Wall-resolved and wall-modeled ILES based on high-order DG

Ralf Hartmann

14 June 2018
Overview

1. DG discretization of
   • flow equations
   • integral quantities and
   • local quantities

2. Discretization settings and results/examples for
   • RANS
   • ILES

3. Wall-modelled ILES (WM-ILES)
   • Approach
   • Results
Discretization of equations, integral and local quantities

Discretization of flow equations:

\[
\int_{\Omega} \left( -F^c(u_h) + F^\nu(u_h, \nabla_h u_h) \right) : \nabla_h v_h \, dx + \ldots + \int_{\Gamma} \left( \hat{h}_{\Gamma,h} - \hat{\sigma}_{\Gamma,h} \right) \cdot v_h \, ds
\]

Adjoint consistent discretization of integral quantities (drag, lift coefficients):

\[
J(u) = \int_{\Gamma_w} (\rho \, n - \tau \, n) \cdot \psi \, ds, \quad J_h(u_h) = \int_{\Gamma_w} \left( \hat{h}_{\Gamma,h} - \hat{\sigma}_{\Gamma,h} \right) \cdot \tilde{\psi} \, ds.
\]

Adjoint consistent discretization of local quantities (surface pressure, skin friction)

\[
c_p(u) = \frac{\rho(u) - \rho_\infty}{\frac{1}{2} \rho_\infty \nu_\infty^2}, \quad c_{p,h}(u_h) = \frac{\hat{h}_{\Gamma,h} \cdot \tilde{n} - \rho_\infty}{\frac{1}{2} \rho_\infty \nu_\infty^2},
\]

\[
c_f(u, \nabla u) = \frac{\tau_W(u, \nabla u)}{\frac{1}{2} \rho_\infty \nu_\infty^2}, \quad c_{f,h}(u_h, \nabla u_h) = -\frac{\hat{\sigma}_{\Gamma,h} \cdot \tilde{t}}{\frac{1}{2} \rho_\infty \nu_\infty^2},
\]

with \( \tilde{n} = (0, n_1, n_2, 0)^T \) for the normal vector \( n = (n_1, n_2)^T \),
and \( \tilde{t} = (0, t_1, t_2, 0)^T \) for the tangential vector \( t = (t_1, t_2)^T \).

Hartmann, Leicht: JCP 300, 754-778, 2015
Computational/meshing challenge
4th Int. Workshop on High-Order CFD Meth.

DLR-F11 high lift configuration
• Config 4 incl. slat tracks and flap track fairings
• Mach=0.175, Re=15.1x10^6, alpha=7.0°
• Fully turbulent (RANS with Wilcox-kω model)

Grid (by Harlan McMorris, CentaurSoft):
• Quadratic curved grid (Centaur)
• Hybrid grid (with prisms, pyramids and tetrahedra)
• 3.52x10^6 elements

Flow solver:
• PADGE solver, fully implicit solver (no multigrid)
• p=0, 1, 2 (3.5, 14.1, 35.2x10^6 DoFs/eqn)
• Convergence of nonlinear residual below 1e-10

Hartmann, McMorris, Leicht: ECCOMAS 2016
The DLR-F11 high lift configuration: Results

Hartmann, McMorris, Leicht: ECCOMAS 2016

<table>
<thead>
<tr>
<th></th>
<th>$C_L$</th>
<th>$C_D$</th>
<th>$C_M$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Exp.</td>
<td>1.9270</td>
<td>0.1615</td>
<td>-0.5390</td>
</tr>
<tr>
<td>TAU*</td>
<td>1.8794</td>
<td>0.1681</td>
<td>-0.5647</td>
</tr>
<tr>
<td></td>
<td>-2.5%</td>
<td>+4.1%</td>
<td>-4.8%</td>
</tr>
<tr>
<td>PADGE</td>
<td>1.8781</td>
<td>0.1649</td>
<td>-0.5704</td>
</tr>
<tr>
<td></td>
<td>-2.5%</td>
<td>+2.1%</td>
<td>-5.8%</td>
</tr>
</tbody>
</table>

