# Quantifying the Effect of the Optimization of an M-fold Transmitter Diversity Scheme with Atmospherically Induced Beam Wander and Scintillation

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***Abstract*—Transmitter diversity is an effective fading mitigation scheme in an optical geostationary satellite feeder uplink. We quantify the performance of the scheme over a slow fading channel where the atmospheric turbulence causes scintillation and wandering of the optical beam resulting in deep fades at the satellite receiver. For given atmospheric conditions and residual beam pointing jitter, we optimize the transmit power of each beam to obtain the minimum overall power scintillation index and achieve a bit error rate gain of 0.46dB considering intensity modulation and direct detection.**

***Keywords—Free-space optical communications, transmitter diversity, atmospheric turbulence, residual beam pointing jitter, power scintillation index, bit error rate***

## I. Introduction

To meet the growing demand of bandwidth hungry applications and to provide internet services to remote areas on Earth, an optical geostationary orbit (GEO) satellite feeder link is seen as a potential candidate in conjunction with a satellite network [1,2]. Optical signals have numerous advantages over traditional RF links due to the high carrier frequency (and thus, available signal bandwidth and, data carrying capacity), secure communications due to the narrow beam divergence, direct line of sight, less power and weight requirements, and no frequency spectrum regulation constraints [3]. This also serves the purpose to solving the bottleneck of the last mile problem [4]. However, the problem in the deployment of free-space optical (FSO) communication links is the phase perturbation of the optical wave as it propagates through the turbulent atmosphere. The turbulence is caused by the large and small scale cells with varying index of refraction produced by difference in temperature and pressure of the atmospheric layers [5]. The most deteriorating effects even in clear weather condition are the random fluctuations of the signal intensity, called scintillation and wandering of the beam at the satellite receiver which results in residual pointing errors. Both show up as fading which means loss of signal power at the satellite receiver. This is crucial at high data rates where millions of bits are lost when the channel coherence time is in the order of milliseconds.

To achieve a stable signal over a long period of time, an effective and practical fading mitigation scheme for a particular scenario has to be used. In this paper, we consider the application of optical geostationary feeder links (OGEOFL) where the data carrying optical signal propagates towards the GEO satellite in the uplink. Since a GEO satellite is located at around 36000km above earth and turbulence is significant in the first 20km, the size of the speckle with intensity and phase distortions is much larger than the receiver telescope which is considered as a point receiver in comparison to the phase coherence diameter of a single speckle. In this situation the receiver is not affected by phase distortions rather it greatly suffers from intensity fluctuations. In this case, the favorable fading mitigation technique is diversity at the transmitter which is the focus of this work. Multiple transmitters which are spatially separated on ground by more than the coherence length of the atmosphere transmit the same copy of the data through uncorrelated atmospheric paths. The physical separation of the transmitters by more than 100cm provides the sufficient condition for the uncorrelated channels. The optical signals are then combined in one receiver at the satellite where the duration and depth of the fades are mitigated due to favorable joint probability density function (PDF) of combination of all the transmitted beams.

In this work, the weak irradiance due to atmospheric scintillation and the residual beam pointing jitter are modeled by the lognormal and beta PDF, respectively. The statistical analysis of the random irradiance fluctuations and residual beam pointing jitter in FSO links is performed through numerical simulations. The multiple-input-single-output (MISO) scheme is analyzed taking the total scintillation index $\sigma_{\mathrm{tot}}^2$ due to the lognormal irradiance and residual beam pointing jitter, and bit error rate (BER) as performance parameters and considering no spectral overlap between the communication beams. Moreover, the transmit power of each of the individual beams is optimized to minimize the $\sigma_{\mathrm{tot}}^2$ value which is the normalized variance of the received signal at the satellite receiver. After the optimization the BER gain is shown to increase as compared to the unoptimized transmit powers.

The paper is organized as follows: Section II describes the channel model considered for the analysis. Section III presents the methodology for the numerical simulations. Section IV shows the results obtained through the optimization of the transmit powers and discusses the reduction in $\sigma_{tot}^2$ value and BER gain due to the application of the transmitter diversity scheme. Finally, section V concludes the paper.

## II. CHANNEL MODEL

In the weak turbulence regime, the PDF of intensity variation $I_{LN}$ at the satellite receiver is modeled as a lognormal distribution $p(I_{LN})$ which is given as in [6]

$$p(I_{LN}) = \frac{1}{\sqrt{2\pi}\sigma_{lnI} I_{LN}} \exp\left\{-\frac{\left[\ln\left(I_{LN}/I_0\right) + 1/2\,\sigma_{lnI}^2\right]^2}{2\sigma_{lnI}^2}\right\}, \quad I_{LN} \geq 0 \quad (1)$$

where $I_0$ is the signal light intensity without turbulence and $\sigma_{lnI}^2$ is the log-irradiance variance. The $\sigma_{lnI}^2$ is related to the atmospheric scintillation index $\sigma_I^2$ due to weak turbulence as given in [5]

