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Experimental Determination of the Aerodynamic Diameters of Particles Across a Shock Wave

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Julian T. Herzog



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## Experimental Determinationof the Aerodynamic Diameters of Particles Across a Shock Wave

## Übersicht:

Im Kontext einer Messkampagne am kryogenen Rohrwindkanal in Göttingen wurden Algorithmen und Softwarewerkzeuge entwickelt, um PTV-Messungen auszuwerten sowie diese mit gegebenen PIV-Strömungsfeldern zu vergleichen. Ziel der Messungen war es, besser zu verstehen, wie sich die Strömung über ein Laminarprofil unter dem Einfluss von Eispartikeln in der Anströmung verändert. Hierbei sollte eine Vergleichbarkeit mit einem kommerziellen Linienflug durch Zirruswolken gegeben sein. Um im Experiment die Durchmesser der Eispartikel zu bestimmen, wurden Partikeltrajektorien an der Stelle ermittelt, an der diese den Verdichtungsstoß auf der Oberseite des Profils überquerten. Schlupfgeschwindigkeiten wurden ermittelt, indem von den PTV-Geschwindigkeiten ein PIV-Strömungsfeld abgezogen wurde. Letzteres wurde mittels eines Windkanaltests mit feinerem Seeding bestimmt. Die PTV-Messung wurde mit acht Pulsen pro Bild und einer einzelnen Kamera aufgenommen, unter Verwendung von Eispartikeln verschiedener Größe als Seeding-Material. Ziel war es, eine höhere Genauigkeit zu erreichen, als mit bestehenden Verfahren möglich ist. Zu den besonderen Herausforderungen hierbei gehörte unter anderem, Abstände zwischen Partikelbildern präzise zu bestimmen, welche sowohl größer als wünschenswert als auch asymmetrisch sind. Auch die korrekte Korrelation der Spuren aus acht Laserpulsen über lange Distanzen und in unterschiedlich dichtem Seeding erforderte einen spezifisch entwickelten Ansatz. Die hier entwickelte Methode zur Bestimmung von Partikelbildabständen basiert auf einer Optimierung der aufsummierten quadrierten Intensitätsdifferenz zwischen zwei Partikelbildern, welche durch B-Splines fünften Grades interpoliert werden. Hierbei wird die relative Position der Bilder, welche die Kostenfunktion minimiert, mit Subpixelgenauigkeit gefunden. Die Ergebnisse der Validierung zeigen eine deutlich bessere Genauigkeit der Methode im Vergleich zur Intensitätsschwerpunktsmethode und zum Fitting einer Gaußfunktion. Um PTV-Spuren zu finden wird eine Wahrscheinlichkeitsverteilung des Geschwindigkeitsvektors an der Position jedes Partikelbildes generiert, welche auf den Partikelbildpositionen und -intensitäten in der direkten Umgebung basiert. Das Ergebnis wird verwendet, um über große Entfernungen und auf Basis grober Partikelpositionen die Wahrscheinlichkeit zu ermitteln, mit der ein anderes Partikelbild zur selben Spur gehört. Mithilfe eines Algorithmus zum Fitting der Schlupfgeschwindigkeiten an ein Modell der Partikelbewegung wurden für mehrere Beispielbilder Partikeldurchmesser bestimmt. Eine Beurteilung der Fehlerquellen ermöglicht es, die Parameter für die Messung und die Analyse in Zukunft weiter zu verbessern.

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# Experimental Determination of the Aerodynamic Diameters of Particles <br> <br> Across a Shock Wave 

 <br> <br> Across a Shock Wave}

Master's thesis by<br>Julian T. Herzog

Institute of Aerodynamics and Flow Technology
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University of Stuttgart

Göttingen, December of 2017

# Experimental Determination of the Aerodynamic Diameters of Particles 

## Across a Shock Wave

Master's thesis by<br>Julian T. Herzog

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Master's Thesis/Objective<br>for Julian T. Herzog



Institute of Aerodynamics
and Flow Technology

## Experimental Determination of the Aerodynamic Diameters of Particles Across a Shock Wave

The investigation of key technologies for the development of transonic wing profiles with reduced drag is part of the Low Drag Aircraft in Operation (LDAinOP) project. Among other aspects, this involves research to better understand the negative effects of atmospheric ice particles in clouds on the effectivity of low drag laminar profiles in flight. To identify the relevant parameters, a model of a two-dimensional laminar profile is analyzed in the cryogenic wind tunnel DNW-KRG at Reynolds and Mach numbers relevant for airline flight operations. For characterizing the ice particles that are artificially generated in the wind tunnel, Particle Tracking Velocimetry (PTV) is used. Using the PTV data, the aerodynamic diameter of the particles is determined by analyzing the particle trajectories across a shock wave. This analysis is performed using two software algorithms. The first is used to determine the particle positions while the second solves the equations of motion for each particle. The accuracy of determining the position of the particles is decisive for the precision of this method.
As part of a wind tunnel measurement campaign, the thesis will seek to determine the aerodynamic diameter of ice particles in the flow across a shock wave using PTV measurement data. Additionally, the goal is to improve the precision of the results by increasing the accuracy of existing software solutions for determining the particle positions.
The project is divided into the following tasks:

- Literature review.
- Investigation of the precision that can be achieved based on existing measurements.
- Revision of the current software tools to improve the precision of the particle position determination.
- Creation of an interface between the two existing software algorithms.
- Installation and calibration of the measuring equipment for PTV.
- Assistance in performing the PTV measurements as part of the wind tunnel measurement campaign.
- Analysis of the particle trajectories that were recorded during the campaign.
- Documentation of the results of the PTV measurements.
- Documentation of the changes made to the analysis software.

The work is performed at the German Aerospace Center (DLR) site in Göttingen. The candidate is referred to the leaflet for master's theses at the IAG (see the IAG homepage).

Begin of work: 01.06.2017
Handed in: 31.12.2017


Julian T. Herzog


Dr. -Ing. Robert Konrath (DLR)

## Declarations

## Erklärung

Hiermit versichere ich, dass ich diese Masterarbeit selbstständig mit Unterstützung der Betreuer angefertigt und keine anderen als die angegebenen Quellen und Hilfsmittel verwendet habe.

Die Arbeit oder wesentliche Bestandteile davon sind weder an dieser noch an einer anderen Bildungseinrichtung bereits zur Erlangung eines Abschlusses eingereicht worden.

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Göttingen, den 31. Dezember 2017

Julian Herzog

## Statement of Originality

This thesis has been performed independently with support of my supervisors. It contains no material that has been accepted for the award of a degree at this or any other university. To the best of my knowledge and belief, this thesis contains no material previously published or written by another person except where due reference is made in the text.

I further declare that I have performed this thesis according to the existing copyright policy and the rules of good scientific practice. In case this works contains contributions of someone else (e.g. pictures, drawings, text passages etc.), I have clearly identified the source of these contributions, and, if necessary, have obtained approval from the originator for making use of them in this thesis. I am aware that I have to bear the consequences in case I have contravened these duties.


#### Abstract

As part of a measurement campaign at the cryogenic Ludwieg tube type wind tunnel in Göttingen, algorithms and software tools were developed for the analysis of particle tracking velocimetry (PTV) measurements as well as the comparison to a given particle image velocimetry (PIV) flow field. The aim of the measurements was to better understand how the performance of a laminar airfoil changes under the influence of ice particles in the flow, in conditions comparable to a commercial airliner cruise flight through cirrus clouds. For determining the diameters of the ice particles in the experiment, they were tracked as they crossed the recompression shock wave on the upper airfoil model surface. Slip velocities were derived from the PTV velocities by subtracting a PIV-determined flow field.

The goal was to achieve a higher accuracy than is possible using existing methods for a single-camera PTV measurement, with eight illumination pulses per image, using ice particles of varying sizes as the seeding material. Among the unique challenges was the determination of distances between particle images that were larger than desirable as well as non-symmetrical in shape. Correctly correlating eight-pulse traces across long distances in variable seeding densities required a custom approach as well.

The particle image distance determination method that was developed uses a squared intensity difference sum minimization approach in which both particle images are interpolated using fifth degree B-splines. Then, the particle image offset which minimizes the cost function is found with subpixel accuracy. Validation results indicate significantly superior accuracy compared to centroid and Gaussian peak fitting methods. For the assembly of PTV traces, a flow vector probability distribution is generated at each particle image position using nearby particle image positions and intensities. The result is used to determine a likelihood for any nearby particle to belong to the same trace, across long distances, based on an approximate initial search vector and area.

Particle diameters have been determined for several examples by fitting the slip velocities to a model for the particle motion. An evaluation of error sources can be used to improve measurement parameters and analysis techniques in the future.


## Kurzfassung

Im Kontext einer Messkampagne am kryogenen Rohrwindkanal in Göttingen wurden Algorithmen und Softwarewerkzeuge entwickelt, um PTV-Messungen auszuwerten sowie diese mit gegebenen PIV-Strömungsfeldern zu vergleichen. Ziel der Messungen war es, besser zu verstehen, wie sich die Strömung über ein Laminarprofil unter dem Einfluss von Eispartikeln in der Anströmung verändert. Hierbei sollte eine Vergleichbarkeit mit einem kommerziellen Linienflug durch Zirruswolken gegeben sein. Um im Experiment die Durchmesser der Eispartikel zu bestimmen, wurden Partikeltrajektorien an der Stelle ermittelt, an der diese den Verdichtungsstoß auf der Oberseite des Profils überquerten. Schlupfgeschwindigkeiten wurden ermittelt, indem von den PTV-Geschwindigkeiten ein PIV-Strömungsfeld abgezogen wurde. Letzteres wurde mittels eines Windkanaltests mit feinerem Seeding bestimmt.

Die PTV-Messung wurde mit acht Pulsen pro Bild und einer einzelnen Kamera aufgenommen, unter Verwendung von Eispartikeln verschiedener Größe als Seeding-Material. Ziel war es, eine höhere Genauigkeit zu erreichen, als mit bestehenden Verfahren möglich ist. Zu den besonderen Herausforderungen hierbei gehörte unter anderem, Abstände zwischen Partikelbildern präzise zu bestimmen, welche sowohl größer als wünschenswert als auch asymmetrisch sind. Auch die korrekte Korrelation der Spuren aus acht Laserpulsen über lange Distanzen und in unterschiedlich dichtem Seeding erforderte einen spezifisch entwickelten Ansatz.

Die hier entwickelte Methode zur Bestimmung von Partikelbildabständen basiert auf einer Optimierung der aufsummierten quadrierten Intensitätsdifferenz zwischen zwei Partikelbildern, welche durch B-Splines fünften Grades interpoliert werden. Hierbei wird die relative Position der Bilder, welche die Kostenfunktion minimiert, mit Subpixelgenauigkeit gefunden. Die Ergebnisse der Validierung zeigen eine deutlich bessere Genauigkeit der Methode im Vergleich zur Intensitätsschwerpunktsmethode und zum Fitting einer Gaußfunktion. Um PTV-Spuren zu finden wird eine Wahrscheinlichkeitsverteilung des Geschwindigkeitsvektors an der Position jedes Partikelbildes generiert, welche auf den Partikelbildpositionen und -intensitäten in der direkten Umgebung basiert. Das Ergebnis wird verwendet, um über große Entfernungen und auf Basis grober Partikelpositionen die Wahrscheinlichkeit zu ermitteln, mit der ein anderes Partikelbild zur selben Spur gehört.

Mithilfe eines Algorithmus zum Fitting der Schlupfgeschwindigkeiten an ein Modell der Partikelbewegung wurden für mehrere Beispielbilder Partikeldurchmesser bestimmt. Eine Beurteilung der Fehlerquellen ermöglicht es, die Parameter für die Messung und die Analyse in Zukunft weiter zu verbessern.

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## Nomenclature

## Latin Symbols

| Symbol | Description | Unit |
| :--- | :--- | :--- |
| $a$ | acceleration | $\left[\mathrm{m} \mathrm{s}^{-2}\right]$ |
| $C$ | concentration in a volume | $\left[\mathrm{kg} \mathrm{m}^{-3}\right]$ |
| $c, C$ | coefficient | $[1]$ |
| $c$ | chord length | $[\mathrm{m}]$ |
| $c_{\mathrm{d}}$ | dimensionless drag coefficient | $[1]$ |
| $c_{\mathrm{i}}$ | integration constant | $[1]$ |
| $c_{1}$ | dimensionless lift coefficient | $[1]$ |
| $C_{n}^{m}$ | Zernike coefficient | $[1]$ |
| $C_{p}$ | pressure coefficient | $\left[\mathrm{Jkg}^{-1} \mathrm{~K}^{-1}\right]$ |
| $c_{p}$ | specific isobaric heat capacity | $\left[\mathrm{Jkg}^{-1} \mathrm{~K}^{-1}\right]$ |
| $c_{v}$ | specific isochoric heat capacity | $[\mathrm{m}]$ |
| $D$ | diameter | $\mathrm{from} \mathrm{context}$, |
| $d$ | distance between two entities | $\mathrm{usually}[\mathrm{px}]$ |
|  |  |  |
| $E$ | expected value of a random variable | $[\mathrm{N}]$ |
| $F$ | force | $\left[\mathrm{s}^{-1}\right]$ |
| $f$ | frequency | $\left[\mathrm{N} \mathrm{kg}^{-1}\right]$ |
| $f_{\mathrm{pdf}}$ | Gaussian probability density function | $[\mathrm{m}]$ |
| $\mathbf{g}$ | gravitational field | $[1]$ |
| $H$ | horizontal dimension of an object | $\left[\mathrm{s}^{-1} \mathrm{~m}^{-2}\right]$ |
| $I$ | intensity | particle image |
| I | flux |  |
| $j$ |  |  |


| Symbol | Description | Unit |
| :---: | :---: | :---: |
| $k$ | degree of a polynomial or spline |  |
| $L$ | length (most extended dimension of an object) | [m] |
| $l_{\text {f }}$ | focal length | [m] |
| M | Mach number | [1] |
| $m$ | mass | [kg] |
| $m$ | angular frequency of a Zernike polynomial | [1] |
| $n$ | (natural) number, usually designating the number of entities of a given category or type |  |
| $n$ | radial order of a Zernike polynomial | [1] |
| $n_{\mathrm{r}}$ | refractive index | [1] |
| $\mathrm{OO}^{\prime}$ | object to image distance | [m] |
| $p$ | pressure | [Pa] |
| $p$ | particle index variable |  |
| $R$ | average roughness | [m] |
| R | Reynolds number | [1] |
| $R$ | specific gas constant | [ $\mathrm{Jkg}^{-1} \mathrm{~K}^{-1}$ ] |
| $r$ | resolution in the focal plane | [m] |
| $r$ | radius | from context: $[\mathrm{m}],[\mathrm{px}] \text { or }[1]$ |
| $R_{n}^{m}$ | radial function within a zernike polynomial | [1] |
| $r_{f()}$ | random sample picked from distribution function $f()$ | [1] |
| S | Stokes number | [1] |
| $s$ | scaling factor | [1] |
| $s_{\text {A }}^{\prime}$ | flange focal length | [m] |
| T | temperature | [K], if specified: $\left[{ }^{\circ} \mathrm{C}\right]$ |
| T | optical (modulation) transfer function | [1] |
| $t$ | time | [s] |
| $u$ | velocity | [ $\mathrm{m} \mathrm{s}^{-1}$ ] |
| $u^{\prime}$ | radial position on the projected image | [m] |
| $u_{\text {s }}$ | slip velocity | [ $\mathrm{m} \mathrm{s}^{-1}$ ] |


| Symbol | Description | Unit |
| :--- | :--- | :--- |
| $V$ | vertical dimension of an object | $[\mathrm{m}]$ |
| $w$ | (spatial) width | $[\mathrm{m}]$ |
| $w_{\mathrm{s}}$ | span width | $[\mathrm{m}]$ |
| We | Weber number of the fluid | $[1]$ |
| $W$ | weight, i.e. a number that represents an estimate of the | $[1]$ |
| $x$ | relative quality of a tested connection between two particles |  |
| $x$ | position in the first spatial direction | generally $[\mathrm{m}]$ |
| $x_{\text {FOV }}$ | independent variable in a function |  |

## Greek Symbols

| Symbol | Description | Unit |
| :--- | :--- | :--- |
| $\alpha$ | angle relative to the optical axis | $[\mathrm{rad}]$ |
| $\alpha_{\{a, b\}}$ | angle between some element $a$ and a second element $b$ | $[\mathrm{rad}]$, if specified: |
| $\beta$ |  | $\left[{ }^{\circ}\right]$ |
|  | angle representing the overall non-straightness of the trace | $[\mathrm{rad}]$, if specified: |
| $\epsilon$ | rotation of the target relative to the sensor plane | $\left[{ }^{\circ}\right]$ |
| $\gamma$ | adiabatic index | $[\mathrm{rad}]$ |
| $\kappa$ | non-dimensional wavenumber | $[1]$ |
| $\lambda$ | dynamic viscosity of the fluid | $[1]$ |
| $\mu_{\mathrm{f}}$ | mean/expectation/mode in a normal distribution | $[\mathrm{m}]$ |
| $\mu$ | normalized spatial frequency | $[1]$ |
| $\nu$ | spectroscopic wavenumber (number of wavelengths per unit | $\left[\mathrm{m}^{-1}\right]$ |
| $\tilde{\nu}$ | distance) | $\left[\mathrm{m}^{2} \mathrm{~s}^{-1}\right]$ |
| $\nu_{\mathrm{f}}$ | kinematic viscosity of the fluid | $\left[\mathrm{kg} \mathrm{m}^{-3}\right]$ |
| $\rho$ | density | standard deviation |


| Symbol | Description | Unit |
| :--- | :--- | :--- |
| $\tau$ | relaxation time | $[\mathrm{s}]$ |
| $\theta$ | angle relative to the refracting surface | $[\mathrm{rad}]$ |
| $\vartheta$ | angular resolution | $[\mathrm{rad}]$ |
| $\varphi$ | image smoothing factor | $[1]$ |

## Modifiers

| Modifier | Description |
| :---: | :---: |
| $G(1)$ | Two-dimensional Gaussian PDF convolution kernel |
| $D_{n}(\square)$ | distortion function |
| $f(-)$ | function |
| $\operatorname{sgn}(\square)$ | signum/sign function |
| std ( ) | standard deviation function |
| - | arithmetic mean |
| $\cdots$ | derivative with respect to $t$ |
| ^ | value given in the image coordinate system, relative to the respective maximum extent from the center of the image in the given dimension |
| ~ | median |
| $\Delta$ | deviation of the given quantity |
| $\nabla$ | gradient of the given quantity |
| L-1 | nearest integer of the given value |
| \{ $\quad$ \} | vector (generally Euclidean) |
| \| 1 | absolute value |
| \|| - || | Euclidean norm/ $L^{2}$ norm |
| x | Euclidean vector |

## Subscript Indices

| Index | Description |
| :--- | :--- |
| $\infty$ | in the undisturbed flow |

Continued on the following page

| Index | Description |
| :---: | :---: |
| - | stagnation/total state variable |
| 0 | cutoff point |
| 1 | first image dimension (horizontal) |
| 1 | before refraction |
| I | ahead of the shock wave |
| a | first instance |
| 2 | second image dimension (vertical) |
| 2 | after refraction |
| II | behind the shock wave |
| b | second instance |
| c | third instance |
| A | regarding the aperture |
| a | regarding the airfoil |
| abs | absolute (for pressure values: including atmospheric pressure) |
| C | charge state variable (Ludwieg tube) |
| ${ }^{\text {c }}$ | chord-referenced |
| c | regarding a connection between two particle images |
| cr. | critical |
| d | regarding greyscale dilation |
| e | effective |
| f | regarding the fluid |
| G | regarding Gaussian blur |
| I | regarding the overall particle intensity |
| ${ }_{i}$ | dimension |
| 1 | iterator variable, or individual element within a set |
| i | value in image coordinates |
| ${ }_{j}$ | iterator variable, or individual element within a set |
| k | regarding a kernel |
| M | regarding test/weight maps in general |
| m | wind tunnel model case |
| $m$ | regarding a single, discrete map entry |


| Index | Description |
| :---: | :---: |
| ${ }_{\text {max }}$ | maximum |
| 1 min | minimum |
| n | noise |
| $n$ | radial order of a Zernike polynomial |
| ${ }^{\text {NF }}$ | regarding the NFT camera |
| np | regarding a neighbor particle |
| ${ }_{\text {obj }}$ | objective, cost, penalty in the context of minimization |
| -obs. | regarding the point being observed |
| p | regarding a particle (or particle image) |
| p | regarding a pixel |
| ${ }^{1 / d}$ | determined through a distance determination method |
| 1 ref. | reference case |
| s | search area neighbor candidate particle |
| s | regarding the shock wave |
| 1 sat. | saturation limit |
| 1 st. | in standard atmospheric conditions |
| ${ }_{\text {T }}$ | total value |
| t | regarding a trace, or traces in general |
| tp | regarding a test particle |
| WF | regarding the WFT camera |
| ${ }_{y}$ | position in the second spatial direction |
| $1{ }_{z}$ | position in the third spatial direction |

