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Prandtl's flow visualization film C1 revisited

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Abstract

We would like to report on the techniques used around 1930 for the creation of a film consisting of sequences visualizing various forms of flow separation produced by L. Prandtl and his colleagues O. Tietjens and W. Müller in open-surface water channels. After first experiments with cinematography in the 1910s, Prandtl began to produce flow visualisation films in the 1920s, in order to better capture the dynamics of the unsteady flow phenomena. However, due to the short exposure times provided by the film camera the particles appeared as dots rather than the anticipated streaks, which were desired as representations of the streamlines. Therefore Prandtl and his colleagues first decided to work with a modified camera to provide longer exposure time and consequently streaked images. At this point, the real potential of animated film, for instance for the demonstration of flow separation, was realized and resulted in a flow visualization film, originally named "The Production of Vortices by Bodies Travelling in Water", that Prandtl showed 1927 in London and while travelling the globe from 1929 to 1930. The sequences of this film were found to be highly instructive from an educational point of view such that the film, now named C1, was made available in 1936 through the Reich Office for Teaching Films and in the 1950s by the Institute of Scientific Film.

During the preparation of this paper, new movies recorded by Prandtl and his colleagues have been found in the archives. They are discussed in this paper in order to demonstrate the feasibility of such flow visualization movies for evaluation with modern PIV correlation and tracking analysis methods in a broader frame. While the film sequences were mainly intended to illustrate the temporal evolution leading to boundary layer separation and separated flow, numerous references confirm that Prandtl and his colleagues were fully aware of the quantitative nature of the acquired imagery. With much of the material preserved over the decades, it nowadays is possible to use advanced particle image velocimetry techniques such as PIV and Lagrangian particle tracking to retrieve this quantitative information and make a clear statement on the importance of proper data archival.

1 Introduction

There are several motivations for this article. Compared to photography, the role of moving images in scientific research only very gradually was accepted as a valuable tool for the visualization of dynamic processes. In part, this is attributed to the simple fact, that, until a few decades ago, moving images were more difficult to convey via printed media, the de facto standard method of conveying scientific results to a wider audience. In contrast, a byproduct of the increased digitization of media is the nowadays widespread use of moving images, also within scientific publication. On the other side, as Schulze and Waltenspül (2019, p. 163) state, "the historical research on scientific moving images is remarkably underrepresented." For the specific case of Ludwig Prandtl's flow visualization, the transition from streamtrace photography to recording time-resolved motion of unsteady flow was a gradual one. Beyond this, we intend to shed light on the instrumentation and facilities used for the production of the films and show that, even today, the image data can still be of value.

2 Steps that made C1 possible

2.1 Flow visualization techniques

The visualization methods utilized for the film C1, such as small particles (lycopodium seeds) sprinkled onto the water surface, were already developed much earlier by Friedrich Ahlborn, a botanist and expert of bird flight from Hamburg (Ahlborn, 1902b). Around 1900, Ahlborn already operated highly sophisticated facilities (Fig. 1, left) that allowed the photographic capture of streak images of various bodies towed through the water. He even devised an imaging setup that involved a light sheet configuration in order to demonstrate that the vortical structures observed in surface flows were equally present below the water surface. Ahlborn's efforts most likely were influenced by prior work of Étienne-Jules Marey, an expert on photographic and chronophotographic[1] recording, who previously successfully visualized the subsurface motion to translating and stationary waves using neutrally buoyant particles (Marey, 1893; Didi-Huberman and Mannoni, 2004). Only recently, the value of Ahlborn's work with regards to advancing flow visualization has been put into perspective (Eckert, 2006a; Bloor, 2008; Hinterwaldner, 2015; Fermigier, 2017).

In comparison to the facilities that Prandtl used to underline his seminal work on boundary layer theory, the facilities of Ahlborn were already much more sophisticated. His setup, shown in Fig. 1, featured a self-driven carriage with a camera to obtain particle streak images capturing the flow around objects. At the same time Prandtl was using flakes of mica mixed into the water to visualize the fluid flow, as the thin flakes orient themselves along with the strain in the fluid (Prandtl, 1905, see e.g. Figs. 11-20 in Prandtl, 1927a). In the following years Prandtl and his colleagues began to adopt Ahlborn's visualization approach, in part by an increased mutual exchange of information (Eckert, 2006a, p. 40).

