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PolInSAR Ground and Volume Response Separation

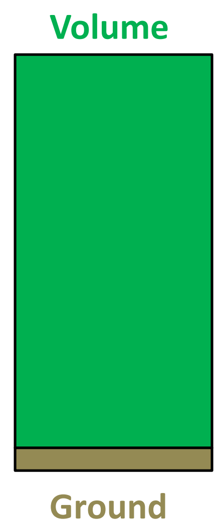
Alberto Alonso-Gonzalez, Kostas Papathanassiou

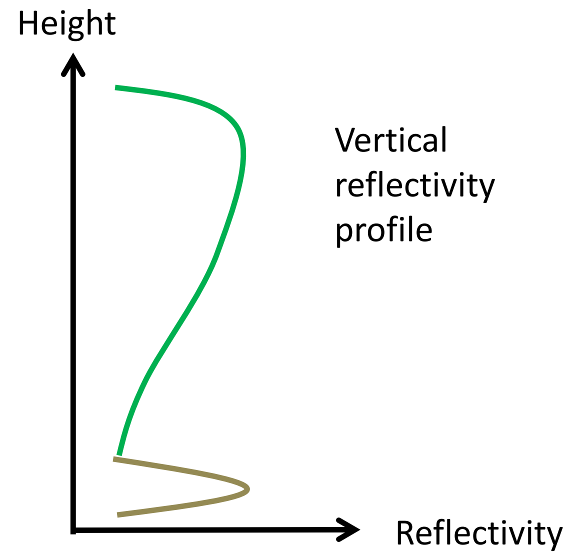
German Aerospace Center, DLR

PolInSAR Two Layer Model

- Polarimetric SAR Interferometry (PolInSAR) is sensitive to the vertical structure of vegetation

  - Vegetation scattering might be modelled as a superposition of two layers: **Ground** and **Volume**
    - Some assumptions are made:
      - Ground and volume have different vertical profile
      - Ground and volume have different polarimetric behaviour





- The **goal** is to use PolInSAR to **separate the response** of **Ground** and **Volume** components

PolInSAR Two Layer Model – Formulation

- The MB PolInSAR matrix may be defined as:

$$\mathbf{T} = \langle \mathbf{k}\mathbf{k}^{\mathrm{H}} \rangle = \begin{bmatrix} \mathbf{T}_{11} & \mathbf{\Omega}_{12} & \cdots & \mathbf{\Omega}_{1N} \\ \mathbf{\Omega}_{12}^{H} & \mathbf{T}_{22} & & \mathbf{\Omega}_{2N} \\ \vdots & & \ddots & \vdots \\ \mathbf{\Omega}_{1N}^{H} & \mathbf{\Omega}_{2N}^{H} & \cdots & \mathbf{T}_{NN} \end{bmatrix}$$

$$\mathbf{k} = [\mathbf{k}_1^T \mathbf{k}_2^T \ldots \mathbf{k}_N^T]^T$$

$$\mathbf{k}_i = \frac{1}{\sqrt{2}} \left[S_{hh}^i + S_{vv}^i, S_{hh}^i - S_{vv}^i, S_{hv}^i + S_{vh}^i \right]^T$$

- With the two layer model the coherences may be interpreted as a function of the volume and ground contributions:

$$\gamma_{ij}(\mathbf{w}) = \frac{\mathbf{w}^H \mathbf{\Omega}_{ij} \mathbf{w}}{\sqrt{\mathbf{w}^H \mathbf{T}_{ii} \mathbf{w} \cdot \mathbf{w}^H \mathbf{T}_{jj} \mathbf{w}}}$$

$$\gamma_{ij}^{l} = \frac{\int F^l(z) e^{j k_{z_{ij}} z} dz}{\int F^l(z)\, dz} \quad \text{for } l \in (g, v)$$

$$\mu(\mathbf{w}) = \frac{\mathbf{w}^H \mathbf{T}_g \mathbf{w}}{\mathbf{w}^H \mathbf{T}_v \mathbf{w}}$$

- This formulation separates polarimetric and interferometric components
    - This also encapsulates two additional assumptions:
        - Vertical profiles of ground and volume are independent of polarization
        - Ground and Volume components are constant over baselines

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PolInSAR Two Layer Model – Formulation

- With this formulation the coherency and PolInSAR matrices may be defined as

$$\mathbf{T} = \langle \mathbf{k}\mathbf{k}^{\mathrm{H}} \rangle = \begin{bmatrix} \mathbf{T}_{11} & \boldsymbol{\Omega}_{12} & \cdots & \boldsymbol{\Omega}_{1N} \\ \boldsymbol{\Omega}_{12}^{H} & \mathbf{T}_{22} & & \boldsymbol{\Omega}_{2N} \\ \vdots & & \ddots & \vdots \\ \boldsymbol{\Omega}_{1N}^{H} & \boldsymbol{\Omega}_{2N}^{H} & \cdots & \mathbf{T}_{NN} \end{bmatrix}$$

$$\mathbf{T}_{ii} = \mathbf{T}_g + \mathbf{T}_v$$

$$\boldsymbol{\Omega}_{ij} = \gamma_{ij}^{g}\mathbf{T}_g + \gamma_{ij}^{v}\mathbf{T}_v$$

- … or equivalently, in the multibaseline case as a sum of Kronecker products:

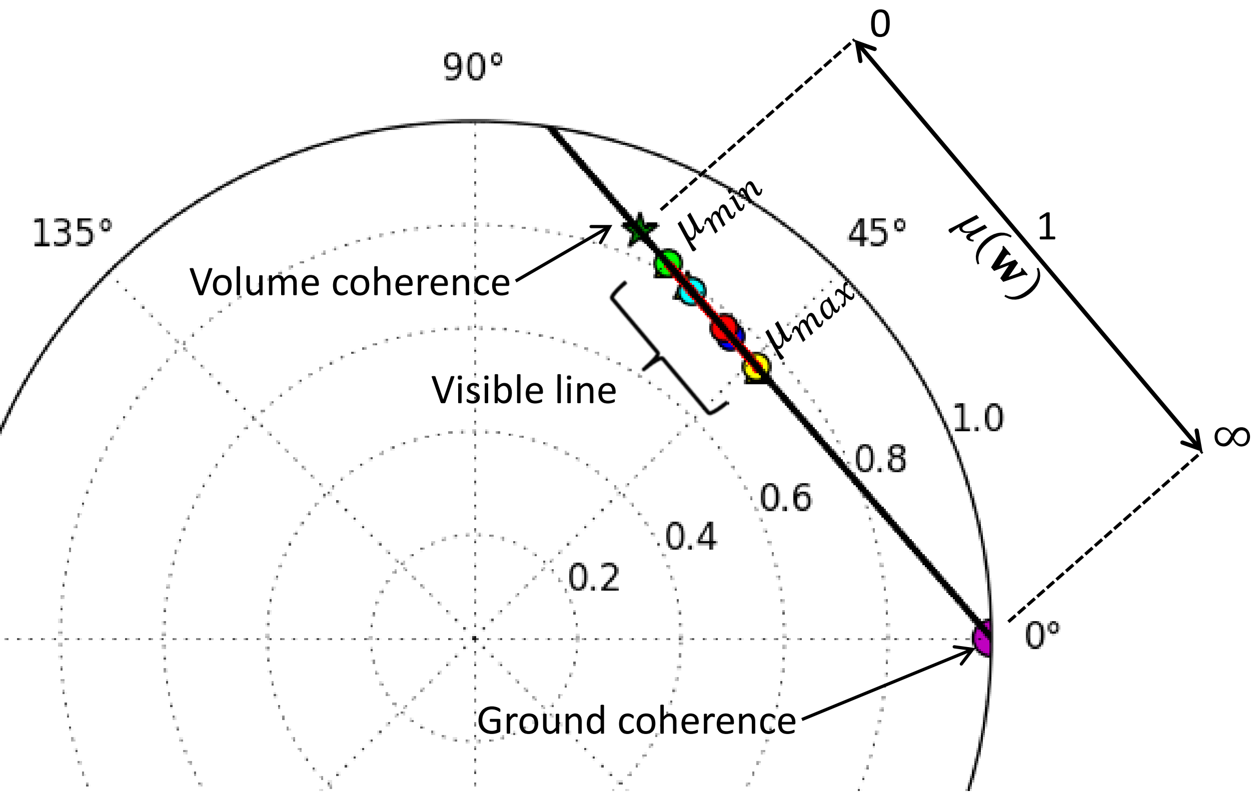
$$\mathbf{T} = \mathbf{R}_g \otimes \mathbf{T}_g + \mathbf{R}_v \otimes \mathbf{T}_v$$

$$\mathbf{R}_g = \begin{bmatrix} 1 & \gamma_{12}^{g} & \cdots & \gamma_{1N}^{g} \\ \left(\gamma_{12}^{g}\right)^{*} & 1 & & \gamma_{2N}^{g} \\ \vdots & & \ddots & \vdots \\ \left(\gamma_{1N}^{g}\right)^{*} & \left(\gamma_{2N}^{g}\right)^{*} & \cdots & 1 \end{bmatrix}$$

$$\mathbf{R}_v = \begin{bmatrix} 1 & \gamma_{12}^{v} & \cdots & \gamma_{1N}^{v} \\ (\gamma_{12}^{v})^{*} & 1 & & \gamma_{2N}^{v} \\ \vdots & & \ddots & \vdots \\ (\gamma_{1N}^{v})^{*} & (\gamma_{2N}^{v})^{*} & \cdots & 1 \end{bmatrix}$$

PolInSAR Two Layer Model – Coherence linearity

- The observed coherence region follows a line between the ground and volume coherences



$$\gamma_{ij}(\mathbf{w}) = \frac{\gamma_{ij}^{v} + \gamma_{ij}^{g} \cdot \mu(\mathbf{w})}{1 + \mu(\mathbf{w})}$$

$$\mu(\mathbf{w}) = \frac{\mathbf{w}^{H}\mathbf{T}_{g}\mathbf{w}}{\mathbf{w}^{H}\mathbf{T}_{v}\mathbf{w}}$$

$$\gamma_{ij}^{l} = \frac{\int F^{l}(z) e^{jk_{z_{ij}}z} dz}{\int F^{l}(z)\, dz} \quad \text{for } \mathrm{l} \in (g, v)$$

PolInSAR Two Layer Model – Ground and Volume extraction

- A pre-whitening will be applied

$$\tilde{\mathbf{T}} = \mathbf{N}_T^{-\frac{1}{2}} \mathbf{T} \mathbf{N}_T^{-\frac{1}{2}} = \begin{bmatrix} \mathbf{I} & \mathbf{\Pi}_{12} & \cdots & \mathbf{\Pi}_{1N} \\ \mathbf{\Pi}_{12}^H & \mathbf{I} & & \mathbf{\Pi}_{2N} \\ \vdots & & \ddots & \vdots \\ \mathbf{\Pi}_{1N}^H & \mathbf{\Pi}_{2N}^H & \cdots & \mathbf{I} \end{bmatrix} \qquad \mathbf{N}_T = \begin{bmatrix} \mathbf{T}_{11} & \mathbf{0} & \cdots & \mathbf{0} \\ \mathbf{0} & \mathbf{T}_{22} & & \mathbf{0} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{0} & \mathbf{0} & \cdots & \mathbf{T}_{NN} \end{bmatrix}$$

- Then the definition of the individual matrices of the whitened coherency matrix

$$\mathbf{I} = \mathbf{T}_{gw} + \mathbf{T}_{vw}$$

$$\mathbf{T}_{ii} = \mathbf{T}_{ii}^{\frac{1}{2}} (\mathbf{T}_{gw} + \mathbf{T}_{vw}) \mathbf{T}_{ii}^{\frac{1}{2}}$$

$$\mathbf{\Pi}_{ij} = \gamma_{ij}^{g} \mathbf{T}_{gw} + \gamma_{ij}^{v} \mathbf{T}_{vw}$$

$$\mathbf{\Pi}_{ij} = \mathbf{T}_{ii}^{\frac{1}{2}} \mathbf{\Omega}_{ij} \mathbf{T}_{jj}^{\frac{1}{2}}$$

$$\tilde{\mathbf{T}} = \mathbf{R}_g \otimes \mathbf{T}_{gw} + \mathbf{R}_v \otimes \mathbf{T}_{vw}$$