## Superscript Indices

| Index | Description |
| :--- | :--- |
| $\square^{\prime}$ | value derived form the respective quantity |
| virtual or abstract |  |


| Index | Description |
| :--- | :--- |
| $(-1)$ | regarding the pixel next to the respective pixel position, in the negative <br> direction of the given dimension $i$ |
| L | lower bound |
| $m$ | angular frequency of a Zernike polynomial |
| U | upper bound |

## Acronyms

| Notation | Description |
| :---: | :---: |
| $k$-d tree | $k$-dimensional binary tree. |
| ADC | analog-to-digital converter. |
| AGARD | Advisory Group for Aerospace Research and Development. |
| AIAA | American Institute of Aeronautics and Astronautics. |
| AOA | angle of attack. |
| ASCII | American Standard Code for Information Interchange character encoding standard. |
| BBO equation | Basset-Boussinesq-Oseen equation. |
| BNC | Bayonet Neill-Concelman. |
| BSD | Berkeley Software Distribution. |
| CIV | in the vicinity of clouds. |
| CL | Camera Link. |
| CONSAVE | Constrained Scenarios on Aviation and Emissions. |
| CPI | cloud particle imager. |
| CSV | comma-separated values. |
| DIRECT | Dividing Rectangles. |
| DLR | German Aerospace Center (Deutsches Zentrum für Luft- und Raumfahrt e. V., DLR). |
| DNW | German-Dutch Wind Tunnels. |
| DOF | depth of field. |
| EU | European Union. |


| Notation | Description |
| :---: | :---: |
| FL | flight level (altitude at standard pressure in hundreds of feet). |
| FOV | field of view. |
| FWHM | full width at half maximum. |
| GASP | Global Atmospheric Sampling Program. |
| GCC | GNU Compiler Collection. |
| GmbH | Gesellschaft mit beschränkter Haftung. |
| GUI | graphical user interface. |
| IDLSS | interpolated derivative least squares shift. |
| IEA | International Energy Agency. |
| IEEE | Institute of Electrical and Electronics Engineers. |
| IILSS | interpolated intensity least squares shift. |
| IPCC | Intergovernmental Panel on Climate Change. |
| IPG | ice particle generator. |
| ISDLSS | interpolated smoothed derivative least squares shift. |
| ISILSS | interpolated smoothed intensity least squares shift. |
| IWC | ice water content (cloud ice mass per unit volume of air). |
| KRG | cryogenic Ludwieg tube Göttingen. |
| LDAinOp | Low Drag Aircraft in Operation. |
| LED | light-emitting diode. |
| LEFT | Laminar Flow Control Leading Edge Glove Flight Test Article Development. |
| LF | laminar flow. |
| LFC | laminar flow control. |
| LOSU | level of scientific understanding. |
| MTF | modulation transfer function. |
| NA | numerical aperture. |
| NASA | National Aeronautics and Space Administration. |
| NATO | North Atlantic Treaty Organization. |
| NC | neighbor candidate. |
| Nd:YAG | neodymium-doped yttrium aluminium garnet. |
| NFT camera | narrow field tracking camera. |
| NLF | natural laminar flow. |


| Notation | Description |
| :---: | :---: |
| NPP | no-parallax point. |
| OpenCV | Open Source Computer Vision. |
| PD5 | concentration of particles larger than $3 \mu \mathrm{~m}$ in diameter. |
| PDF | probability density function. |
| PI | particle image. |
| PIC | particle image center. |
| PIV | particle image velocimetry. |
| PSD | particle size distribution. |
| PSF | point spread function. |
| PTV | particle tracking velocimetry. |
| RF | radiative forcing. |
| rpm | revolutions per minute. |
| $\mathrm{Ru}($ trpy $)$ | Di(tripyridyl)ruthenium(II). |
| SA | Simulated Annealing. |
| SAGE | system for assessing aviation's global emissions. |
| SAS | simulated airline service. |
| SNR | signal-to-noise ratio. |
| SQP | Sequential Quadratic Programming. |
| TIC | time in clouds. |
| TICIV | fraction of time in clouds when in the vicinity of clouds. |
| TSP | temperature sensitive paint. |
| URL | Uniform Resource Locator (web address). |
| US | United States. |
| USAF | US Air Force. |
| USCS | US customary system. |
| WFT camera | wide field tracking camera. |

## 1

## Background

This chapter aims to offer some context for the motivation behind the developments that were made as part of this thesis. This includes arguments for the importance and relevance of the goal to better understand the influence of ice particles on the laminar flow across an airfoil at flight-relevant Reynolds numbers. Beyond that, previous work relating to this topic, both historical and relatively recent, is summarized and discussed. Additionally, the theoretical work and assumptions which constitute the foundation for the analysis steps are described.

### 1.1 Environmental Impact of Commercial Aviation

Aviation has a significant impact on the climate, both in the short term as a result of, most notably, water vapor, particles and nitrogen oxides $\left(\mathrm{NO}_{\mathrm{x}}\right)$, as well as in the long term, mainly as a result of $\mathrm{CO}_{2}$ emissions. Here, short term effects refer to effects lasting several decades while long term effects are likely to affect mean surface temperatures for many centuries (D. S. Lee, Pitari, et al., 2010). As estimated by D. S. Lee, Fahey, et al. (2009), the total amount of radiative forcing as a result of aviation in the year 2005 was at approximately $55 \mathrm{~mW} \mathrm{~m}^{-2}$ not considering, and $78 \mathrm{~mW} \mathrm{~m}^{-2}$ including the effect of cirrus cloud enhancement. This is estimated to correspond with $3.5{ }_{-2.2}^{+6.5} \%$ and $4.9_{-2.9}^{+9.1} \%$ of total anthropogenic forcing, respectively. According to D. S. Lee, Pitari, et al., there is a high remaining spread between models - especially regarding the effects of high-altitude $\mathrm{NO}_{\mathrm{x}}$ emissions on ozone, the effect of contrails and the general influence of aviation on cloud formation. The influence and confidence of different emission types is summarized in Figure 1.1a by IPCC working group I (2007), followed by an overview over the contributions that have so far been significant in aviation (Figure 1.1b). These estimates show that the overall effect is very likely to be a significant positive contribution to radiative forcing, with the largest influence from $\mathrm{CO}_{2}$ and $\mathrm{NO}_{\mathrm{x}}$ emissions.

Due to the projected increase in the overall volume of air transport in the coming decades, even moderate improvements in fuel efficiency are likely to result in an increase in the overall
greenhouse gas emissions from the aviation industry. Figure 1.2, taken from D. S. Lee, Pitari, et al. (2010), gives an overview over different projections for fuel-based emissions from aviation. In the European Union (EU), current policy aims to reduce greenhouse gas emissions by $40 \%$ by the year 2040 compared to the 1990 levels (Delbeke and Vis, 2016). Based on the previously mentioned projections, aviation will then significantly increase its share of greenhouse gas emissions in the near future. This means that airlines and aircraft manufacturers find themselves under pressure to achieve very significant reductions in emission levels. This is accentuated by the dependence of aviation on fossil fuels due their the superior energy density compared to most other means of energy storage, which is likely to be the case far into the future. The IPCC working group III (2007, pp. 353-355) highlights that engine developments to reduce emissions are limited, partly by the increase in $\mathrm{NO}_{\mathrm{x}}$ emissions as a result of increased pressure ratios that would further improve the fuel efficiency and reduce $\mathrm{CO}_{2}$ emissions. The report lists structural weight savings due to composite materials, drag reduction through laminar flow control (LFC), as well as airframe and engine technology developments as promising development elements for long-term fuel burn reductions. Additionally, the usage of alternative fuels, procedural changes in air traffic control as well as lower flight speeds are named as potential contributing measures not directly related to the aircraft system.


Figure 1.1: Global average radiative forcing (RF) estimates and ranges in 2005, by component. LOSU: level of scientific understanding
a) Global average total RF estimates between 1750 amd 2005. Figure by IPCC working group I (2007, p. 4) (AR4: Fourth Assessment Report).
b) Global aviation impact, from preindustrial times until 2005. Figure by D. S. Lee, Pitari, et al. (2010, p. 4714).


Figure 1.2: Different projections of the $\mathrm{CO}_{2}$ fuel emissions from aviation until 2050. Filled symbols indicate projections, open symbols indicate estimates for past emissions. Figure from D. S. Lee, Fahey, et al. (2009, p. 5) with minor formatting adjustments. Projections: AERO-2K (Eyers et al., 2004), ANCAT/EC2 (Gardner et al., 1998), CONSAVE (Berghof et al., 2005), FAST (Owen and D. S. Lee, 2006), IPCC (Joyce E Penner, 1999), NASA (Baughcum, Henderson, et al., 1996; Baughcum, Sutkus, et al., 1998; Sutkus Jr et al., 2001), SAGE (Kim et al., 2007).

### 1.2 Laminar Flow Control

The greatest potential for increasing the overall aerodynamic performance of transonic transport aircraft lies in achieving a greater surface area with laminar flow across the wings, flight control surfaces and nacelles. This has been known for decades, and the fundamental mechanisms for increasing the percentage of laminar flow across these surfaces are understood well. Yet the implementation of this technology in commercial aircraft is only progressing slowly, with the Boeing 787-9 being the first aircraft delivered to an airline customer (in June 2014) with an implementation of LFC, in this case suction type LFC on the vertical stabilizer.

The goal of the Low Drag Aircraft in Operation (LDAinOp) research project ${ }^{1}$ is to investigate key technologies for achieving a low-drag transonic wing and to transfer these technologies into a complete system using a multi-disciplinary approach (Deutsches Zentrum für Luft- und Raumfahrt e. V., 2013). LDAinOp is partitioned into multiple projects, with subproject no. 1 concerning laminar wing operation and, among other issues, the influence of ice particles on the laminar-turbulent transition within a Mach number and Reynolds number regime relevant for cruise flight. The experiments with which this thesis is concerned form an element of the LDAinOp project.

### 1.2.1 Incidence of Clouds at Cruise Altitudes

Ice particles are prevalent in the upper troposphere and lower stratosphere, where cirrus clouds composed purely of ice particles can form due to a temperature range of between $-40^{\circ} \mathrm{C}$ and $-80^{\circ} \mathrm{C}$. Nastrom et al. (1981) offer a very detailed summary and analysis of cloud encounter and particle concentration data from the NASA Global Atmospheric Sampling Program (GASP). Within this program, several commercial airliners were equipped with a range of meteorological and atmospheric sensors (Perkins and Gustafsson, 1975). Data was then collected on routine airline flights between 1975 and 1977.

Within this report, time in clouds (TIC) represents the fraction of any single observation that was spent in clouds, with any single measurement of TIC $>0$ being described as in the vicinity of clouds (CIV), such that the CIV number represents the fraction of time that the aircraft spent in the vicinity of clouds, of which not necessarily all time was spent actually in a cloud. From these values, the fraction of time in clouds when in the vicinity of clouds (TICIV) is derived ( $\overline{\mathrm{TICIV}}$ if averaged over multiple recordings), as well as $\overline{\mathrm{TIC}}=\sum_{i=1}^{n} \frac{\mathrm{TIC}_{i}}{n}$, which denotes the total fraction of time spent in an actual cloud whether or not the aircraft was in the vicinity of clouds. The latter number is therefore the most relevant for this discussion. PD5 denotes the concentration of particles larger than $3 \mu \mathrm{~m}$ in diameter.

[^0]The presented data shows that in $13.2 \%$ of all measurements above FL 335 ( 10.21 km ), the respective aircraft detected being in cloudy conditions (CIV). For the data above FL 385 ( 11.75 km ), about 9.4 \% of measurements were in CIV. Figures $1.3 \mathrm{a}-\mathrm{c}$ give a more detailed representation of the more useful $\overline{\mathrm{TIC}}$ values for various altitudes, as well as atmospheric temperatures and humidity values for reference. Figure 1.4 shows the measured particle concentrations from the same program. Jasperson et al. (1985) offer additional information on the variance of these values due to different factors.

This indicates that, while the majority of cruise flight conditions are not within cirrus clouds, the share of time spent in cloudy conditions is large enough to have, for example, a significant impact on the overall fuel needed for a flight if laminar flow cannot be achieved in these conditions. What is known about the influence of these particles on laminar flow, however, is so far mostly limited to observations during flight test campaigns.

### 1.2.2 Flight Experiments Showing the Effect of Ice Particles on Laminar Flow

Likely the earliest documentation of an observed influence of ice particles on a laminar flow was published by Hall (1964): At Northrop Corporation, the X-21A flight test aircraft was used to observe the performance of LFC in various conditions over many flight tests. The LFC was implemented as span-wise suction slots on $30^{\circ}$ swept wings, with suction provided from the engines of the aircraft (Kosin, 1965). A loss or degradation of laminar flow across the wing was observed whenever the aircraft was flown through or close to clouds, or in hazy conditions. Tests were performed at a free stream Mach number of $\mathrm{M}=0.75$, and observations of LFC performance as a result of ice particle flux were qualitative only.

For an approximated X-21A airfoil and $0.9 \times 10^{3} \mathrm{~kg} \mathrm{~m}^{-2}$ density ice particles, Hall (1964) found that in order for particles to impinge on the surface, they have to be greater than about $4 \mu \mathrm{~m}$ in diameter. Above a diameter of $\sim 50 \mu \mathrm{~m}$, the boundary layer was found to have little effect on the particle velocity before impact. A local collection efficiency by particle diameter, flight altitude and y coordinate was calculated in order to relate free stream particle flux measurements with the airfoil incident particle flux (see Figures 6 and 7 in Hall, 1964, pp. 32-33). Free stream particle flux values were estimated from visibility observations in flight, with no particle measurement device installed on the aircraft. According to Hall, the particle flux in conditions of "horizontal visibility of the order of $5000-10000 \mathrm{ft}$ " in cirrus clouds is sufficient to explain total loss of LFC, and the flux in light haze is sufficient to explain degraded LFC performance.

The results of the flight tests were summarized in diagrams of complete, partial and no loss of LFC depending on the observed particle dimensions and free stream particle flux, as shown in Figure 1.5. In these results, ice particles in clouds are assumed to be hexagonal prisms, with an aspect ratio of 2.5 (length to diameter), of varying size. No observations were available with concentrations higher than $1.0 \mathrm{~g} \mathrm{~m}^{-3}$ or particle diameters beyond approximately $110 \mu \mathrm{~m}$. The LFC performance degradation is bounded by a minimum particle flux as well as a minimum
particle size, with both bounds indicated in the diagrams. As is discussed later, the critical particle size here is purely a result of a critical particle diameter model and not derived from the observations.

The extent of degradation of the laminar flow is mostly dependent on the free-stream particle flux, i.e. the particle diameters are either sufficient for causing a change of the laminar flow, or, below a certain critical particle size, they have no effect. Provided that it is above the critical value of $D_{\mathrm{p}} \approx 33 \mu \mathrm{~m}$ and $D_{\mathrm{p}} \approx 17 \mu \mathrm{~m}$ at 40000 ft and 25000 ft respectively, the particle size only plays a relatively minor role. In both altitude cases, typical cirrus cloud particle concentrations and sizes give a strong degradation of LFC performance, and maximum expected values for particle concentrations result in a complete loss of laminar flow according to this model. The conclusions that are drawn by Hall from this data, as well as some information on how the model was developed, are discussed in Section 1.3.

Davis, Maddalon, and Wagner (1987) and Davis, Maddalon, Wagner, et al. (1989) describe experiments using the NASA Lockheed C-140 JetStar aircraft within the Laminar Flow Control Leading Edge Glove Flight Test Article Development (LEFT) program (Wagner et al., 1990) with different laminar flow airfoil sections on each of the wings. The right wing of this aircraft was equipped with a laminar flow glove provided by the Douglas Aircraft Company and using electron-beam-drilled $\sim 60 \mu \mathrm{~m}$ diameter perforations on the upper surface, as well as a Kruger flap. The left wing laminar flow glove, provided by the Lockheed Aircraft Corporation, used suction through $\sim 0.1 \mathrm{~mm}$ diameter spanwise slots on both the upper and lower surface. Both installations featured systems for anti-icing and insect protection systems. Measurements of the conditions were obtained using a Knollenberg probe as well as a charging patch. The former is a a laser shadowgraphy particle imaging device with a size determination accuracy of $20 \mu \mathrm{~m}$ in the range from $60 \mu \mathrm{~m}$ and $600 \mu \mathrm{~m}$. Lower sizes could not be detected due to high air speeds beyond the design conditions of the probe. The latter is able to detect free-stream particles through frictional charge exchange with the patch, with a sensitivity down to $20 \mu \mathrm{~m}$. The percentage of laminar flow across the respective wing sections was determined using arrays of pitot tubes determining the thickness of the boundary layer downstream of the test area.

The aircraft performed 19 simulated airline service (SAS) flights, of which 10.28 hours were analyzed in Davis, Maddalon, Wagner, et al. (1989). A significant effect of cloud and haze particles on the performance of the laminar flow control devices was confirmed, as were the qualitative observations from the $\mathrm{X}-21 \mathrm{~A}$ studies. Figure 1.6 shows a comparison of the results from the $\mathrm{X}-21 \mathrm{~A}$ studies with those from the LEFT data analysis. The data is not directly comparable because the airfoil shape, Mach number conditions and altitudes were not identical between the two test campaigns. Additionally, only the Knollenberg probe was used to obtain this data as the charging patch does not indicate particle sizes, limiting the minimum particle diameter to $60 \mu \mathrm{~m}$ for the LEFT data.


Figure 1.3: Cloudiness parameters measured during routine airline service as part of the NASA GASP program. Data and figures from Nastrom et al. (1981).
a) Cloudiness parameters, by distance from the tropopause (global annular means).
b) Cloudiness parameters, by pressure altitude (global annular means).
c) Relative humidity and temperature measurements by distance from the tropopause.



Figure 1.4: Particle concentrations measured during routine airline service as part of the NASA GASP program. Plots show the percentage of observations with values of PD5 larger than the respective value on the horizontal axis. Data and figures from Nastrom et al. (1981).
a) Measurements by pressure altitude.
b) Measurements by distance from the tropopause.


Figure 1.5: Estimated regions of degraded LFC performance over a range of particle sizes and free stream particle flux levels, derived from Hall (1964, pp. 36-37). Particle lengths are 2.5 times the given diameter. Orange lines indicate particle mass densities in the cloud volume, purple lines show visibility in US customary system (USCS) units.
a) Flight altitude: $12.19 \mathrm{~km}(40000 \mathrm{ft}) \quad$ b) Flight altitude: $7.62 \mathrm{~km}(25000 \mathrm{ft})$


Figure 1.6: Comparison of the observations from a single representative flight in the LEFT SAS experiments (marked areas with laminar flow (LF) percentages, see Davis, Maddalon, Wagner, et al., 1989) with those from the X-21A flight tests (lines, see Hall, 1964). Figure from Davis, Maddalon, Wagner, et al. (1989, p. 16).

### 1.3 A Model for the Loss of Laminar Flow

Hall (1964) offers extensive thoughts on the mechanism by which ice particles in these clouds might cause transition across the airfoil. He finds that the condition for such a transition is likely that a particle passing through the boundary layer of the wing produces a small turbulent spot, which can then initiate transition downstream. A comparison is drawn to the transition caused by two-dimensional and three-dimensional roughness elements in a boundary layer. Hall points to experiments in previous studies showing that the wake transition Reynolds numbers for roughness elements also resemble the critical roughness Reynolds numbers ${ }^{2}$.

This simplifies the problem such that, in order for a particle to cause transition in the laminar boundary layer of an airfoil, it is only necessary that:

1. Transition is occurring in the wake of the particle as it travels through a laminar boundary layer.
2. The time that a particle spends within the boundary layer is sufficient for the wake turbulence to occur at all.
3. There are enough particles in the incident flow such that the number of turbulent spots has a significant effect on the average laminar flow distance across the airfoil over time.

As an alternative approach to explain the transition caused by ice particles, Hall (1964) also describes a model of an equivalent adverse pressure gradient due to the effect of particles decelerating the flow in the boundary layer when impinging the boundary layer at steep angles. This effect would be a result of their drag force alone. The possibility that this explanation is sufficient is rejected based on numerical calculations which show that, as stated by Hall, "the particle flux is far too low to result in a significant momentum interchange, or adverse pressure gradient."

### 1.3.1 Wake Transition

Analyzing the flow field around an ice particle which is suspended in a volume of accelerating air, Hall (1964) finds that the important parameters for whether the particle produces a turbulent wake are the particle Reynolds number and the geometry of the particle. In this instance, the particle Reynolds number refers to the Reynolds number based on the particle slip velocity, i.e.

$$
\begin{equation*}
\mathrm{R}_{D_{\mathrm{p}}}=\frac{\left|\mathbf{u}_{\mathrm{s}}\right| D_{\mathrm{p}}}{\nu_{\mathrm{f}}} \tag{1.1}
\end{equation*}
$$

with

$$
\begin{equation*}
\mathbf{u}_{\mathrm{s}}=\mathbf{u}_{\mathrm{p}}-\mathbf{u}_{\mathrm{f}} \tag{1.2}
\end{equation*}
$$

[^1]Here, $\mathbf{u}_{\mathrm{s}}$ is the slip velocity, $D_{\mathrm{p}}$ is the diameter of the particle, $\nu_{\mathrm{f}}$ is the kinematic viscosity of the fluid, and $\mathbf{u}_{f}$ and $\mathbf{u}_{p}$ are the velocities of the fluid and particle, respectively. Hall also notes that the production of turbulence by the particles is independent of parameters that generally affect boundary layer stability, such as "boundary layer Reynolds number, free stream turbulence, pressure gradient, surface condition, suction quantity, vibration, heat input etc., within the usual limits of these parameters". The Mach number is suggested to have an effect in the supersonic flow regime.