$$\sigma_{lnI}^2 = \ln(\sigma_I^2 + 1)$$

The PDF of the received intensity $I_{PE}$ in the presence of only residual beam pointing jitter is modeled as a beta distribution $p(I_{PE})$ which is given as in [6]

$$p(I_{PE}) = \beta I_{PE}^{\beta-1}, \quad 0 \leq I_{PE} \leq 1 \quad (2)$$

where $\beta = \omega_0^2 / 4\sigma_j^2$, $\omega_0$ is the $1/e^2$ beam divergence half-angle, $\sigma_j$ is the rms value of the residual beam pointing jitter error due to the combined effect of the atmospherically induced beam wander, tracking errors, and mechanical vibrations of the transmitter and receiver terminals.

The combined effect of weak atmospheric turbulence and residual beam pointing jitter in the atmospheric channel for a single-input-single-output (SISO) scheme is modeled as the PDF given in [6]. This joint PDF is given as

$$p(I) = \frac{\beta}{2I_0} \operatorname{erfc}\left\{\frac{\ln(I/I_0) + \sigma_{lnI}^2\left(\beta + \frac{1}{2}\right)}{\sqrt{2}\sigma_{lnI}}\right\} \cdot \exp\left(\frac{\sigma_{lnI}^2}{2}\beta(\beta+1)\right)\left(\frac{I}{I_0}\right)^{\beta-1} \quad (3)$$

## III. NUMERICAL SIMULATIONS

The normalized random intensity due to weak turbulence is $I_{LN} = \exp(\mu + \sigma_{lnI} X_{LN})$, where $\mu = -\frac{\sigma_{lnI}^2}{2}$ and $X_{LN}$ is a normally distributed random numbers. The normalized intensity due to pointing errors assuming circularly symmetric normally distributed jitter is $I_{PE} = \exp\left(-2\frac{\sigma_j^2(X_{PE}^2 + Y_{PE}^2)}{\omega_0^2}\right)$, where $X_{PE}$ and $Y_{PE}$ are zero mean Gaussian random variables in *x* and *y* axis of the circular aperture, respectively. The normalized received irradiance $I_r$ of the Gaussian beam after the optical wave has been propagated through the turbulent atmosphere in a FSO link can be expressed as

$$I_r = P_t \cdot \exp\left(-2\frac{\sigma_j^2(X_{PE}^2 + Y_{PE}^2)}{\omega_0^2}\right)\exp(\mu + \sigma_{lnI} X_{LN}) \quad (4)$$

where $P_t$ is the transmit power of the beam from the OGS. To find the optimized transmit power of each beam, numerical iterations of the individual transmit power of each of the beam is performed to obtain the overall minimum $\sigma_{tot}^2$ of the combination of all the beams. As an example, four uplink beams are simulated in this analysis.

## IV. RESULTS AND DISCUSSION

The value of $\sigma_I^2$ due to lognormal fading distribution is taken as 0.065 for all the atmospheric paths as it was measured in the experiment for the uplink signal between optical ground station (OGS) in Tenerife and OPALE terminal onboard the GEO satellite ARTEMIS [7] and $\omega_0$ for all beams is fixed to 6.4μrad as in [7]. For the analysis, three cases are presented in Table I.

In case I, the transmitted power of each beam is equally reduced by the number of transmitted beams *M* and equal residual beam pointing jitter value of 2.17μrad is assumed. This case is taken as a benchmark. The sum of powers of the transmitted beams in all cases is same as the individual beam without transmitter diversity for a fair comparison.

In case II, the residual beam pointing jitter values are varied keeping the transmit powers of all the beams the same as in case I to find the combined effect on $\sigma_{tot}^2$. This is the case of the unoptimized transmit powers.

In case III, transmit powers are optimized as described in section III to minimize the $\sigma_{tot}^2$ value when all the beams have propagated through unequal atmospheric channels. The effect of the power and residual beam pointing jitter values on the selected performance parameters is presented next:

### A. *Total Scintillation Index, $\sigma_{tot}^2$*

In the first case, $\sigma_{tot}^2$ of a multi-beam system reduces by a factor equal to the number of transmitters because of the combination of equal irradiance from all beams at the receiver. The $\sigma_{tot}^2$ value for a MISO system changes due to the change in the irradiance statistics when multiple incoherent beams are combined. As shown in case III, the overall scintillation index can be minimized by selecting optimum transmitted power according to the residual beam pointing jitter associated with each beam.