- The ground and volume coherency matrices $\mathbf{T}_{gw}$ and $\mathbf{T}_{vw}$ may be extracted by knowing γ_{ij}^{g} and γ_{ij}^{v}

$$\mathbf{T}_{gw} = \frac{\mathbf{\Pi}_{ij} - \gamma_{ij}^{v} \mathbf{I}}{\gamma_{ij}^{g} - \gamma_{ij}^{v}} \qquad \mathbf{T}_{vw} = \frac{\mathbf{\Pi}_{ij} - \gamma_{ij}^{g} \mathbf{I}}{\gamma_{ij}^{v} - \gamma_{ij}^{g}}$$

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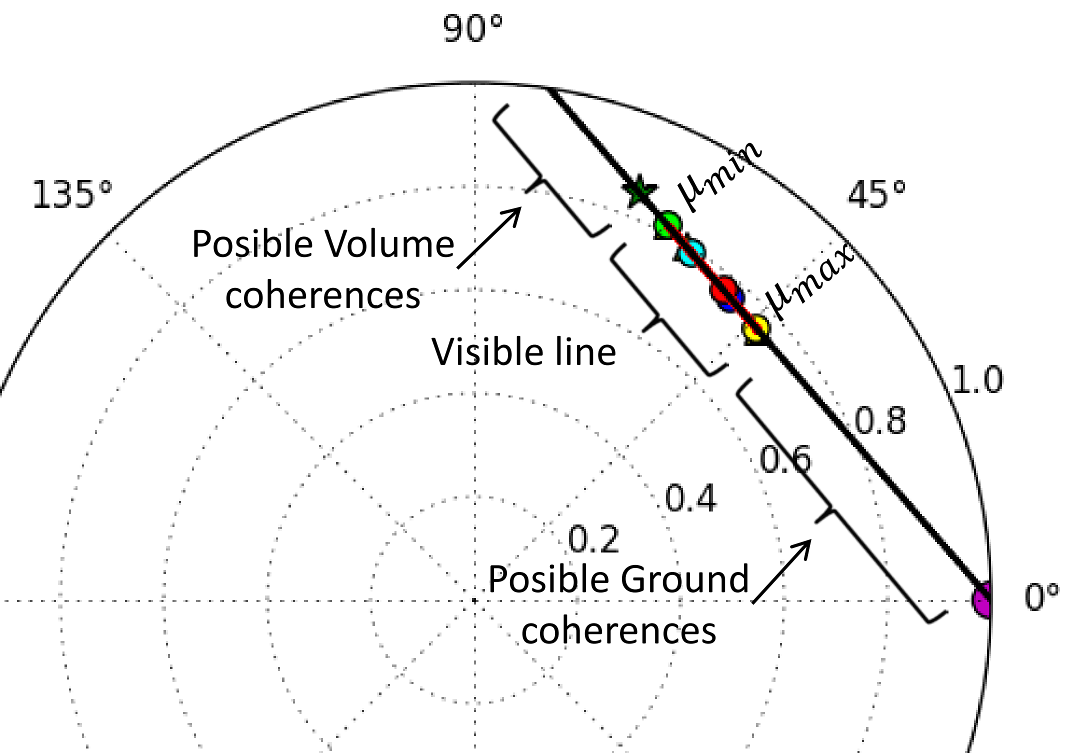
PolInSAR Two Layer Model – Ground and Volume extraction

- The Ground and Volume components depend on the selected coherences

Matrix

$$\mathbf{T}_{gw} = \frac{\boxed{\mathbf{\Pi}_{ij} - \gamma_{ij}^{v}\mathbf{I}}}{\boxed{\gamma_{ij}^{g} - \gamma_{ij}^{v}}} \qquad \mathbf{T}_{vw} = \frac{\boxed{\mathbf{\Pi}_{ij} - \gamma_{ij}^{g}\mathbf{I}}}{\boxed{\gamma_{ij}^{v} - \gamma_{ij}^{g}}}$$

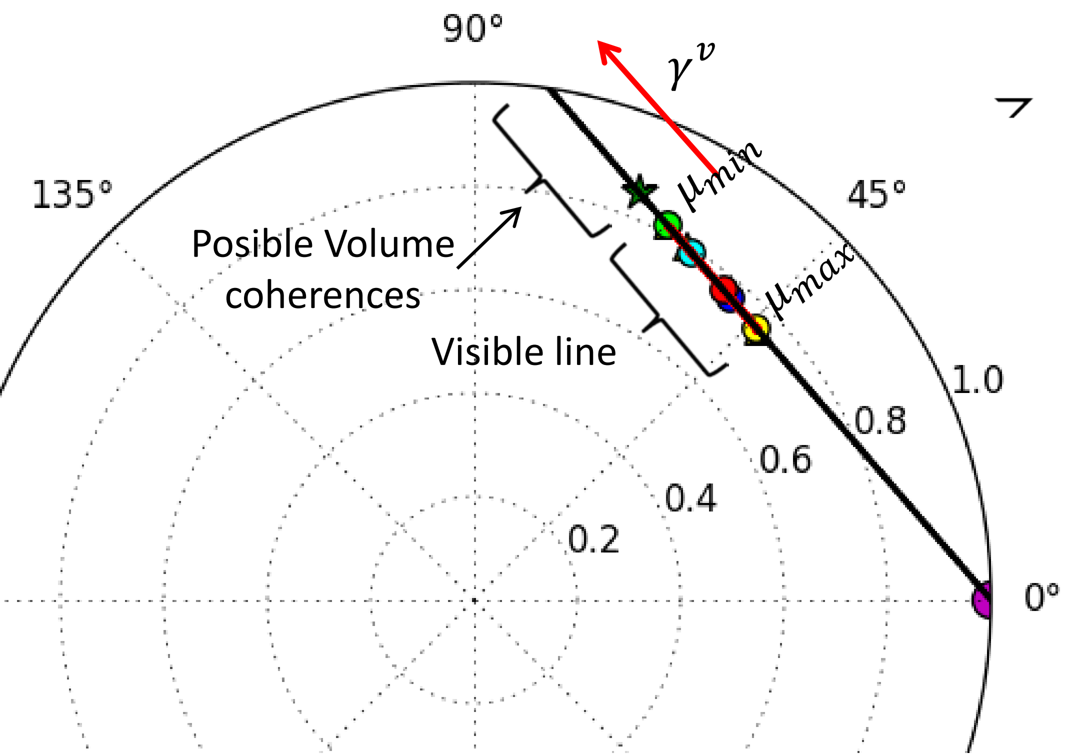
Scalar



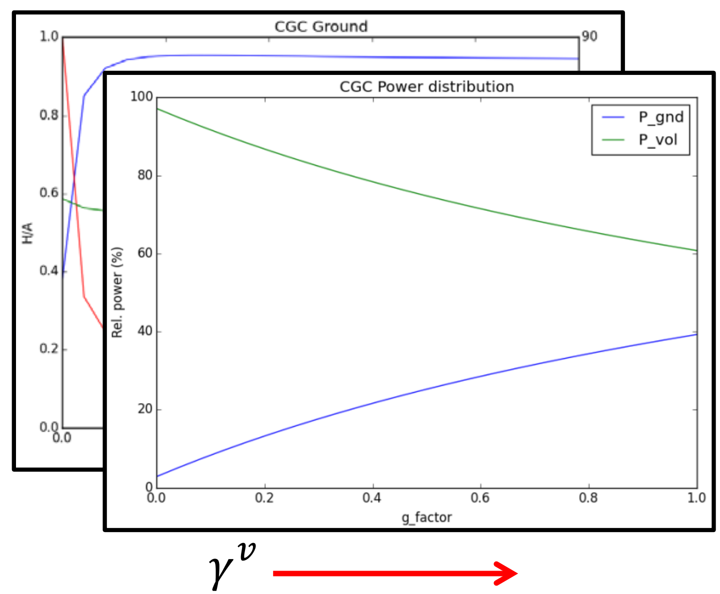
PolInSAR Two Layer Model – Ground and Volume extraction

- The Ground and Volume components depend on the selected coherences

$$\mathbf{T}_{gw} = \frac{\mathbf{\Pi}_{ij} - \gamma_{ij}^{v}\mathbf{I}}{\gamma_{ij}^{g} - \gamma_{ij}^{v}} \qquad \mathbf{T}_{vw} = \frac{\mathbf{\Pi}_{ij} - \gamma_{ij}^{g}\mathbf{I}}{\gamma_{ij}^{v} - \gamma_{ij}^{g}}$$



- Changing Volume coherence in the line
  - ... affects the Ground polarimetry
  - ... and the power of both components

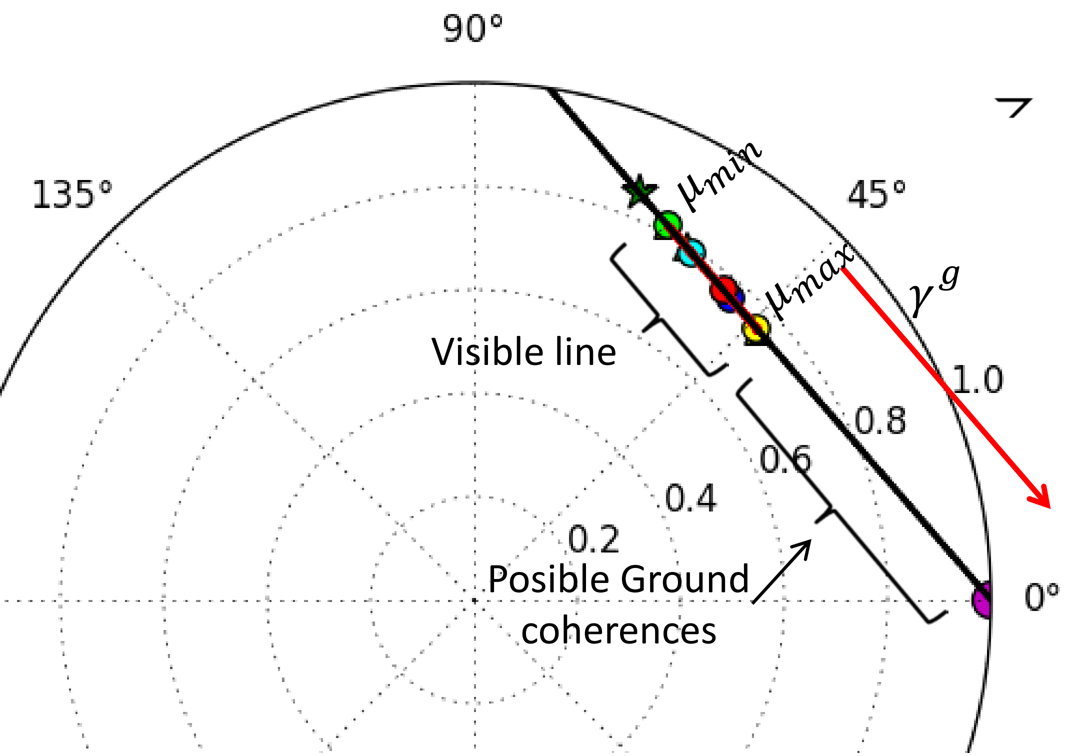


Fixing γ^g determines $\mathbf{T}_v$ up to a scalar factor

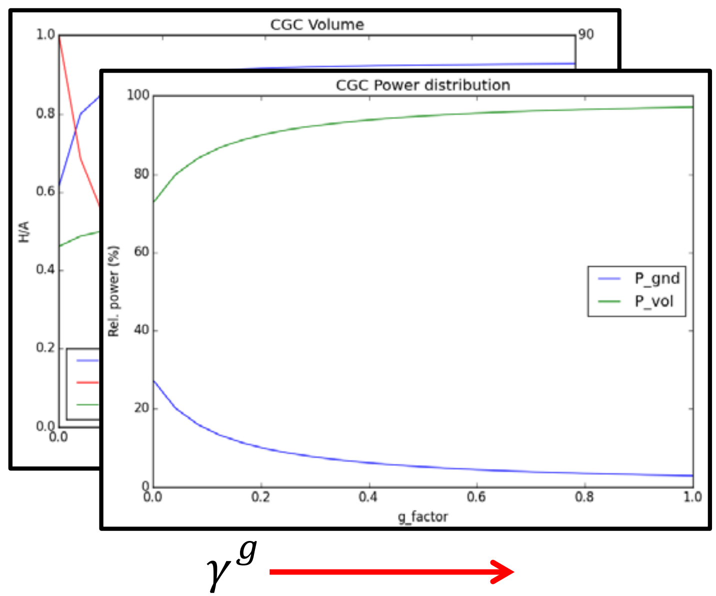
PolInSAR Two Layer Model – Ground and Volume extraction