Under the assumption that a hexagonal prism is similar to a cylinder in the critical particle Reynolds number, Hall, p. 14 finds a value of $\mathrm{R}_{\text {crit }}=150$, resulting in $D_{\mathrm{p}, \text { crit }}=17 \mu \mathrm{~m}$ at 25000 ft , $32 \mu \mathrm{~m}$ at 40000 ft for $\mathrm{M}_{\infty}=.75$. he critical particle size limits in Figure 1.5 are a representation of this model.

### 1.3.2 Time in the Boundary Layer

At a $M a_{\infty}=0.75$, Hall (1964) finds that a particle spends about $1 \times 10^{-5} \mathrm{~s}$ in the boundary layer when impinging within 1 ft from the leading edge (measured along the airfoil surface). This estimate is based on an average boundary layer thickness over the first 1 ft of 0.03 in , and on the assumption of elastic impact of the particle on the airfoil surface. The Strouhal number has also been determined experimentally to be dependent only on the particle Reynolds number and geometry, giving a vortex shredding frequency of about $1 \times 10^{6} \mathrm{~s}^{-1}$. As this is one order of magnitude higher than the time spent in the boundary layer, Hall expects the volume of turbulent fluid from these vortices to be sufficient for potentially initiating a turbulent spot.

### 1.4 Ice Particles in Cirrus Clouds

In order to model and predict the laminar behaviour on a cruise transport aircraft flight, it is necessary to understand which sizes, shapes and concentrations of ice particles are dominant in the relevant altitude and temperature regimes. An extensive study by Lawson et al. (2006) offers data, including recordings from a cloud particle imager (CPI), from 104 flights totaling over 15000 km in clouds at temperatures between $-28^{\circ} \mathrm{C}$ and $-61^{\circ} \mathrm{C}$. Most of the data was recorded in the range between $-35^{\circ} \mathrm{C}$ and $-50^{\circ} \mathrm{C}$. The particle size distributions (PSDs) were found to be bimodal with a main maximum at around $30 \mu \mathrm{~m}$ and a secondary, lower maximum around $200-300 \mu \mathrm{~m}$. Figure 1.7a shows the different classifications of ice particles used in the publication. Shapes for particles greater than $50 \mu \mathrm{~m}$ are dominated by rosettes and polycrystals ( $>50 \%$ of the total surface area and mass) as well as irregulars ( $\sim 40 \%$ ). As shown in Figure 1.7b, small irregulars and spheroids dominate among smaller particles, with spheroids being more dominant below $20 \mu \mathrm{~m}$ and almost absent for sizes above $40-50 \mu \mathrm{~m}$. Lawson et al. found that of the total number concentration, $99 \%$ were particles with diameters $<50 \mu \mathrm{~m}$.

For this experiment, the focus lies on cruise conditions where aircraft spent most of their flight time on longer routes. For this reason, the lowest of the three temperature ranges in the study,
the one between $-50^{\circ} \mathrm{C}$ and $-63^{\circ} \mathrm{C}$, is the most relevant here - this range is most common at optimum performance cruise altitudes (FL 350 and above). Given that the critical ice particle size (according to Hall (1964)) is at around $33 \mu \mathrm{~m}$, the relevant particle shapes to be tested are spheroids and small irregular shapes. For creating monodisperse seeding, creating spheroids in the ice particle generator (IPG) appears to be much more achievable than creating small irregular shapes of a controlled and constant size. For this reason, the goal in this experiment is to create a monodisperse seeding of spherical ice particles. Their particle diameters should cover as much of the range between $33 \mu \mathrm{~m}$ and $110 \mu \mathrm{~m}$ that was tested by Hall as possible - after appropriate scaling for a comparable particle Reynolds number.


Figure 1.7: Results of the analysis of in-flight particle measurements and imaging of cirrus clouds. Data and figures from Lawson et al. (2006).
a) CPI particle classifications.
b) Percentage mass PSD with particle classifications as a function of particle size for a single cirrus flight. IWC: ice water content (cloud ice mass per unit volume of air).
c) Histogram with results from a automatic particle classification by number, area, and mass, in cirrus clouds.
d) Comparison of CPI images in a wave cloud (left) and a deep cirrus cloud (right). FL: flight level (altitude at standard pressure in hundreds of feet).

### 1.5 Motivation for this Measurement Campaign

Being able to predict the effect of ice particles in the upper atmosphere on the LFC characteristics of an airfoil is important in order to predict the performance and range of an aircraft employing this technology. Without a good prediction of the performance on average over the lifetime of the system, it is not possible to guarantee fuel savings to a customer airline. Similarly, prediction for a specific upcoming flight is necessary in order to know with a reasonable uncertainty the amount of fuel that will be needed in order to reach the destination, as well as optimal flight paths and altitudes. Better prediction models can reduce unnecessary fuel reserves and therefore decreases actual fuel burn and emissions.

For these reasons, the general goal of the measurement campaign at the cryogenic Ludwieg tube Göttingen (KRG) is to investigate the influence of ice particles similar to those encountered in cruise flight of commercial aircraft on a natural laminar flow (NLF) wing profile. This includes facilitating the measurement environment with comparable parameters, most notably a high Reynolds number transonic wind tunnel. For a better understanding of the influence of the ice particle diameter on the transition behavior, it is also necessary to have an IPG which is able to produce particles of various sizes, consistently, and with little size variation. For checking the output of the IPG, which is especially important as there is no prior experience with the design, particle sizes need to be determined in order to verify the output characteristics.

The cryogenic Ludwieg tube in Göttingen, provided by the German-Dutch Wind Tunnels (DNW), facilitates tests at Reynolds numbers of up to $\mathrm{R}=20 \times 10^{6}$. For the chord length in this experiment and a fast repeat time of the wind tunnel runs, Reynolds numbers of up to $\mathrm{R}=9 \times 10^{6}$ are achieved in this experiment. It also offers the laminar base flow and lack of contaminations in the nitrogen test gas that are necessary to get meaningful results regarding the transition characteristics. Because the test gas is cryogenic, a Ludwieg tube is suitable for injecting ice particles into the test gas ahead of the actual experiment. It should be possible to guarantee that all particles entering the test section are indeed entirely frozen and that there are no droplets or partially frozen water particles entering the test section. The latter would result in the formation of water ice on the wing surface, which has happened in some previous experiments. This leads to a disruption of the laminar flow around the wing profile while also rendering the temperature sensitive paint (TSP) measurement invalid by locking in a fixed temperature on the airfoil surface now covered in water ice.

### 1.6 Modeling Aerodynamic Diameters

While retaining the assumption that the turbulence behind the ice particles is the origin of the shift in the point of transition, it is possible to refine the model by Hall (1964). Specifically, the assumption of the particle as a non-moving sphere can be replaced by equations of motion of a free-flying particle entering the boundary layer of the airfoil model in question.

A complete model for the particle motion, as well as its critical Reynolds number, allows for the calculation of a critical diameter of the ice particles based on the change in the surrounding flow velocity as the particle enters the boundary layer of the wing. Two different models for the motion of the ice particles were developed by Girnth (2017) as part of the LDAinOp project. Both models are based on the BBO equation (Boussinesq, 1885; Basset, 1888; Oseen, 1927) describing the motion of a small particle in unsteady flow. Additional external forces in the models include gravity (hydrostatic lift), the Saffman lift force (Saffman, 1965), as well as the Magnus force (Magnus, 1853) (formulations taken from Nikolas, 2009, pp. 14-17). The two models only differ in the modeling of the interaction between the Saffman and Magnus forces in a shear layer, with the second model taking into consideration the change in the direction of the Saffman force. Girnth found that, for the Saffman and Magnus forces and their interaction at Reynolds numbers greater than 400 , the validity of existing models is limited, and that they differ considerably in their predictions for $c_{1}$ and $c_{\mathrm{d}}$. Models that were considered for lift and drag forces for particles in the shear flow of a boundary layer and the resulting induced rotation include those by Kurose and Komori (1999), S. Lee and Wilczak (2000), Cao and Tamura (2009), Dandy and Dwyer (1990), Hölzer (2007), Cheng et al. (2007), and Legendre and Magnaudet (1998).

### 1.7 Experimental Determination of the Critical Diameters

Preceding the work presented in this thesis, a wind tunnel campaign in the same facility has already been performed, with preliminary results summarized by Girnth (2017). The goal was the same as in the new experiments discussed here: To gain a better understanding of the conditions, especially the particle sizes, shapes and densities, under which the laminar flow across an airfoil is disturbed. To this end, an airfoil model was placed inside of the cryogenic Ludwieg tube in Göttingen under somewhat realistic Mach and Reynolds number conditions and subjected to ice particles embedded in the flow that were being generated using an IPG. The experimental setup was largely identical, with the exception of a different IPG setup as well as some minor differences in the measurement setup that was used. Cameras were installed to observe the particles in the IPG (using shadowgraphy) and downstream of the test section (using holographic shadowgraphy). For the observation of the laminar flow and the position of the recompression shock wave on the upper surface of the airfoil model, TSP was used. A two-dimensional particle image velocimetry (PIV) and particle tracking velocimetry (PTV) measurement setup allowed for the observation of the flow field and individual ice particle traces in the area of the recompression shock wave.

In these experiments, ice buildup on the airfoil model prevented the recording of reliable TSP-based observations of changes in the laminar flow across the airfoil. Additionally, the production of a monodisperse ice particle seeding proved challenging. Instead, polydisperse water droplets were likely present in most of the test runs. This was largely due to limitations of the
previous iteration of the IPG. In the analysis of both the PTV and holographic shadowgraphy images, a higher quality was deemed to be desirable. The former is used to determine the position of the particle over time after crossing the shock wave in order to determine the response of the particles to the deceleration of the flow. Increasing its accuracy is the main focus of this thesis. The latter is used to determine the number of seeding particles in a measurable volume of the flow (giving a measurement of the seeding density), while ideally also offering a means of determining sizes and shapes of the particles.

### 1.7.1 Scaling

For a comparison of the results in this campaign with earlier and future experimental and theoretical work, appropriate scaling needs to be applied. For the particles, this is most importantly the particle Reynolds number $\mathrm{R}_{D_{\mathrm{p}}}$, with the Mach number also being significant. According to the transition model that is being tested, the airfoil chord Reynolds number $\mathrm{R}_{c}$ is not as significant.

The values and equations in the following sections were taken from Konrath (2014).

## Airfoil

For some context, the Northrop X-21A aircraft used by Hall was equipped with a modified NACA 65A210 airfoil with a mean aerodynamic chord length of 14.66 ft ( 4.468 m , Nayfeh, 1988, p. 665), with flight (chord-referenced) Reynolds numbers likely at around $R_{c, X-21 \mathrm{~A}}=22.5 \times 10^{6}$ (Kaups and Cebeci, 1977, p. 666) and Mach numbers of $\mathrm{M}=0.75$ (Davis, Maddalon, and Wagner, 1987, p. 164). The NASA Lockheed C-140 JetStar LEFT aircraft used by Davis, Maddalon, and Wagner was also flown at Mach numbers of $\mathrm{M}=0.75$, equipped with two different laminar flow test sections (one on each wing).

For the LDAinOp program, the decision was made to assume realistic test parameters of $\mathrm{M}_{\text {ref. }}=0.75, \mathrm{R}_{c, \text { ref. }}=16 \times 10^{6}$ to $18 \times 10^{6}$ and $c_{\text {ref. }}=2.8 \mathrm{~m}$. These values are chosen for the entire project to come close to those encountered by a transport aircraft in cruise flight, although Reynolds number will likely be slightly higher in that case. As such, they were not modified here in spite of possible benefits for achieving more realistic particle Reynolds numbers.

The relation between the Reynolds number of the airfoil model chord and the reference chord is

$$
\begin{equation*}
\mathrm{R}_{c}=\frac{u_{\infty} c_{\text {ref. }}}{\nu_{\text {ref. }}}=\frac{u_{\infty} s_{\mathrm{m}} c_{\mathrm{m}}}{s_{\mathrm{m}} \nu_{\mathrm{m}}} . \tag{1.3}
\end{equation*}
$$

Here, $\nu_{\text {ref. }}$. is the kinematic viscosity in the reference case, with $c_{\mathrm{m}}$ and $\nu_{\mathrm{m}}$ as the chord length and kinematic viscosity of the wind tunnel model case, respectively. For the condition that $s_{\mathrm{m}} c_{\mathrm{m}}=c_{\text {ref. }}$, this leads to a scaling factor $s_{\mathrm{m}}=18.67$ based on the differences in chord length between the model and a full-scale transport aircraft. Due to limitations in keeping the tunnel and airfoil model entirely clean, Reynolds numbers around $\mathrm{R}_{c, \text { ref. }}=16 \times 10^{6}$ proved to be too
high for keeping a significant section of the airfoil in the wind tunnel laminar. For this reason, the tests were performed at $\mathrm{R}_{c}=9 \times 10^{6}$. This lower Reynolds number does not affect the scaling factor $s_{\mathrm{m}}$, but it does change the value of $\nu_{\mathrm{m}}$ for a given Mach number.

## Particle Diameters

For the equivalence of the particle Reynolds numbers, the relation is

$$
\begin{equation*}
\mathrm{R}_{D_{\mathrm{p}}}=\frac{\left(u_{\mathrm{p}}-u_{\mathrm{f}}\right)_{\mathrm{ref}} . D_{\mathrm{p}, \text { ref. }}}{\nu_{\text {ref. }}}=\frac{\left(u_{\mathrm{p}}-u_{\mathrm{f}}\right)_{\mathrm{m}} s_{\mathrm{p}} D_{\mathrm{p}, \mathrm{~m}}}{s_{\mathrm{m}} \nu_{\mathrm{m}}} \tag{1.4}
\end{equation*}
$$

Here, $u_{\mathrm{s}, \mathrm{m}}$ is the slip velocity in the wind tunnel case, $s_{\mathrm{p}}$ is the particle diameter scaling factor and $D_{\mathrm{p}, \mathrm{m}}$ is the corresponding particle diameter (for spherical particles) in the wind tunnel experiment. The first constraint for this similarity is a particle Reynolds number in the range of $\mathrm{R}_{D_{\mathrm{p}}}=100-1000$. Somewhere in this range, the development of a turbulent wake behind particles can be expected. Hall (1964, p. 10) specifically gives a critical Reynolds number for a spherical particle of around $\mathrm{R}_{\text {cr. }} \approx 600$, and $150 \leq \mathrm{R}_{\text {cr. }} \leq 300$ for a cylinder. Beyond that, the scaling factor $s_{\mathrm{m}}$ and kinematic viscosities $\nu_{\mathrm{ref}}$. and $\nu_{\mathrm{m}}$ are already fixed by the previous relation. For the validity of the right side, it is further necessary that $s_{\mathrm{m}}=s_{\mathrm{p}}$.

To achieve similarity of the particle sizes, $s_{\mathrm{p}} D_{\mathrm{p}, \mathrm{m}}$ must then be equal to $D_{\mathrm{p}, \text { ref. }}$. i.e. the size of the particles must scale proportionally to the scaling of the airfoil chord length. Hall (1964, p. 37) observed a lower limit for the loss of LFC at a diameter of $D_{\mathrm{p}, \text { ref. }} \approx 33 \mu \mathrm{~m}$, which then corresponds to a size of $D_{\mathrm{p}, \mathrm{m}} \approx 1.77 \mu \mathrm{~m}$ in the wind tunnel experiment. As mentioned in Section 1.4, the maximum expected particle size is at $\sim 500 \mu \mathrm{~m}$, which equates to $\sim 27 \mu \mathrm{~m}$ in the experiment. The largest size shown in the Hall criterion is a cylinder diameter of $110 \mu \mathrm{~m}$, or $\sim 5.9 \mu \mathrm{~m}$ in this experiment.

Due to difficulties in the control of the ice particle diameters, these small sizes were not achieved in the measurement campaign discussed here.

## Particle Concentrations

The volumetric concentration of particle in the flow, $C_{\mathrm{p}}$, scales with the third power of the other scaling factors, such that

$$
\begin{equation*}
C_{\mathrm{p}, \text { ref. }}=s_{\mathrm{m}}^{3} C_{\mathrm{p}, \mathrm{~m}} . \tag{1.5}
\end{equation*}
$$

### 1.7.2 Model for the Motion Across the Shock Wave

The characteristic dimensionless number describing the behavior of a particle suspended in a flow of changing velocity is the Stokes number S, defined as

$$
\begin{equation*}
\mathrm{S}=\frac{\tau u_{\mathrm{f}}}{D_{\mathrm{p}}} \tag{1.6}
\end{equation*}
$$

where $\tau$ is the relaxation time of the particle, i.e. the time after a sudden change in the fluid velocity surrounding the fluid that it takes for the particle to reduce its slip velocity $u_{\mathrm{s}}$ to $\frac{1}{e}$ of
its original value. This threshold is used because, for a constant drag coefficient and no change in the flow velocity, the slip velocity decreases exponentially with time. Determining the Stokes number along with the particle Reynolds number and observations of the particle effects on transition allows for a comparison between experiments conducted under different conditions.

A shock wave presents an ideal and sudden change of flow velocity, thereby providing a simplified system for observing the aerodynamic behavior of a particle surrounded by an accelerated flow. Figure 1.8 shows the basic concept of the expected particle response as it encounters the shock wave. An observation of the slip velocity $u_{\mathrm{s}}$ at multiple points in time can


Figure 1.8: Illustration of the particle velocity $\left(u_{\mathrm{p}}\right)$ response as it crosses an idealized recompression shock wave. Here, $u_{\mathrm{f}}$ represents the flow velocity surrounding the particle, and $u_{\mathrm{s}}$ represents the particle slip velocity. The flow velocities in front of and behind the shock are indicated by $u_{1}$ and $u_{2}$, respectively. The particle crosses the shock at the time $t_{\mathrm{s}}$.
be fitted to a model for this slip velocity response.
For the particle motion across a fast, shear-free change in fluid velocity, as is nearly ideally the case for a compression shock wave, Saffman and Magnus forces are negligible, such that the complete BBO equation can be written in the form $m \mathbf{a}=\sum_{i} \mathbf{F}_{i}$ as (see Girnth, 2017, pp. 15-17)

$$
\begin{align*}
\frac{\pi D_{\mathrm{p}}{ }^{3}}{6} \rho_{\mathrm{p}} \dot{\mathbf{u}}_{\mathrm{s}}= & -\frac{\pi D_{\mathrm{p}}{ }^{3}}{6} \nabla p  \tag{1.7a}\\
& +\frac{\pi D_{\mathrm{p}}{ }^{3}}{6} \rho_{\mathrm{f}} c_{\mathrm{vm}} \dot{\mathbf{u}}_{\mathrm{s}}  \tag{1.7b}\\
& +\frac{\pi D_{\mathrm{p}}^{2}}{8} c_{\mathrm{d}} \rho_{\mathrm{f}}\left|\mathbf{u}_{\mathrm{s}}\right| \mathbf{u}_{\mathrm{s}}  \tag{1.7c}\\
& -\frac{3}{2} \sqrt{\rho_{\mathrm{f}} \pi \mu_{\mathrm{f}}} D_{\mathrm{p}}{ }^{2}\left(\int_{t_{0}}^{t} \frac{\dot{\mathbf{u}}_{\mathrm{p}}\left(t^{\prime}\right)-\dot{\mathbf{u}}_{\mathrm{f}}\left(t^{\prime}\right)}{\sqrt{t^{\prime}-t_{0}}} \mathrm{~d} t^{\prime}+\frac{\left(\mathbf{u}_{\mathrm{p}}-\mathbf{u}_{\mathrm{f}}\right)_{0}}{\sqrt{t}}\right)  \tag{1.7d}\\
& -\frac{\pi D_{\mathrm{p}}{ }^{3}}{6}\left(\rho_{\mathrm{p}}-\rho_{\mathrm{f}}\right) \mathbf{g} . \tag{1.7e}
\end{align*}
$$

Here, $\rho_{\mathrm{p}}$ is the density of the particle, and $\dot{\mathbf{u}}_{\mathrm{p}}$ is the derivative of the particle velocity with respect to $t$. The right side of Equation 1.7a describes the forces as a result of the external
pressure gradient $\nabla p$. Equation 1.7b describes the force as a result of a virtual mass resulting from the fact that, for any acceleration of the particle, some volume of surrounding fluid needs to be accelerated as well. For this force, $c_{\mathrm{vm}}$ represents a dimensionless coefficient and $\rho_{\mathrm{f}}$ is the density of the fluid. Equation 1.7c describes the drag force on the particle, with $c_{\mathrm{d}}$ as the dimensionless drag coefficient. Equation 1.7d describes the Basset force, with the dynamic viscosity $\mu_{\mathrm{f}}$. It is a result of viscous forces from the fluid around the particle due to the past motion of the particle within the fluid. Equation 1.7 f finally describes the hydrostatic lift force as a result of the gravitational field $\mathbf{g}$.

