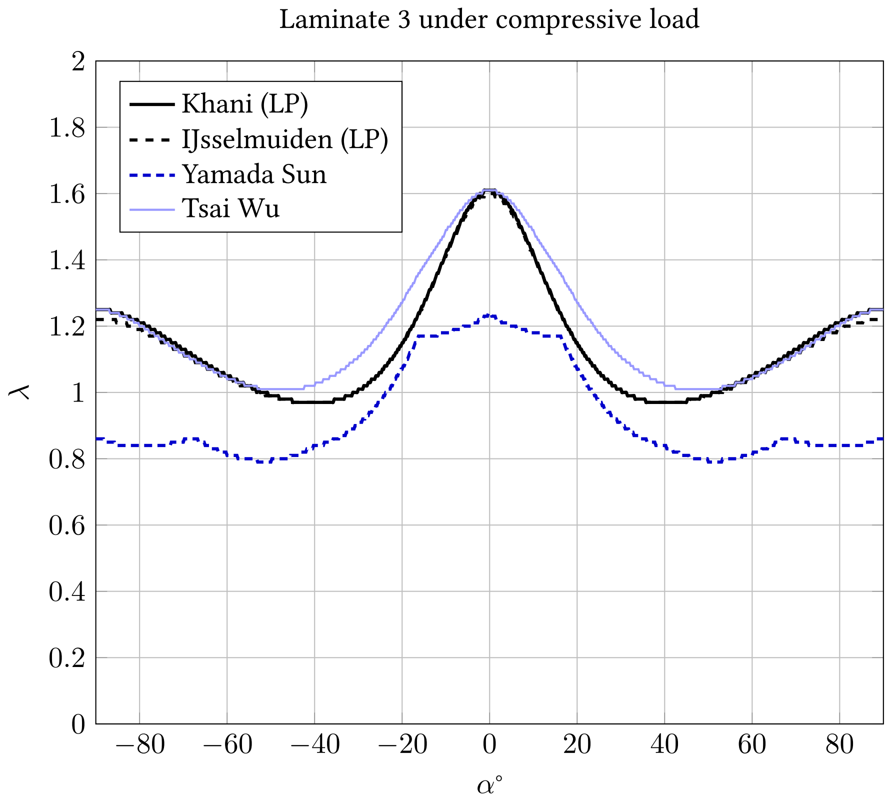

Figure A.1: Strength criteria safety factor comparison for a laminate under compressive load

Appendix B

Test Panel Optimisation Results

Tabel B.1 and *Tabel B.2* show the constraints development for thickness and symmetric balanced lamination parameters and for thickness and all lamination parameters respectively. In both cases the stability constraint of load case 3 (LC3) is the active constraint preventing a further mass reduction of the test panel. It is remarkable, that the final values of all other constraints between the two optimisations are very similar as well. A good explanation for the resemblance are the similar structural parameters.

	Stability			Strength		
i	LC2	LC3	LC6	LC2	LC3	LC6
0	−0.892960	−0.847010	−0.862060	−0.890320	−0.587710	−0.961680
1	−0.933330	10.000000	10.000000	10.000000	10.000000	10.000000
2	−0.929270	1.006420	−0.462230	−0.691630	0.155010	−0.892660
3	−0.917920	−0.514750	−0.834670	−0.813720	−0.299770	−0.934920
4	−0.928820	0.799340	−0.519200	−0.703160	0.115790	−0.896300
5	−0.926960	0.258830	−0.667900	−0.737640	−0.013810	−0.908340
6	−0.924940	−0.058320	−0.754190	−0.763010	−0.109160	−0.917210
7	−0.925420	0.002310	−0.738300	−0.757710	−0.089240	−0.915350
8	−0.925410	0.001470	−0.738520	−0.757810	−0.089610	−0.915390
9	−0.925400	−0.000200	−0.738960	−0.757950	−0.090150	−0.915440
10	−0.925400	−0.000100	−0.738940	−0.757920	−0.090050	−0.915430
11	−0.925390	−0.001420	−0.739330	−0.757800	−0.089590	−0.915390
12	−0.925390	−0.001400	−0.739380	−0.757550	−0.088650	−0.915300
13	−0.925250	−0.016360	−0.743960	−0.756000	−0.082820	−0.914760
14	−0.925160	−0.008850	−0.746100	−0.740040	−0.098980	−0.916290
15	−0.925130	−0.005730	−0.746620	−0.725820	−0.159230	−0.921720
16	−0.925070	−0.004520	−0.748150	−0.667470	−0.332770	−0.935070
17	−0.925060	−0.001820	−0.748280	−0.602160	−0.402790	−0.937100
18	−0.925050	−0.000800	−0.748300	−0.546800	−0.431950	−0.936550
19	−0.925060	0.000000	−0.748090	−0.547240	−0.432640	−0.936280
20	−0.925060	0.000330	−0.748060	−0.546200	−0.431460	−0.936640
21	−0.925060	0.000250	−0.748080	−0.546680	−0.432000	−0.936480
22	−0.925060	0.000020	−0.748090	−0.547180	−0.432580	−0.936300
23	−0.925060	0.000000	−0.748090	−0.547230	−0.432640	−0.936290
24	−0.925060	0.000000	−0.748090	−0.547240	−0.432640	−0.936280
25	−0.925060	0.000000	−0.748090	−0.547240	−0.432640	−0.936280
26	−0.925060	0.000000	−0.748090	−0.547240	−0.432640	−0.936280
27	−0.925060	0.000000	−0.748090	−0.547240	−0.432640	−0.936280
28	−0.925060	0.000000	−0.748090	−0.547240	−0.432640	−0.936280