- The Ground and Volume components depend on the selected coherences

$$\mathbf{T}_{gw} = \frac{\mathbf{\Pi}_{ij} - \gamma_{ij}^{v}\mathbf{I}}{\gamma_{ij}^{g} - \gamma_{ij}^{v}} \qquad \mathbf{T}_{vw} = \frac{\mathbf{\Pi}_{ij} - \gamma_{ij}^{g}\mathbf{I}}{\gamma_{ij}^{v} - \gamma_{ij}^{g}}$$



- Changing Ground coherence in the line
  - … affects the Volume polarimetry
  - … and the power of both components



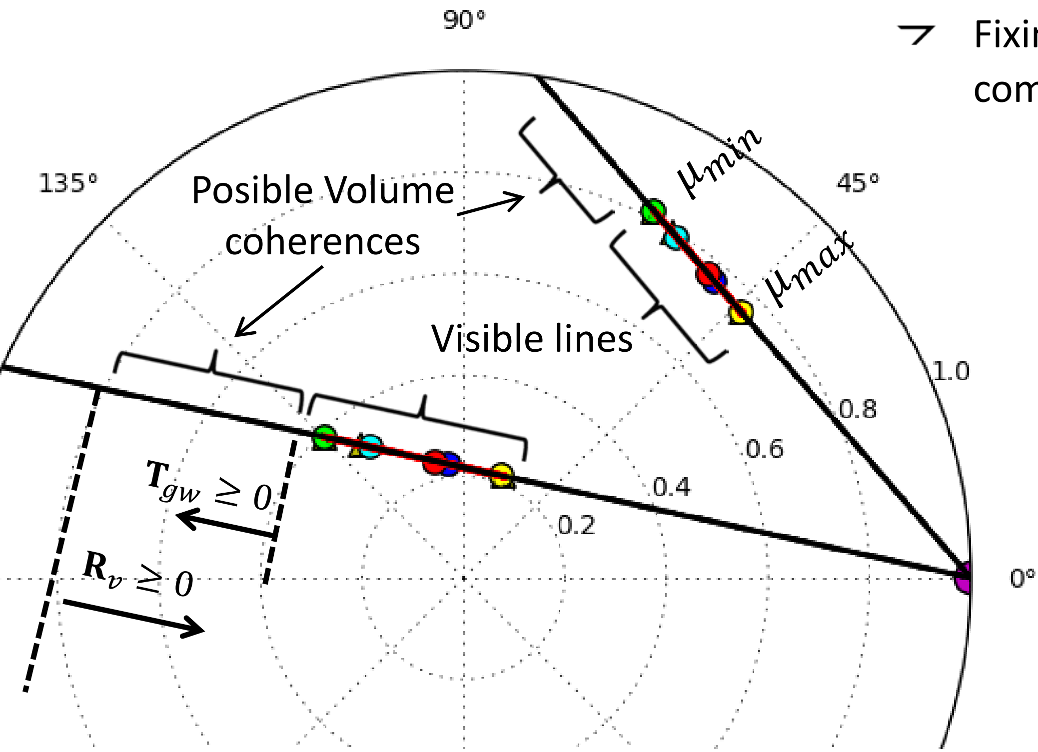
Fixing γ^v determines $\mathbf{T}_g$ up to a scalar factor

PolInSAR Two Layer Model – Multibaseline case

- Adding more baselines does not help to resolve the ambiguity

$$\tilde{\mathbf{T}} = \mathbf{R}_g \otimes \mathbf{T}_{gw} + \mathbf{R}_v \otimes \mathbf{T}_{vw}$$

$$\tilde{\mathbf{T}} - \mathbf{R}_g \otimes \mathbf{I} = (\mathbf{R}_v - \mathbf{R}_g) \otimes \mathbf{T}_{vw}$$



- Fixing Structure of the ground $\mathbf{R}_g$, the principal component of the remainder may be estimated

$$\min_{\mathbf{R}_0, \mathbf{T}_0} \left\| \left(\tilde{\mathbf{T}} - \mathbf{R}_g \otimes \mathbf{I} \right) - \mathbf{R}_0 \otimes \mathbf{T}_0 \right\|_F$$

$$\mathbf{R}_v = a\mathbf{R}_0 + \mathbf{R}_g$$

$$\mathbf{T}_{vw} = \frac{1}{a} \mathbf{T}_{vw0}$$

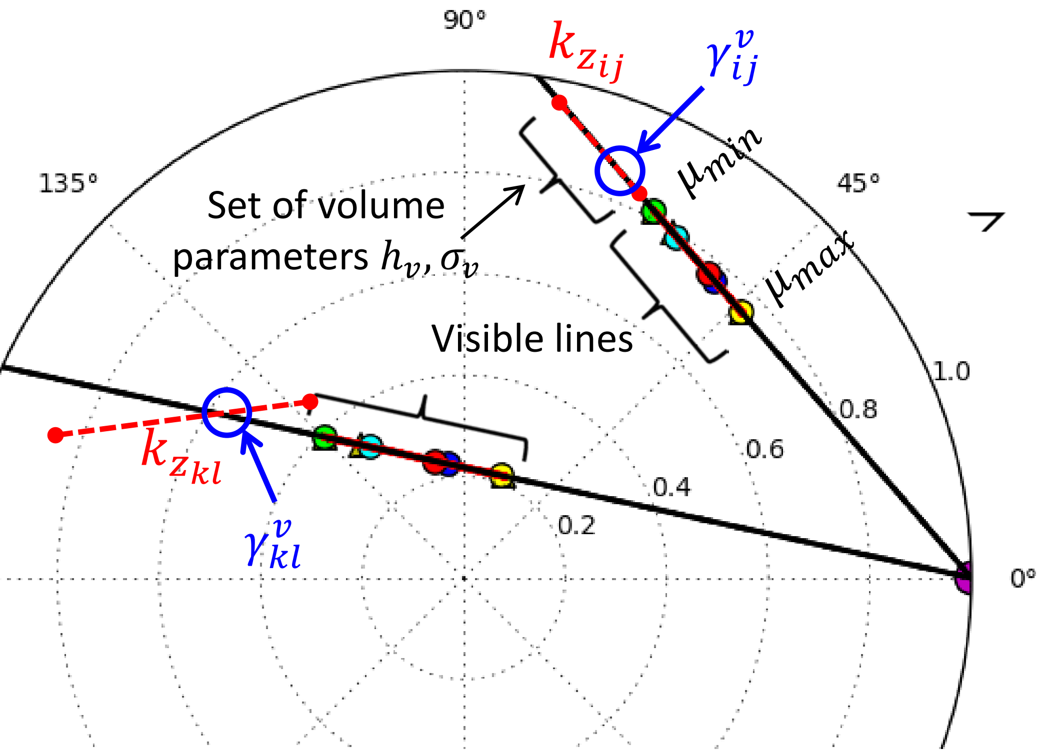
$$\mathbf{T}_{gw} = \mathbf{I} - \mathbf{T}_{vw}$$

- Structure $\mathbf{R}_g$, $\mathbf{R}_v$ and covariance matrices $\mathbf{T}_{gw}$ and $\mathbf{T}_{vw}$ are positive semidefinite

Multi-baseline PolInSAR reduce the ambiguity region, but estimation is still ambiguous

PolInSAR Two Layer Model – Model based approach

- Is it possible to estimate ground and volume components unambiguously?



- To solve the ambiguity, the coherences of different baselines need to be linked through a model

$$\gamma_{ij}^{l} = \frac{\int F^{l}(z) e^{jk_{z_{ij}}z} dz}{\int F^{l}(z)\, dz} \quad \text{for } \mathrm{l} \in (g, v)$$

- E.g. Ground as a delta and Volume as an exponential profile with height h_v and extinction σ_v

$$F^{v}(z, h_v, \sigma_v) = e^{-2\sigma_v(h_v + h_0 - z)/\cos\theta}$$

$$\gamma_{ij}^{v} = \frac{2\sigma_v e^{jk_{z_{ij}}h_0}}{\cos\theta\left(e^{-\frac{2\sigma_v h_v}{\cos\theta}} - 1\right)} \int_0^{h_v} e^{jk_{z_{ij}}z}\, e^{2\sigma_v z/\cos\theta}$$

$$\min_{h_0, h_v, \sigma_v} \sum_{i,j|i\neq j} \left\| \mathbf{\Pi}_{ij} - \left(\gamma_{ij}^{g}\mathbf{T}_{gw} + \gamma_{ij}^{v}\mathbf{T}_{vw}\right)\right\|_F^2$$

➡ Volume vertical profile model may be assumed in MB PolInSAR to solve the ambiguity

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PolInSAR Two Layer Model – Separation methods

- The observed coherence region follows a line between the ground and volume coherences

Ground and Volume Separation	No Assumptions	Assumption Ground delta	Assumption Ground delta and Volume profile
Single-baseline	Arbitrary γ_g and γ_v on the coherence line $\mathbf{T}_g$ and $\mathbf{T}_v$ ambiguous	$\lvert\gamma_g\rvert = 1$ γ_v on the coh. Line $\mathbf{T}_g$ ambiguous $\mathbf{T}_v$ up to a scalar factor	<u>RVOG</u> $\lvert\gamma_g\rvert = 1$ γ_v defined by model $\mathbf{T}_g$ ambiguous $\mathbf{T}_v$ up to a scalar factor
Multi-baseline	<u>SKP</u> Arbitrary $\mathbf{R}_g$ and $\mathbf{R}_v$ $\mathbf{T}_g$ and $\mathbf{T}_v$ ambiguous	<u>Hybrid KP decomp.</u> Rank-1 $\mathbf{R}_g$ Arbitrary $\mathbf{R}_v$ $\mathbf{T}_g$ ambiguous $\mathbf{T}_v$ up to a scalar factor	<u>MB-RVOG</u> Rank-1 $\mathbf{R}_g$ $\mathbf{R}_v$ defined by model $\mathbf{T}_g$ unambiguous $\mathbf{T}_v$ unambiguous

➡ Ambiguity can be avoided assuming $\mu_{min} = 0$, but this may not be true for longer wavelenghts

➡ Ambiguity can be solved with Volume model in MB PolInSAR

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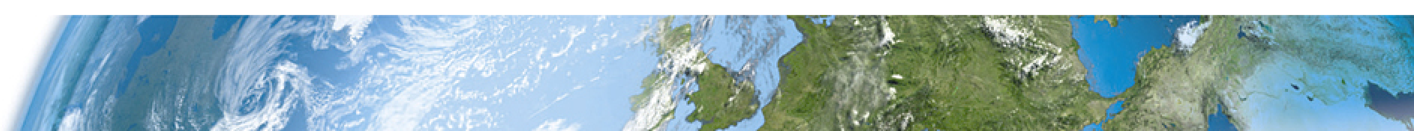
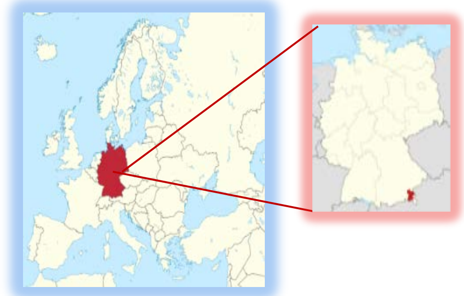
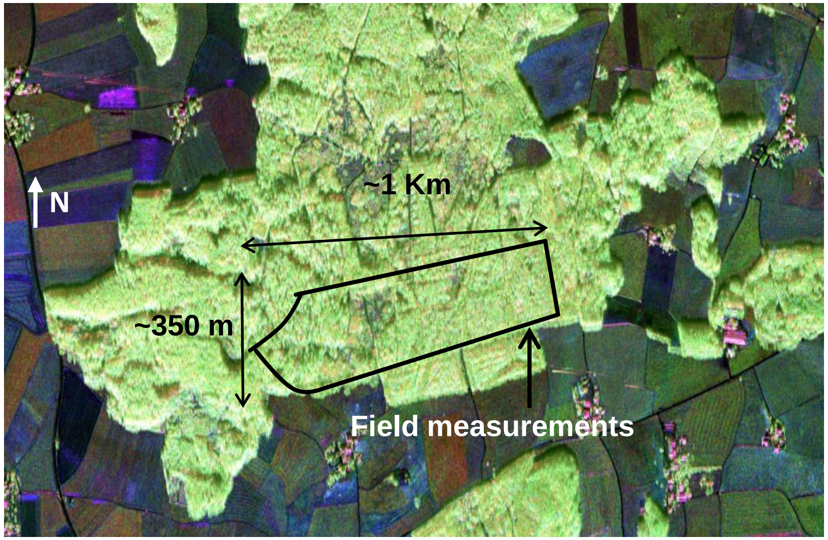
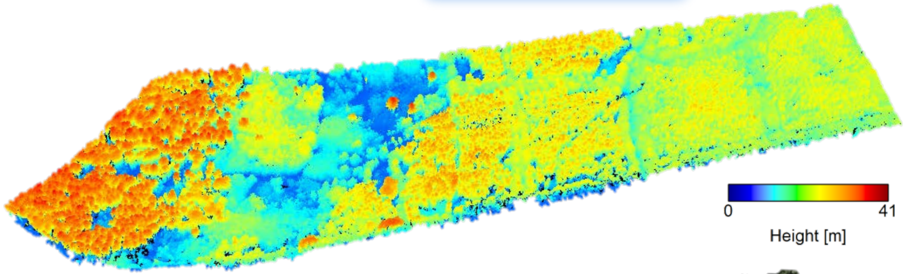