The forces of the virtual mass (Equation 1.7b) and the Basset force (Equation 1.7d) are negligible for a large ratio of the particle density to the density of the fluid surrounding it. This ratio is approximately $3 \times 10^{3}$ here, which is why they will not be considered here. Further, Girnth (2017, p. 50) found that the forces due to gravity, hydrostatic lift and external pressure are much smaller than the drag force. This, in addition to only regarding the one-dimensional movement of the particle across a vertical shock wave, greatly simplifies the description of the movement, which is now

$$
\begin{equation*}
\dot{u}_{\mathrm{s}}=-\frac{3}{4} \frac{\rho_{\mathrm{f}}}{\rho_{\mathrm{p}}} \frac{c_{\mathrm{d}}}{D_{\mathrm{p}}} u_{\mathrm{s}}{ }^{2} . \tag{1.8}
\end{equation*}
$$

As this is a description of only the slip velocity, this allows for the calculation of the particle motion from any point in time at which both the velocity of the particle and the velocity of the surrounding flow are known. The initial point is not required to be exactly at the position of the shock wave, which is why this position does not need to be determined, as long as the starting point is indeed behind the shock wave and further changes in the surrounding flow velocity are small compared in comparison. Ideally, any analysis is still started as close to the shock wave as possible in order to benefit from the highest possible slip velocity, resulting in a higher accuracy of the results.

In order to solve this equation for the position as a function of $t$ or for the diameter based on a given set of positions, an approximation of the drag coefficient $c_{\mathrm{d}}$ is required. For a perfect sphere, the most simple and well known analytic approximation is the one developed by Stokes (1851), which is

$$
\begin{equation*}
c_{\mathrm{d}}=\frac{24}{\mathrm{R}_{D_{\mathrm{p}}}}, \tag{1.9}
\end{equation*}
$$

and is valid for $\mathrm{R}_{D_{\mathrm{p}}}<0.2$ only (Liao, 2002, pp. 2-3). Because Reynolds numbers up to $1 \times 10^{3}$ need to be approximated reasonably well, the additional terms by Kaskas (1964) (see also Molerus, 2013, p. 3) were used to achieve a good fit for the actual drag up to this Reynolds number regime. The degree to which the approximation matches the measured drag is shown in Figure 1.9. The resulting approximation then becomes

$$
\begin{equation*}
c_{\mathrm{d}}=\frac{24}{\mathrm{R}_{D_{\mathrm{p}}}}+\frac{4}{\sqrt{\mathrm{R}_{D_{\mathrm{p}}}}}+0.4 \tag{1.10}
\end{equation*}
$$

which still allows for an analytical solution of the differential equation. For finding the particle

…..... (a)
----- (b)

- (c)
-.-.- (d)

Figure 1.9: Comparison of different correlations of the drag coefficient of a sphere.
a) Stokes, 1851 (Equation 1.9)
b) Kaskas, 1964 (Equation 1.10)
c) Clift et al., 1978, p. 112 (near-perfect piecewise approximation of the measured drag curve, as evaluated in Brown and Lawler, 2003, p. 225)
d) Constant approximation as $c_{\mathrm{d}}=\frac{4}{10}$ for high Reynolds numbers.
diameter, the solution requires values for $\rho_{\mathrm{p}}, \rho_{\mathrm{f}}$ (approximately constant after the shock wave) and $u_{\mathrm{s}}$. The particle density $\rho_{\mathrm{p}}$ is the density of water ice ( $\approx 917 \mathrm{~kg} \mathrm{~m}^{-3}$ ) for spherical particles, but needs to be set to a lower value for non-spherical shapes. The slip velocity $u_{\mathrm{s}}$ has to be determined by taking the measured $u_{\mathrm{p}}$ values and either subtracting the theoretical, calculated $u_{\mathrm{f}}$ from the pressure distribution over the airfoil, or subtracting the PIV $u_{\mathrm{f}}$ at the corresponding position in the image.

For the Kaskas drag model, Figure 1.10 shows the relaxation curves of the slip velocities of particles for different particle diameters. As eight illumination pulses with a time difference of $10 \mu \mathrm{~s}$ each were observed, the total useful time period from the first to the last connection within a given PTV trace amounts to $70 \mu \mathrm{~s}$. The relaxation curves demonstrate that, for small particles, the amount of relaxation in the observed time period is strongly dependent on the diameter of the particles. For particle diameters greater than about $20-40 \mu \mathrm{~m}$, the curves here already indicate that the velocity measurements need to be very accurate in order to retrieve useful particle diameters from a parameter fit. For particle diameters below about $3 \mu \mathrm{~m}$, the slip velocity will likely be indistinguishable from zero within the observed time period.


Figure 1.10: Slip velocity relaxation model results for different particle diameters. Calculated using Maple for $\rho_{\mathrm{p}}=900 \mathrm{~kg} \mathrm{~m}^{-3}$ and $\rho_{\mathrm{f}}=1.25 \mathrm{~kg} \mathrm{~m}^{-3}$.

## 2

## Experimental Setup

This chapter describes the technical setup and processes involved in conducting the experiments at the DNW-KRG wind tunnel. The setup, including the measurement aspects and general method of ice crystal production, was largely identical to experiments that preceded this work, and have been described in Konrath (2015). The information here focuses on the measurement techniques that are the most relevant to the analysis methods that were developed. Other measurement techniques necessary for the overall usefulness of the experiment are mentioned briefly, with more technical information given in Appendix A.1.

See Appendix A. 2 for a description of the sequence of measurements and events both across a whole day of testing as well as within a single wind tunnel run. This includes notes regarding the repeatability of PTV and PIV measurements.

### 2.1 The Cryogenic Wind Tunnel (DNW-KRG)

The DNW KRG ${ }^{1}$ is a Ludwieg tube type blow-down wind tunnel located in Göttingen, Germany. Figure 2.1 shows the tunnel with its main components in a simplified schematic. As described by Rosemann et al. (1995), the storage tube is 130 m in length and has a diameter of 0.8 m . This tube, along with the test section, can be pressurized up to 1.25 MPa using vaporized liquid nitrogen. The dump tank and the entire volume behind a fast acting valve is evacuated in order to maximize the difference in pressure. To start a run, the fast-acting valve is opened. The minimum time this valve can remain open is $\Delta t<0.1 \mathrm{~s}$ (Rosemann, 1997).

An expansion wave then travels from the position of the valve into the storage tube, passing the test section, while a shock wave travels into the dump tank. In the test section, this expansion wave is followed by an approximate steady state flow in the direction towards the dump tank, which lasts for about 0.6 to 1 s (Rosemann et al., 1995). The measurement time is limited by the arrival of the reflected expansion wave from the far end of the storage tube.

[^2]

Figure 2.1: Schematic diagram of the cryogenic Ludwieg tube DNW-KRG in Göttingen, based on Rosemann et al. (1995) and DNW German Dutch Wind Tunnels (n.d.).

The test section itself has dimensions of 400 mm in width, 350 mm in height and approximately 2 m in length. An illustration of the test section, with approximate positions and fields of view of various measurement techniques employed in this experiment, is shown in Figure 2.2. It features adaptive upper and lower walls in order to allow the walls to conform to the flow around a two-dimensional airfoil. As a result, testing with minimal wall interference is possible even when changing the angle of attack or switching out the test subject. A number of test shots is required in order for the adaptive walls to match the flow around an airfoil for a given set of parameters. The optimal shape is computed using pressure distribution measurements on the top and bottom that are fed into a single step algorithm.

The standard mounting method for airfoils in the test section allows for a maximum of 200 mm in chord length. In the case of this experiment, an airfoil model with a chord length of 150 mm is used. Using liquid nitrogen as the source of the test gas allows for the operation at temperatures between 100 K and 280 K . Using the maximum stagnation pressure of 1.0 MPa , and a temperature of 100 K , a Reynolds number of up to $60 \times 10^{6}$ is theoretically achievable for a 150 mm reference length (Rosemann et al., 1995).

Mach numbers between 0.3 and 0.95 are possible by varying the opening size of the control valve sonic throat, defining the area ratio relative to the test section. The control cone has a positioning accuracy of 0.01 mm , leading to a Mach number accuracy of $\Delta \mathrm{M}<0.001$ (Rosemann, 1997).


Figure 2.2: Cut through the test section of the DNW-KRG, with schematic illustrations of the measurement technique positions. Camera field of views are approximately to scale. Adaptive wall actuators not shown.
a) Side view, flow from left to right.
b) Top view, flow from left to right.

Labelled items:
1: Holographic shadowgraphy.
2: Wide field tracking camera.
3: Narrow field tracking camera.
4a, 4b: TSP cameras.
5: Illuminating laser sheet.

### 2.2 Ice Particle Generation

An ice particle generator (IPG) was built specifically for this experiment in order to provide a batch of ice particles that are similar in size and shape, with repeatable output. For the purpose of this work, the IPG is regarded as a black box. The ice particles that are being produced are adjustable, within certain limits, both in their diameters and in their total amount. For all experiments discussed here, the ice particles entering the test section consist entirely of frozen purified water.

As indicated in Figure 2.1, the IPG is positioned vertically on top of the storage tube next to the gate valve and is connected to the storage tube via an induction pipe. Different storage gas circulation procedures can be used to position the particle seeding material at different points within the storage tube, thereby making them travel through the test section at different times during the run. Alternatively, the particles can be inserted into the active flow of the run itself, but the resulting distribution, both vertically and in time, is harder to predict in this case. For this reason, this alternative method was rarely used.

### 2.3 Airfoil Model

All tests were conducted using an unswept two-dimensional model of the laminar-type LV2F airfoil. The model was manufactured from austenitic stainless steel with a chord length of $c=150 \mathrm{~mm}$. It features a TSP coating as well as pressure taps for measuring the surface pressure distribution and surface temperature differences (see Section 2.4.3 for details on the TSP measurements). The contour of the airfoil, including the positions of pressure taps installed for this experiment, is shown in Figure 2.3. The span width of the model is $w_{\mathrm{s}}=500 \mathrm{~mm}$, which exceeds the 400 mm width of the KRG test section. This allows for it to be clamped on both sides. The installation allows for remotely controlled changes in the angle of attack. Observation windows are located symmetrically on both sides, both on top of the airfoil model and behind it in the direction of the flow. This can be seen in the photo in Figure 2.4.

The steel surface is coated with a primer which facilitates adhesion, a layer of white paint and the temperature sensitive paint (TSP). The thickness of the TSP layer is approximately $120 \mu \mathrm{~m}$ and was polished to an average surface roughness of about $R_{\mathrm{a}}=0.05 \mu \mathrm{~m}$ (Costantini et al., 2015, p. 1174).


Figure 2.3: Contour and pressure tap locations of the laminar-type LV2F airfoil model.


Figure 2.4: Photo of the airfoil model in the test section, as seen from behind the test section in the direction of the flow. The windows on either side closer to the camera were used for the holographic shadowgraphy measurements.

### 2.4 Measurement Techniques

Several techniques are used to characterize the flow across the airfoil as well as the size, shape and flight paths of the ice particles at different positions. Ideally, the experiment relies on all of these to work in multiple successive wind tunnel runs in order to establish the following information.

1. The size distribution and seeding density of ice particles that have passed the airfoil in the test section
2. The transition point on the upper airfoil surface, relative to the point of transition for a run without any ice particles in the flow but otherwise identical parameters
3. The position of the recompression shock wave on top of the airfoil
4. The velocity of the flow in the entire area behind the shock wave during a run without
large ice particles, but with otherwise identical flow conditions
For each of these, a separate measurement technique is used in and around the wind tunnel. The following sections give an overview over those techniques.

### 2.4.1 Particle Tracking Velocimetry (PTV)

The goal of this measurement is to determine the velocity of particles at multiple times behind the shock wave, enabling a comparison with the velocity of the surrounding flow as determined using a PIV measurement. Autocorrelation of repeatedly illuminated particles on a single image is used to determine the flight path of individual particles. From such a path (or trace), the average velocity of the particle in-between two illuminations can be derived by dividing the distance that the particle has travelled in each interval by the known time delay between the illumination pulses.

## Technical Setup

In order to achieve a bright illumination of the particles at very precise time intervals for very short periods of time, a laser light source is necessary. Because illuminating particles in front of and behind the focal plane of the camera would make analysis of the images difficult or impossible, a thin sheet of light is necessary. Ideally, this sheet should be approximately as wide as the depth of focus of the imaging system. This is achieved in this experiment by introducing a laser beam into the test section and then expanding it vertically in order to illuminate the entire field of view of the cameras.

Specifically, four InnoLas SpitLight 1000 flashlamp-pumped double pulse laser systems are used (see Table 2.1 for detailed specifications). The lasers feature harmonic generating assemblies to convert their (infrared light) beams into second harmonics to achieve a wavelength of 532 nm (green light).

The beams from the four lasers (in pairs of two, with one power supply for each pair) are combined into a single beam and their flashlamps are synchronized in order to produce a total of eight laser pulses. As two pulses are necessary to provide one measurement of average velocity in-between the pulses, using eight evenly-spaced pulses gives seven measurements of velocity per particle. The pulsing order of the lasers is such that in each pair, the master laser first emits one double pulse, followed by a double pulse from the slave laser. After the first laser pair has fired their four pulses, the same sequence is repeated by the second laser pair.

For this set of measurements, the time delay was kept at $10 \mu \mathrm{~s}$ both within each double pulse and between double pulses. This was determined to be the best pulse delay for particle seeding with $15 \mu \mathrm{~m}$ diameters as a compromise between maximum accuracy of particle diameter determination and the ease of finding traces in the resulting images, as described by Girnth (2017, pp. 52-53). For better accuracy, delays should, to some extent, be longer: The velocity precision is limited by the accuracy of the particle location determination. However, delays
should also not be too long in order to avoid convergence of the particle velocities to the flow velocity before the end of the entire 8 -pulse trace.

| Type designation | SpitLight 1000 <br> Manufacturer |
| :--- | ---: |
| InnoLas Laser GmbH |  |

Table 2.1: Specifications for the InnoLas SpitLight 1000 laser system (InnoLas Laser GmbH, 2013) used in PTV and PIV measurements.

The beam enters the wind tunnel behind the test section in an area that can be moved in order to open the tunnel and access the inside of the test section (see Figure 2.2). As both the laser and the test section itself stay in position, this presents the challenge of providing some flexibility for the laser beam path before entering the tunnel. This is achieved through a laser arm with three joints-one in the center and one on each mounting point. Ideally, this arm keeps the exit point and angle of the laser beam approximately constant when being articulated. However, as this is not the case to the precision that is needed here, a simple observation camera is located at the point where the beam from the laser arm hits the bottom mirror of the periscope. This camera shows the point on this mirror which the beam hits, such that the mirrors at the entry point of the laser arm can be adjusted to keep this point constant. This arm is attached to the bottom of the wind tunnel between the test section and the control valve. The beam enters the tunnel itself through a periscope that is located within the vertical model support structure behind the airfoil (which is not used for supporting any model here). The beam exits this support structure horizontally in the direction against the flow, through a mechanical device which is built to hold a sting. A cylindrical diverging lens at the tip of this
structure expands the beam vertically from an approximate Gaussian beam into a sheet of light that then illuminates the area above the airfoil. The horizontal focal point of the laser beam is located between the diverging lens and the model, such that the sheet of light has a finite width of a few mm in the area of interest.

The illuminated particles in the flow are recorded by two cameras located on either side of the TSP camera in the starboard side camera box. Both cameras that are used for this are of the Basler acA2040-180km type (see Table 2.2 for specifications).

| Type designation | Basler acA2040-180km |
| :--- | ---: |
| Basler AG |  |

Table 2.2: Specifications for the Basler acA2040-180km camera (Basler AG, 2012) and CMOSIS CMV4000 sensor (CCMOSIS BVBA, 2015; CMOSIS BVBA, 2017) used in IPG shadowgraphy as well as in PTV, PIV and holographic shadowgraphy measurements.

One of them, the wide field tracking camera (WFT camera), is equipped with a 16 mm wide angle lens in order to give an overview of the entire flow across the airfoil. Its lens is centered 37 mm upstream of the mount rotation axis, i.e. at $25.3 \% \frac{x_{a}}{c}$. The camera is mounted such that it can be moved horizontally relative to the lens in order to shift within the image circle and look further downstream than would otherwise be possible while retaining a focal plane that is aligned with the longitudinal tunnel axis. Details of the WFT camera lens are listed in Table 2.3. Figure 2.5b shows a sample image with PTV seeding.

The narrow field tracking camera (NFT camera), on the other hand, is equipped with a 50 mm lens providing a more detailed view of a small section of the flow across the airfoil. In
this section, the recompression shock wave is expected to be located, as well as some additional downstream distance. Its lens is centered 37 mm downstream of the mount rotation axis, i.e. at $74.7 \% \frac{x_{\mathrm{a}}}{c}$ and downstream of the recompression shock wave. The field of view of the NFT camera overlaps fully with the field of view of the WFT camera in the focal plane. Details of the NFT camera lens are listed in Table 2.4, and Figure 2.5 a shows a sample image with PTV seeding. Figure 5.5 in the calibration chapter visualizes the overlap in the fields of view from the two cameras.


Figure 2.5: Sample images from the NFT camera and WFT camera, with seeding material. The images show ice particles from $100 \%$ water seeding and were taken simultaneously. Both images have been mirrored horizontally in order to achieve a flow direction from left to right.
a) Narrow field tracking camera
b) Wide field tracking camera

Some areas within the field of view, such as the out-of-focus background as well as the airfoil in the case of the WFT camera, are illuminated by the TSP LEDs. Neither the LEDs nor the WFT camera or NFT camera feature any optical filter in order to reduce the amount of unwanted light spillage for the PTV measurement. This does not cause any major problems due to the short exposure time of the PTV cameras $(2500 \mu \mathrm{~s})$ and the much higher light output of the SpitLight laser relative to the TSP LEDs.

## Pulse Brightness Adjustments

For the full analysis of PTV images, it is necessary that all eight laser pulses are visible in a single exposure, which requires them to be within a certain brightness variation relative to each other. Beyond that, it is helpful for the pulses to be very similar in brightness in order to

| Type designation | Cinegon $\mathbf{1 . 8} / \mathbf{1 6}$ Ruggedized |
| :--- | ---: |
| Manufacturer | Schneider Kreuznach |
| Focal length | 16.4 mm |
| Relative aperture | $1 / 1.8$ |
| Image Circle | 16 mm |
| Interface | C-Mount |
| Adjustability | Aperture $\left(l_{\mathrm{f}} / D_{\mathrm{A}, \mathrm{e}}=1 / 1.8^{-1 / 22}\right)$ |
| Internal design | 7 elements in 7 groups |
| Dimensions | L: $44.8 \mathrm{~mm}, \mathrm{D}: 34 \mathrm{~mm}$ |
| Filter thread | $30.5 \mathrm{~mm}(\mathrm{M} 30.5 \times 0.5)$ |
| Weight | 102 g |

Table 2.3: Specifications for the Schneider Kreuznach Cinegon 1.8/16 WFT camera lens (Schneider Kreuznach, 2013; Schneider Kreuznach, 2009) used in PTV and PIV measurements.
be able to correlate bright particle images with large particles, rather than with an unusually bright laser pulse. In some cases, it might also be possible to improve the correlation of the different pulses on the image by discarding correlations of very bright particle images with very dim particle images - if all laser pulses are indeed very similar in brightness.

A photodetector ${ }^{2}$ is used to record the brightness profile of the last eight pulses of each run. The detector is placed in the vicinity of the laser beam coming from the SpitLight 1000 lasers and records scattered light instead of direct light. As a result, it does not interfere with the beam and can be used to measure the pulse brightness whenever the laser is used at a high power setting.

After the first test runs of the day, and occasionally in-between measurement runs, both the images from the NFT camera as well as the voltage profile from the photodetector are used to gauge the evenness of the pulses and adjust single laser pulses if necessary. This is done by adjusting the pulse delay relative to the pumping source.

### 2.4.2 Particle Image Velocimetry (PIV)

In order to evaluate the behavior of ice particles across a shock wave using PTV, it is necessary to also know the flow velocity that these particles are surrounded by, enabling the calculation of slip velocities. To enable this, PTV images are recorded in separate wind tunnel runs using a very fine particle or droplet seeding. This seeding material can also be generated using the IPG. Additionally, determining the location of the shock wave should be possible with higher precision compared to the PTV method thanks to the smaller seeding material Stokes number

[^3]| Type designation | Apo-Xenoplan 2.8/50 Ruggedized |
| :--- | ---: |
| Manufacturer | Schneider Kreuznach |
| Focal length | 50.2 mm |
| Relative aperture | $1 / 2.8$ |
| Image Circle | $\sim 24 \mathrm{~mm}$ |
| Interface | C-Mount |
| Adjustability | Aperture (f/2.8-f/22), |
| Internal design | 6 elements in 4 groups |
| Dimensions | L: $52.7 \mathrm{~mm}, \mathrm{D}: 34 \mathrm{~mm}$ |
| Filter thread | $30.5 \mathrm{~mm} \mathrm{(M} 30.5 \times 0.5)$ |
| Weight |  |

Table 2.4: Specifications for the Schneider Kreuznach Apo-Xenoplan 2.8/50 NFT camera lens (Schneider Kreuznach, 2008; Schneider Kreuznach, 2009) used in PTV and PIV measurements.
and higher seeding density. Schlieren photography is not feasible as there is no large enough window on either side of the airfoil to be able to provide the background illumination that would be necessary for this.

