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An energy based peridynamic state-based failure criterion

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The paper presents a verification and convergence study of an enhanced energy based failure model based on Foster et al. [2]. The failure model has been implemented in the open source software Peridigm. The study is performed with a virtual double cantilever beam test. To verify the implementation an energy release rate is virtually measured and compared with the input data. Time and load of the crack initiation are used as the convergence criteria.

1 Introduction

Motivated by ideas of molecular dynamics and to overcome the deficits of fracture mechanics, Stewart Silling developed the fundamental Peridynamics theory in the early 2000's as an alternative approach to the classical continuum mechanical modeling. In this theory the fundamental partial differential equations of the momentum conservation is replaced by an integral equation. Singularities at discontinuities are avoided. Within the neighborhood $\mathcal{H}$ with the volume V_q, defined by a spherical domain if the horizon δ, the force volume density state $\underline{\mathbf{T}}$, the external force $\mathbf{b}$, the mass density ρ and the acceleration $\ddot{\mathbf{u}}$ we get for the interaction bond interaction between the positions $\mathbf{x}$ and $\mathbf{x}'$ the integral balance of momentum.

$$\int_{\mathcal{H}} (\underline{\mathbf{T}}(\mathbf{x},t)\langle\mathbf{x}'-\mathbf{x}\rangle - \underline{\mathbf{T}}(\mathbf{x}',t)\langle\mathbf{x}-\mathbf{x}'\rangle)dV_{\mathbf{q}} + \mathbf{b} = \rho\ddot{\mathbf{u}} \tag{1}$$

To model damages within a material, specific criteria for the initiation are needed, which utilize measurable parameters. In many publications a criterion is used which evaluates the relative change in distance between two points in the neighborhood. If this relative change of distance or stretch exceeds a certain value an irreversible crack occurs and there are no longer interactions between these two points. This damage model is called critical stretch model [1]. The critical stretch is not a purely physically-based parameter and can not be measured directly. Therefore, the value is recalculated by measuring the energy release rate of the material.

2 Theory

Foster et al. [2] described an energy-based failure criterion which is valid for state-based peridynamic analysis. The assumption is that each bond is capable of a maximum elastic potential. Foster determined the critical bond potential value based on the energy release rate G_0 and the horizon δ shown in equation 2 (b). If the critical bond energy potential w_c is exceeded the bond breaks and a damage occur. This criterion has been implemented in the open source peridynamic code Peridigm [6]. In Rädel et al. [3] it has been shown that the convergence of the damage initiation could not be reached for the critical stretch damage model. Therefore, in this publication the convergence of the presented damage model is tested. A simple virtual double cantilever beam experiment is used to study the convergence of the crack initiation and propagation. To determine the bond energy based on the bond extension scalar state $\underline{e}$ in the ordinary state-state based peridynamics we can use the following equation

$$w_{bond} = 0.25\underline{\omega}\left(\underline{t}[\mathbf{x},t] - \underline{t}[\mathbf{x}',t]\right]\underline{e} \qquad \text{(a)} \qquad w_c = \frac{4G_0}{\pi\delta^4} \qquad \text{(b)} \tag{2}$$

Following Silling et al. [5] the force density scalar state $\underline{t}$ can be determined as

$$\underline{t}[\mathbf{q},t] = \chi(\underline{e},t)\omega\left(\frac{3K[\mathbf{q},t]\theta[\mathbf{q},t]}{m_v[\mathbf{q},t]}\underline{x} + \frac{15G[\mathbf{q},t]\theta[\mathbf{q},t]}{m_v[\mathbf{q},t]}\underline{e}^d[\mathbf{q},t]\right) \tag{3}$$

with

$$\theta[\mathbf{q},t] = \frac{3}{m_v[\mathbf{q},t]}\int_{\mathcal{H}(\mathbf{q})} \underline{\omega x e} dV \quad \text{and} \quad \underline{e}^d[\mathbf{q},t] = \underline{e} - \frac{\theta[\mathbf{q},t]\underline{x}}{3} \tag{4}$$

By replacing $\mathbf{q}$ with $\mathbf{x}$ and $\mathbf{x}'$ the force density scalar state for the corresponding opposite point can be obtained. Therein, K is the compression modulus, G the shear modulus, $\underline{e}^d$ the deviatoric part of the bond extension scalar state, m_v the weighted volume, θ the dilatation, t the time and $\underline{x}$ the undeformed scalar state.