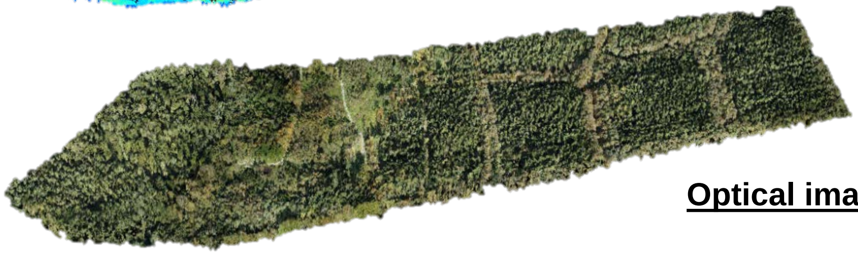
Froschham dataset and Ground Truth

- Fully polarimetric F-SAR @ L-band over Traunstein, Germany
- 8 baselines, all acquired the same day in May 2017
- Field measurements over 25ha of all trees (DBH, position and specie)

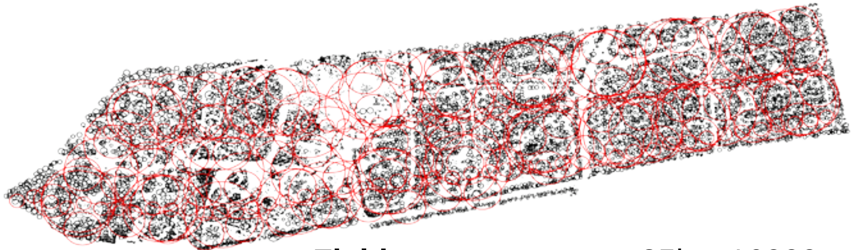








Optical image

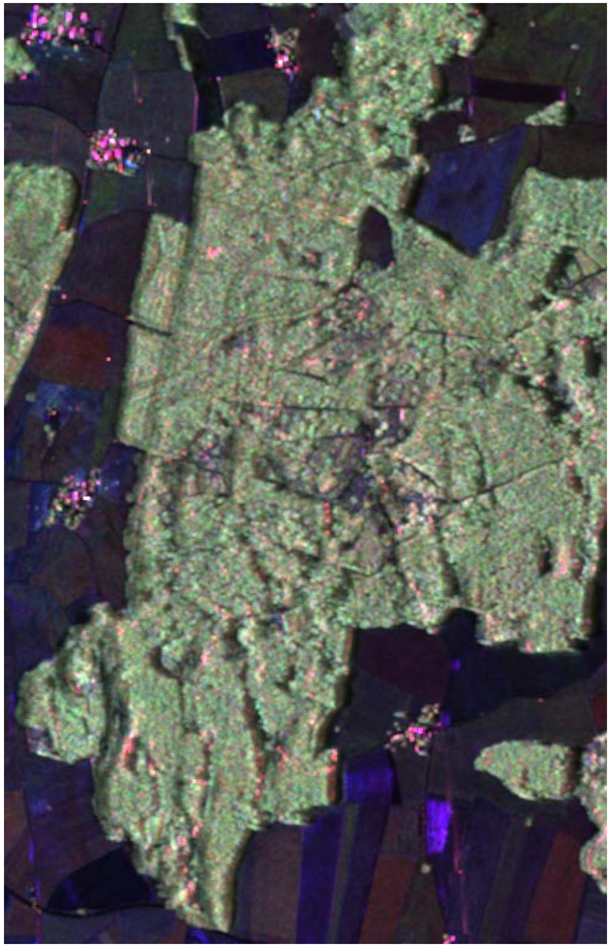


Field measurements: 25ha, 16000 trees.

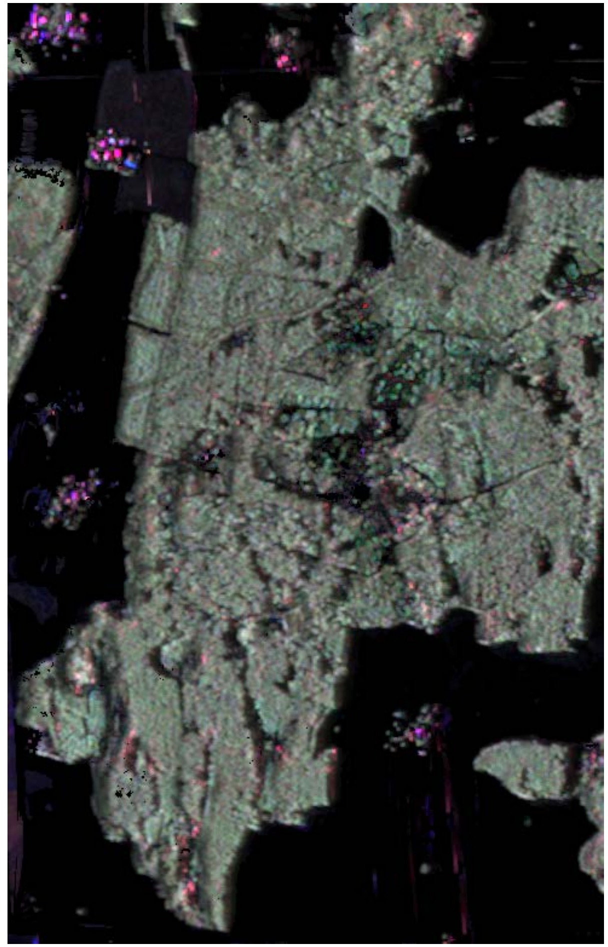
Diameter at breast height and position

Froschham dataset Separation Results

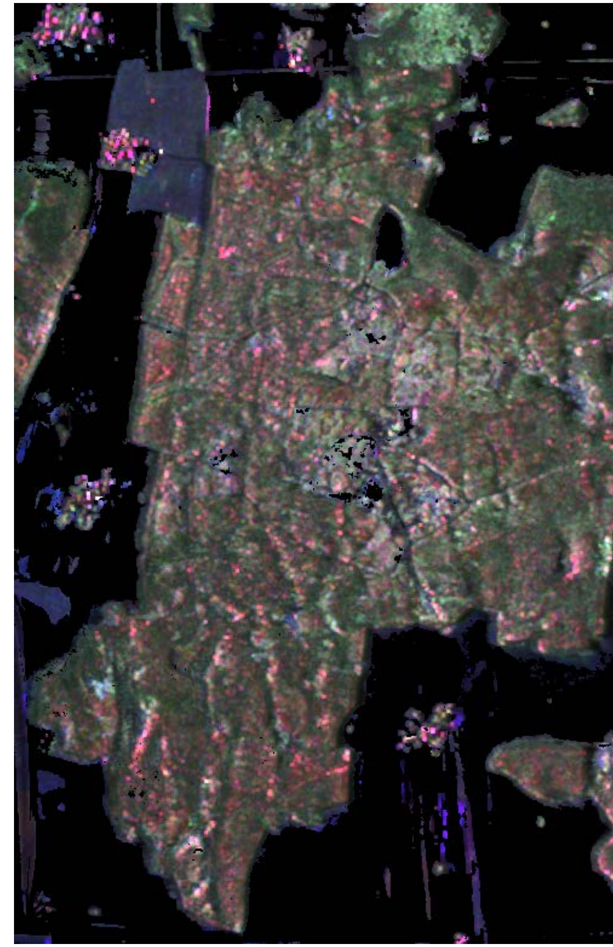
- Obtained unambiguous Ground and Volume components fixing exponential profile



Original



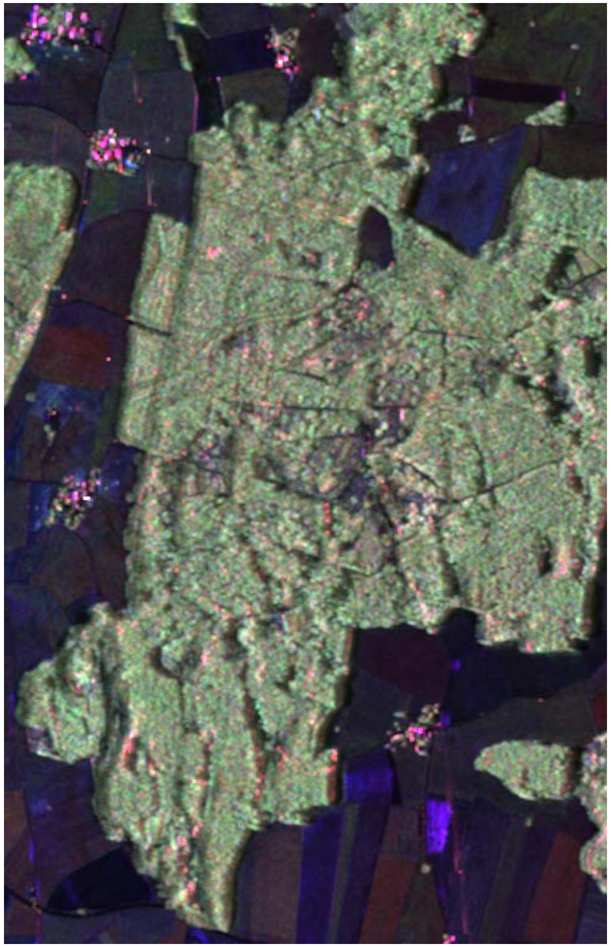
$\mathbf{T}_v$



$\mathbf{T}_g$

Froschham dataset Separation Results

- Obtained Ground and Volume components with arbitrary volume profile and fixing $\mu_{min} = 0$