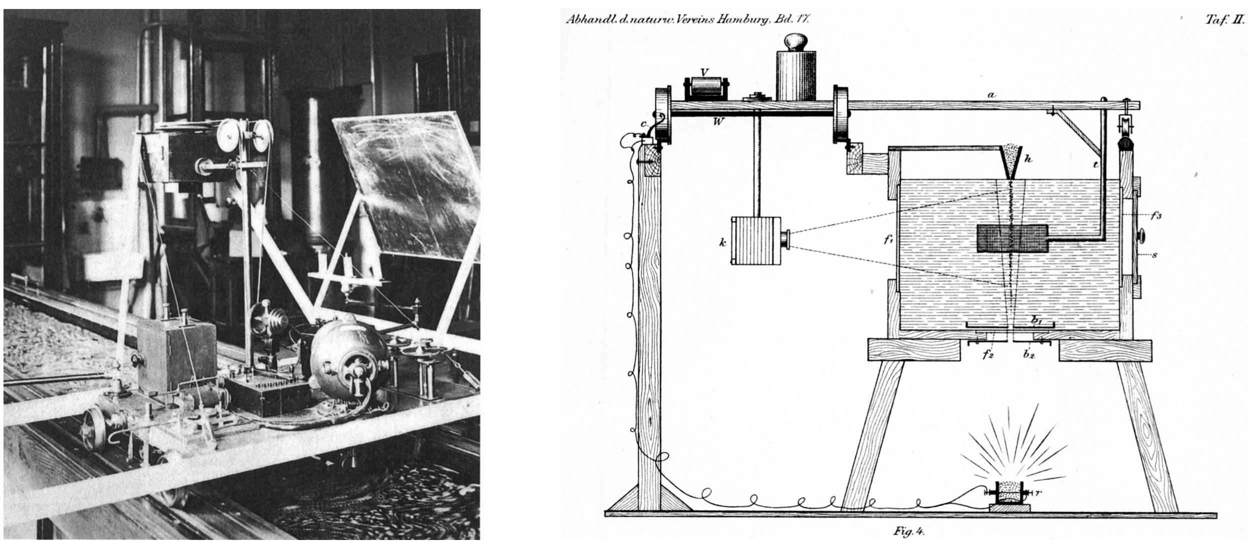

Figure 1: Photograph of towing tank with traversing camera rig of Friedrich Ahlborn in Hamburg in 1904 (reproduced from Eckert, 2006b). Right: cross section of Ahlborn's towing tank illustrating light sheet visualization for submerged objects (reproduced from Ahlborn, 1902a).

2.2 From photography toward cinematography

The facilities utilized for flow visualization were typically open channel flumes with recirculating flow. For his work on boundary layers on cylinders K. Hiemenz built a 3500 mm long by 1100 mm wide basin that contained a recirculating channel flow with a 400 mm wide, 1150 mm long and 290 mm deep test section (Hiemenz, 1911). H.L. Rubach later added a carriage that supported a camera travelling with the flow to study the wake of cylinders (Rubach, 1916). In an annual report of 1912 Prandtl already states that the facility was being used for both photographic and cinematographic visualizations (Prandtl, 1912/13).

[1]Chronophotography is a form of multi-exposure recording to capture the dynamics of motion on a single frame.

The work of Rubach and von Kármán (1912) on cylinder wake flows motivated Nayler and Frazer (1917) to perform time resolved measurements of the cylinder wake flow using streak imaging and light sheet illumination. The light sheet, created by two arc lamps, passing through slits from either side of the water channel, was shuttered by a pendulum mechanism such that multi-pulse streak images could be generated and subsequently (manually) analyzed to retrieve velocity information.

To best of the authors' knowledge, the first documented use of cinematography at the "Kaiser Wilhelm Institute for Aero- and Hydrodynamics" in Göttingen was for the investigation of the Magnus effect on rotating cylinders (Prandtl and Tietjens, 1925). The aim of their effort was to record a sequence of streak recordings, called "Linienbilder" (line images or "long-exposure moving pictures" in Prandtl and Tietjens, 1934, p. 273), rather than producing individual snapshots containing frozen particle images, such that the recorded images could be studied individually. With the length of the streaks indicative of the particles' velocity the viewer was given the impression of streamline patterns. In their article on "cinematographic flow images" (Kinematographische Strömungsbilder) they explain that the images produced by their Liesegang camera of "dated design", initially operating at 16 fps, resulted in streaks too short to provide the desired streak recordings. A modification of the camera reduced the transport speed of the film such that images were recorded for 7/8 of the time followed by a corresponding frame change of 1/8 of a frame. This resulted in image recordings at about 3 fps with the desired effect of providing streak images, which, however were regarded as unsuited for movie projection (Schulze and Waltenspül, 2019). An example of these time resolved streak images in comparison to images captured by the unmodified camera are provided in Fig. 2.