As the same laser and camera system is used that is optimized for the PTV measurements, it is not possible to record two separate images within a short time interval as is usually the case for PIV measurements. Instead, the multiple pulses of the InnoLas SpitLight 1000 lasers are used here as well, resulting in images that are not ideal for PIV analysis as they contain all eight illuminations. Using two identical images with a horizontal shift for compensation of the long distances between pulses, autocorrelation can nevertheless be used using conventional PIV analysis tools.

## Technical Setup

All technical systems used for PIV are exactly the same as used for the PTV measurement technique. Those have been described in Section 2.4.1.

### 2.4.3 Temperature Sensitive Paint (TSP)

The experiment relies on temperature sensitive paint (TSP), applied to the airfoil's upper surface, for non-intrusively identifying the transition point. This paint contains molecules that, when excited with a certain wavelength of light, temporarily emit light at a different wavelength. These molecules are called luminophores, and the amount of light that they emit after excitation decreases with increasing temperature. Therefore, areas of lower temperature on the TSP surface will show up as brighter in an image taken during and shortly after a period of illumination. In order to isolate the light emitted by the luminophores from the light used for excitation (as
well as from unrelated light sources), an optical bandpass filter is employed that is tailored to the specific emission wavelength range of the TSP. The general arrangement of such a TSP measurement is illustrated in Figure 2.6.


Figure 2.6: Illustration showing the basic principle of a TSP measurement.
As described by Costantini et al. (2015), the luminophores in this experiment are a ruthenium complex ${ }^{3}$ embedded in a commercial polyurethane clear coat binder. The excitation wavelength for this material is in the range of $420 \mathrm{~nm}<\lambda<580 \mathrm{~nm}$ with a peak around 480 nm (blue, towards cyan), the emission occurs in the range of $580 \mathrm{~nm}<\lambda<670 \mathrm{~nm}$ with a peak around 605 nm (orange). These values are taken from Egami et al. (2007, p. 7) and Iijima et al. (2003, p. 74 ) at $\gtrsim 25 \%$ normalized intensity. As shown in Figure 2.7, the usable temperature range for detecting transition using this material is roughly between $T_{\text {min }}=100 \mathrm{~K}$ and $T_{\max } \approx 220 \mathrm{~K}$, with the upper limit mostly being set by the drop in absolute intensity of the emission. The temperature resolution that can be achieved with this setup is of the order of $1 \times 10^{-1} \mathrm{~K}$ (Costantini et al., 2015, p. 1173).

The temperature difference between laminar and turbulent boundary layer sections at the surface of the airfoil due to the difference in recovery factors for the two boundary layer states is less than 1 K for a transonic flow and cryogenic temperatures (Costantini et al., 2015, p. 1173; Fey, Egami, and Engler, 2006, p. 5). This is not sufficient for visualization through the TSP technique. However, the heat transfer by convection in a turbulent boundary layer is much stronger than in a laminar boundary layer. For this reason, a difference in the temperature between the flow and the airfoil surface will result in a temporary wall temperature difference between laminar and turbulent boundary layer sections. This difference is only significant if it is not quickly equalized by thermal conduction between the TSP layer on the surface and the material of the model itself (Fey, Egami, and Engler, 2006, p. 5). The airfoil model material is, as usual for cryogenic wind tunnel models, metallic and therefore provides good heat transfer. The model therefore features a white screening layer on top of the primer layer for thermal

[^4]

Figure 2.7: Emission intensity of Ru (trpy) single-component TSP between 100 K and 300 K after excitation at $500 \pm 40 \mathrm{~nm}$. Emission was recorded at $630 \pm 50 \mathrm{~nm}$, with the surface subject to 100 kPa of pressure. Plots and data from Egami et al. (2007, p. 8).
insulation. This layer also acts as a diffuse background layer for the TSP measurement, which is applied on top of this screening layer.

Creating such a temperature difference can be achieved by heating or cooling the airfoil or by creating a temperature step in the flow. In a cryogenic Ludwieg tube, such a temperature step is naturally created: As the high-pressure nitrogen gas in the storage tube quickly expands into the test section, it drops in temperature by about $\Delta T_{0} \approx 15 \mathrm{~K}$ at a Mach number of $\mathrm{M}=0.7$ and stays approximately constant for the remaining test time (Costantini et al., 2015; Fey, Egami, and Klein, 2007). Figure 2.8 shows for parameters that are comparable to this test scenario that the temperature change of the airfoil model is very small within the measurement time. As noted by Costantini et al. (2015, p. 1173), any change in the wall thermal boundary condition (which is required for any transition detection using TSP) can affect the boundary layer stability.

Three rows of black tick markers are used to give a reference of where on the airfoil transition is occurring, separated by steps of $\frac{x_{\mathrm{a}}}{c}=0.1$ from front to back. Additionally, black circular dots markers are used for automatic TSP image correlation and reprojection of the results into a flat coordinate system.

Additionally, the TSP image gives an indication of where the recompression shock wave is located on the airfoil, although this information is also contained in the PIV and PTV data, with some limitations for either method. Most importantly, the TSP measurement only indicates where the shock wave meets the upper surface of the airfoil, which is not trivial to correlate with its position higher above the airfoil where the PTV detail measurements are recorded and the position is more interesting in this experiment. The precision is also not very high compared to what is useful for the PTV analysis.

See Appendix A.1.1 for more detailed notes on the technical setup used in the TSP measure-


Figure 2.8: Development of $T_{\text {model }}, T_{0}$ and $p_{0}$ in the test section during a test run in a cryogenic wind tunnel, taken from Fey, Egami, and Klein (2007, p. 3). Measured at a Mach number of $\mathrm{M}=0.79$, Reynolds number of $\mathrm{R}=10 \times 10^{6}$ and charge temperature of $T_{\mathrm{C}}=200 \mathrm{~K}$.
ment.

### 2.4.4 Holographic Shadowgraphy

The composition of the particle population that leaves the IPG into the storage tube is not necessarily identical to the one crossing the airfoil in the test section, and the latter can vary greatly at different times during a single measurement interval. It is also known that the transition behavior depends not only on the aerodynamic diameters of particles, but also on the density of the particle population, which is nearly impossible to gauge based on the IPG observations alone.

For these reasons, a measurement of the density and sizes of particles in the flow as close to the airfoil as possible is desirable. A setup returning information on these parameters was implemented about 350 mm aft of the airfoil trailing edge, using the measurement principle of holographic shadowgraphy.In a regular shadowgraphy measurement, a collimated widened laser beam is sent through the test area and directly into a lens and camera system which is focused to the area of interest. Any objects traversing in-between the illumination source and the camera are recorded by the camera in the form of a shadow. What differentiates a holographic shadowgraphy setup is the usage of coherent rather than incoherent laser light. This results in an image on the sensor which shows the pattern that results from interference of the direct illumination from the laser with the diffracted light from the particles in the flow. Using digital hologram processing with a wavelet-like Fresnelets reconstruction as described by Liebling et al. (2003), it is then possible to reconstruct the shapes of particles from the interference pattern
not only in the immediate focal plane but also in planes slightly ahead and behind.
As a result, counting the number of particles and looking at their shapes is possible within a well-defined volume of space in the center of the flow. Due to very fine background seeding, condensation on the observation windows and turbulence in the test gas, the evaluation of these measurements in this case is very challenging and not always possible (see Appendix A.1.2 for more information). This increases the importance of determining ice particle diameters using the PTV slip velocity relaxation technique.

Also see Appendix A.1.2 for a description of the technical setup and calibration methodology used in this experiment.

### 2.5 Test Parameters

The wind tunnel and airfoil can be adjusted to achieve certain flow parameters around the airfoil. This allows for the simulation of different flight conditions.

### 2.5.1 Scenarios

For the parameters of the test itself, two different scenarios were planned for the experiment. The corresponding dimensionless parameters are shown in Table 2.5. Figure 2.9 contains the

| Parameter | Scenario A | Scenario B |
| :--- | :---: | :---: |
| Mach number | 0.80 | 0.76 |
| Reynolds number | $9 \times 10^{6}$ | $10 \times 10^{6}$ |
| AOA | $0.9^{\circ}$ | $2.0^{\circ}$ |

Table 2.5: Parameters for the two test cases in this experiment.
corresponding simulated pressure coefficient $\left(C_{p}\right)$ distributions across the upper and lower airfoil surface in each scenario.

Scenario A provides a case that is close to realistic for airliner cruise flight, within the range that is feasible to test in the DNW-KRG for our experimental setup. It provides no adverse pressure gradient on the upper surface up to a chord position of around $\frac{x_{a}}{c} \approx 70 \%$ and is therefore more likely to remain laminar up to that point even in unfavorable conditions such as a high ice particle density.

Scenario B gives a stronger shock wave and an adverse pressure gradient very early in the boundary layer of the upper surface. As such, its laminar boundary layer is much more sensitive and the transition point is more likely to move forward. The increased Reynolds number makes it additionally susceptible to an earlier transition.

This second scenario is also useful in that it provides a strong and clearly defined shock wave at $\frac{x_{a}}{c} \approx 64 \%$. The shock wave is used in the determination of aerodynamic particle diameters as it provides a sudden change in the flow velocity within the visible window for the NFT camera.


Figure 2.9: Pressure coefficients across the LV2F airfoil for both test cases. Data taken from an MSES (Drela, 2007) simulation by M. Costantini.

### 2.5.2 Flow and Storage Conditions

For this experiment, Table 2.6 gives an overview over the target values for the storage conditions as well as the expected flow conditions in the measurement section. Conditions for Scenario B, which were intended to be used for the purposes of this thesis, were not used exactly as planned beforehand. For this reason, the most commonly used parameters are the ones shown in the third column labelled as Scenario C. Figure 2.10 shows the pressure coefficient values derived from the pressure tap pressures measured during one of the measurement runs. Pressure coefficients here are very similar to Scenario B.

| Property | Scenario A | Scenario B | Scenario C | Units |
| :--- | :--- | :--- | :--- | :--- |

## Test section target conditions

| Mach number $M$ | 0.80 | 0.76 | 0.76 |
| :--- | ---: | ---: | ---: |
| Reynolds number $\mathrm{R}_{c}$ | $9.0 \times 10^{6}$ | $10.0 \times 10^{6}$ | $9.0 \times 10^{6}$ |

## Test section flow conditions

| Velocity $u_{\infty}$ | 207.5 | 198.2 | 203.5 | $\mathrm{~m} \mathrm{~s}^{-1}$ |
| :--- | ---: | ---: | ---: | :---: |
| Temperature $T_{\infty}$ | 162.4 | 164.2 | 173 | K |
| Pressure $p_{\infty}$ | $1.565 \times 10^{5}$ | $1.859 \times 10^{5}$ | $1.842 \times 10^{5}$ | Pa |
| Density $\rho_{\infty}$ | 3.257 | 3.826 | 3.596 | $\mathrm{~kg} \mathrm{~m}^{-3}$ |
| Dynamic pressure $q_{\infty}$ | $70.118 \times 10^{3}$ | $75.178 \times 10^{3}$ | $74.46 \times 10^{3}$ | Pa |
| Dynamic viscosity $\mu_{\infty}$ | $11.2630 \times 10^{-6}$ | $11.3769 \times 10^{-6}$ | $11.9237 \times 10^{-6}$ | Pas |
| Kinematic viscosity $\nu_{\infty}$ | $3.4586 \times 10^{-6}$ | $2.9736 \times 10^{-6}$ | $2.62516 \times 10^{-6}$ | $\mathrm{~m} \mathrm{~s}^{-2}$ |

## Storage conditions

| Temperature $T_{0}$ | 183.15 | 183.15 | 193 | K |
| :--- | ---: | ---: | ---: | :---: |
| Pressure $p_{0}$ | $2.386 \times 10^{5}$ | $2.726 \times 10^{5}$ | $2.7 \times 10^{5}$ | Pa |
| Density $\rho_{0}$ | 4.401 | 5.028 | 4.54 | $\mathrm{~kg} \mathrm{~m}^{-3}$ |

Table 2.6: Storage and flow conditions in the DNW-KRG wind tunnel during the experiment, for the different testing scenarios. Test section flow conditions are approximate, especially static and dynamic pressures.


Scenario C
Figure 2.10: Measured pressure coefficients across the LV2F airfoil for Scenario C.
Corresponding measured flow conditions:
$\mathrm{M}=0.7610, \mathrm{R}=9.03 \times 10^{6}, p_{0, \infty}=217072 \mathrm{~Pa}, T_{0, \infty}=172.3 \mathrm{~K}, \mathrm{AOA}: 2.0^{\circ}$.


## Error Analysis for Relative Particle Locations

It is essential for analyzing the results of the PTV experiment to know the precise location of the particles' centers of mass relative to each other. The goal of this chapter is to understand roughly which factors contribute how much to the overall error in this distance determination. This is useful for two reasons: First, it gives an indication, in advance of the actual construction of the algorithms to analyze the images, where the potential to eliminate errors is the greatest. Second, such an analysis is helpful to estimate the error that can be expected in general. This is especially relevant as the ability to ascertain the actual measurement error in this experiment is very challenging.

Before discussing the contributions to the location determination error, a quick overview over common particle image location determination methods is necessary as most sources mentioned below refer to these methods, and the choice of method can have a significant influence on the accuracy.

### 3.1 Location Determination Methods

There is a large amount of existing research on algorithms for finding the positions of particles or targets or the relative displacement between two particle or target images, both in one single image (with multiple pulses of illumination) as well as in an image series with each image showing one illumination. The experiment described in this thesis is unusual in that many of the particle images are larger than the recommended size for PIV and PTV analysis (which is generally between 1.5 px and 3 px ). Additionally, the particle image intensity profiles are distinctly non-Gaussian, occasionally overexposed and not consistent between different particles in the same image. Therefore, many of the assumptions from existing research do not apply - most notably the assumption that the intensity distributions of the particle images are Gaussian or resemble, in some cases, Airy disks. With particle images that come very close to fulfilling this criterion, most existing research shows very precise center determination results
with Gaussian peak fitting (examples for this can be found in Westerweel (1997), Westerweel (2000), Shortis et al. (1994), and Nobach (2004)). In methods that require image upsampling, the same is true for using Gaussian reconstruction (see, for example, Nobach et al. (2005)).

### 3.1.1 Intensity Based Methods

The following methods use the intensities of all pixels deemed to belong to the given particle or, more frequently, only the five pixels including and surrounding the brightest one in a given particle image. The latter group of methods are often referred to as "three-point estimators" because they use three pixels in each image dimension.

## Gaussian Distribution Fit

This method, as described by Shortis et al. (1994, p. 7), assumes a two-dimensional Gaussian distribution with separate standard deviations in $x_{1}$ and $x_{2}$ with an additional parameter each for rotation and scaling. This gives a total of four degrees of freedom to iteratively fit to each particle image. The center of the Gaussian distribution with the best fit is then assumed to be the center of the particle.

Alternatively, a simpler implementation as described by Westerweel (1993, p. 77) uses a three-point estimator around the brightest pixel, such that

$$
\begin{equation*}
{ }_{\mathrm{i}} x_{i}=\frac{\ln \left({ }^{(-1)} I_{\mathrm{p}}\right)-\ln \left({ }^{(+1)} I_{\mathrm{p}}\right)}{2\left(\ln \left({ }^{(-1)} I_{\mathrm{p}}\right)+\ln \left({ }^{(+1)} I_{\mathrm{p}}\right)-2 \ln \left({ }^{(0)} I_{\mathrm{p}}\right)\right)} \text {, } \tag{3.1}
\end{equation*}
$$

where ${ }_{\mathrm{i}} x_{i}$ is the centroid position in the image dimension $i \in\{1,2\}$ and ${ }^{(-1)} I_{\mathrm{p}}$ is the pixel intensity of the pixel next to the brightest pixel in the negative $i$ direction.

## Centroid (Center-of-Mass)

Weighted average of the location of the pixels that are considered to be part of the particle image, as described by Shortis et al. (1994, p. 7). In the case of the simple centroid, they are weighted by their respective intensity, such that

$$
\begin{equation*}
{ }_{\mathrm{i}} x_{i}=\frac{\sum_{\mathrm{p}} I_{\mathrm{p} \mathrm{i}} x_{i, \mathrm{p}}}{\sum_{\mathrm{p}} I_{\mathrm{p}}}, \tag{3.2}
\end{equation*}
$$

where $p$ are the pixels belonging to the particle image and $I_{\mathrm{p}}$ are the intensities of those pixels.
Reduced to a three-point estimator, this gives

$$
\begin{equation*}
{ }_{\mathrm{i}} x_{i}=\frac{{ }^{(+1)} I_{\mathrm{p}}-{ }^{(-1)} I_{\mathrm{p}}}{2\left({ }^{(+1)} I_{\mathrm{p}}+{ }^{(0)} I_{\mathrm{p}}+{ }^{(-1)} I_{\mathrm{p}}\right)} \tag{3.3}
\end{equation*}
$$

## Square-Weighted Centroid

This method is identical to the centroid method, except for using squared intensities to give a higher weight to larger values and therefore reduce the influence of noise and dark current effects in the image.

## Parabolic Distribution Fit

This method determines the center of a parabola fitted to the three pixel intensities with the highest intensity pixel in the center (Westerweel, 1993, p. 77), i.e.

$$
\begin{equation*}
{ }_{\mathrm{i}} x_{i}=\frac{{ }^{(-1)} I_{\mathrm{p}}-{ }^{(+1)} I_{\mathrm{p}}}{2\left({ }^{(-1)} I_{\mathrm{p}}+{ }^{(+1)} I_{\mathrm{p}}-2^{(0)} I_{\mathrm{p}}\right)} \text {. } \tag{3.4}
\end{equation*}
$$

### 3.1.2 Threshold Based Methods

These methods do not use the intensity information of the particle image beyond an initial selection of a binary region covering the area of the particle. Consequently, particle images larger than, for example, 2 px are required for these to work well and give results with subpixel precision.

## Binary Centroid

This method is the same as centroid method, but with $I_{\mathrm{p}}$ set to either 0 or 1 for each pixel depending on a given threshold intensity (Shortis et al., 1994, p. 6). Equation 3.2 is therefore simplified to be

$$
\begin{equation*}
{ }_{\mathrm{i}} x_{i}=\frac{\sum_{\mathrm{p} \mathrm{i}} x_{i, \mathrm{p}}}{n_{\mathrm{p}}} \tag{3.5}
\end{equation*}
$$

where $n_{\mathrm{p}}$ is the number of pixels in the thresholded region.

## Average Co-ordinates (Average of Perimeter)

Just like in the binary centroid, a number of pixels is averaged in their coordinates to obtain the center of the particle. Here, however, the pixels are taken from the perimeter of the thresholded region only (Shortis et al., 1994, p. 7). Equation 3.5 applies here as well, if p indices refer the pixels in the particle image perimeter.

## Ellipse Fit

Described by Shortis et al. (1994, p. 7), this method aims to fit an ellipse to a set of perimeter points of a thresholded particle. Using an iterative least squares estimation, the degrees of freedom for the fit are the center coordinates, semi-major axis, semi-minor axis and rotation.

### 3.2 Error Contributions

The following sections cover contributing factors to the overall error of the determination of the relative distance (i.e. length in the flow across the airfoil) between two illuminations of a particle in the test section. Relative, in this case, means in relation to a different determination of such a distance in a different part of the image, i.e. the first derivative of the distance in the image space. The goal is to get a very general idea of which effects deserve the greatest
attention in improving the location determination algorithm for the experimental data that has already been collected.

Existing research has been used as a basis of this estimation, but can only provide a very rough estimate of the errors because Gaussian or Airy-disk-like peaks are used almost exclusively and conclusions are not necessarily applicable between different algorithms for peak-fitting and displacement computation. This is especially a problem as the intent here is to perform the estimation of errors somewhat algorithm-agnostic.

Nevertheless, there should be some value in the trials that were preformed by researchers in the field at least for estimating the order of magnitude of the error from different effects. Additionally, the overview is aimed at highlighting steps and algorithmic properties that are crucial for keeping each of the error contribution factors within an acceptable range.

For a sensitivity analysis of the particle diameter fitting method to errors in the distance determination, see the following Section 3.3.

### 3.2.1 Quantization, Sampling and Sensor Errors

The resolution of both PTV cameras is $2048 \times 2048$ pixels (see Table 2.2). For each of these pixels, a given amount of illumination will result in a corresponding amount of charge for each pixel which is converted into a 10 bit digital value by an analog-to-digital converter (ADC). This results in a representation of the sensor illumination that is sampled in square two-dimensional pixel areas as well as quantized in intensity. Consequently, even the illumination of a pixel array with a perfect Gaussian intensity distribution would not allow for an exact reconstruction of the position of this Gaussian distribution on the pixel array. Sampling alone, on the other hand, does not necessarily degrade the result in the determination of the particle correlation peak if done correctly (Willert, 1996, p. 95; Westerweel, 1997, p. 1387).