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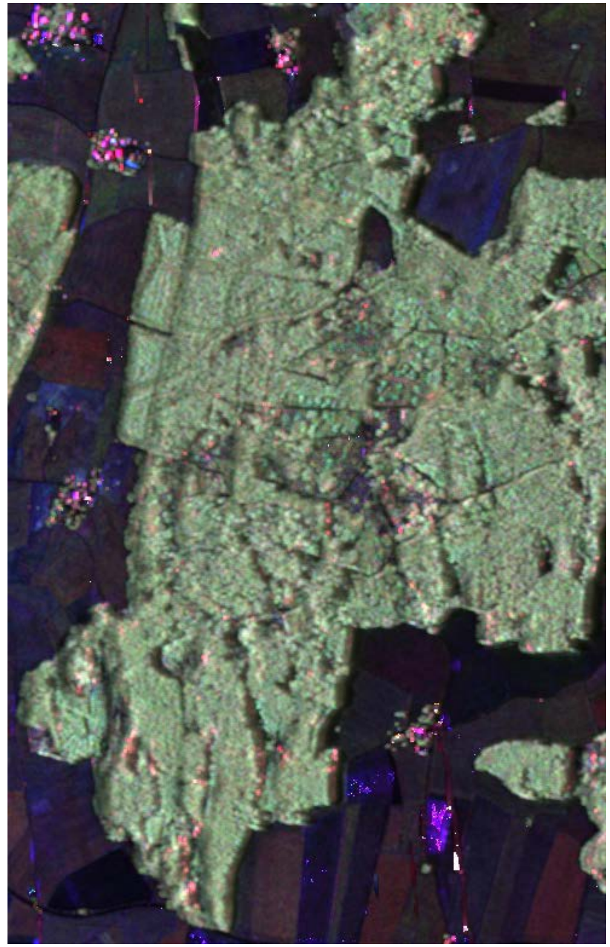
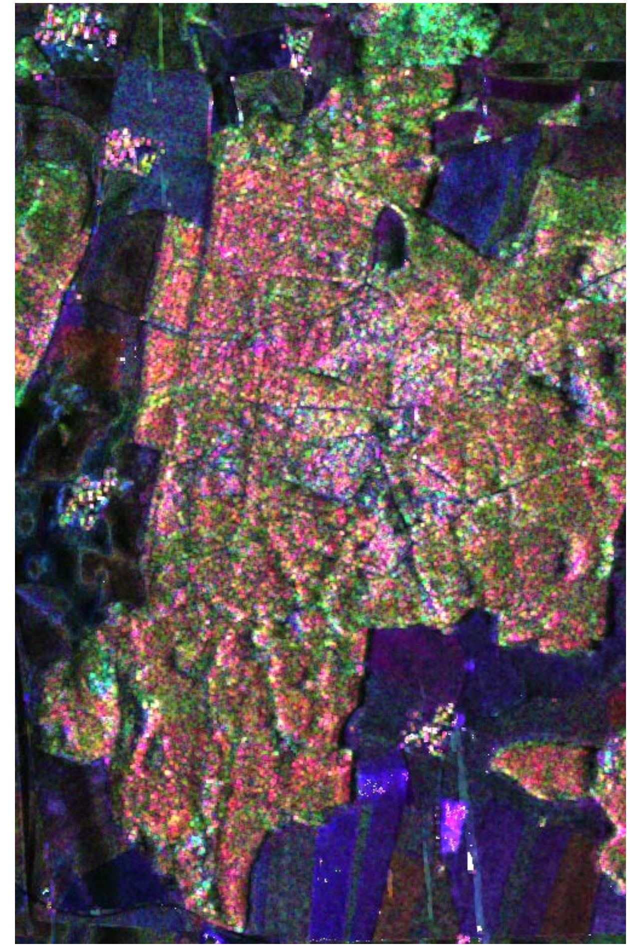


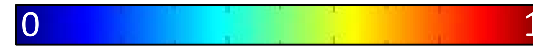
$\mathbf{T}_v$

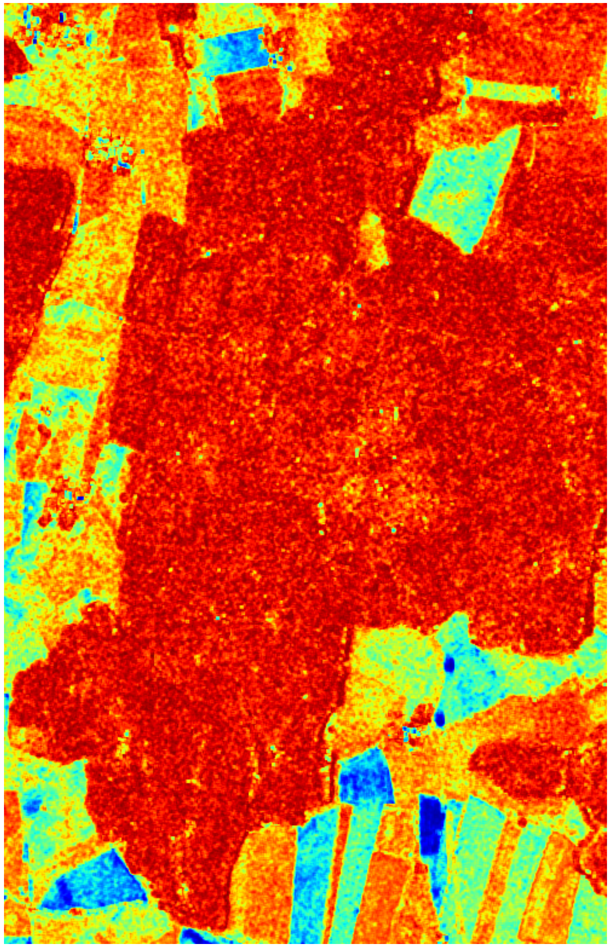


$\mathbf{T}_g$

Froschham dataset Separation Results

- Entropy of the Volume component for the two techniques





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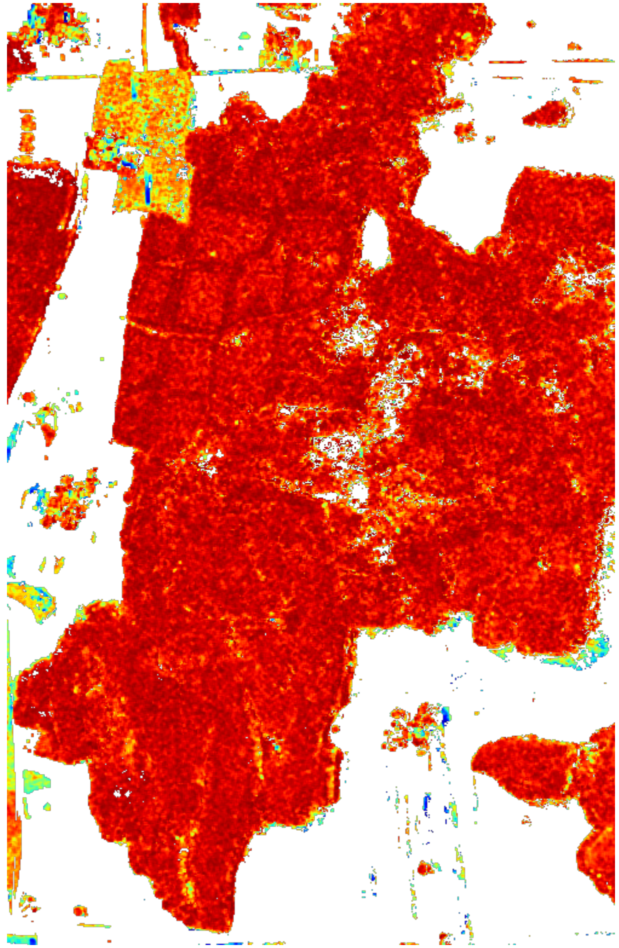
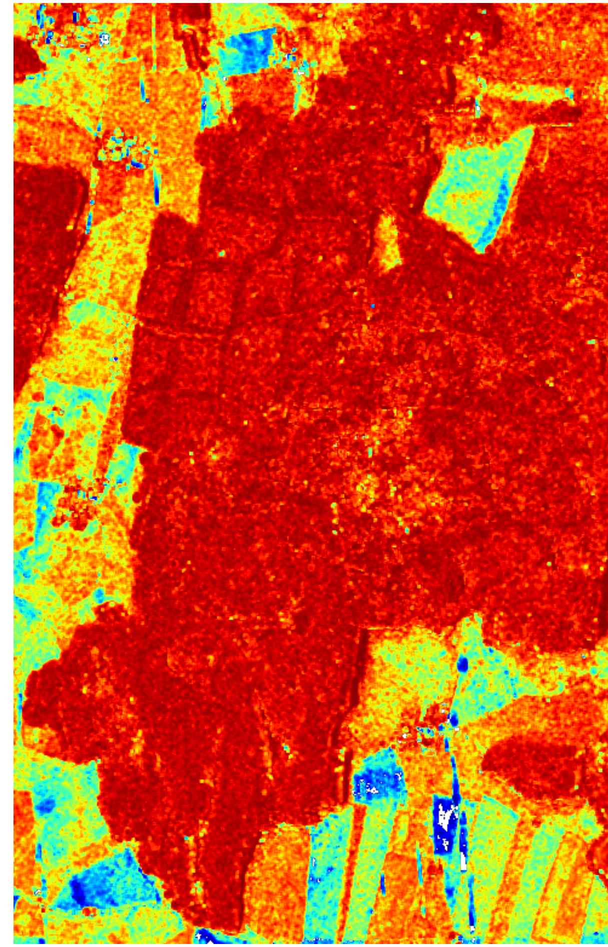