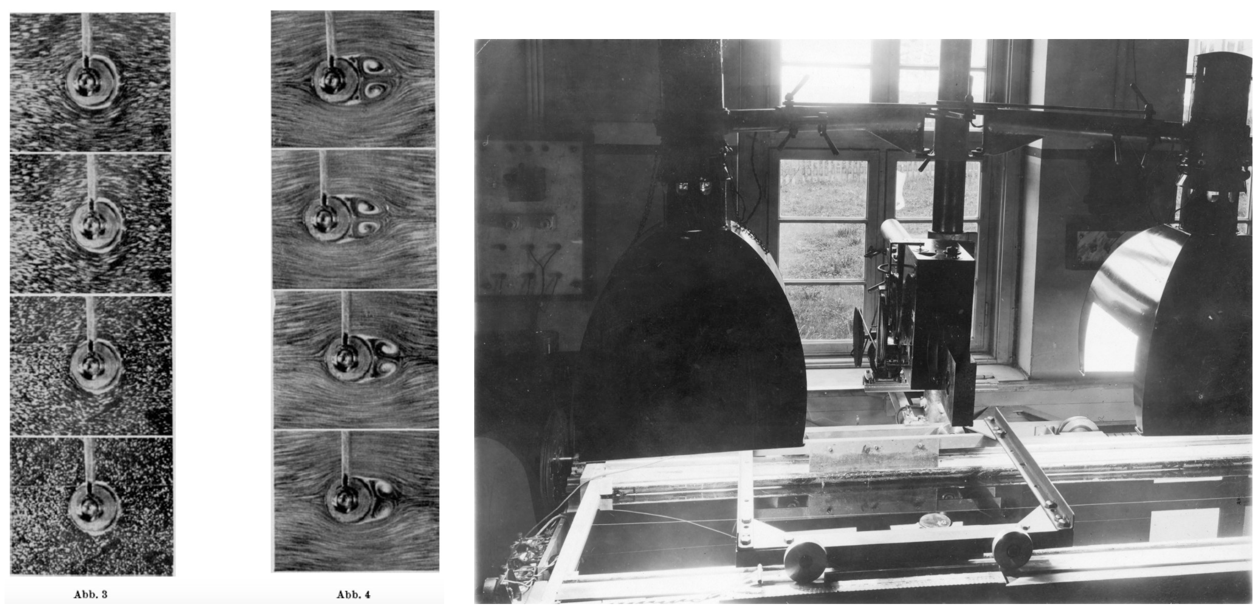

Figure 2: Cinematic recordings of wake flow behind the cylinder obtained by Prandtl & Tietjens illustrating the loss of information due to the short exposure of each frame (left column) compared to images recorded by their modified camera shown in the right column (figures reproduced from Prandtl and Tietjens, 1925). Right: Water channel and camera setup used to record images shown on the left (FS-0480, Central Archive of DLR).

Particle streak imaging was also used by Prandtl's student Johann Nikuradse in course of his thesis work on the formation of turbulence in channel flow (Nikuradse, 1923, 1926, see also Fig. 4, left). Here the camera could be translated along the channel at different speeds to visualize the turbulent structures with different frames of reference. While his Ph.D. thesis makes no reference to the use of cinematography, in the years following, Nikuradse produced films of the channel flow he had been investigating. Following a presentation on the subject by Prandtl at the 2nd International Congress on Technical Mechanics held 1926 in Zürich, a movie of the turbulent channel flow was shown, from which stills are provided in Fig. 3. Prandtl commented that these recordings so far could only be used to determine the temporal mean of the velocity and the size of the velocity fluctuations, but little could be learned regarding the turbulence itself (Prandtl, 1926).

2.3 Genesis of Film C

Prandtl realized the potential of motion picture as a means of conveying the concepts of boundary layer formation and flow separation and, with the help of Tietjens, produced a film consisting of a number of short sequences. This film, titled "Production of Vortices by Bodies Travelling in Water", was first shown in May 1927 as part of Prandtl's Wilbur Wright Memorial Lecture in London (Prandtl, 1927a) and then in September 1927 at the Physics and Mathematics Meeting in Kissingen, Germany (Prandtl, 1927b). In the following years, he showed the nearly eight minute long film on several conferences and took it on a "round-the-world trip", showing it in Moscow, Kobe, Tokyo, Pasadena, Los Angeles ... until part of his baggage including the film reel was stolen at the Detroit train station.

Due to its educational value, Prandtl proceeded to further improve the original film and augmented it by a second film that focussed on flow separation within curved channels, cavities and junctions. Now respectively named C1 and C2 and a combined playing time of about 20 minutes, both films were made available in 1936 for an increased audience by the Reichsstelle für den Unterrichtsfilm (RfdU, Reich Office for Teaching Films) and later digitized by the Institut für Wissenschaftlichen Film (IWF, Institute of Scientific Films) of Göttingen (Prandtl, 2009). It is these sequences that were then processed by Willert and Kompenhans (2010) using correlation-based PIV algorithms. Due to limitations introduced by the (JPEG) compression artifacts of the available DVD, the film was recently digitized at higher resolution for the work presented herein.

One particular set of sequences, named "Strömungsaufnahmen kleiner Zylinder" (flow visualizations of small cylinders) came to the attention of the authors through the scientific estate material of Prandtl available at the Archives of the Max-Planck-Society. Although not part of the of C1 and C2, these sequences are both aesthetically appealing and of a high recording quality that lends itself for PIV and PTV processing as described in the following section.