## Intensity Quantization

For intensity based methods (Gaussian fit, centroid and squared centroid), this limitation only occurs as a result of the combination of the sampling and intensity quantization. It can be regarded only as a function of the intensity quantization level given that the particle images are larger than about 3 px in full width at half maximum (FWHM) or $\sigma=1.25$, according to T. A. Clarke et al. (1993, p. 7) and Shortis et al. (1994, p. 9). Shortis et al. (1994) also give quantization errors for Gaussian fit and Centroid methods of particle images of known size and shape. These are at or below $1 \times 10^{-2}$ px for quantization levels above 100 , and below $4 \times 10^{-2}$ px for quantization levels above 10. For edge threshold methods tested by Shortis et al. (1994) (namely ellipse fit, average coordinates and binary centroid), quantization is shown to not significantly influence the error. The threshold methods never reach the level of accuracy of the intensity based methods, even in cases where the latter is heavily degraded by quantization error. This might explain their lack of sensitivity to quantization as a result of not being precise
enough for this effect to matter.

## Sampling

Results from Shortis et al. (1994, p. 10) show no sampling error for intensity based methods, but a significant error for threshold methods, explained by the fact that they use no intensity information. As a result, ellipse fit and binary centroid methods have been shown to benefit somewhat from target sizes greater than $\sigma=1.25$, with error reductions by a factor of $\sim 2$ with $\sigma=3.5$ both for the binary centroid and the ellipse fit. The total error from spatial quantization alone can then be assumed to be around 0.1 px with $1.25<\sigma<2.25$ and below 0.07 px for $\sigma>2.25$ for methods not using intensity information. The $\sigma$ size values refer to the standard deviation for the Gaussian probability density function (PDF) that was used to generate the particle images.

Many peak finding methods suffer from a (non-random) bias towards integer displacement values that is often called "peak locking", as described by Scarano and Riethmuller (2000, p. 53). Their work shows that for a PIV analysis with, for example, 10 particles per window, this bias is reduced simply as a statistical result of having a zero mean and uniform amplitude distribution across particles. Given a fitting function with a relatively small peak locking tracking error, for example the Gaussian peak-fitting function, this leaves an error that is negligible compared to the random error. Since the goal of this investigation is the analysis of single particles, even Gaussian peak-fitting would, according to the results by Scarano and Riethmuller (2000, p. 53), result in a maximum location error of the order of 0.1 px .

## Gray Value Offset

As shown by T. A. Clarke (1995), a mis-adjustment of the black level in the ADC or afterwards can result in an oscillating error in the subpixel location of, in this specific case, a centroiddetermined particle position. This bias is similar in nature to the previously mentioned sampling error, but can be much stronger, possibly exceeding 0.2 px total error for a single particle (Nobach, 2004, p. 7).

The reason for this is an asymmetrical influence of the black level change on the relative brightness of pixels on one side of a particle compared to the other, depending on the subpixel location of the particle. This is likely to affect all intensity-based methods that fit some predefined intensity function. The squared centroid method should be affected less due to a stronger weight of large values. How heavily this affects threshold methods should depend on whether the specific method relies heavily on relatively dark pixels around the perimeter of the particle, i.e. on how low the chosen threshold is. Here, the target will shrink with an increased black level-unless the threshold is adjusted, in which case there is no effect at all. These assumptions are confirmed by results from Shortis et al. (1994, p. 10).

Given the source of the effect, it is likely to become more pronounced with decreased particle
intensity relative to the black level error, as well as with smaller particles where the edges between particle and background contribute more to the overall position analysis result. An increase of the error with decreased particle size can also be presumed, as the edge of a larger particle partially covers more pixels such that any bias regarding single pixel coverages should average out to some extent (but not necessarily entirely).

The effect of a nonlinear response curve of the sensor or ADC to the incoming light intensity is similar in nature to the effect due to an incorrectly set black level.

Overall, it can be concluded that these errors are small (probably $\sim 1 \times 10^{-2} \mathrm{px}$ ) as long as no modifications to the original linear signal from the sensor are applied and particles are recorded in front of a truly black background. In the given experiment, the background is unfortunately neither perfectly black (the opposing wall does reflect some light as it is made of stainless steel) nor is the gas fully transparent as it often contains small amounts of very fine fog, essentially increasing the base intensity in the image locally in a non-uniform manner.

For the final subpixel determination of particle offset, any noise background level should not be subtracted globally but instead only from areas outside of detected particle areas. For the fog component, the increase in intensity may originate from gas in front of or behind the particle, and any decision to subtract or not subtract the background level will be correct only in some cases. Hence, some error will always remain from this gray value offset, depending on how exactly it is handled. Details on how this topic is treated in the final implementation can be found in Section 4.5.1.

## Saturation

Some of the images from our PTV cameras contain pixels with saturated intensities, i.e. the saturation limit of the pixel or ADC was reached and the maximum image value was stored regardless of the amount of additional incident light. An overexposed particle image contains less information especially around the central peak that would usually be most influential in determining the location of the particle image relative to the corresponding partner image. It may also slightly increase peak-locking as it can concentrate much of the decline of the illumination at the point where the saturation stops.

Gui and Wereley (2002, p. 513) show results of overexposed PIV seeding with their continuous window shift correlation algorithm. With a number of particles of the order of $2 \times 10^{1}$ in their test windows, the results are not directly applicable to a single particle distance determination. While the higher number of particles allows for some averaging of errors, it also means that the overexposure covers a very significant portion of the entire frame and connects some single particles, which is very likely a contributing factor to the loss of accuracy in these results. It is shown, however, that the random error is greater than the peak-locking error, and the overall error is increased only by a factor of about two compared to the non-overexposed image.

Shortis et al. (1994, pp. 9-10) show a similar analysis for a wide range of algorithms, with overexposure intensities up to a factor of 2 . Due to the saturation, the Gaussian fit reaches an
error of $\sim 2 \times 10^{-2}$ px while the threshold methods do not appear to be affected negatively far beyond their already reduced accuracy.

The conclusion that can be drawn from this is that the error from pixel saturation can potentially reach $1 \times 10^{-1}$ px but is likely much lower, largely depending on the exact characteristics of the algorithm. This experiment may be less susceptible to this than general PIV because the particles are not Gaussian to begin with, nor will any specific shape be assumed in the fitting algorithm. Nevertheless, it seems advisable to discard heavily overexposed particle images simply because losing a few particle tracks is not problematic for the given analysis, and the vast majority of particles images that were recorded are do not contain saturated pixels.

If the goal of the analysis is specifically to find the diameters of the largest particles that were recorded, which are likely to also produce the brightest particle images, discarding overexposed particle images may not be possible.

## Noise

The error due to noise that can be expected from fitting any peak shape to a respective particle image should be comparable to fitting a Gaussian peak to a Gauss-like particle image, provided that the fitted function matches the particle image intensity distribution reasonably well.

Marxen et al. (2000, p. 148) present errors for least-square Gaussian peak fitting and threepoint Gaussian peak fitting with varying levels of the standard deviation of added Gaussian noise. The error for both methods is very similar and approximately proportional to the noise standard deviation. While the error is approximately constant with increasing particle sizes from diameters of 3 px to 5 px , it eventually increases and becomes approximately proportional to particle size for large diameters. For an upper limit of 64 px in absolute particle size in our case (which corresponds to a particle diameter of 9 px for round particles), we get a position error of approximately

$$
\begin{equation*}
0.13 \mathrm{px} \frac{\sigma_{\mathrm{n}}}{I_{\max , \mathrm{p}}} \frac{200}{12} \tag{3.6}
\end{equation*}
$$

where $\sigma_{\mathrm{n}}$ is the standard deviation of the noise and $I_{\mathrm{p}}$, max is the peak of the intensity distribution of the ideal Gaussian particle image.

To compare this to the measurements at hand, it is necessary to estimate the noise standard deviation as well as the peak intensity in the recorded images. Figure 3.1 shows the intensities of the brightest pixel in each respective particle that was identified, both for the entire set of particles as well as for the limited number that end up in a final trace. The latter is intended as a general indication of how the distribution changes in detected traces and may vary significantly between different trace identification settings and recorded images. Based on these histograms, it is safe to say that the vast majority of particles in traces will have a peak intensity of at least $10^{4}$. After all, the peak intensity of the true brightness distribution for each particle is always higher than the brightest recorded pixel.

To find a value for the noise floor in the PTV measurement, the standard deviation across


Figure 3.1: Brightest pixel intensity histograms for the particle images of a PTV measurement image. Particle images were selected with a mean threshold displacement of 6 , minimum particle image size of 3 px and maximum size of 64 px . In both cases, the y axes are normalized such that the area under the histogram equals 1 . The saturation limit for the given image was 65344 or $10^{4.815}$.

5 images without (significant) seeding was calculated. For each pixel location, the standard deviation is based on the image values across all images and across the surrounding $3 \mathrm{px} \times 3 \mathrm{px}$ area, making it a standard deviation across a total of 45 pixel values. The local standard deviation of the pixel, $\sigma_{\mathrm{p}}$, is

$$
\begin{equation*}
\sigma_{\mathrm{p}}=\sqrt{E\left(I_{\mathrm{p}}^{2}\right)-\left[E\left(I_{\mathrm{p}}\right)\right]^{2}} . \tag{3.7}
\end{equation*}
$$

Here, $I_{\mathrm{p}}$ are the pixel intensities of the image, and $E\left(I_{\mathrm{p}}\right)$ is the expected value of $I_{\mathrm{p}}$, calculated here from the average value of the surrounding pixel values and across the given images. Figure 3.2 shows the results of this computation on the 5 dark frames as well as the 5 images with ice particle seeding in the form of a histogram for the image standard deviation across all image pixels. Figure 3.3 contains the corresponding visualizations of the standard deviation in image form, helping to put into context which image features correspond to which standard deviations.

The dark frame case shows that the noise in areas that are not illuminated is negligible, as $\sigma_{\mathrm{p}}$ values are around $10^{-4}$ giving a signal-to-noise ratio (SNR) of around $10^{8}$. Values are higher in some background areas that reflect some light, with the outline of the opposite side camera box being visible in the standard deviation analysis. There, values of $\sigma_{\mathrm{p}}$ approach $10^{2}$ in many areas.

Looking at the image with seeding, it becomes clear that the dark frame noise is not dominant in the actual measurement. Unwanted background seeding of very small particles adds a noise


Figure 3.2: Local standard deviation histograms for two sets of 5 images from the NFT camera. Both are normalized such that the area under the histogram equals 1. Corresponding image plots for $\sigma_{\mathrm{p}}$ are shown in Figure 3.3.
Standard deviations have been capped at $10^{-13}$, including values of zero-these pixels are essentially black with no noise. Values around $10^{-6}$ correspond with what appear to be sensor readout artifacts (single vertical and horizontal lines).


Figure 3.3: Local standard deviation in two images from the NFT camera. Again, the saturation limit of these images is at 65344 or $10^{4.8152}$. Corresponding histograms are shown in Figure 3.2.
a) Dark frame with very little seeding (taken before the first seeding of the day, with full laser illumination. A few very faint particles are still visible.
The median value of $\sigma_{\mathrm{p}}$ is $10^{1.12}$.
b) Measurement frame with seeding, otherwise close to identical flow conditions as above.
The median value of $\sigma_{\mathrm{p}}$ is $10^{2.92}$.
component that is much stronger, with $\sigma_{\mathrm{p}}$ showing values of $10^{2.6}-10^{3.0}$ across most of the image. This specific set of images was chosen as a relatively bad scenario for the background seeding - the amount varies significantly between different runs but is rarely entirely negligible. Values in excess of $10^{3.1}$ only appear in image regions containing large seeding particles, which are not relevant for this discussion (they are signal rather than noise). Between relatively faint seedings particles and the seeding noise background, the worst case for the peak intensity to noise background ratio is then at slightly below one order of magnitude, or

$$
\begin{equation*}
\frac{\sigma_{\mathrm{n}}}{I_{\mathrm{max}, \mathrm{p}}} \approx \frac{1}{10^{0.9}} \tag{3.8}
\end{equation*}
$$

Unlike a random noise addition to the image, approximately half of this noise will likely originate from behind the seeding particles, such that the impact on the precision determination on the particle image (which is cut out from the background) will be slightly less severely affected (from the noise aspect that is-the impact from the grey value offset was discussed above). Ignoring this, the worst case error that would be derived from equation 3.6 amounts to $\sim 0.273 \mathrm{px}$.

It is reasonable to assume that only some of the fine material that is causing this noise contribution is in front of the particles that are being analyzed. If it was only half, a factor of $\sqrt{2}$ could be gained in the noise standard deviation, giving a "half-noise" error of $\sim 0.193 \mathrm{px}$. However, for a conservative estimate, the assumption is that the particle in question is at the very far end of the illuminated volume and affected by all of the fine material in the test section.

Furthermore, the maximum particle size $A_{\mathrm{p}}$ of 64 px was chosen as it produces the largest error, but the largest particles are likely to have higher peak intensities. Particle images with peak intensities of less than $10^{4.3}$ almost always have sizes no greater than $24 \mathrm{px}\left(A_{\mathrm{p}}<2^{4.585}\right.$, corresponding with a 5.5 px diameter). Figure 3.4 shows that this statement is true with very few exceptions (less than $2.5 \%$ of the particles). For the smaller particle images, the approximate error from Marxen et al. (2000, p. 148) then becomes

$$
\begin{equation*}
0.08 \mathrm{px} \frac{\sigma_{\mathrm{n}}}{I_{\max , \mathrm{p}}} \frac{200}{12} \tag{3.9}
\end{equation*}
$$

resulting in a relative position error of 0.168 px . For the larger ones,

$$
\begin{equation*}
\frac{\sigma_{\mathrm{n}}}{I_{\max , \mathrm{p}}} \approx \frac{1}{10^{1.2}} \tag{3.10}
\end{equation*}
$$

then results in a relative position error of 0.137
In either case, this is a conservative estimate and most particles have a much better SNR. Still, it represents a very significant addition to the overall error that is difficult to mitigate for the existing measurement.

### 3.2.2 Resolution of the Lens

Any lens system has a number of aberrations that limit the achievable maximum resolution in addition to the resolving limitations of the sensor. A quick look into these aberrations is meant to estimate any possible error contributions from these aberrations.


Figure 3.4: Particle size histogram for all particles (after size and saturation filtering) and those with low peak intensities, specifically $I_{\text {max,p }}<10^{4.3}$. Size filtering removed all particles with sizes greater than $64 \mathrm{px}\left(2^{6}\right)$ and lower than $3 \mathrm{px}\left(2^{1.58}\right)$.

## Diffraction Limit

The resolution of a perfect lens with a circular aperture is limited by diffraction of light at the aperture. The result of this diffraction is that a point light source in the focal plane corresponds to an Airy pattern in the image (Airy, 1835, p. 287).

Commonly, the achievable resolution based of a diffraction limited optical system is described by the Rayleigh criterion (Rayleigh, 1879, p. 262), which assumes that two objects cannot be separated if the central disks of the Airy patterns of two light sources are overlapping. This corresponds to a resolution of

$$
\begin{equation*}
\vartheta_{\text {Rayleigh }}=1.21967 \frac{\lambda}{D_{\mathrm{A}, \mathrm{e}}} \tag{3.11}
\end{equation*}
$$

where $\vartheta_{\text {Rayleigh }}$ represents the angular resolution, $\lambda=532 \mathrm{~nm}$ the wavelength of the observed light source, and $D_{\mathrm{A}, \mathrm{e}}$ the effective diameter of the aperture of the lens. More usefully, the resolution in the focal plane, $r_{\text {Rayleigh }}$, is therefore

$$
\begin{equation*}
r_{\text {Rayleigh }}=\arctan \left(\vartheta_{\text {Rayleigh }}\right) z_{\text {obs. }} \tag{3.12}
\end{equation*}
$$

where $z_{\text {obs. }} \approx 230 \mathrm{~mm}$ is the depth from the no-parallax point (NPP) of the lens to the focal plane.

For the two lenses used in the PTV and PIV measurements, this results in the values shown in Table 3.1. The aperture sizes were set such that the Rayleigh criterion resolution would

| Lens | Cinegon 1.8/16 | Apo-Xenoplan 2.8/50 |
| :--- | ---: | ---: |
| Application | WFT camera | NFT camera |
| Lens specifications | Table 2.3 | Table 2.4 |
| Focal length | 16.4 mm | 50.2 mm |
| Relative aperture (max.) | $1 / 1.8$ | $1 / 2.8$ |
| Relative aperture (set) | $1 / 8$ | $1 / 8$ |
| Absolute aperture (set) | 2.05 mm | 6.275 mm |
| Pixel size in focal plane ${ }^{\mathrm{i}}$ | $72.80 \mu \mathrm{~m}$ | $23.78 \mu \mathrm{~m}$ |
| $r_{\text {Rayleigh }}$ | $77.13 \mu \mathrm{~m}$ | $25.20 \mu \mathrm{~m}$ |

${ }^{\text {i }}$ Calculated based on field of view from nominal focal length, which does not match the calibration results exactly. This is at least partly a result of flange distance modifications.
Table 3.1: Lens resolutions of the WFT camera and NFT camera based on the Rayleigh criterion.
approximately match the pixel resolution. This is desirable because a smaller aperture size mitigates some of the lens other aberrations, and an Airy disk that is much smaller than the pixel pitch does not enable distinguishing additional particles in the flow.

Unlike some other aberrations, the diffraction resolution does not significantly limit the potential for determining exact differences in particle positions. Especially for small particles, the Airy pattern can actually increase the visual size of very small particles into a regime that is more suitable for peak finding - a particle size smaller than or around 1 px does not permit subpixel analysis as only one pixel intensity value is potentially available for a given particle. So at least for radially symmetrical particle brightness distributions, Airy patterns of $\sim 1 \mathrm{px}$ do not reduce the accuracy for a given particle pair - they may however hinder the separation of particles with overlapping intensity distributions in an image with very high seeding density.

## Coma, Astigmatism and Spherical Aberration

Schneider Kreuznach provides modulation transfer function (MTF) curves for the lenses that were used, which can be referenced in order to estimate the error introduced through the non-diffractive aberrations from the lens (Schneider Kreuznach, 2008; Schneider Kreuznach, 2013). These curves represent the contrast in the image plane (i.e. on the sensor) for a given frequency of detail. MTFs include the reduction in contrast at high frequencies due to the Airy pattern, but this reduction is constant across the image plane and results in a flat MTF upper limit from the center of the image towards the edge. The diffraction limited MTF is known to be (Johnson, 1972)

$$
\begin{equation*}
\mathrm{T}=\frac{2}{\pi}\left(\arccos (\nu)-\nu \sqrt{1-\nu^{2}}\right) \tag{3.13}
\end{equation*}
$$

with

$$
\begin{align*}
\nu & =\frac{f}{f_{0}}  \tag{3.14}\\
f_{0} & =\frac{D_{\mathrm{A}, \mathrm{e}}}{\lambda} . \tag{3.15}
\end{align*}
$$

Here, T is the $\mathrm{MTF}, \nu$ is the spatial frequency $f$ normalized to the cutoff frequency $f_{0}$. This gives an ideal, diffraction limited MTF for each frequency as shown in Figure 3.5. The curve is identical for the two lenses because the cutoff frequency only depends on the wavelength and the relative aperture, which are the same in both cases.


Figure 3.5: Diffraction limited MTF curve for the two PTV lenses at a relative aperture of $1 / 8$. $l_{\mathrm{f}}=50.2 \mathrm{~mm}:$ Apo-Xenoplan 2.8/50 Ruggedized, see Table 2.4. $l_{\mathrm{f}}=16.4 \mathrm{~mm}$ : Cinegon 1.8/16 Ruggedized, see Table 2.3.

Unfortunately, the charts given by Schneider Kreuznach only offer information for frequencies of 10,20 and 40 cycles $/ \mathrm{mm}$, which are relatively low given that the sensor has pixels at a frequency of $\sim 182 \mathrm{px} \mathrm{mm}^{-1}$ and could therefore resolve around 90 cycles $/ \mathrm{mm}$. Nevertheless, the information gives some indication regarding whether there are strong aberrations towards the edge of the image compared to the center, as the MTF curves would then have to drop off towards the outer edges.
In general, problematic aberrations are those that produce point spread functions (PSFs) which are not radially symmetrical or that depend significantly on the position on the image plane. Asymmetric PSFs mean that the distribution of the light intensity can shift relative to the actual position of the particle, usually depending on the distance from the center of the image.

This would cause an immediate error in the comparison of the relative positions. Asymmetric PSFs like coma and astigmatism necessarily get worse towards the outer part of the image, and affect sagittal and tangential resolution differently. PSFs that are radially symmetrical but still vary greatly between the center of the image and the outer edge make comparisons of the particle images between different positions on the image difficult and are therefore likely to increase the error as well. Spherical aberration, similar to the loss of resolution through diffraction, produces radially symmetrical aberrations that do not severely inhibit the analysis of the position of a particle.

Therefore, the focus of the analysis lies on the variability of the MTF across away from the center and on the difference between sagittal and tangential values.

The charts for the NFT camera lens at an object to image distance of $\mathrm{OO}^{\prime}=223 \mathrm{~mm}$ (which is very close to this length in the experiment) and a relative aperture of $1 / 8$ show MTF values between $49 \%$ in the center and $56 \%$ (sagittal) at the outer corner of our sensor. The outer corner of both camera sensors is at $u^{\prime} \approx 8 \mathrm{~mm}$, corresponding to $72 \%$ on the x -axis of the given charts. Sagittal and tangential contrast losses are always within $5 \%$ of each other, which suggests very little coma or astigmatism.