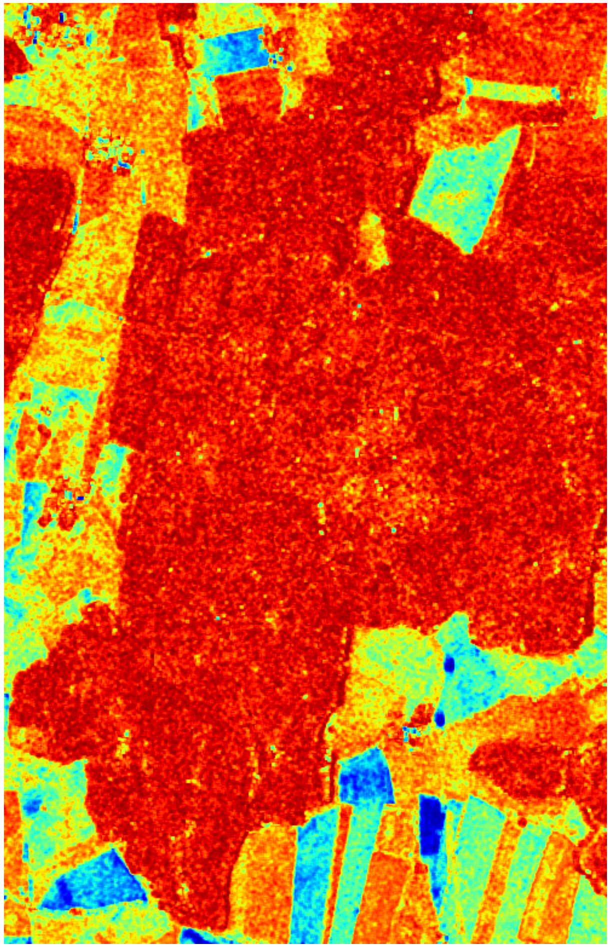
Volume H exp. volume



Volume H arbitrary volume

Froschham dataset Separation Results

- Entropy of the Ground component for the two techniques



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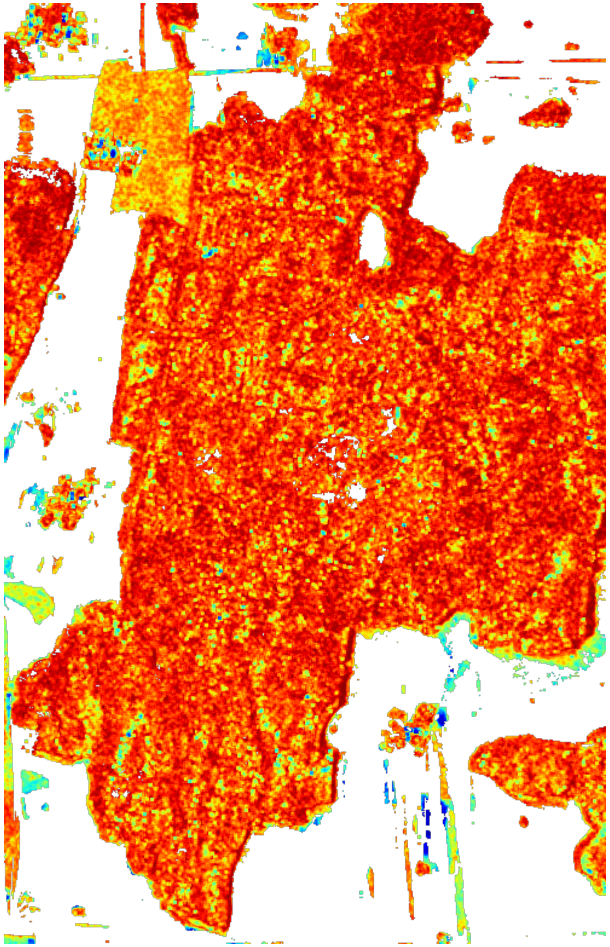
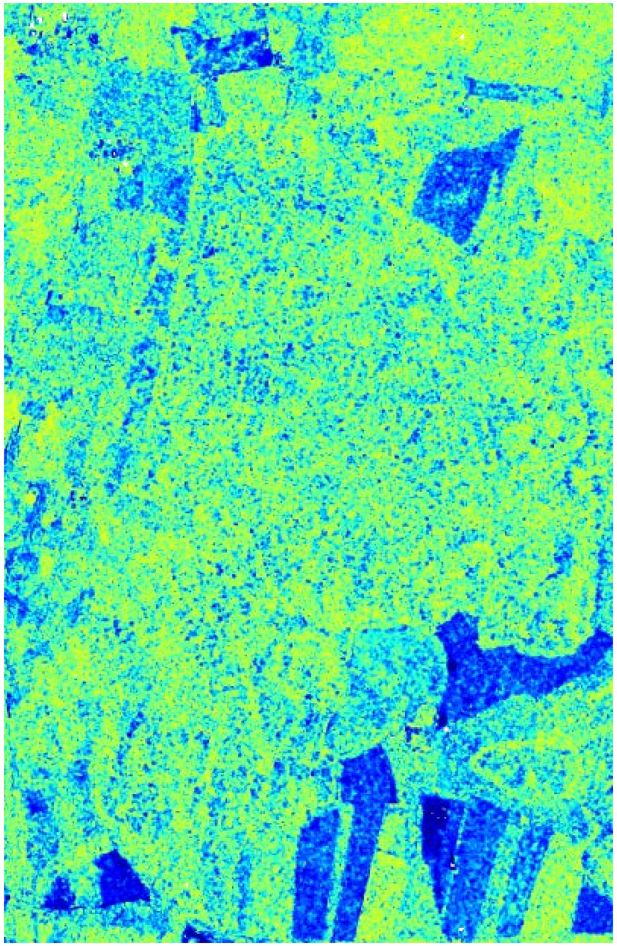


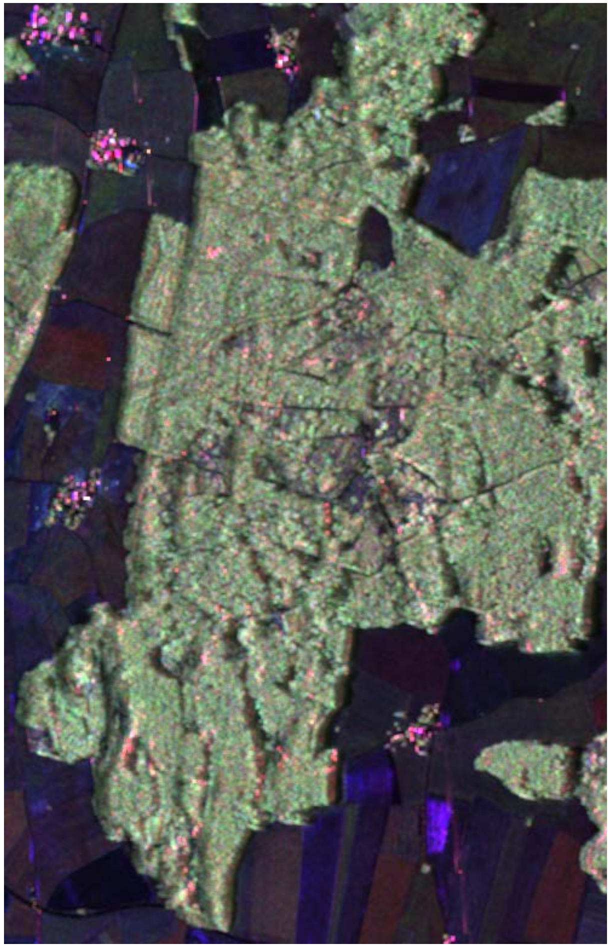
Ground H exp. volume



Ground H arbitrary volume

Froschham dataset Separation Results

- Ground to Volume ratio of the two techniques compared



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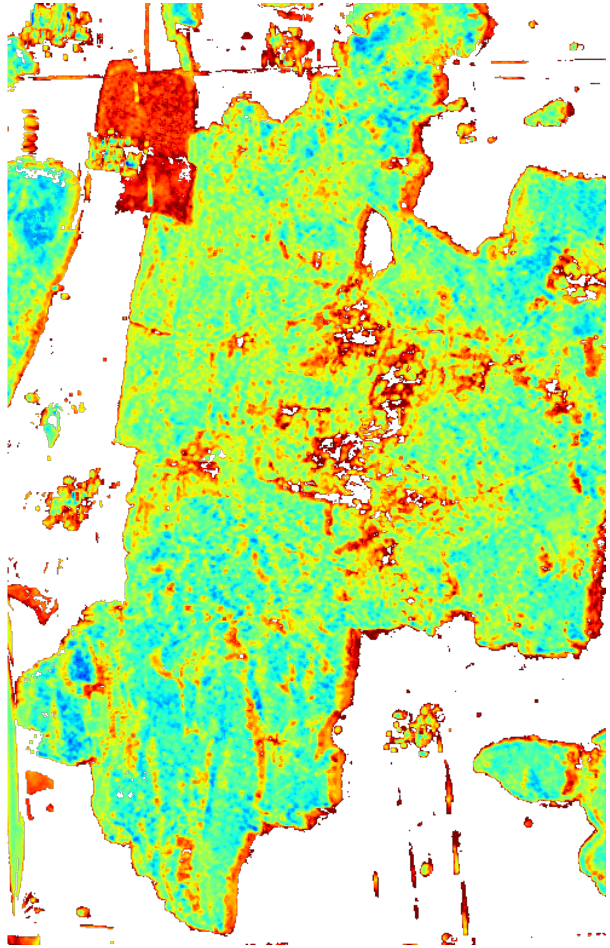
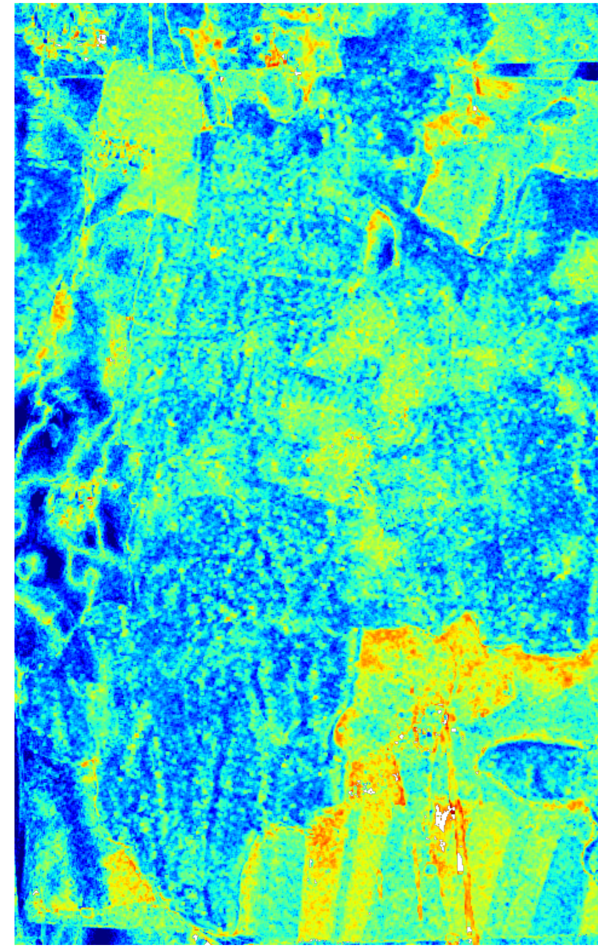


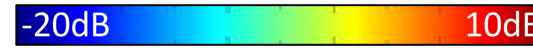
G2V ratio exp. volume

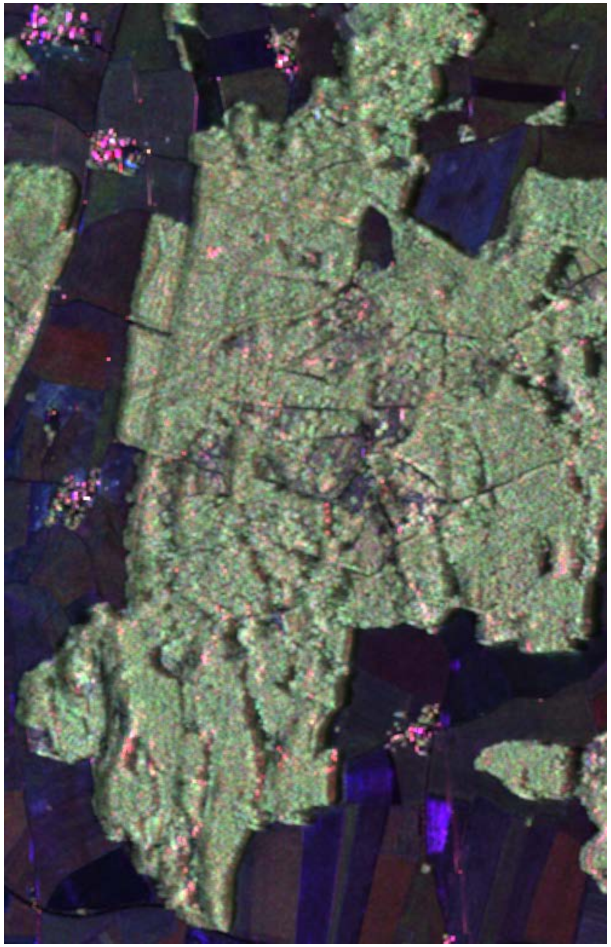


G2V ratio arbitrary volume

Froschham dataset Separation Results

- Ground to Volume ratio of the two techniques compared





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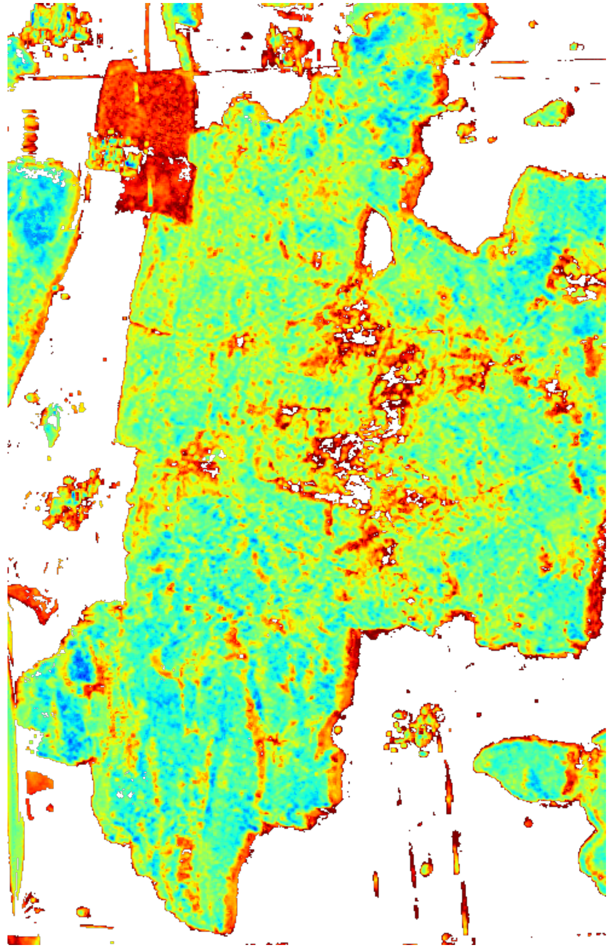
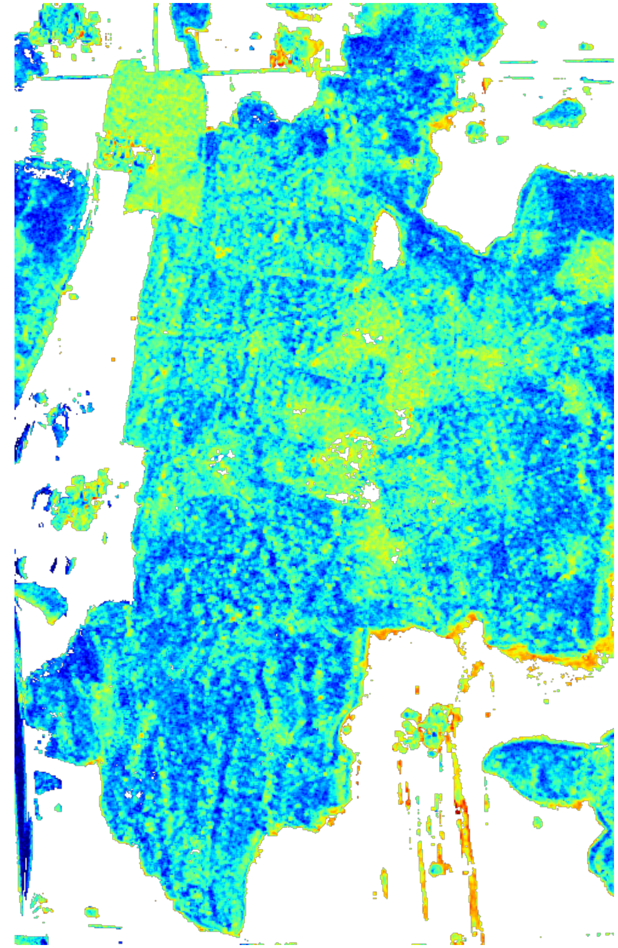