2.4 Recording equipment

The sequences in films C1 and C2 were recorded on 35 mm film using a Universalkinamo 5401 by ICA AG camera with a Zeiss-Tessar 1:3.5 objective lens. Originally designed for manual crank transport, the camera was fitted with a motor to automate the film transport while mounted on a traverse. Precise information on the actual recording frequencies are difficult to verify. Whereas the inlays of the films state a recording frequency of 20 frames per second, the films were most likely recorded at 16 Hz, what can be deduced from Prandtl's recommendations for the projection speed (Prandtl, 1927b). In some cases, the film transport was even slowed to increase the desired streakiness of the particles. The exposure and transport time are of similar duration (actual ratio is $3 \div 2$), such that streaks have a gap of about one exposure frame between the exposures. Illumination was continuous and based on strong light bulbs or arc lamps. The arc lamps shown in Fig. 2, right, could be operated between 50 and 550 Volt at a maximum current of 130 Ampere.

3 Analysis of the film by modern PIV and PTV

The original film C1 along with several other film sequences were newly digitized at a higher resolution (2048×1536 pixel) than was available for the initial investigation by Willert and Kompenhans (2010). The loss-less scanned images now are more suitable for analysis using state-of-the-art PIV methods as well as multi-frame particle tracking velocimetry. With a few exceptions, the assignment of actual physical velocity to the processed data is not possible because of unknown length scales and unknown recording speeds.

Here it is worth noting that, in spite of the streakiness of the recorded particle images, the sequences produced by Prandtl and Tietjens are suitable for correlation-based PIV processing (see Fig. 6), yet this very streakiness makes them less suited for PTV processing (see e.g. Fig. 7). The tendency of the particles to cluster into groups further complicates the PTV analysis whereas PIV is less reliant on images of discretely images particles. In principle, the 35 mm film material investigated herein could have supported a much higher seeding concentrations, that is, the particles images could have been much finer and thereby even better suited for the analysis with state-of-the-art PIV and PTV algorithms. Whereas F. Ahlborn strived to achieve the highest possible detail in his particle streak photographs, L. Prandtl and his colleagues were more focussed on providing clear visualizations of flow dynamics that could be projected during presentations, even after reproduction onto 16 mm film which was the common media for teaching films.

In the following, a short sequence from the recently discovered "Strömungsaufnahmen kleiner Zylinder" (flow visualizations of small cylinders) is used to demonstrate the suitability of the nearly century-old film material for both PIV and PTV processing.

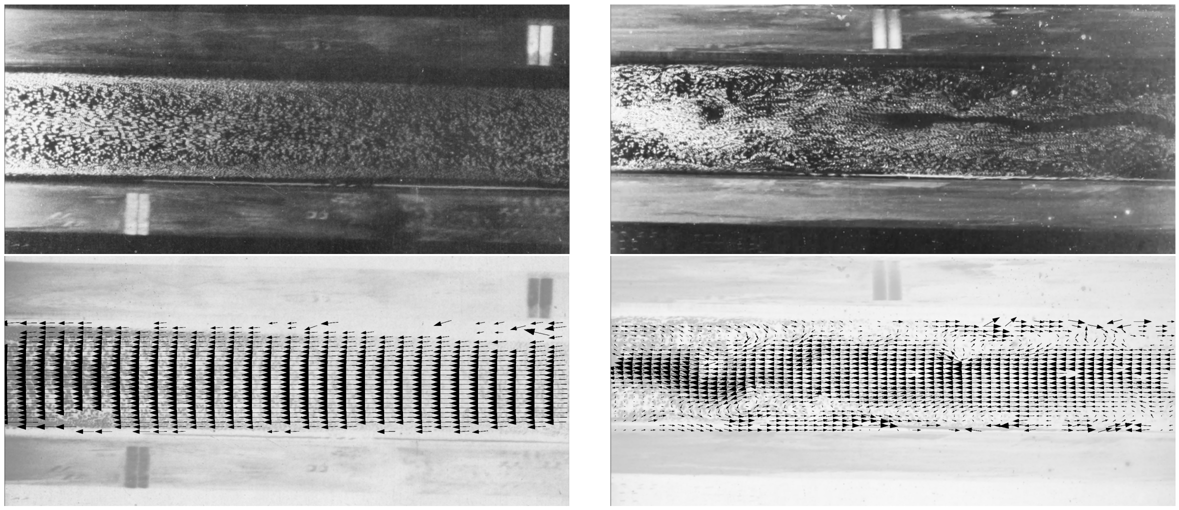

Figure 3: Top: Image frames reproduced from a film recorded by J. Nikuradse between 1923 and 1926 capturing the development of turbulence in a channel flow (channel dimensions: 15 cm width, 25 cm depth, ≈ 6 m length). Two consecutive images are overlaid to visualize the relative particle displacement. Bottom: PIV data extracted from the image pairs. (Film material from Archives of Max- Planck-Society, Abt. VII, Rep. 1, F142)

3.1 PIV processing

Per image segmentation about 30,000 "particles" (including clusters) could be detected in the image shown in the top-left of Fig. 8. This corresponds to a seeding density of approximately 0.01 particles per pixel. At this density a sample of 32×32 pixels contains 10 particles in the mean, reducing to about 6 particles for a sample of 24×24 pixel used for the present analysis. The image sequence was processed pair-wise with 50% sample overlap using a coarse-to-fine processing algorithm with validation rates exceeding 99 percent. PIV analysis mainly fails in the vortex cores and in area of strong gradients, a indicated by red vectors in the bottom left sub-figure of Fig. 8. Following Adrian (1997), the dynamic range of the PIV data can be estimated: With displacements up to 50 pixels and an uncertainty of 0.1 pixels, based on a particle diameter of ≈ 5 pixels, the dynamic velocity range (DVR) is $50 \div 0.1 = 500$. The corresponding dynamic spatial range (DSR) is $2048 \div 24 \cong 85$. Overall the resolution is similar to what was achievable with 1-2 MPixel PIV cameras that started to appear in the late 1990s. As mentioned above, the resolution of the film could have supported a much higher seeding density, but this was not the aim of Prandtl's flow visualizations.