Spherical aberration may explain the difference between the diffraction limited ideal MTF of around $78.5 \%$ and the actual values. If the MTF drops approximately linearly with increasing frequencies (which is a very roughly true when looking at many MTF curves), doubling the loss of contrast correlates to half the resolution. ${ }^{1}$ Therefore, the overall sharpness with an MTF between $56 \%$ and $49 \%$ is slightly less than half of what would be possible in the diffraction limited case, and we can expect a blur radius of the particle images around 2 px . Given that the particle images should be around that size anyway for optimal conditions after sampling (see Section 3.2.1), this overall resolution is at the lower limit of what is ideal but should not cause significant problems. A difference in the loss of contrast $(1-\mathrm{T})$ of around $10 \%$ between the best and worst point in the image (and less between sagittal and tangential values) suggests a distortion of the particle images of approximately that much, i.e. around 0.2 px .

The charts for the WFT camera lens at an object to image distance of $\mathrm{OO}^{\prime}=211 \mathrm{~mm}$ (which is close to this length in the experiment) and a relative aperture of $1 / 8$ show sagittal MTF values between $73 \%$ in the center and $81 \%$ about two thirds of the way out to the edge of our sensor. At the outer edge, the value falls back down to about $77 \%$. Tangential MTF values, on the other hand, drop down to around $62 \%$ on the very edge, and do so pretty linearly.

While the sagittal resolution is close to or at the diffraction limit across the image, the loss of contrast in the tangential plane is almost $90 \%$ greater than in the sagittal plane at the point of greatest difference. This indicates some problematic aberrations for our analysis with sizes of the order of 1 px .

It is not likely that an aberration of the order of 1 px will result in a deviation of a particle image distance by that amount, but it will likely prevent a precision of an order of magnitude

[^5]less, which is why a conservative estimate for now is that the measurement will deviate by half that amount. This gives an expected error of 0.5 px for the WFT camera and of 0.1 px for the NFT camera from those aberrations.

## Focus

The lenses have been focused manually at full pressure and final measurement temperature, with particles in the flow illuminated by the laser sheet. Similar to the effect of spherical aberration, particles that are out of focus increase their size without any change to the center of their distribution of sensor illumination. Therefore, the effect of a slight deviation from perfect focus for some of the particles on the analysis is negligible.

## Distortion

The given distortion curve for the NFT camera lens (Schneider Kreuznach, 2008) suggests a maximum distortion within our imaging range ( $\pm \sim 8 \mathrm{~mm}$ ) and at an object to image distance of $\mathrm{OO}^{\prime}=223 \mathrm{~mm}$ of between $0.005 \%$ and $0.01 \%$ (the charts are not readable to a higher precision). This corresponds to a deviation of less than 0.10 px at the outer edges, and about an order of magnitude less across a single particle distance which is approximately 100 px long. If the point distance determination does reach this level of precision, some of this distortion can be corrected based on the calibration, but even without any correction, this error does not present a major concern.

The given distortion curve for the WFT camera lens (Schneider Kreuznach, 2013) shows a maximum distortion within our imaging range ( $\pm \sim 8 \mathrm{~mm}$ ) and at an object to image distance of $\mathrm{OO}^{\prime}=211 \mathrm{~mm}$ of $-2.16 \%$. This corresponds to a deviation of about 22.1 px at the outer edges. Given that our object to image distance does not exactly match the one in the chart that is available, and the camera is possibly not perfectly centered on the central axis of the lens, this information is not directly usable for a correction of this distortion. When using the image from the WFT camera, it is therefore clear that the distortion must be taken into account and the calibration must measure the exact amount of distortion in order to approach subpixel accuracy in particle image distance measurements.

Given that the displacement calibration returns four distance values across the relevant area, a curve fit to this data should achieve a distortion correction accuracy within the precision of the calibration itself. The precision of the calibration is in turn higher than the precision of the measurement, because several error contributions such as the shock wave or the presence of non-spherical particles do not apply. As such, the error from distortion can be neglected if a correction from a multi-point calibration is applied.

### 3.2.3 Particle Reflection Profile and Center of Mass

Whereas the velocity of a particle can be taken exactly from the measurement of two positions of the center of mass of the particle at different times, the intensity of the scattered light from a given particle does not necessarily align with its center of mass. Given that the particles are illuminated from one side only, at an angle of approximately $90^{\circ}$ with slight variation depending on the position in the image, it is likely that the position of the center of mass is shifted relative to the intensity maximum, or any peak fitting result. This does not cause a problem if this shift is constant across the image, as only the particle image distance is relevant for this experiment.

While the reflective behavior of water droplets is very well understood and described (Van de Hulst and Wang, 1991; Hess and L'Esperance, 2009), a precise prediction is likely not possible for very small ice particles given their irregularity. Nevertheless, the reflective behavior for a single particle as it crosses the imaged area is unlikely to change dramatically, as this is neither the case for water droplets nor for diffuse or specular spheroids or cylinders in most orientations. For the very narrow orientation range in which, for example, a specular cylinder would produce a direct reflection which could shift the illumination center dramatically, the intensity is likely to also change very dramatically.

It is therefore assumed that restricting the combination of particle pairs to particles within a few orders of magnitude in brightness, and using diffusely reflecting white spheres for the reproductions scale and distortion calibration (see Section 5.3), resolves any potential issues with shifts in the reflective behavior across the frame.

### 3.2.4 Perspective Depth

Particles that are closer to the camera or further away from it than the calibration distance travel a different real-world distance for the same displacement on the image as a consequence of perspective. More precisely, the distance that a particle is displaced in the image is inversely proportional to its distance from the NPP of the lens. The laser sheet in the test section has a width (or, from the perspective of the camera, a depth) of approximately 3 mm . The distance to the lens NPP is approximately 230 mm , which gives a maximum deviation of $0.65 \%$, or $1.1 \%$ assuming that the calibration depth is only precise to 1 mm . A deviation of $1.1 \%$ corresponds to a shift by 11 px at the very edge of the frame.

This effect, however, is entirely linear across the frame under the assumption that the particles are traveling on straight lines parallel to the sensor plane and the laser sheet. Because the measurement is performed in the exact center of the symmetrical test section, the assumption of parallel flight paths is likely valid. Consequently, the absolute velocity measurement is affected by this, while the relative change in velocity across and after the shock wave does not incur any major error.

### 3.2.5 Refractive Effects of the Shock Wave

The PTV measurement is conducted with a camera which is positioned on one side of the channel looking in perpendicularly, at a location just behind the compression shock wave. This shock wave causes an increase in density in the nitrogen gas within the test section. For a constant chemical composition, the refractive index $n_{r}$ of an ideal gas depends on the density of the gas $\rho$ with the relation

$$
\begin{equation*}
\rho \propto\left(n_{\mathrm{r}}-1\right) . \tag{3.16}
\end{equation*}
$$

Therefore, light reflected off of particles ahead of the shock wave will undergo refraction at the surface of the shock wave before arriving at the camera. This means that the position of the particles as seen in the camera image is distorted compared to the actual location of those particles. Similarly, the shapes of the particles themselves are also distorted, and some PTV images show this effect very clearly in areas where particles are observed such that the light passes through the shock wave at a very acute angle.

Within this chapter, ${ }_{\mathrm{i}} \hat{x}$ always implicitly refers to the horizontal axis, i.e. ${ }_{\mathrm{i}} \hat{x}_{1}$. While $x$ generally represents absolute distances from the center in the focal plane, ${ }_{\mathrm{i}} \hat{x}$ represent distances relative to the field of view, where ${ }_{\mathrm{i}} \hat{x}=-1.0$ is the very left edge of the image and ${ }_{\mathrm{i}} \hat{x}=1.0$ represents the right edge.

A detailed description of the assumptions and calculation steps to obtain the following results can be found in Appendix A.3.

## Analysis Results

The amount of the distortion that can be expected for a non-oblique idealized shock wave of a strength comparable to that in the experiment is shown in Figure 3.6 as well as Figures 3.7 a and 3.7 b . For a shock wave position further back on the airfoil than $\frac{x_{a}}{c}=72 \%$, total reflection at the shock wave occurs and effects such as duplicate particle images make any particle image analysis infeasible in the affected area.

As a point of comparison when looking at the charts, one pixel represents around $20 \mu \mathrm{~m}$ in the focal plane. The strong shock wave in the given measurements from scenario B wind tunnel runs is usually located between ${ }_{\mathrm{i}} \hat{x}=-0.5$ at the very bottom and ${ }_{\mathrm{i}} \hat{x}=-0.6$ around the vertical center of the image. Any higher positions are not relevant because the shock wave becomes too weak at the increased distance from the airfoil model. This is consistent with a shock wave position around $\frac{x_{a}}{c} \approx 65 \%$ that is expected based on the airfoil pressure coefficient measurements (see Figure 2.10 in Section 2.5.2), given that the shock curves forward at increasing distances from the model surface.

The results indicate that, for these shock wave positions, the highest expected distortion of a single trace connection about $10^{2} \mathrm{px}$ in length (corresponds with $\delta_{\mathrm{i}} \hat{x} \approx 0.1$ ) could be around $5 \%$. This corresponds with an error of about 5 px , which is beyond any reasonably expectation


Figure 3.6: Deviation of ${ }_{\mathrm{i}} x_{\text {perceived }}$ compared to ${ }_{\mathrm{i}} x_{\text {actual }}$ due to refractive effects of the shock wave, depending on the particle location on the image ${ }_{\mathrm{i}} \hat{x}_{\text {perceived }}$ and the shock wave position ${ }_{i} \hat{x}_{\mathbf{s}, 1}$.
Note that the coordinates are inverted horizontally compared to what the camera would actually see in order to achieve consistency with images that show a flow direction from left to right.


Figure 3.7: Distortion of $\delta_{\mathrm{i}} x_{\text {perceived }}$ compared to $\delta_{\mathrm{i}} x_{\text {actual }}$ due to refractive effects of the shock wave, depending on the particle location on the image ${ }_{i} \hat{x}_{\text {perceived }}$ and the shock wave position ${ }_{\mathrm{i}} \hat{x}_{\mathbf{s}, 1}$. Coordinates are inverted horizontally compared to what the camera would actually see in order to achieve a flow direction from left to right.
a) Overview over the entire distortion regime, logarithmic in $y$.
b) Detailed view of smaller distortions, linear in y.
of accuracy for this measurement.

## Discussion

Applying a correction for this issue is non-trivial. For a known non-oblique ideal shock wave, it would be relatively easy to warp the image coordinates using the data shown above such that the distortion would be compensated well enough for most of the frame. However, the shock wave is slightly oblique, curved, and neither its strength nor position can be determined precisely. There are also additional flow features close to the wall, especially in the interaction of the shock wave with the wall boundary layer, that have optical effects on the observation. These effects should, however, be much weaker because the refraction angles are likely less acute and the density differences smaller. Nevertheless, a correction in post processing of the resulting images would require a three-dimensional numerical simulation of the flow in the tunnel and a subsequent optical simulation using a raytracing engine. Even then, precision would be lost due to a lack of knowledge about the properties and position of the shock wave at any point during the test, as these are known to fluctuate.

Calibration in the actual experiment is also very challenging. As the shock wave is only present in the high-speed flow, calibration should ideally be performed during a wind tunnel run. The tunnel, however, does not provide good mounting positions for placing a calibration target in the center of the flow perpendicular to the airfoil (in the focal plane of the camera). Mounting it on the airfoil itself would be challenging without disturbing the flow around it or damaging the TSP coating, as any mounting mechanism would have to withstand the forces from flow speeds of $\sim 200 \mathrm{~m} \mathrm{~s}^{-1}$ at $\sim 186 \mathrm{kPa}$ of pressure. Such a calibration would then still cost several days of measurement time for closing, cooling, running, reheating and reopening the tunnel. Due to these difficulties, this approach was rejected.

In order to verify how well a simplified refraction model matches the actual distortion in the experiment, it would be helpful to at least have a calibration line on the tunnel wall opposite to the camera. However, using this would require a change in focus of the lens that is beyond what can easily be adjusted in the current lens setup.

The WFT camera is located further upstream in the test section, such that it is reliably ahead of the shock wave and has a field of view entirely overlapping that of the NFT camera. This camera should not suffer significant distortion in its observation of particles ahead of the shock wave, at least not from the main shock wave. It is calibrated together with the narrow field camera ahead of the test run and can therefore be used to determine the velocity of particles ahead of the shock wave if it is triggered (approximately) simultaneously with the narrow field camera such that it records the same laser pulses. It should then be possible to use, within each track, the WFT camera for the particle positions ahead of the shock wave and the NFT camera for positions behind the shock wave, with a given known projection of both camera images onto a common coordinate system based on the calibration. The wider field of view relative to the NFT camera results in a loss of precision by a factor of about 3 due to quantization and sensor
errors alone (see Section 3.2.1). A major challenge in this is the separation of the particles in the lower resolution image of the WFT camera, and the reliable correlation of the tracks between the two cameras. For achieving subpixel accuracy in the velocity measurement that is taken in the transition between the two cameras, it would be necessary to perform the mapping onto a common coordinate system with very high accuracy.

Beyond a compensation of the error due to refraction, which would be the primary benefit of this method, it would also be possible to derive some information about the shock wave itself from the difference in particle positions between the two images. This would be based on a first order approximation of the optics of the shock wave like the one done above, and consequently not very precise.

The reason for discarding this approach is the lack of optical quality of the WFT camera lens according to the results of Section 3.2.2 as well as problems with noise found in Section 3.2.1 which are amplified through the wider focal length. To conclude, it is necessary to find a way to analyze the given results without making any use of the particle images located ahead of the shock wave.

### 3.2.6 Refractive Effects of the Changing Medium

There is additional distortion caused by the fact that the camera is looking through a different medium during wind tunnel runs (nitrogen at a low temperature and high pressure) compared to the calibration conditions (air at atmospheric pressure and room temperature). This distortion is linear in nature and results in a slight change in the effective focal length compared to what was measured during calibration.

Given that both the camera boxes and the interior of the wind tunnel contain nitrogen exclusively, the difference in refractive index comes from the difference in density only. Assuming a temperature of $0^{\circ} \mathrm{C}$ as well as standard atmospheric pressure in the camera box (which is heated), and a density of approximately $3.596 \mathrm{~kg} \mathrm{~m}^{-3}$ in the flow, the distortion only affects the reproduction scale with a factor of 1.00054 (increasing the actual distance relative to the measured distance by about 540 ppm ). This influence is below the uncertainty of the reproduction scale measurement during calibration, which is why it is not considered further.

### 3.2.7 Shock Wave Position

Any attempt at correcting for the effects described in Section 3.2.5 requires some knowledge about the position of the shock wave in both the NFT camera as well as the WFT camera image. As described in Section 1.7.2, knowing the exact shock wave position is not necessary to scientifically evaluate the results from the experiments.

While the TSP method returns information about the position of the shock wave on the airfoil surface, that location information is not very precise and does not directly correspond to the position of the shock wave higher up in the airflow. The PIV measurement, on the other hand,
provides a relatively precise shock wave position, but for a different wind tunnel run that was performed with very similar parameters. This cannot easily be determined on the same run as PIV requires different seeding characteristics. Because the shock wave is moving during the measurement time and does not position itself very reliably at the exact same position for each run, this information is also not highly accurate for any given PTV analysis.

Therefore, only an approximate shock wave position can be determined from the PTV data itself or be inferred with some loss of precision from the PIV results of a different run. This does not negatively affect the slip velocity relaxation analysis itself, as long as traces are selected that start well after the approximate shock wave position.

### 3.2.8 Particle Matching

When evaluating multiple particle tracks from a number of images and deriving conclusions from these results, it can be expected that tracks will occasionally contain mismatched pairs of particles, meaning that tracks switch from one (true) particle track to another over the course of an eight pulse sequence. In the current work, this risk is very manageable mostly because every single result can be evaluated individually, and velocity curves that do not match the bulk of those that have been observed can be discarded or at least checked manually. It is also not critical for this work to analyze a very high percentage of all possible particle tracks in the images that are available - instead, the goal is to find a small number of high quality measurements for particles that appear to be of a usual size for a given wind tunnel run. As a consequence of that, it is possible to choose particle tracks without competing tracks in the immediate vicinity, if those are available in the given measurement.

Any error from mismatched traces is therefore not considered for this evaluation. More information on work that has been done to improve the selection of correct traces can be found in Chapter 4.4.

### 3.2.9 Summary

Table 3.2 summarizes the error contributions discussed in this chapter. Some more complex dependencies are reduced down to a single value, which is why this mainly provides a first-glance estimate of where the biggest difficulties are located and which areas deserve increased attention.

This analysis focuses only on the errors in the determination of relative particle image distances, accepting certain error in the determination of the absolute velocity of the particles. The latter is caused by effects such as the refractive effect of cooling down the test fluid compared to the time of calibration, a possible slight shift of the laser sheet depth between calibration and runs due to the geometry of the laser arm, and the perspective effect of tracks being closer to or further away from the lens.

The summary of the results for the NFT camera shows that, apart from the area ahead of

| Contributing factor | NFT <br> Error <br> [px] | amera <br> timate <br> [ $\mu \mathrm{m}$ ] | WFT <br> Error <br> [px] | camera <br> stimate <br> [ m ] |
| :---: | :---: | :---: | :---: | :---: |
| Quantization and sensor |  |  |  |  |
| Intensity quantization | $10^{-4}$ | $2 \times 10^{-3}$ | $10^{-4}$ | $8 \times 10^{-3}$ |
| Sampling | $10^{-1}$ | 2 | $10^{-1}$ | 8 |
| Gray value offset | $10^{-2}$ | $2 \times 10^{-1}$ | $10^{-2}$ | $8 \times 10^{-1}$ |
| Saturation | $5 \times 10^{-2}$ | $10^{-1}$ | $5 \times 10^{-2}$ | 4 |
| Noise | $10^{-1}$ | 2 | $1 \times 10^{-1}$ | 8 |
| Resolution of the lens |  |  |  |  |
| Diffraction limit | - | - | - | - |
| Coma, astigmatism, sph. aberr. | $10^{-1}$ | 2 | $5 \times 10^{-1}$ | $4 \times 10^{1}$ |
| Focus | - | - | - | - |
| Distortion | $10^{-2}$ | $2 \times 10^{-1}$ | s.o.e. ${ }^{\text {i }}$ | s.o.e. ${ }^{\text {i }}$ |
| Center of mass correlation | - | - | - | - |
| Perspective depth | - | - | - | - |
| Refractive effects of the shock wave ${ }^{\text {ii }}$ | 5 | $10^{2}$ | $\mathrm{n} / \mathrm{a}^{\text {iii }}$ | $\mathrm{n} / \mathrm{a}^{\mathrm{iii}}$ |
| Shock wave position | - | - | - | - |
| Particle matching | - | - | - | - |

${ }^{i}$ See other errors: The error here depends on the precision of the calibration, which is in turn a function of the general precision of the distance determination for this camera view.
ii Only particles ahead of the shock wave are affected.
iii The WFT camera is only potentially used for the purpose of not having to use particles that would be imaged through the shock wave. For this reason, an analysis of this camera regarding this factor is not relevant.

- Deemed not significant in comparison with other error contributions.

Table 3.2: Conributions to the overall error in a single particle PTV offset determination. All numbers represent rough estimates for the purpose of evaluating the most useful areas of focus for improvements. Errors are given in pixels and $\mu \mathrm{m}$ for the distance between a single pair of particle images.
the shock wave (which is essentially unusable for the purposes of this work), the main sources of error are the sampling error, noise, and the resolution limit of the lens due to optical aberrations. All of these have been estimated to be around $10^{-1} \mathrm{px}$, which corresponds to about $2 \mu \mathrm{~m}$.

Regarding the WFT camera, the results look much less promising. Especially the expected error due to optical aberrations is about a factor of 20 higher than in the NFT camera, at around $4 \times 10^{1} \mu \mathrm{~m}$. Beyond this largest error, the sampling and noise errors are increased by the difference in the reproduction ratio, and therefore still four times higher, at around $8 \mu \mathrm{~m}$.

It can be concluded that it is very desirable to analyze the data in such a way that neither the pre-shock wave particles in the NFT camera nor any information from the WFT camera are used. Even reducing, for example, the usable initial slip velocity by a factor of two by disregarding some region directly around the shock wave is preferable to using particle images recorded in either of the aforementioned areas.

### 3.3 Particle Response Fitting Error Sensitivity

In order to put into context how large or small the error in the distance determination needs to be, the subsequent step in the analysis, although not optimized as part of this thesis, is important. The determined distances are converted into slip velocities and then fitted to a particle response curve that corresponds to a certain particle diameter. For a given ideal signal, this will return a diameter, assumed here to be $D_{\mathrm{p}, \mathrm{fit}}=D_{\mathrm{p}, \mathrm{true}}$. In other words, for a signal without noise, the fit is assumed to return the exact particle diameter. With added Gaussian noise or a simple pixel offset, this result $D_{\mathrm{p}, \mathrm{fit}}$ changes. Figure 3.8 shows by how much, relative to the true value, it changes depending on the particle diameter itself and the initial slip velocity. These results are based on traces generated using a numerical integration of the Kaskas (1964) drag approximation across shocks of the respective initial slip velocity, with a post-shock velocity of $200 \mathrm{~m} \mathrm{~s}^{-1}$, to obtain connection velocities. Areas with $100 \%$ error or greater are shown in red with no further information, as these values are not useful. Often, the fit has failed to return any result at all in these cases. Even if a valid result was returned with greater than $100 \%$ deviation, any error of this magnitude is considered unusable. The Gaussian noise results were obtained numerically with 128 trials. As a result, some variation can be seen in the error in Figures 3.8 I-III a.