TABLE I. Comparison between $\sigma_{tot}^2$ for SISO and MISO system

| Beam # | $P_t$ (%) | $\sigma_j$ (μrad) | $\beta$ | $\sigma_{tot}^2$ SISO | $\sigma_{tot}^2$ MISO |
|---|---|---|---|---|---|
| **Case I** | | | | | |
| **1** | 25 | 2.17 | 2.17 | 0.1823 | 0.0455 |
| **2** | 25 | 2.17 | 2.17 | 0.1821 | |
| **3** | 25 | 2.17 | 2.17 | 0.1821 | |
| **4** | 25 | 2.17 | 2.17 | 0.1823 | |
| **Case II** | | | | | |
| **1** | 25 | 2.17 | 2.17 | 0.1822 | 0.0887 |
| **2** | 25 | 3.01 | 1.13 | 0.3655 | |

| *Beam* # | $P_t$ ( %) | $\sigma_j$ ($\mu$rad) | $\beta$ | $\sigma_{tot}{}^2$ SISO | $\sigma_{tot}{}^2$ MISO |
|---|---|---|---|---|---|
| **3** | 25 | 2.87 | 1.24 | 0.3293 | |
| **4** | 25 | 4.57 | 0.49 | 0.9369 | |
| **Case III** | | | | | |
| **1** | 36 | 2.17 | 2.17 | 0.1822 | |
| **2** | 24 | 3.01 | 1.13 | 0.3655 | 0.0812 |
| **3** | 26 | 2.87 | 1.24 | 0.3293 | |
| **4** | 14 | 4.57 | 0.49 | 0.9369 | |

### *B. BER*

The BER is calculated using a receiver model and a commercially available 10Gb/s avalanche photodiode from [8]. The BER for the SISO and MISO systems in the presence of lognormal scintillation and residual beam pointing jitter is obtained using the fading statistics given in (4). The BER curves for the three cases from Table I are plotted in Fig. 1-3.

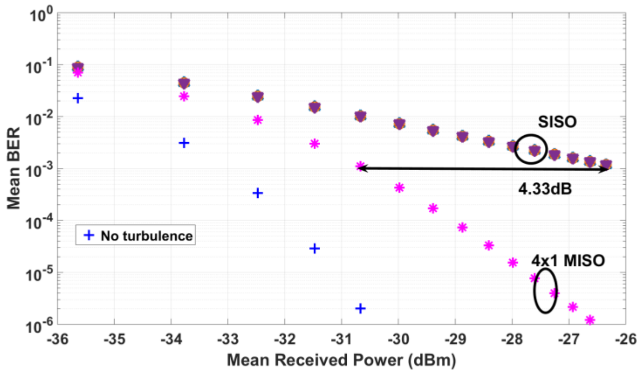


Fig. 1: Case I – Equal transmit powers and equal β values

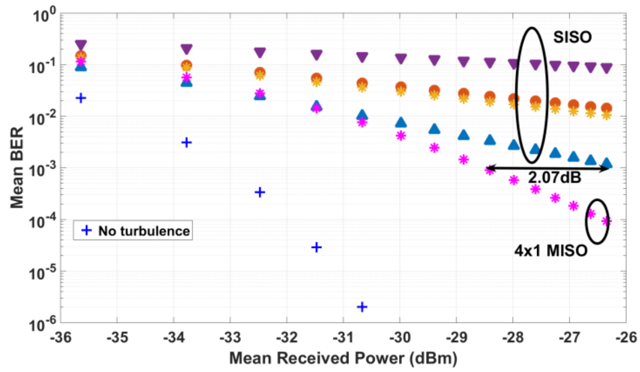


Fig. 2: Case II – Unoptimized transmit powers and unequal β values

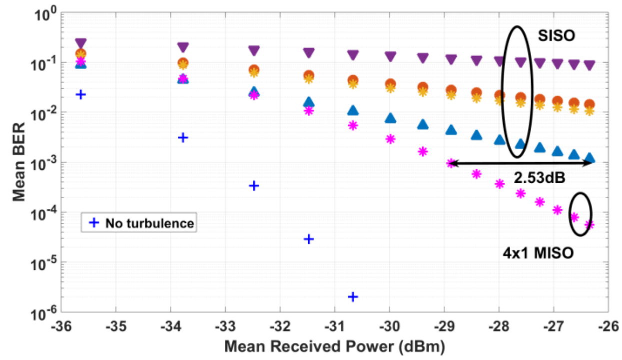


Fig. 3: Case III- Optimized transmit powers and unequal β values

We see an improvement by 0.46dB at the BER value of 1E-3 when the transmit powers of the individual beams are optimized as compared to the unoptimized transmit powers.

## V. CONCLUSION

In this paper, we have shown the benefit of transmit diversity in reducing the overall $\sigma_{tot}{}^2$ and increasing the BER gain as compared to the SISO system. Moreover, optimization of the transmit power per beam according to the channel conditions is shown to maximize the performance of the FSO communications link. The total $\sigma_{tot}{}^2$ and BER gain for the optimized transmit power can be improved further as the number of beams are increased. The presented analysis is helpful for the free-space optical communications designers to quantify the performance of MISO schemes for fading mitigation in the atmospheric turbulent channels with varying residual beam pointing jitter at the satellite.

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