3.2 Lagrangian Particle Tracking using 2D-STB

The Lagrangian Particle Tracking (LPT) evaluation was performed using the temporal predictor-corrector scheme of the Shake-The-Box method (STB, Schanz et al., 2016), which was originally developed for LPT in three dimensions using multiple, simultaneous camera views. For this case, the scheme was adapted for the two-dimensional temporal tracking of particle peaks, as captured by a single camera. To this end a peak detection with subsequent position optimization using an analytic image matching approach ("shaking": Jahn, 2017) was performed on the first four images of a sequence. From the recovered peaks, four-step tracks are searched for by defining a fixed search radius for subsequent frames and accepting all four-step candidates which show a low average deviation from a Wiener filter fit. The identified tracks are then extended to the next time-step by predicting a new position (again using a Wiener filter). The predicted positions are then corrected using the shake-approach and residual images are created by subtracting the virtual images of the now correctly positioned tracked particles from the original images. A calibration of the particle imaging properties (e.g. using the Optical Transfer Function: Schanz et al., 2012) could not be carried out due to the macroscopic imaging of the particles (in contrast to the diffraction limited imaging that is typically found in modern PIV and LPT experiments) and the temporal blurring/streaking. Instead, a constant Gaussian particle image shape was assumed, scaled by the fitted peak intensity. Peaks are identified on the created residual images and more iterations of shaking are carried out on all peaks (predicted and newly found). Finally, new four-step tracks are searched within the clouds of so far untracked particles of

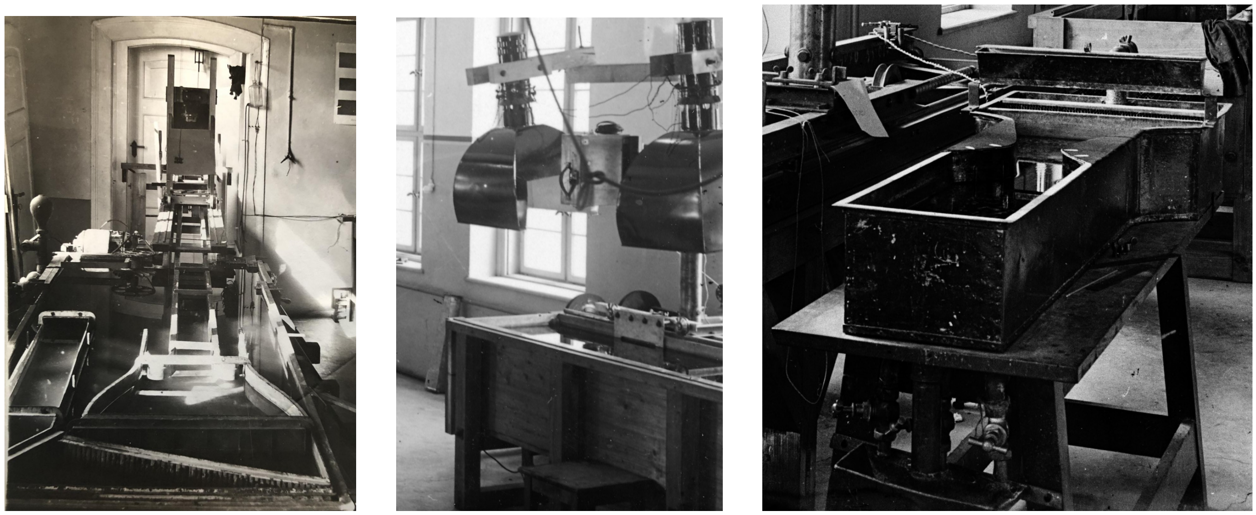

Figure 4: Left: photograph of Nikuradse's facility used for the investigation of channel turbulence (ca. 1923). Middle: photograph of camera in between two arc lamps above channel with towing mechanism (ca. 1925, FS-0172, Zentrales Archiv DLR). Right: photograph of diffuser channel used to visualize boundary control through suction (ca. 1925, GK-0425, Central Archive of DLR, see also Ackeret, 1926).

the last four time-steps. Then the algorithm advances to the next time-step, again predicting all tracked particles.

