Verification and Validation of a 2D energy based peridynamic state-based failure criterion

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Motivation – State-of-the-art design process in aeronautics

Design criteria

Fatigue
Stability

*Damage tolerance*
Plain and bearing strength

\[ [45 \, -45 \, 0 \, 90]_s \]
\[ [0 \, 60 \, -60 \, 0]_s \]
Motivation - Summary

• Micromechanical or damage models are not directly used in the design process
• These models can be used to verify simplified criteria
• Robustness of damaged structures can be evaluated
• Reduction of cost-intensive experiments

• A better understanding of damage initiation can be used to improve criteria and avoid expensive experiments.
Physically motivated material modeling

- If the conservation equations are fulfilled + if the material behaviour is described, it is a physically motivated modeling
Peridynamics

1. The medium is continuous
2. Internal forces are contact forces (interaction only with the neighbourhood)
3. Deformations are twofold continuously derivable (in the weak formulation only simple)
4. The conservation equations are fulfilled

\[ \text{div}(\sigma) + b = \rho \ddot{u} \]

\[ \int_H (T(x, t)(q - x) - T(q, t)(x - q))dV + b = \rho \ddot{u} \]

\[ \lim_{H \to 0} \int_H (T(x, t)(q - x) - T(q, t)(x - q))dV = \text{div}(\sigma) \]
Peridynamics – ordinary state based formulation

\[ \rho \left( x \right) \ddot{u} \left( x, t \right) = \int_{\mathcal{H}} \left( T \left[ x, t \right] \left( x' - x \right) - T \left[ x', t \right] \left( x - x' \right) \right) \, dV + b \left( x, t \right) \]
Peridynamics – 2D - ordinary state based formulation

\[ t\langle \xi, t \rangle = \frac{\omega \langle \xi \rangle}{m_V} \left[ 3K \theta x + 15G \varepsilon^d \right] \]

\[ t_{\text{planestress}}\langle \xi, t \rangle = \frac{\omega \langle \xi \rangle}{m_V} \left[ \frac{4KG}{3K + 4G} \theta x + 8G \varepsilon^d \right] \]

\[ t_{\text{planestrain}}\langle \xi, t \rangle = \frac{\omega \langle \xi \rangle}{m_V} \left[ \frac{4(3K - G)}{9} \theta x + 8G \varepsilon^d \right] \]

\[ T = t \frac{\overrightarrow{Y}}{|\overrightarrow{Y}|} \]
\[ G = 2 \int_{0}^{\delta} \int_{\frac{\delta}{2\pi}}^{{\delta}} \int_{\frac{\delta \cos^{-1} \left( \frac{z}{\xi} \right)}{\delta}}^{2\pi} w_C dV_{\xi} \]

\[ = \frac{4G}{\pi \delta^4} \]

\[ w_{c3d} = \frac{4G}{\pi \delta^4} \]

\[ w_{c2d} = \frac{3G}{2 \delta^3 h} \]
Verification

<table>
<thead>
<tr>
<th>Geometry</th>
<th>a</th>
<th>h</th>
<th>L</th>
<th>B</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0.005m</td>
<td>0.02m</td>
<td>0.05m</td>
<td>0.003m</td>
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</tbody>
</table>

<table>
<thead>
<tr>
<th>Material</th>
<th>Bulk Modulus</th>
<th>Shear Modulus</th>
<th>Density</th>
<th>G₀</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1.75E+09 Nm⁻²</td>
<td>8.08E+08 Nm⁻²</td>
<td>2000 kgm⁻³</td>
<td>12 Nm⁻¹</td>
</tr>
</tbody>
</table>
Validation - Experiment – ISO 13586:2000(E)
Range of geometrical data and energy release rate

- Energy Release Rate [N/m]:
  - Minimum: 0.243 N/m
  - Maximum: 0.311 N/m

- Pre Crack [mm]:
  - Minimum: 16.72 mm
  - Maximum: 21.56 mm

- Thickness [mm]:
  - Minimum: 5.13 mm
  - Maximum: 5.48 mm
Crack propagation
Crack front
Diagram showing the relationship between force [N] and displacement [mm]. The graph includes lines for Min. Value, Probe_4, Probe_7, and Probe_4 20% rE.
Prognosis
Crack initiates by 0.85 mm
Conclusion

- 2D energy criterion has been verified
- Validation has started and the order of magnitude is reachable for KIC test
- Prognosis failed, because of missing plasticity
  - Overestimation within the KIC is maybe explainable
Acknowledgement

We would like to thank Wibke Exner (DLR - wibke.exner@dlr.de) for providing the experimental data.
Thank you!

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All presented models and source code can be found here
Rädel, M. & Willberg, C. PeriDoX Repository
https://github.com/PeriDoX/PeriDoX
Doi: 10.5281/zenodo.1403015

Knowledge for Tomorrow