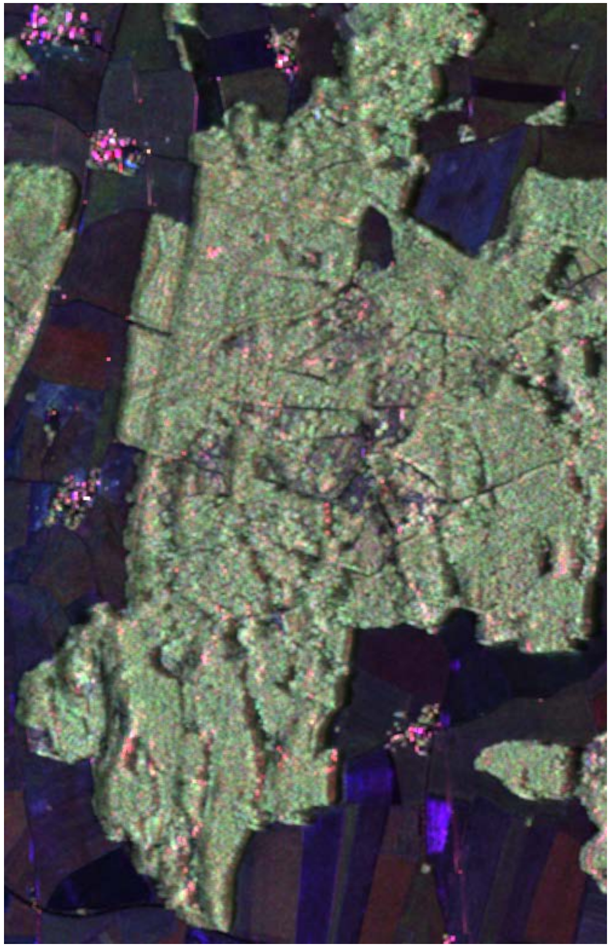
G2V ratio exp. volume



G2V ratio arbitrary volume

Froschham dataset – Results compared with Ground Truth

- Obtained Volume model height vs Lidar DSM



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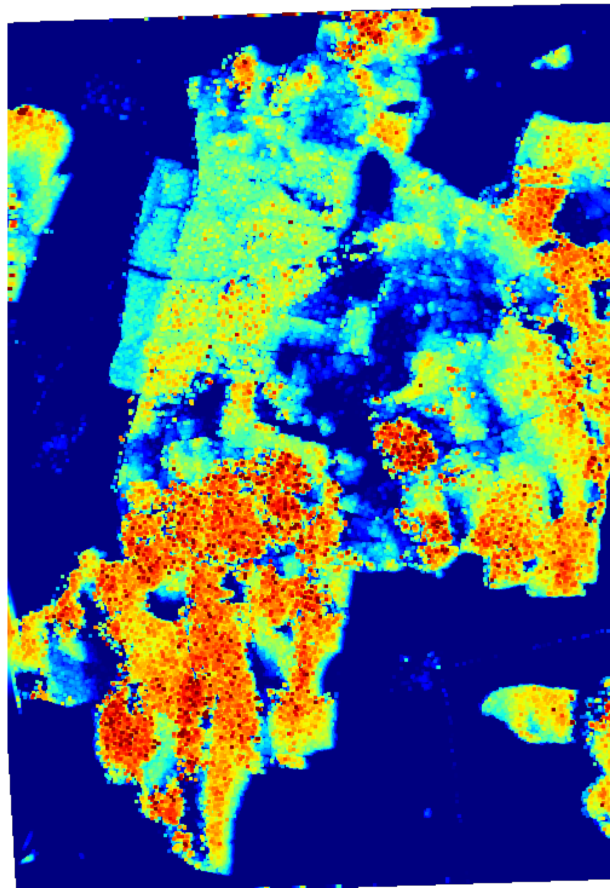
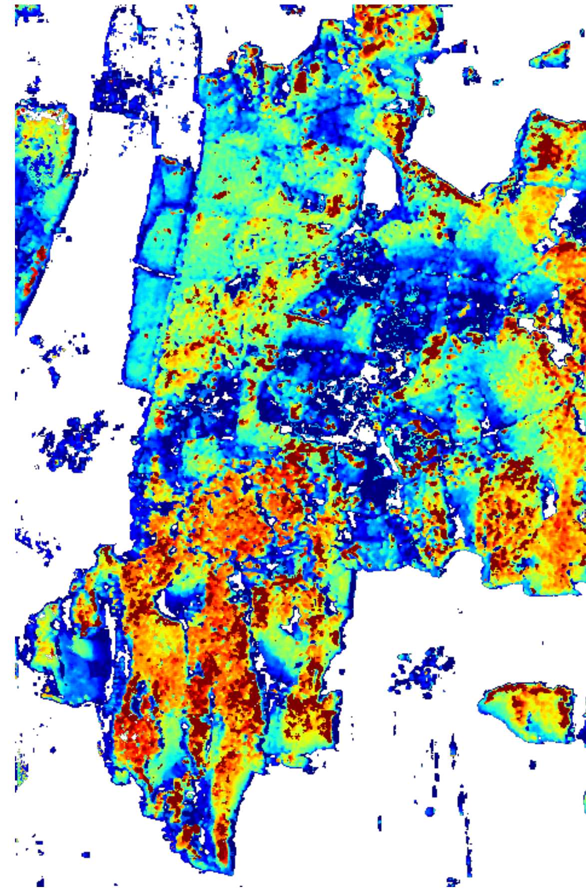


Lidar DSM



Exp. Volume h_v parameter

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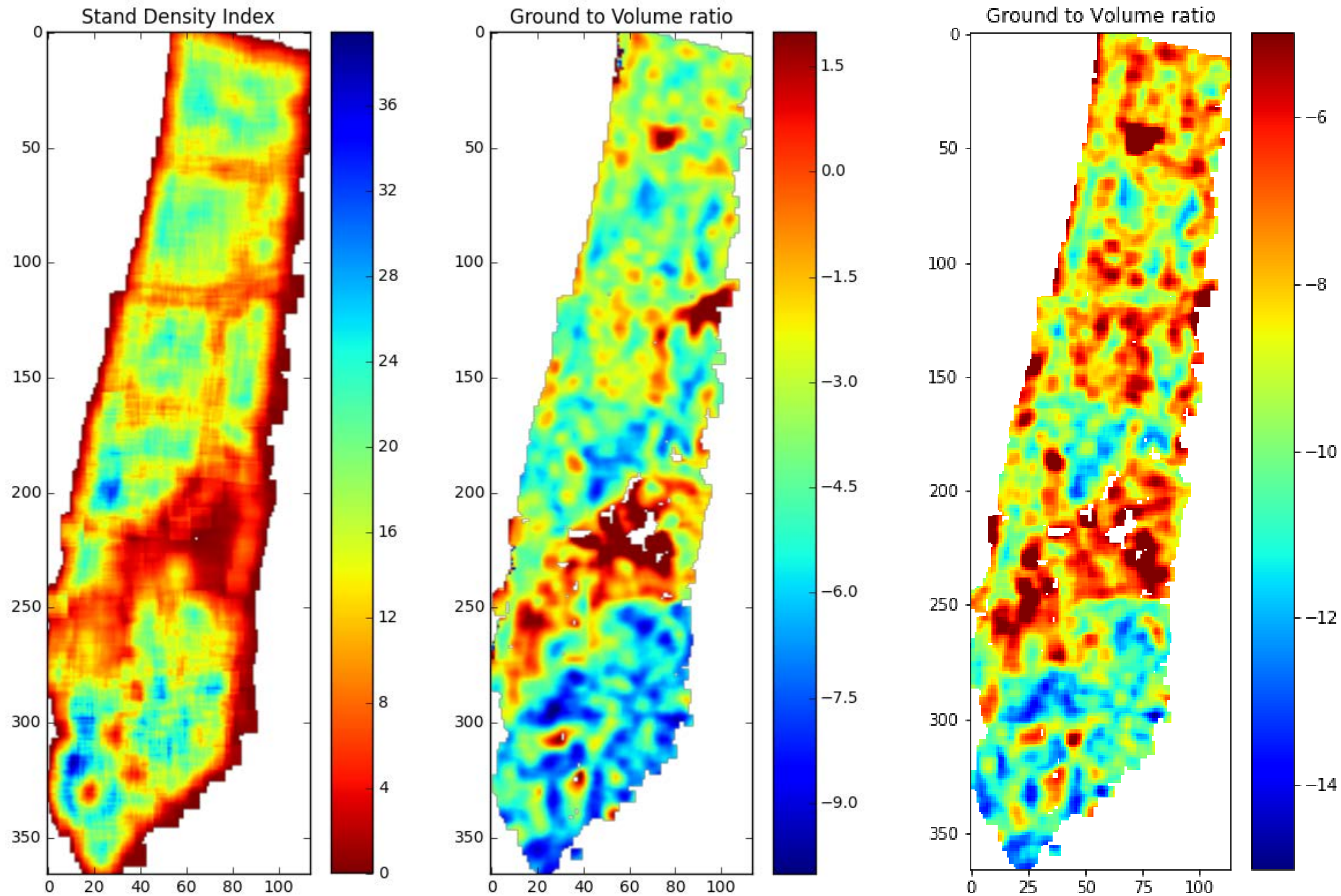
Froschham dataset – Results compared with Ground Truth

- Ground to Volume ratio of the two techniques compared with Stand Density Index $SDI = N\left(\frac{dg}{25}\right)^{1.605}$



SDI

Exp. Volume

Arbitrary Volume

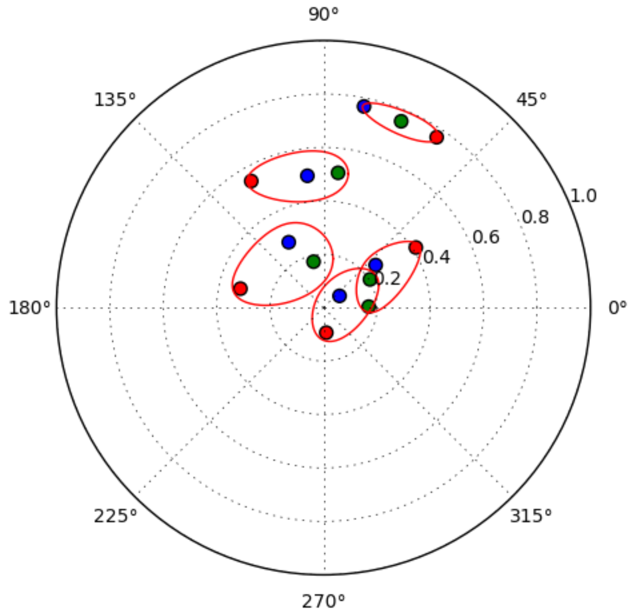
Conclusions

- The PolInSAR 2-layer model has been employed to separate ground and volume components
  - The coherence region follows a line under the 2-layer model assumption
  - There is an ambiguity in the separation of ground and volume components which cannot be solved adding more baselines
  - Fixing one of the layer coherences defines the covariance matrix of the other layer up to a constant factor
  - When $\mu_{min} = 0$ is assumed a rank-2 ground and an overestimated volume power is obtained
  - This ambiguity may be solved by defining a layer model and linking the coherences of the different baselines under this model
- This technique has been employed to process a real dataset over Forest
  - The volume component presents very high entropy and low anisotropy, following the expected response of a random volume
  - Ground component represents a mixture of surface and double bounce, having most of the details of the image in the forest areas
  - The estimated model parameters forest height h_v and G2V ratio present a good match with lidar and ground measurements

Thank you for your attention!