The above described scheme was applied to sequence “Strömungsaufnahmen kleiner Zylinder”, which visualizes the wake of an oscillating cylinder. An initial search radius of 15 pixels was chosen, identifying nearly 2 million potential four-step tracks. By accepting only those that show an average deviation of less than 1.5 pixels from a Wiener fit, this number is reduced to about 13.500 simultaneous tracks. Within the next time-steps, the track-system saturates, until approximately 24.000 particles are tracked per time-step. Fig. 9 shows an exemplary result from the evaluated time-series. The particle images of the original camera image are color-coded with the particle tracking results using either the displacement magnitude or the instantaneous particle acceleration. The oscillating cylinder is exiting the field-of-view to the right and leaves a system of vortices in its wake. Successful particle tracking is possible in most regions, apart from the strongest vortices directly trailing the cube, where velocities and accelerations are maximal. The distinct streaking of particle images in these regions inhibits a reliable positioning of the particles.

4 Summarizing comments

The cinematographic material recorded by Prandtl and his team in the 1920s up to the 1930s was mainly intended for visualization purposes and less so for quantitative analysis, for which suitable processing equipment anyhow was not available at the time. In general, in the early days of PIV development, that is in 1980s and 1990s, the use of cinematography was not used for the acquisition of time-resolved PIV image sequences. At that time ‘snap-shot’ PIV relied on the illumination of the flow field by two short light pulses that were recorded on a single photographic recording. The displacement of the particle images, and hence of the velocity, was estimated by optical-digital autocorrelation methods, facing an ambiguity problem regarding the sign of the velocity vector. Being able to record the illumination of each light pulse on a different recording, much like the images in Prandtl's films, would have allowed the use of digital cross-correlation evaluation methods and thus avoiding the velocity ambiguity issue. However, no affordable equipment for digitizing photographic films with sufficiently high spatial resolution along with the required storage and processing capabilities was available at that time. Moreover, photographic recording provided superior spatial image resolution compared to analog video equipment.

To our knowledge, the concept of film based time-resolved PIV only found application in combination with drum cameras at considerably higher framing rates but with a limited number of total frames. Vogel and Lauterborn (1988) recorded at 10 kHz whereas Hartmann et al. (1996) used a copper vapor laser to capture sequences at 30 kHz. In both cases recording was on 35 mm film which was digitized and processed using

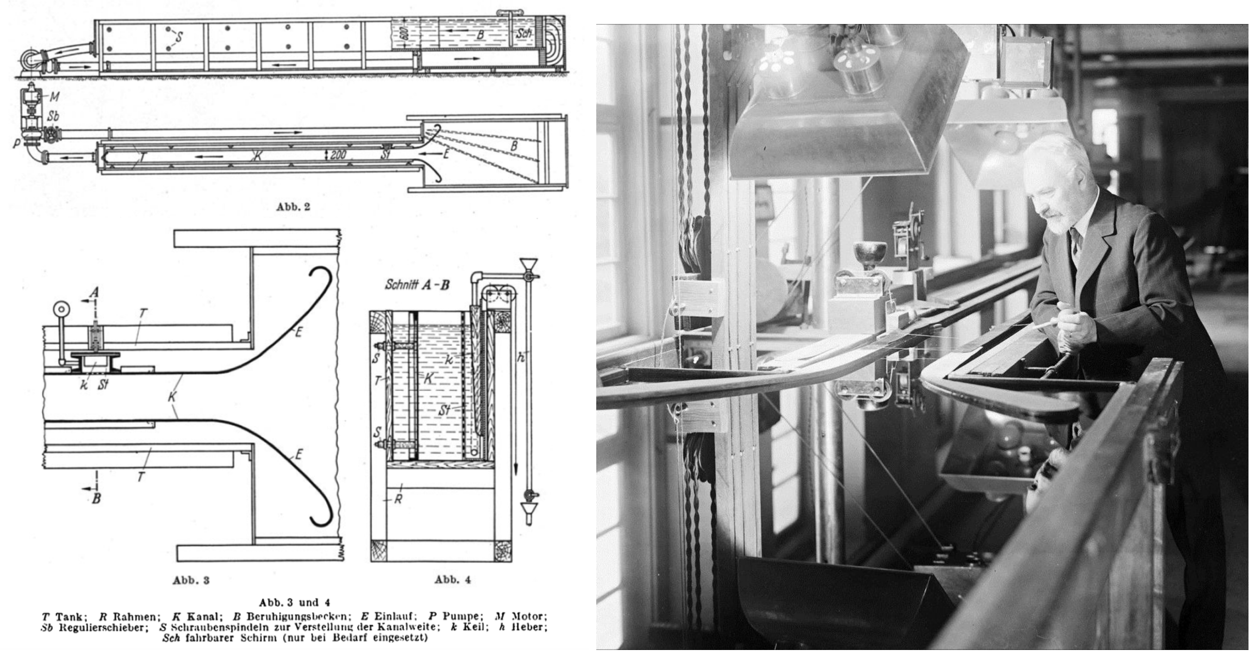

Figure 5: Left: Drawing of water flume with a 200 mm wide and 6 m long test section specifically built in 1931 for the investigation of turbulence at the “Kaiser Wilhelm Institute for Aero- and Hydrodynamics” in Göttingen. Right: photograph of Ludwig Prandtl next to the facility.

cross-correlation approaches.