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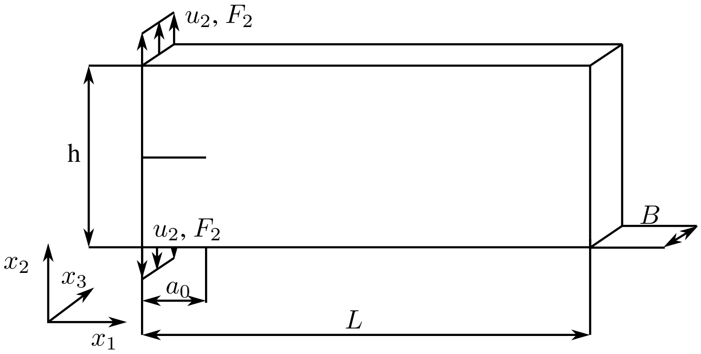
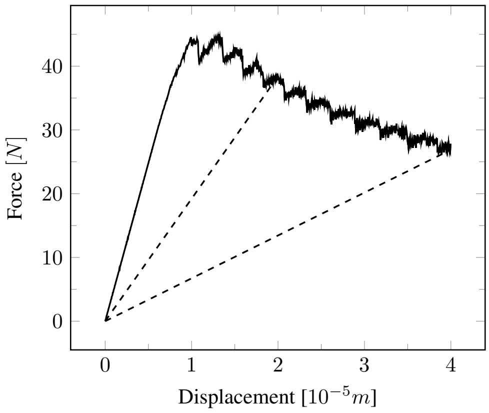
3 Results

To determine the energy release rate the area between the force displacement curve and an arbitrary linear function has to be calculated, cf. figure 1b. The area corresponds to the dissipated energy. To get the energy release rate, this value has to be divided by the crack surface $l_{crack} \cdot B$. The results for several horizons for a fixed discretization are given in table 2a. Inaccuracies in the crack length determination lead to the differences to the reference value of $G_0 = 12N/m$. Figure 2a shows



(a) Double cantilever beam ($a = 0.005m$, $h = 0.02$, $L = 0.05m$, $B = 0.003$, $K = 1.75 \cdot 10^9 Nm^{-2}$, $G = 8.08 \cdot 10^9 Nm^{-2}$, $\rho = 2000kgm^{-3}$ and $G_0 = 12Nm^{-1}$.



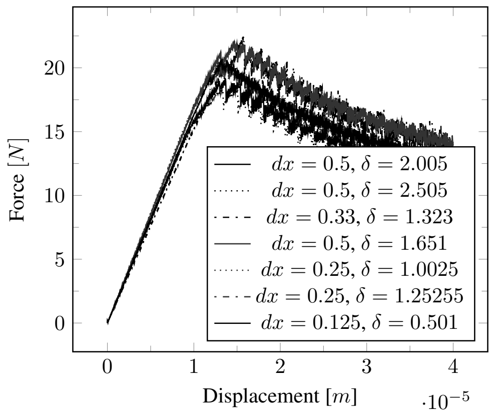
(b) Force-displacement curve for $\delta = 0.004m$ and average point distance $dx = 0.001$.

Fig. 1: Double cantilever model and results of the virtual experiment.

the result of the convergence analysis. Convergence for the double cantilever beam model is reached for horizons $4 - 5dx$, with a structured point mesh discretization of $L/dx = 200$. All models shown in the presentation and here in the publication as well as the used source code can be found here [4].

δ $[m]$	l_{crack} $[m]$	G_0 $[N/m]$
$2.015e-3$	0.003	12.8
$3.015e-3$	0.005	13.1
$4.015e-3$	0.004	11.1
$5.015e-3$	0.006	11.2

(a) Results of the verification.



(b) Force-Displacement Curves for several discretizations and horizons (dx and δ in mm).

Fig. 2: Results.

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