Ground and Volume separation – Real vs modelled coherences

- Some modelled coherences vs. original data for different baselines for a sample pixel



Original coherence regions $\mathbf{\Pi}_{ij}$

$$\tilde{\mathbf{T}} = \mathbf{N}_T^{-\frac{1}{2}} \mathbf{T} \mathbf{N}_T^{-\frac{1}{2}} = \begin{bmatrix} \mathbf{I} & \mathbf{\Pi}_{12} & \cdots & \mathbf{\Pi}_{1N} \\ \mathbf{\Pi}_{12}^H & \mathbf{I} & & \mathbf{\Pi}_{2N} \\ \vdots & & \ddots & \vdots \\ \mathbf{\Pi}_{1N}^H & \mathbf{\Pi}_{2N}^H & \cdots & \mathbf{I} \end{bmatrix}$$

Modelled coherence regions

$$\widehat{\mathbf{\Pi}}_{ij} = \gamma_{ij}^g \mathbf{T}_{gw} + \gamma_{ij}^v \mathbf{T}_{vw}$$

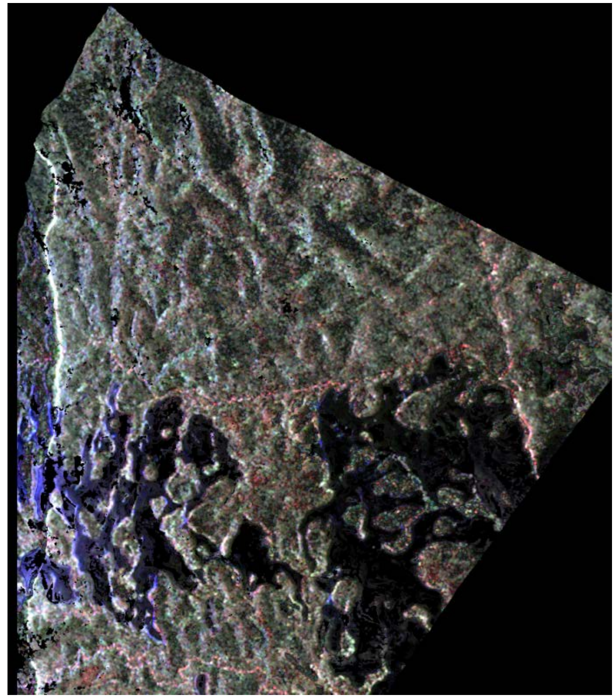
$$\hat{\tilde{\mathbf{T}}} = \mathbf{R}_g \otimes \mathbf{T}_{gw} + \mathbf{R}_v \otimes \mathbf{T}_{vw}$$

Ground and Volume separation – AfriSAR16 dataset

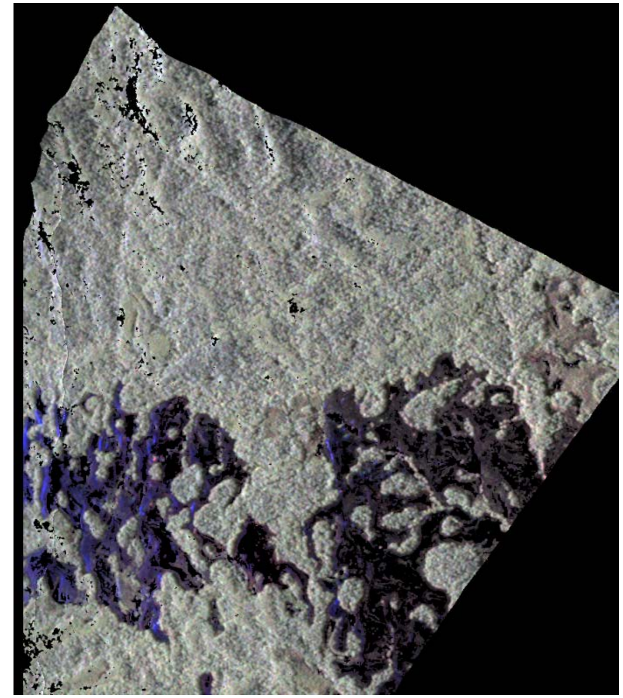
- AfriSAR campaign. Lope test site, F-SAR L-band dataset

Original

Ground



Volume

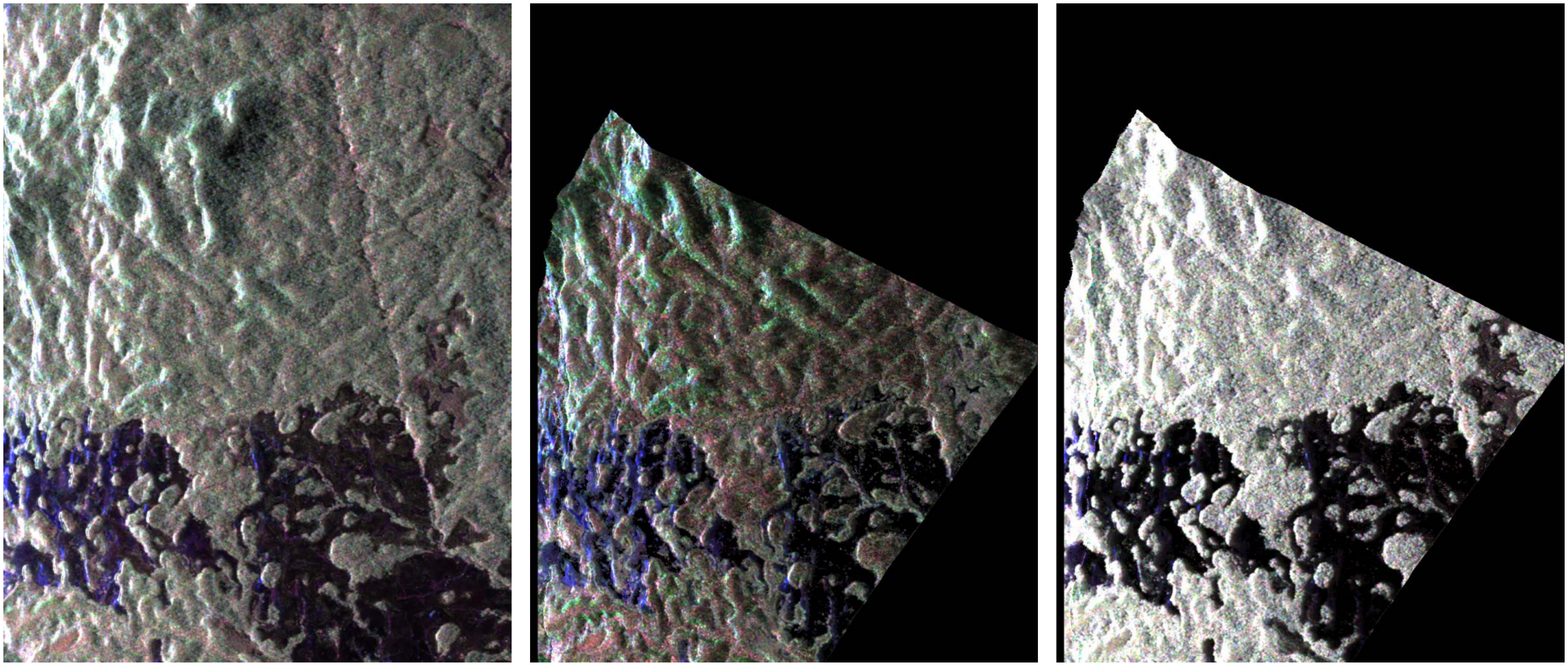


|HH+VV|
|HH-VV|
|HV+VH|

Ground and Volume separation – AfriSAR16 dataset

- AfriSAR campaign. Lope test site, F-SAR P-band dataset

Original Ground Volume

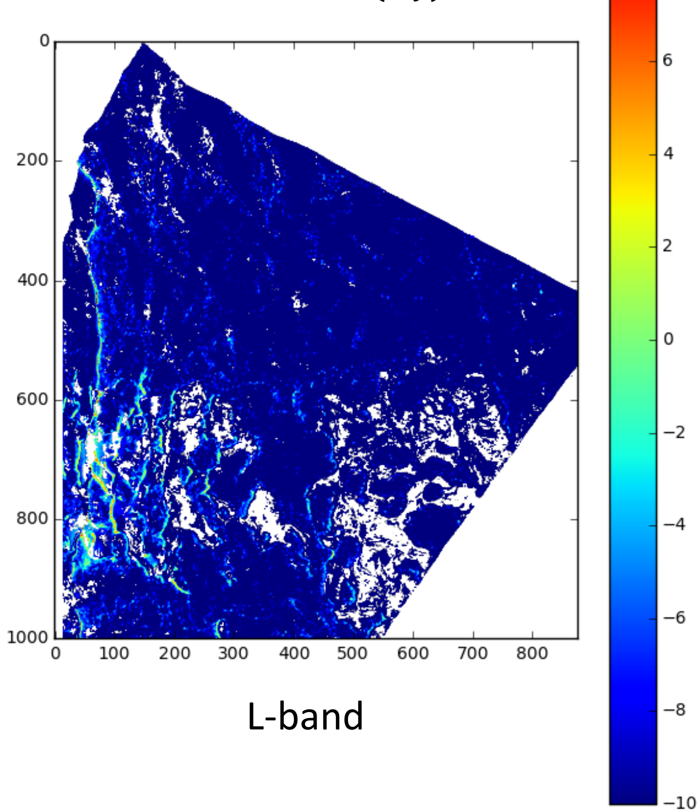


|HH+VV|
|HH-VV|
|HV+VH|

Ground and Volume separation – AfriSAR16 dataset

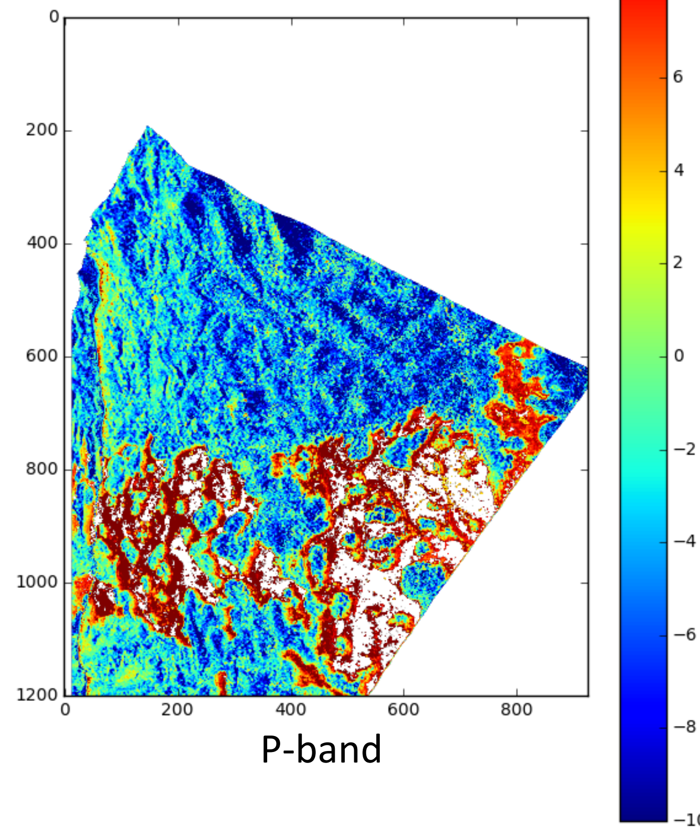
- AfriSAR campaign. Lope test site, L-band vs, P-band ground to volume ratio

$$10\log_{10}\frac{tr(\mathbf{T}_g)}{tr(\mathbf{T}_v)}$$



L-band

$$10\log_{10}\frac{tr(\mathbf{T}_g)}{tr(\mathbf{T}_v)}$$



P-band

DLR