Within a few years the situation changed: consumer grade video equipment was rapidly displacing film based recording, a process that was to continue in the entire field of photography with a complete transition toward electronic/digital imaging. Now the immediate availability of a video signal provided a clear advantage over photographic film which first required elaborate wet-processing. Over the years equipment became easily available which allowed digital recording and evaluation for PIV, as well as digitization of photographic films. Today sophisticated algorithms and large computer memory space allow data analysis and data presentation as used in this paper.

Acknowledgements

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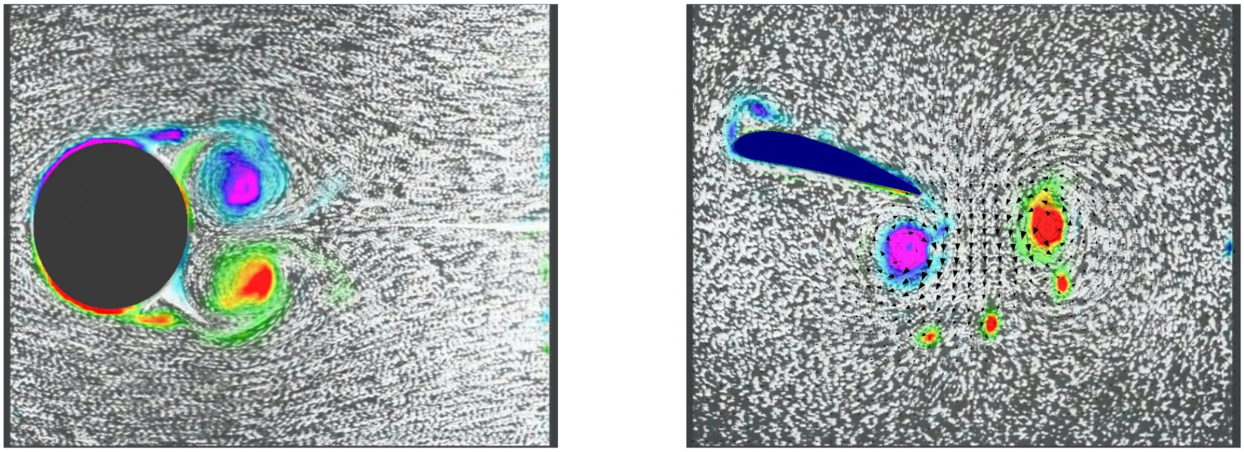

Figure 6: Vorticity fields obtained by processing digitized images (from Willert and Kompenhans, 2010) of Flow Visualization Film C1 originally recorded in facilities similar to those shown in Fig. 4. Animations are available at `https://doi.org/10.3203/IWF/C-1` (C1) and `https://doi.org/10.5446/12719` (PIV evaluation).

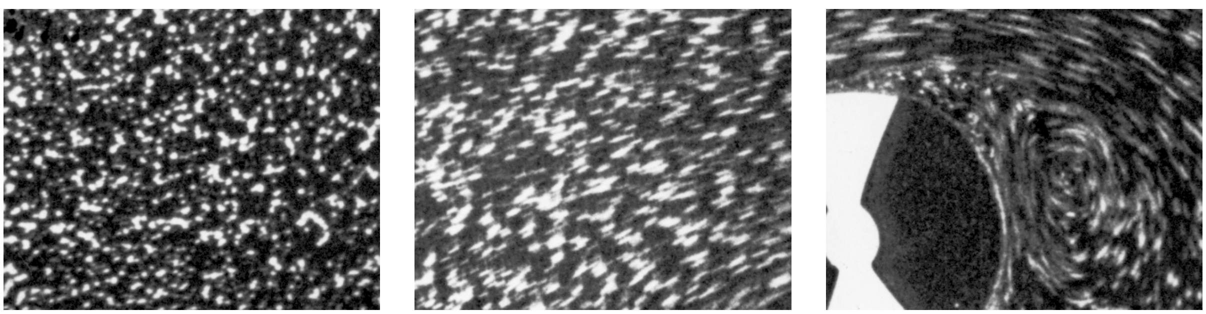

Figure 7: Details of a single frame of Flow Visualization Film C1 from sequence of spinning cylinder: quiescent region below the cylinder (left), streaked particle images due to particle motion upstream of the cylinder (middle), area of recirculating flow immediately downstream of the cylinder (right).

