An energy based peridynamic state-based failure criterion

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Outline

1. Motivation

2. Peridynamics

3. Damage model

4. Convergence

5. Comparison – Critical Stretch

6. Example

7. Conclusion
Motivation

- Challenges:
  - Exploitation of fiber reinforced plastics (FRP) lightweight potential limited
  - Missing reliability of failure predictions

- Goals:
  - Increase understanding of failure mechanisms
  - Reduce number of experiments
  - Derive improved failure criteria for design process of structures
Motivation - Continuum mechanics vs. Peridynamic approach

1. The medium is continuous (a continuous mass density field exists)
2. Internal forces are contact forces (material points interact only if they are separated by zero distance)
3. The deformation is twice continuously differentiable (this assumption is relaxed)
4. The conservation laws of mechanics apply (conservation of mass, linear momentum, and angular momentum)\(^1\)

\[
\operatorname{div}(\sigma) + b = \rho \ddot{u}
\]

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\[
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\]

\[
\lim_{H \to 0} \int_H \left( \mathbf{T}(\mathbf{x}, t)\mathbf{(x' - x)} - \mathbf{T}(\mathbf{x'}, t)\mathbf{(x - x')} \right) dV + \mathbf{b} = \rho \ddot{\mathbf{u}}
\]

\[
\lim_{H \to 0} \int_H \left( \mathbf{T} (\mathbf{x}, t)\mathbf{(x' - x)} - \mathbf{T}(\mathbf{x'}, t)\mathbf{(x - x')} \right) dV = \text{div}(\mathbf{\sigma})
\]

Peridynamics
Peridynamics – ordinary state based

\[
\begin{align*}
\mathbf{T}[\mathbf{x}', t](\mathbf{x} - \mathbf{x}') = \\
= \int_{\mathcal{H}} \left( \mathbf{T}[\mathbf{x}, t](\mathbf{x}' - \mathbf{x}) - \mathbf{T}[\mathbf{x}', t](\mathbf{x} - \mathbf{x}') \right) dV + \mathbf{b}(\mathbf{x}, t)
\end{align*}
\]
Peridynamics – ordinary state based

\[ W_{CM} = \frac{1}{2} K \left[ \epsilon_{kk} \right]^2 \delta_{ij} + 2G \left[ \epsilon_{ij}^d \right]^2 = W_{PD} \]

\[ Y\langle \xi \rangle = F\xi = F\langle x' - x \rangle \quad \forall \xi \in \mathcal{H} \]

- For small deformations and isotropic material

\[ x = |X\langle \xi \rangle| \quad \quad y = |Y\langle \xi \rangle| \quad \quad e\langle \xi \rangle = y - x \]

\[ e\langle \xi \rangle = |F\xi| - |\xi| = \epsilon_{ij} \xi_i \frac{\xi_j}{|\xi|} \]

\[ e^d\langle \xi \rangle = \epsilon_{ij}^d \xi_i \frac{\xi_j}{|\xi|} \quad \quad e^i\langle \xi \rangle = \epsilon_{ii} \xi_i \frac{\xi_i}{|\xi|} \]

\[ W_{PD} = \frac{A}{2} \int_{\mathcal{H}} \omega\langle \xi \rangle \left[ \epsilon_{ij}^d \xi_i \frac{\xi_j}{|\xi|} \right]^2 dV_\xi + \frac{B}{2} \int_{\mathcal{H}} \omega\langle \xi \rangle \left[ \epsilon_{ii} \xi_i \frac{\xi_i}{|\xi|} \right]^2 dV_\xi \]
Peridynamics – ordinary state based

\[
A = \frac{3K}{m_V} \quad \text{and} \quad B = \frac{15G}{m_V}
\]

\[
m_V = \int_{\mathcal{H}(x)} \omega \langle \xi \rangle xx \, dV_{\xi}
\]

\[
\theta = \frac{3}{m_V} \int_{\mathcal{H}(x)} \omega \langle \xi \rangle xe \langle \xi \rangle \, dV_{\xi}
\]

\[
t \langle \xi, t \rangle = \frac{\omega \langle \xi \rangle}{m_V} \left[ 3K \theta x + 15Ge^d \right]
\]

\[
T = t \frac{Y}{|Y|}
\]
Damage model

- Could be included via the influence function
- For programming reasons the history dependent scalar value representing the damage function is split from the influence function

\[ \chi(\xi, t) = \begin{cases} 
1 & \text{no failure} \\
0 & \text{failure} 
\end{cases} \]

- Critical stretch model

\[ s_C = \sqrt{\frac{G_0 C}{3G + \left(\frac{3}{4}\right)^4 (K - \frac{5G}{3})}} \delta \]

- Critical energy model by Foster et al.