Table B.1: Constraints of the test panel thickness optimisation

	Stability			Strength		
i	LC2	LC3	LC6	LC2	LC3	LC6
0	−0.892960	−0.847010	−0.862060	−0.890320	−0.587710	−0.961680
1	−0.933330	2,382,828.244170	655,192.259020	31.904490	122.686030	10.495220
2	−0.929270	1.008220	−0.461730	−0.691530	0.154500	−0.892650
3	−0.917910	−0.514820	−0.834680	−0.813730	−0.299810	−0.934920
4	−0.928820	0.799870	−0.519060	−0.703130	0.115910	−0.896290
5	−0.926960	0.258840	−0.667900	−0.737640	−0.013810	−0.908340
6	−0.924940	−0.058340	−0.754190	−0.763010	−0.109170	−0.917210
7	−0.925420	0.002310	−0.738300	−0.757710	−0.089240	−0.915360
8	−0.925410	0.001470	−0.738520	−0.757810	−0.089610	−0.915390
9	−0.925400	−0.000200	−0.738960	−0.757950	−0.090150	−0.915440
10	−0.925400	−0.000010	−0.738910	−0.757930	−0.090080	−0.915430
11	−0.925400	−0.000010	−0.738910	−0.757930	−0.090080	−0.915430
12	−0.925400	−0.000100	−0.738940	−0.757920	−0.090040	−0.915430
13	−0.925390	−0.001420	−0.739330	−0.757800	−0.089570	−0.915390
14	−0.925250	−0.016380	−0.743950	−0.756240	−0.083710	−0.914840
15	−0.925090	−0.020270	−0.748540	−0.740050	−0.108230	−0.917150
16	−0.925140	0.000580	−0.746010	−0.698320	−0.280660	−0.931160
17	−0.925130	−0.000650	−0.746240	−0.702160	−0.267780	−0.930360
18	−0.925310	0.036300	−0.739330	−0.507370	−0.408800	−0.937650
19	−0.925090	0.000800	−0.747360	−0.642870	−0.365740	−0.936280
20	−0.925080	−0.000270	−0.747560	−0.654590	−0.351830	−0.935700
21	−0.925070	−0.000340	−0.747850	−0.642000	−0.381200	−0.935400
22	−0.925080	0.000780	−0.747670	−0.624020	−0.394220	−0.935830
23	−0.925070	−0.000180	−0.747870	−0.629790	−0.390900	−0.935730
24	−0.925060	−0.000190	−0.748090	−0.566440	−0.423040	−0.936770
25	−0.925060	−0.000030	−0.748100	−0.556930	−0.428690	−0.936360
26	−0.925060	0.000210	−0.748040	−0.556700	−0.428730	−0.936340
27	−0.925060	0.000030	−0.748090	−0.556860	−0.428730	−0.936350
28	−0.925060	0.000010	−0.748090	−0.556880	−0.428730	−0.936350
29	−0.925060	0.000000	−0.748090	−0.556910	−0.428710	−0.936350
30	−0.925060	0.000000	−0.748090	−0.556920	−0.428690	−0.936350
31	−0.925060	0.000000	−0.748090	−0.556930	−0.428690	−0.936350
32	−0.925060	0.000000	−0.748090	−0.556930	−0.428690	−0.936350
33	−0.925060	0.000000	−0.748090	−0.556930	−0.428690	−0.936350
34	−0.925060	0.000000	−0.748090	−0.556920	−0.428690	−0.936350
35	−0.925060	0.000000	−0.748090	−0.556930	−0.428690	−0.936350

Table B.2: Constraints of the test panel thickness optimisation