* Rudnik, Melber(SAO)
  HiLiftPW-2, 2013
## Discretization details: RANS vs. ILES

<table>
<thead>
<tr>
<th></th>
<th>RANS</th>
<th>ILES</th>
</tr>
</thead>
<tbody>
<tr>
<td>Convective numerical flux</td>
<td>Roe with entropy fix</td>
<td>Roe without entropy fix</td>
</tr>
<tr>
<td>BR2 constant (on hexes)</td>
<td>6</td>
<td>2</td>
</tr>
<tr>
<td>DG basis functions</td>
<td>parametric</td>
<td>non-parametric</td>
</tr>
<tr>
<td></td>
<td>[difference due to different flow solvers]</td>
<td></td>
</tr>
</tbody>
</table>
Taylor Green Vortex at Re=1600

DG, p=3 on 64³ mesh,
Comparison explicit vs. implicit RK
ERK vs. SDIRK

[Single coefficient diagonally implicit Runge-Kutta]

<table>
<thead>
<tr>
<th>Scheme</th>
<th>t. step size</th>
<th>Run time</th>
<th>Note</th>
</tr>
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<tbody>
<tr>
<td>ERK-4</td>
<td>0.001</td>
<td>1.0</td>
<td>reference</td>
</tr>
<tr>
<td>SDIRK-4</td>
<td>0.1</td>
<td>20.1</td>
<td>baseline</td>
</tr>
<tr>
<td></td>
<td></td>
<td>1.3</td>
<td>freeze Jacobian for 1 time step, tol = 10⁻¹²</td>
</tr>
<tr>
<td></td>
<td></td>
<td>0.5</td>
<td>freeze Jac. for 1 time step, tol = 10⁻⁴</td>
</tr>
<tr>
<td></td>
<td></td>
<td>0.4</td>
<td>freeze Jac. for 2 time steps, tol = 10⁻⁴ („optimized“)</td>
</tr>
<tr>
<td>ESDIRK-3</td>
<td>0.05</td>
<td>0.8</td>
<td>„optimized“</td>
</tr>
<tr>
<td>ESDIRK-2</td>
<td>0.025</td>
<td>1.3</td>
<td>„optimized“</td>
</tr>
</tbody>
</table>
ILES for the 2D Periodic Hill test case at $Re_b=2800$

Quadratic grid
64x32x32 elem.
Global $p$-adaptation
ERK-4

<table>
<thead>
<tr>
<th>$p$</th>
<th>DoFs/eqn</th>
<th>tc</th>
<th>dtc</th>
<th>#steps</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>66 K</td>
<td>0 - 2</td>
<td>2e-4</td>
<td>10 K</td>
</tr>
<tr>
<td>1</td>
<td>262 K</td>
<td>2 - 32</td>
<td>1e-4</td>
<td>300 K</td>
</tr>
<tr>
<td>2</td>
<td>656 K</td>
<td>32 - 62</td>
<td>5e-5</td>
<td>600 K</td>
</tr>
<tr>
<td>3</td>
<td>1.3 M</td>
<td>62 - 122</td>
<td>3.33e-5</td>
<td>1.8 M</td>
</tr>
</tbody>
</table>

64x32x32 = 65536 cells

$p = 3$  
$tc = 122.0000$
ILES for the 2D Periodic Hill test case at $Re_b=2800$

After global h-refinement:

Quadratic mesh
128x64x64 elem.
SDIRK-4

<table>
<thead>
<tr>
<th>$p$</th>
<th>DoFs/eqn</th>
<th>$tc$</th>
<th>dtc</th>
<th>#steps</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>10.5 M</td>
<td>122</td>
<td>182</td>
<td>1.66e-3</td>
</tr>
</tbody>
</table>