\[ W_C = \frac{4G_0 C}{\pi \delta^4} \]
Damage model

\[ W_{\text{bond}} = 0.25 \chi(\epsilon(\xi), t) \left\{ t[x, t] - t[x', t] \right\} \epsilon < W_C \]

\[
t[x, t] = \chi(\epsilon(\xi), t) \left( \frac{3K[x, t] \theta[x, t]}{m_V[x, t]} \omega x + \frac{15G[x, t]}{m_V[x, t]} \omega e^d[x, t] \right) \]

\[
t[x', t] = \chi(\epsilon(\xi), t) \left( \frac{3K[x', t] \theta[x', t]}{m_V[x', t]} \omega x + \frac{15G[x', t]}{m_V[x', t]} \omega e^d[x', t] \right) \]

\[
\theta[x, t] = \frac{3}{m_V[x, t]} \int_{\mathcal{H}(x)} \omega x e \, dV_{\xi} \quad e^d[x, t] = e - \frac{\theta[x, t]}{3} x
\]

\[
\theta[x', t] = \frac{3}{m_V[x', t]} \int_{\mathcal{H}(x')} \omega x e \, dV_{\xi} \quad e^d[x', t] = e - \frac{\theta[x', t]}{3} x
\]
Convergence

Discrete Nonlocal $u^h_\delta \xrightarrow{\delta \to 0} u^h_0$

Continuum Nonlocal $u^0_\delta \xrightarrow{\delta \to 0} u^0_0$

Discrete Local $u^h_0$

Continuum Local PDE $u^0_0$
Convergence

Geometry

<table>
<thead>
<tr>
<th>Geometry</th>
<th>a</th>
<th>h</th>
<th>L</th>
<th>B</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0.005m</td>
<td>0.02m</td>
<td>0.05m</td>
<td>0.006m</td>
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</table>

Material

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<tr>
<th>Material</th>
<th>Bulk Modulus</th>
<th>Shear Modulus</th>
<th>Density</th>
<th>$G_0$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1.75E+09 Nm$^{-2}$</td>
<td>8.08E+08 Nm$^{-2}$</td>
<td>2000 kgm$^{-3}$</td>
<td>12 Nm$^{-1}$</td>
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</tbody>
</table>

Mesh

<table>
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<tr>
<th>Mesh</th>
<th>2.01dx</th>
<th>3.01dx</th>
<th>4.01dx</th>
<th>5.01dx</th>
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<td>0.000251</td>
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<td>0.000501</td>
<td></td>
</tr>
</tbody>
</table>
Convergence
Convergence

\[
\begin{array}{c|cc}
\delta & G_0 & G_0 \\
\hline
2.015 \cdot 10^{-3} & 12.8 & 11.4 \\
3.015 \cdot 10^{-3} & 13.1 & 12.9 \\
4.015 \cdot 10^{-3} & 11.1 & 11.3 \\
5.015 \cdot 10^{-3} & 11.2 & 11.9 \\
\end{array}
\]
Convergence - Results

![Graph showing convergence results with displacement and force values.](chart16.png)
Convergence - Results

![Graph showing convergence results](image)

- Black line: \(dx=0.33\text{mm}; h=0.663\text{mm}\)
- Red line: \(dx=0.33\text{mm}; h=0.993\text{mm}\)
- Dashed line: \(dx=0.33\text{mm}; h=1.323\text{mm}\)
- Dotted line: \(dx=0.33\text{mm}; h=1.1653\text{mm}\)

Displacement [m] vs. summmedForce [N]
Convergence - Results

![Graph showing convergence results with various displacement values for different step sizes and heights.]
Convergence - Results
Convergence - Results

The graph shows the convergence results of a numerical simulation. The x-axis represents the displacement in meters, ranging from 0.0E+00 to 4.0E-05, and the y-axis represents the summed force in Newtons, ranging from 0 to 25.

The graph includes four lines, each representing different discretization steps:
- **dx=0.25mm; h=1.0025mm**
- **dx=0.25mm; h=1.2525mm**
- **dx=0.125mm; h=0.501mm**

The lines show how the summed force changes with displacement, indicating the convergence of the simulation as the discretization step size decreases.
Comparison – Critical Stretch

**Critical Stretch**

\[
s_c = 0.000433593 \\
K = 1.75 \times 10^9 \text{ N/m}^2 \\
G = 8.08 \times 10^8 \text{ N/m}^2 \\
\delta = 0.002505 \text{ m} \\
G_0 = 12 \text{ N/m}
\]

**Critical Energy**

\[G_0 = 0.75-0.84 \text{ N/m}\]
Comparison – Critical Stretch

- Critical Stretch $sc = 0.000433593$
- $G0 = 0.75 \text{ N/m}$
- $G0 = 0.81 \text{ N/m}$
- $G0 = 0.84 \text{ N/m}$

summedForce [N]

Displacement [m]
Example – RVE
Conclusion

- The energy criterion from Foster et al. has been implemented and tested due to its convergence.
- The criterion is able to represent the energy release rate.
- $2dx$ meshes of any discretization lead to overestimation of the crack initiation load.
- $4-5dx$ shows the best results + converge; $<2\%$ difference in results.
- Difference between the standard method (critical stretch) and critical energy has been shown.
- Use case has been shown for complex fiber matrix model.

All presented models (end of March) and source code can be found here.
Rädel, R. & Willberg, C. PeriDoX Repository
https://github.com/PeriDoX/PeriDoX
References


Thank you!

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