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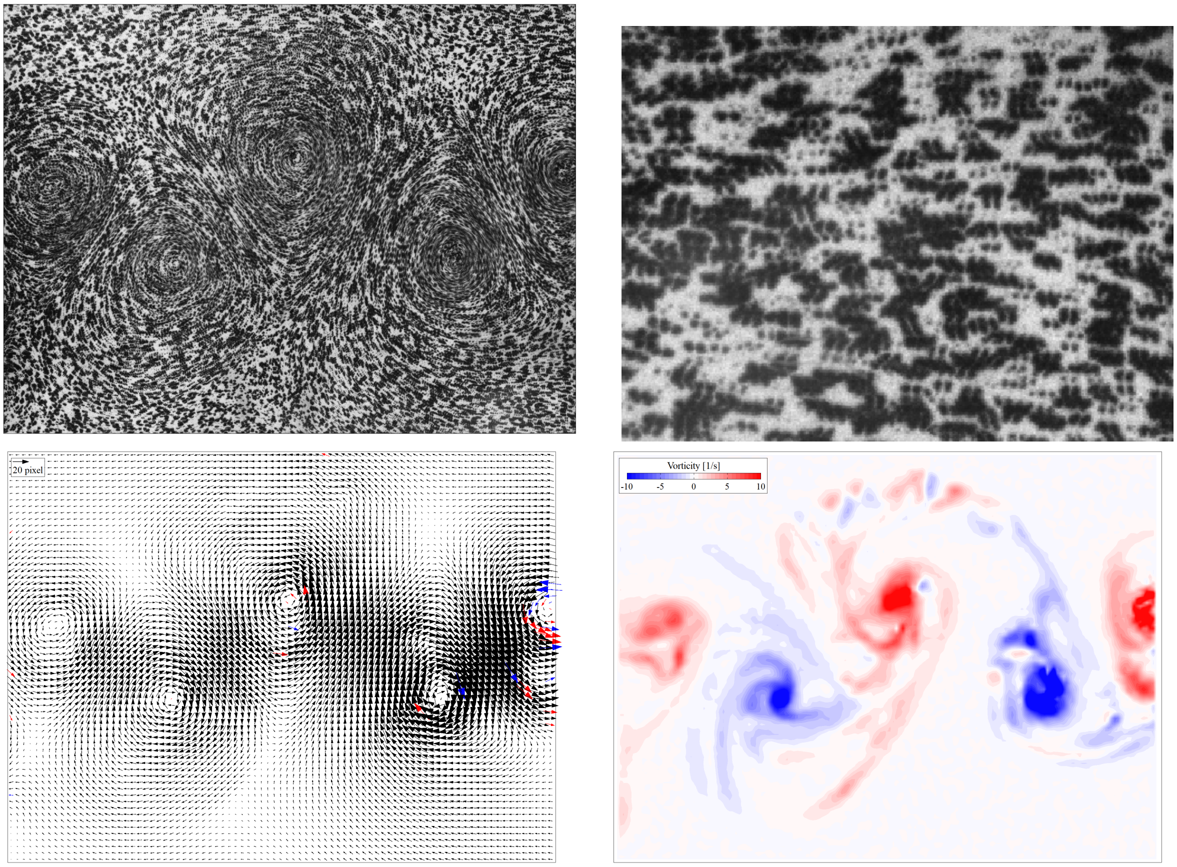

Figure 8: Image pair of sequence of an oscillating cylinder wake: overlaid image pair (top, left), detail of 256×192 pixels from lower right of image pair (top, right), recovered displacement vector field obtain by PIV analysis with only every 4th vector shown (bottom, left), corresponding vorticity map (bottom right), assuming a framing rate of 16 Hz.

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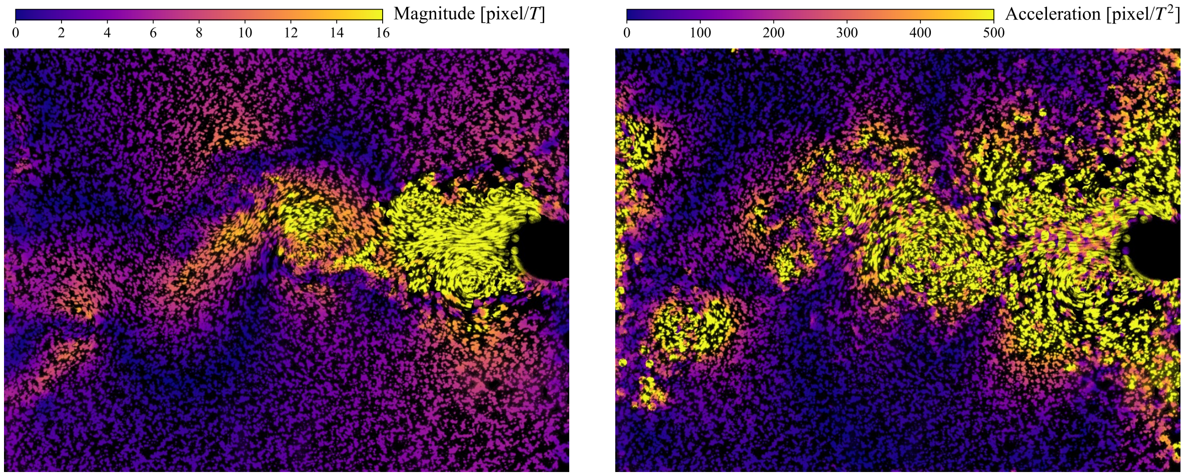

Figure 9: Particle tracking result from sequence of an oscillating cylinder wake color-coded by the displacement magnitude (left) and particle acceleration (right).

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