128x64x64 = 524288 cells
$p = 3$  
$tc = 182.000$
ILES for the 2D Periodic Hill test case at $Re_b=2800$
ILES for the 2D Periodic Hill test case at $Re_b=2800$
ILES for the 2D Periodic Hill test case at $Re_b=2800$
ILES for the 2D Periodic Hill test case at $Re_b=2800$
Channel flow, $\text{Re}_\delta = 6875$, $\text{Re}_\tau$(DNS)=392.24: Wall-resolved ILES

Prescribed data:
- Bulk Reynolds number $\text{Re}_\delta=U_{\text{bulk}} \cdot \delta/\nu$ and $M=0.1$

Measure quality of the (time-averaged) solution
- by comparing against DNS data:
  - friction Reynolds number $\text{Re}_\tau$
  - near-wall velocity profile $u^+(y^+)$

Computational mesh (gen. for hybrid RANS/LES [*]):
- $61 \times 64 \times 64 = 249856$ elements
- $\Delta x^+=41.15$, $\Delta y^+=0.78$, $\Delta z^+=19.61$

<table>
<thead>
<tr>
<th>p</th>
<th>DoFs/eqn</th>
<th>$T_c$</th>
<th>$\text{Re}_\tau$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1.0 M</td>
<td>0 - 420</td>
<td>297.65</td>
</tr>
<tr>
<td></td>
<td></td>
<td>420 - 450</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>2.5 M</td>
<td>450 - 510</td>
<td>380.20</td>
</tr>
<tr>
<td></td>
<td></td>
<td>510 - 540</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>5.0 M</td>
<td>540 - 570</td>
<td>392.30</td>
</tr>
<tr>
<td></td>
<td></td>
<td>570 - 600</td>
<td>391.41</td>
</tr>
<tr>
<td></td>
<td></td>
<td>600 - 630</td>
<td></td>
</tr>
</tbody>
</table>

Channel flow, $Re_\delta = 6875$, $Re_\tau$(DNS)=392.24: Wall-resolved ILES

$Re_\delta = 6875$, DNS[Moser, Kim, Mansour, 1999]: $Re_\tau = 392.24$
Channel flow, $Re_\delta = 6875$, $Re_\tau(DNS)=392.24$: Wall-resolved ILES

$Re_\delta = 6875$, DNS[Moser, Kim, Mansour, 1999]: $Re_\tau = 392.24$
A wall-stress-model approach

For each boundary face integration point $p_{\text{dest}}$:

- Find a point $p_{\text{donor}}$ normal to the wall in a distance of (approx.) $y=0.2\delta$.
- From the solution (instantaneous flow field) at point $p_{\text{donor}}$ take
  - tangential velocity $\mathbf{v}_t = (I - \mathbf{n} \otimes \mathbf{n}) \mathbf{v}$
  - density $\rho$, kinematic viscosity $\nu$, and
  - distance $y = \text{dist}(p_{\text{donor}}, p_{\text{dest}})$.
- Solve $u^+(y^+) = |\mathbf{v}_t|/u_\tau$ with $y^+=yu_\tau/\nu$, i.e.
  $$F(u_\tau) = |\mathbf{v}_t|/u_\tau - u^+(y u_\tau/\nu) = 0, \quad \text{for } u_\tau.$$
- Compute wall shear stress $\tau_w = \rho u_\tau^2$.
- In $p_{\text{face}}$ apply slip-wall bc ($\mathbf{v} \cdot \mathbf{n}=0$) and a viscous numerical flux $\mathbf{n} \cdot F^\nu$ with prescribed $\tau_w$:

Given the normal viscous flux

$$\mathbf{n} \cdot F^\nu(\mathbf{u}, \nabla \mathbf{u}) = (0, (\tau \mathbf{n})_i, \mathbf{n} \cdot (\tau \mathbf{v}) + K \mathbf{n} \cdot \nabla T)$$

we split $(\tau \mathbf{n})_i = \tau_{ij} n_j$ into a wall normal and wall tangential part as follows

$$(\tau \mathbf{n}) = (\tau \mathbf{n})_n + (\tau \mathbf{n})_t \quad \text{with } (\tau \mathbf{n})_t = (I - \mathbf{n} \otimes \mathbf{n}) (\tau \mathbf{n})$$

replace $(\tau \mathbf{n})_t$ by $(\tau \mathbf{n})_{t,wm} = -\tau_w \mathbf{v}_t$, and use the adiabatic condition $K \mathbf{n} \cdot \nabla T=0$. 
Near-wall velocity profiles

\( u(y) \) scaled with the friction velocity \( u_\tau \) and the kinematic viscosity \( \nu \):

\[ u^+(y^+) \quad \text{with} \quad u^+ = \frac{u}{u_\tau} \quad \text{and} \quad y^+ = \frac{y}{u_\tau/\nu} \]

**Algebraic near-wall velocity profiles:**

- **Logarithmic law-of-wall (log-law):**
  \[ u^+(y^+) = \min( y^+, \frac{\ln(y^+)}{\kappa} + c ) \]
  - Von Karman constant \( \kappa \in [0.38,0.41] \) and \( c \in [4.1,5.1] \)
  - Take \( \kappa = 0.38 \) and \( c = 4.1 \) [Österlund et al., 2000]

- **Reichardt’s law-of-wall:**
  \[ u^+(y^+) = \ln(1 + \kappa y^+)/\kappa + A(1 - e^{-y^+/B} - y^+/B e^{-y^+/C}) \]
  - \( A \in [6.6,7.8] \), \( B = 11 \), \( C = 3 \)
  - Take \( A = c - \ln(\kappa)/\kappa \) [Frere, de Wiart, Hillewart et al., 2017]

- **Spalding’s (inverse) law-of-wall:**
  \( y^+(u^+) \)

„**Exact“ near-wall velocity profile:"

- Moser, Kim, Mansour (1999): DNS of turbulent channel flow up to \( Re_\tau = 590 \).
Near-wall velocity profiles

\( u(y) \) scaled with the friction velocity \( u_\tau \) and the kinematic viscosity \( \nu \):

\[
\frac{u(y)}{u_\tau} = \left( \frac{u}{u_\tau} \right) \quad \text{with} \quad \frac{y}{u_\tau} = \frac{y}{u_\tau} / \nu
\]

**Algebraic near-wall velocity profiles:**

- **Logarithmic law-of-wall (log-law):**
  \[
  \frac{u(\nu)}{u_\tau} = \min( \frac{y}{u_\tau}, \ln(\frac{y}{u_\tau}) / \kappa + \epsilon )
  \]
  - Von Karman constant \( \kappa \) ∈ [0.38, 0.41] and \( \epsilon \) ∈ [4.1, 5.1]
  - Take \( \kappa=0.38 \) and \( \epsilon=4.1 \) [Österlund et al., 2000]

- **Reichardt’s law-of-wall:**
  \[
  \frac{u(\nu)}{u_\tau} = \ln(1 + \kappa \frac{y}{u_\tau}) / \kappa + A(1 - e^{-\frac{\nu}{B}} - y^+/B e^{-\frac{\nu}{C}})
  \]
  - \( A \) ∈ [6.6, 7.8], \( B = 11 \), \( C = 3 \)
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„**Exact“ near-wall velocity profile:“**

- **Moser, Kim, Mansour (1999): DNS** of turbulent channel flow up to \( \text{Re}_\tau = 590 \).
Channel flow, $\text{Re}_\delta = 6875$, $\text{Re}_\tau (\text{DNS}) = 392.24$: Wall-modelled ILES vs. wall-resolved ILES

<table>
<thead>
<tr>
<th>p</th>
<th>DoFs/eqn</th>
<th>ILES $\text{Re}_\tau$</th>
<th>WM-ILES $u^+(y^+)$=Reichardt $\text{Re}_\tau$</th>
<th>WM-ILES $u^+(y^+)$=DNS data $\text{Re}_\tau$</th>
<th>DNS $\text{Re}_{\tau,\text{DNS}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1.0e6</td>
<td>293.08 (-25.3%)</td>
<td>415.24 (5.9%)</td>
<td>408.98 (4.3%)</td>
<td>392.24</td>
</tr>
<tr>
<td></td>
<td></td>
<td>291.23 (-25.8%)</td>
<td>418.81 (6.8%)</td>
<td>409.89 (4.5%)</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>2.5e6</td>
<td>379.88 (-3.2%)</td>
<td>403.75 (2.9%)</td>
<td>400.42 (2.1%)</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>379.41 (-3.3%)</td>
<td>404.93 (3.2%)</td>
<td>399.78 (1.9%)</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>5.0e6</td>
<td>391.86 (-0.1%)</td>
<td>404.93 (3.2%)</td>
<td>399.78 (1.9%)</td>
<td></td>
</tr>
</tbody>
</table>

Table 4: Channel flow at $\text{Re}_\delta = 6875$: Comparison of friction Reynolds numbers for ILES and two versions of WM-ILES (once $u^+(y^+)$ is given by Reichardt’s law and once given by DNS data) compared to $\text{Re}_{\tau,\text{DNS}}$ [Moser et al., 1999]. $\text{Re}_\tau$ values are computed from solutions averaged over 60 CTU (for ILES) and 120 CTU (for WM-ILES). Deviations of computed $\text{Re}_\tau$ values from $\text{Re}_{\tau,\text{DNS}}$ are given as percentages.
Channel flow, $Re_\delta = 6875$, $Re_\tau(DNS)=392.24$: Wall-modelled ILES vs. wall-resolved ILES
Boeing Rudimentary landing gear

$U=40\text{m/s},~(M\approx0.12),~Re=UD/v\approx10^{6}$

Grids derived from structured grid (ATAAC)
• coarse (115k), medium (924k), fine (7.4M)

medium quadratic grid with 924k elements
ILES for the Boeing RLG

For \( p=2 \) on medium grid:
- ERK-4: 50e6 steps/CTU
- SDIRK-4: 200 steps/CTU

Computational time (ERK/SDIRK) = 100*

<table>
<thead>
<tr>
<th>grid</th>
<th># el</th>
<th>( p )</th>
<th>DoFs/eqn</th>
<th>( \Delta t_c )</th>
<th>( t_c )</th>
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<tbody>
<tr>
<td>coarse</td>
<td>115K</td>
<td>1</td>
<td>0.5 M</td>
<td>0.01</td>
<td>0-30</td>
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<tr>
<td>medium</td>
<td>924K</td>
<td>1</td>
<td>3.7 M</td>
<td>0.01</td>
<td>30-60</td>
</tr>
<tr>
<td></td>
<td></td>
<td>2</td>
<td>9.2 M</td>
<td>0.005</td>
<td>60-66.6</td>
</tr>
</tbody>
</table>

Medium quadratic grid
\( p=2 \) (3rd order DG)
local Lax-Friedrichs flux

Instantaneous flow field at \( t_c=65.6 \):
- Iso-surfaces of Q criterion
\( Q (D/U_\infty)^2=50 \)
colored with the vorticity magn.

Computation broke after \( t_c=66.6 \)
Next steps:
- BCs based on Riemann invariants
- Sponge layer near outflow?
- WM-ILES
Summary

- DG discretization of equations, integral quantities and local quantities
- Details on discretization settings and results for RANS and ILES:
  - RANS and Wilcox-kω for the DLR-F11 high lift configuration
  - Computational time(ERK/SDIRK)=2.5(TGV), 100*(Boeing RLG)
  - **Wall-resolved ILES** for 2D periodic hill and channel flow
    with results very close to DNS data
  - **Wall-modelled ILES** for channel flow
    with significant improvement over ILES
- Current state of ILES computations for the Boeing rudimentary landing gear

RANS-kω with p=2  
ILES with p=3  
ILES with p=2