Generally, the error increases both for very large and very small diameters. The highest precision is generally achieved for ice particle diameters of between 5 and $20 \mu \mathrm{~m}$, depending on the initial slip velocity. For large diameters, the reason for the loss of accuracy is that the relaxation of the slip velocity becomes very slow, making the difference between the first and the last velocity value in the trace small, thereby emphasizing any small error. As expected, this effect is almost negligible for the global pixel offset test, but even small amounts of added Gaussian noise reduce the accuracy drastically. For small diameters, the slip velocities quickly taper off to small values where, again, a small change makes a big difference. Here, a constant
offset does have a significant impact because the offset will make the taper appear less severe. Additionally, any negative slip velocity values towards the end of the trace will result in a failed fit and return no value at all. For this reason, Figures 3.8 I-III b show failed fits in the negative offset direction for very small diameters.

Larger initial slip velocities increase the accuracy in all cases-with larger slip velocities, the same position error has a smaller effect on the result.


Figure 3.8: Particle diameter output error after particle response fitting, by particle diameter, input location change and initial slip velocity. Grey dots mark sampled parameter combinations.
a) Input modified by changing the particle image positions for each particle in the trace by Gaussian-distributed random values of standard deviation $\sigma_{x}$.
b) Input modified by changing the particle distances by a constant amount for the entire trace, effectively increasing (for positive $\Delta x$ ) or lowering (for negative $\Delta x$ ) the PTV velocity relative to the PIV velocity.
Initial slip velocities:
I) $40 \mathrm{~m} \mathrm{~s}^{-1}$
II) $20 \mathrm{~m} \mathrm{~s}^{-1}$
III) $10 \mathrm{~m} \mathrm{~s}^{-1}$.

## 4

## Analysis Software Development

This chapter describes both the existing algorithms as well as, in greater depth, newly implemented methods that are being used to determine the relative position of particle images produced by the same actual ice particle within PTV images. Beyond that, information about the implementation is also given where this was deemed to be relevant.

The main goal of this work is to create a single application for the analysis of the PTV measurement from the setup described in Section 2.4.1. This application needs to feature an improved trace finding algorithm as well as a higher precision method for determining relative particle image distances within the PTV traces. Finally, it needs to utilize PIV data from an external source in order to calculate the slip velocities of the large particles relative to a PIV velocity field calculated based on much smaller particles.

### 4.1 Existing Software Tools

Previously, two pieces of code were used to perform parts of this analysis: A Python script served the purpose of extracting traces, i.e. locations of multiple particle images that likely show one single ice particle in the test section at different times. Following this analysis, a C ++ program called particleresponse was used to perform a parameter fit to the solution of the BBO equation across a shock wave, as described in Section 1.7.2, using the sphere drag approximation by Kaskas (1964). No tools were available for the comparison of PIV and PTV velocities and the determination of slip velocities.

### 4.1.1 Image Analysis

The image analysis offers significant room for improvement both in its usability as well as in the precision of the analysis and validity of the results. The existing Python script performs the following steps:

1. Read the image file from the file system into a two-dimensional array of greyscale values.
2. Denoise the image using a Gaussian blur filter in order to more easily discern actual particle images from image noise in the following step.
3. Binarize the denoised image using either a fixed threshold or an adaptive threshold, such that, ideally, all pixels of the image showing illuminated particles have an integer value of 1 and all other (background) pixels have a value of 0 . In case of the fixed threshold, perform a binary dilation on the result in order to increase the size of the detected particle image areas.
4. Discard particle images that cover less than a minimum pixel area, or more than a maximum pixel area, in order to remove particle images that are too small or too big to likely yield good results.
5. Compute the center of each group of pixels with a threshold of 1 , using the original (non-denoised) image. This is done using either a three-point gauss estimator around the brightest pixel in each pixel group, or using a center of mass approach based on the pixel intensities within the pixel group.
6. Iteratively connect the resulting center coordinates into traces based on a set of geometric rules. The basis for this is a given, manually entered search vector (in image coordinates) from which connections may only deviate by a certain distance and angle. Additionally, the angle between successive connections in a given trace is limited.
7. The image coordinates for the resulting traces are then output within the console/terminal in which the script is executed, as well as plotted onto the image using matplotlib.

Because a fixed search vector is used here, the change of the velocity vector across and after the shock wave makes it hard to find a value for the search vector which detects traces across the entire image. This method also requires careful and repeated adjustments of the search vector in order to obtain some usable traces. A search area large enough to include all velocity vectors would also include many false positives in any region.

### 4.1.2 Particle Response Fitting

The existing particleresponse application is, within the scope of this project, sufficiently accurate and functional for the analysis and is therefore not part of this development project. Integration into a common front end is necessary such that, after performing the image analysis, the results are formatted in a way that is readable for this application.

The tool performs a least squares fit of a vector of slip velocities, matching it to a slip velocity model for the movement of particles behind an ideal shock wave. This is based on the
velocity model described in Section 1.7.2, specifically Equation 1.8 with the approximation of the coefficient of drag by Kaskas (1964) (Equation 1.10), solved for $D_{\mathrm{p}}\left(u_{\mathrm{s}}(t)\right)$.

Necessary inputs for the tool include the fluid density $\rho_{\mathrm{f}}$, the particle density $\rho_{\mathrm{p}}$ (although fitting of this parameter is also possible), the fluid viscosity $\mu_{\mathrm{f}}$, as well as a vector of timestamped particle slip velocities for each trace. The application returns information in a console output, which can be redirected into a comma-separated values (CSV) file. The output contains fitted particle diameters, the number of iterations required for the fit as well as residuals.

### 4.2 New Application Architecture and GUI

In order to achieve the desired improvements, the overall structure of the application has been re-built with a number of changes. All settings and operations are now combined within a single, graphical user interface (GUI). This includes loading files, selecting output directories, running the different analysis steps and adjusting the parameters for each step. Also included in the GUI is a basic level of user feedback about the success of each step.

The GUI (shown in Figure 4.1) is structured according to the most top-level steps in the analysis, which are the following:
(1) Particle image identification includes the application of a denoise filter and a binarization of the image in order to identify particle image pixel regions, followed by a centroid or Gaussian center finding algorithm which is applied to the identified regions.
(2) Trace finding includes the assembly of the resulting approximate particle image locations into traces, using a newly developed algorithm.
(3) Precise distance determination performs a second, more precise determination of the distances between particle images grouped in the now defined traces, based on the information of which particle images represent the same particle.
(4a) Particle response in-file generation determines the slip velocities at each connection within a given trace by subtracting the PIV velocity from the center of the connection distance from the respective PTV connection velocity.
(4b) Particle response fitting is the final step and passes the slip velocity data through the largely unmodified particleresponse $\mathrm{C}++$ application.

The general data flow and structure of the analysis software is illustrated as a simplified flow diagram in Figure 4.2. The entire application, with the exception of the last step, has been implemented in Python 2.7 using common packages and ensuring full compatibility with both Linux and Windows operating systems.

The following sections cover the considerations behind the exact implementation and algorithms of each of the aforementioned steps, insofar as they were changed as part of this thesis project.


Figure 4.1: Graphical user interface of the analysis application. Global settings are listed in the top row and left column, other settings are grouped according to the processing steps as shown in Figure 4.2. Fields at the bottom provide feedback during and after each step.


Figure 4.2: Flow diagram for the entire analysis pipeline.

### 4.3 Particle Image Identification

The algorithms used in the existing code to isolate particle images from the image and find their approximate centers work reasonably well as were therefore not changed fundamentally. Changes related to particle image identification included the implementation of more user control regarding the parameters, as well as the optimization of the default parameters for good results with the existing set of images. The only change of the algorithm that was implemented was a switch from the previously used adaptive Gaussian thresholding method to an adaptive mean thresholding method. Specifically, the cv2.adaptiveThreshold() method from the OpenCV library ${ }^{1}$ is used with the parameter ADAPTIVE_THRESH_MEAN_C.

The latter takes the mean value of a configurable radius around a pixel and compares the pixel value to this mean. The assignment of the value of 0 or 1 is then performed based on a pre-defined minimum positive intensity difference to this mean. Given the relatively low density of particle images in the existing images, it can be assumed for a mean sampling area much larger than the particle image radii that the mean intensity is at the level of the background intensity. With this method, even relatively dim particle images are detected reliably on images with variable background brightness levels, which is why it was chosen to be the default algorithm for the initial particle image detection.

Figure 4.3 shows the flow diagram for the entire procedure. Some intermediate steps are visualized in Figure 4.4 for a small section of a full camera image.

The horizontal flip of the image after importing it is performed in order to achieve a flow direction from left to right in the output and parameters. This makes setting some parameters, especially the search vector, more intuitive. It can be disabled in the GUI if necessary.

Within the global thresholding method, the binary dilation uses a circular kernel to effectively increase the size of all particle image areas from the thresholding by a given radius. Especially for very small particle images, this guarantees that enough pixel values are available in order to be able to successfully perform a subpixel center estimation. Because the adaptive thresholding technique is sensitive to even quite small values above the background value, such a step is not necessary there. The adaptive method, however, does benefit from using the full dynamic range of an 8 bit integer image by correctly scaling the floating point array to a minimum value of zero and a maximum value of 255 first, as the implementation in OpenCV does not support a higher bitrate image input.

Within the labelling section, functions from the SciPy ${ }^{2}$ library are used to first assign labels to each pixel region with a value of 1 (using scipy.ndimage.label()) and then find out how many pixels are part of each labeled region (using scipy.ndimage.histogram()).
Afterwards, simple logical operators are used to filter out regions that are below the minimum or above the maximum size set by the user. The purpose of being able to set minimum sizes is

[^6]that particle images covering very few pixels may not be part of the main seeding but rather fragments or fog droplets - they are therefore not interesting to analyze. For particle images covering only one or two pixels, it is also not possible to determine precise subpixel positions with any method, due to a lack of surrounding information. Particle images covering too many pixels are filtered out in order to eliminate particles that are abnormally large. These often appear to break apart over the course of the measurement, or represent multiple particles which happen to overlap in one instance. It also appears to be that case that particle images appearing very close to the position of the shock wave can increase in apparent size dramatically due to some optical aberration that smears their image, usually mostly in the horizontal direction.

Similarly, particle images that contain at least one pixel with the saturation value of the image are discarded by default. In most images, this affects a very small number, but as discussed in Section 3.2.1, it often makes sense to not use saturated particle images as their locations are likely to be less precise.

In the center estimation, the centroid (or center of mass) method uses the SciPy method ndimage.center_of_mass for calculating the intensity center of each pixel region associated with a particle. For this, the values from the original, unmodified image are used. The three-point Gauss center estimator on the other hand is implemented in the code (this implementation was taken directly from the existing analysis code) based on the natural logarithm of the intensity of the four pixels adjacent to the brightest one for each particle image, as well as the intensity of the center itself. Both algorithms return a set of floating point particle image positions and function reasonably well in most cases. The centroid method is used by default because the images occasionally contain particle images with multiple overexposed pixels in the center, which causes the Gauss estimator to fail and return a less accurate position as it requires intensity differences in the neighboring pixels. As described in Section 5.4, the centroid method is also much more precise in most cases.


Figure 4.3: Flow diagram for the particle image identification algorithm. Green elements (triangles pointing towards the right, diamonds) indicate the influence of user-defined settings. This includes the choices for the binarization and center estimation methods.


Figure 4.4: Different stages of the particle image identification process on a $512 \times 512 \mathrm{px}$ excerpt from a NFT camera image. Colormap applied to images b) through f).
a) Original image section.
b) Flipped horizontally, colormap applied for better reproduction of low intensities.
c) Denoised using Gaussian blur with a 2 px radius.
d) Binarized image using adaptive thresholding with $\mathrm{a}+8$ mean threshold displacement (relative to the 8 bit image), 8 px area radius for the calculation of the mean value.
e) Binarized image using global thresholding with a $2^{13}$ global threshold displacement (relative to the 16 bit image), 2 px dilation radius.
f) Image with center of mass centers marked with diagonal crosshair-like markers (based on the adaptive thresholding method).

### 4.4 Particle Image Pair and Trace Matching

In this measurement campaign, a single camera image contains the information from eight laser pulse illuminations of the particle seeding in the test section. Unlike in multi-frame PTV, there is no clear indication contained in this image regarding which particle image belongs to which illumination. As usually in PTV, there is also no strong indication available of which particle image belongs to which particle. Therefore, it is challenging to identify the particle images that represent eight illuminations of a single particle.

In order to use the results from this measurement with any confidence, it is necessary to identify and isolate such particle image groups reliably. Such a group will be referred to as a trace in this thesis. The vector from one particle image to the next within a trace will be called a connection. It is not crucial that all identified particle images can be assigned to a trace, especially as some traces will necessarily be clipped at the edges of the frame, while others will contain some particle images below the given threshold for particle image identification. It is, however, important to remove any traces from the results that do not actually show the same real particle.

The goal is to achieve a selection of traces across changing flow directions within a single image with a low number of detected false positives. Based on what is known about the particle trajectories, it is possible to develop a number of criteria for particle images belonging to a single trace:

- A trace should contain as many particle images as there were illumination pulses when the given image was recorded.
- Any particle image can only belong to one trace.
- Deviations of the absolute velocity across and behind the shock wave are moderately small within a small area of the image compared to the absolute velocity, especially in the vertical direction. This is apparent in Figures 4.11 and A.12.
- Deviations of the absolute velocity depend on the size of the particle, and will therefore vary slightly between particles.
- Deviations in the direction of the velocity vector are small along the flight path of any particle. The angle differences that occur tend to mostly deviate in one direction in the given flow field.
- The overall intensity of a particle image depend mostly on the geometric properties of the recorded particle and will therefore not change by orders of magnitude between multiple illuminations of the same particle.

From these criteria, the following set of rules was derived to form the trace generation algorithm:

1. Any neighbor particle image must be within a user-defined search area around a userdefined search vector. The search vector should approximate the global flow direction. All particle images within this area, from the perspective of a given particle image, are considered a neighbor candidate to this particle image.
2. Any neighbor candidate receives a total weight which approximates its likelihood to be the subsequent illumination of the same particle. This total weight is the result of multiplying two weights:

- Intensity weight: A function of the brightness difference between the test particle image and the neighbor candidate particle image.
- Map weight: The number of other test particle image in the region which also have a neighbor candidate at this position relative to their own position.

3. For a complete trace, some combination of the angle between two adjacent connections and the change of this angle along the trace (i.e. between multiple pairs of connections) may not exceed a user defined value.
4. For any particle image which is part of more than one trace, all but the trace with the highest sum of total weights will be discarded.

Figure 4.5 illustrates the search vector, search area and neighbor candidate particle images. While the first and last criterion are very straightforward, the following sections will explain in


Figure 4.5: Illustration of the search vector, search area as well as the neighbor candidate particle images. The particle image at the origin of the search vector is considered the test particle image.
more detail how the selection and angle filtering algorithms were implemented. To given an overview over the trace finding process, the algorithm is illustrated in a simplified flow diagram in Figure 4.6. The algorithm resembles the one developed by Mikheev and Zubtsov (2008), which also features a match probability and the assumption of similar movement of neighboring particles.


Figure 4.6: Flow diagram for the trace finding algorithm. Symbols in the top left corner indicate steps being performed for each particle image center (PIC) or neighbor candidate (NC).

### 4.4.1 Intensity Weight Calculation

Given the intensities $I_{\mathrm{tp}}$ and $I_{\mathrm{np}}$ of the test and neighbor particle image, the intensity weight $W_{\mathrm{I}}$ is derived to be

$$
\begin{equation*}
W_{\mathrm{I}}=f_{\mathrm{pdf}}\left(x=\left|\ln \left(I_{\mathrm{tp}}\right)-\ln \left(I_{\mathrm{np}}\right)\right|, \mu=0, \sigma=\sigma_{\mathrm{I}}\right), \tag{4.1}
\end{equation*}
$$

where

$$
\begin{equation*}
f_{\mathrm{pdf}}(x, \mu, \sigma)=\frac{1}{\sqrt{2 \pi \sigma^{2}}} e^{-\frac{(x-\mu)^{2}}{2 \sigma^{2}}} \tag{4.2}
\end{equation*}
$$

is the normal PDF and $\sigma_{\mathrm{I}}$ is a user-defined value for the tolerance regarding the difference in the natural logarithm of the intensities. The intensities are calculated to be the sum of the pixel value intensities of all pixels that belong to the given particle image. A logarithmic scale was chosen in order to make the criterion independent of absolute exposure and comparable in effect for faint and bright particle images.

Figure 4.8 shows a histogram of the intensity distribution of all detected particle images in a typical PTV measurement image. For comparison, it also shows a histogram of the particle images from the final set of traces in this case. A bias towards larger particle images is apparent in the valid trace results. The lack of very faint particle images in chosen traces is expected as these are more likely to have partners below the size or intensity threshold, in which case a full trace cannot be assembled.


Figure 4.7: Example for traces detected by the algorithm. Some of the most visible traces are not found due to the intentional exclusion of saturated particle images.

### 4.4.2 Map Weight Calculation

The map weight is a representation of the percentage of nearby particle images that have a corresponding neighbor candidate in a similar relative position. This methodology is similar to the one described by Cowen and Monismith (1997, p. 201), creating a "hybrid" PTV method hat includes aspects that resemble a PIV analysis. In fact, as shown in Section 4.4.7, the results of this analysis can be used to determine a full velocity field. Establishing this information is done in two steps:

## Neighbor Maps

First, a value array is created for each particle image, called a neighbor map. It is initialized to be zero and each element within this neighbor map represents a small section of the search area. Because the search area is a square, this array is equally square, with a resolution that can be set by the user ( $16 \times 16$ pixels by default). Within the neighbor map, the array value is increased based on the vicinity to any other particle images in the area. Specifically, the value $T_{m}$ of each element $m$ in the neighbor map is set to

$$
\begin{equation*}
T_{m}=W_{\mathrm{I}} \sum_{p=1}^{n_{\mathrm{s}}} f_{\mathrm{pdf}}\left(x=\left\|\mathbf{x}_{m}-\mathbf{x}_{p}\right\|_{2}, \mu=0, \sigma=\sigma_{\mathrm{M}}\right) \tag{4.3}
\end{equation*}
$$



- Identified particle images --- Particle images in valid traces

Figure 4.8: Intensity histograms for the particle images of a PTV measurement image. Particle images were selected with a mean threshold displacement of 6 , minimum particle image size of 3 px and maximum size of 64 px . Values are normalized such that the area under each histogram equals 1 .
for $n_{\mathrm{s}}$ neighbor candidate particle images in the search area, with $\mathbf{x}_{m}$ representing the position vector of the respective map pixel mapped onto the image (i.e. in image coordinates, not in map coordinates) and $\mathbf{x}_{p}$ representing the position vector of the respective particle image. The value of $\sigma_{\mathrm{M}}$ can be set by the user to control the smoothness of the resulting maps. The default value is $\sigma_{\mathrm{M}}=3.4$, which represents about a $3 \%$ tolerance for the absolute velocity of the particles given the search vector length of about $1 \times 10^{2} \mathrm{px}$. This value has been found to work well in practice. A good value of $\sigma_{\mathrm{M}}$ for a specific image can be found by evaluating the the PIV-like output described in Section 4.4.7-it should be smooth with few or no artifacts.

The multiplication of $W_{\mathrm{I}}$ aims to increase the contribution of particle images of similar intensities to the neighbor map, as those are more likely to represent the true neighbor. Figure 4.9 shows a few evenly selected samples of neighbor maps for eight different particle images.

## Weight Maps

With all of these neighbor maps populated, a second value array is created for each particle image in the following step. This so-called weight map is the result of a summation of all neighbor maps of particle images in the vicinity of the respective particle image. Specifically, a user-defined number of particle images' neighbor maps are collected, in order of increasing distance from the test particle image. The distance is distorted by a user-defined aspect ratio to take into account the fact that the flow velocity gradients are much smaller in the vertical direction than


Figure 4.9: Neighbor maps from a PTV measurement image. The resolution of the maps is $16 \mathrm{px} \times 16 \mathrm{px}$. They cover a search area of $40 \mathrm{px} \times 40 \mathrm{px}$ and were created with a $\sigma_{\mathrm{M}}$ value of 3.4 and a $\sigma_{\mathrm{I}}$ value of 2.0 . All of these neighbor maps were scaled such that 1.0 represents the highest possible neighbor map value based on $\sigma_{\mathrm{I}}$ and $\sigma_{\mathrm{M}}$.
they are in the horizontal direction. This is achieved by first scaling all coordinates, by default with a horizontal divisor of 4 , and then building a $k$-d tree from those coordinates. By using the $k$-d tree, the $n_{\text {np }}$ nearest neighbors (including the test particle image itself) can be retrieved efficiently for each test particle image.

As a result, the weight map value $W_{m}$ for each element $m$ in the weight map (which has the same pixel dimensions as the neighbor maps) ends up being

$$
\begin{equation*}
W_{m}=\sum_{p=1}^{n_{\mathrm{np}}} T_{m, p} . \tag{4.4}
\end{equation*}
$$

Figure 4.10 shows samples of weight maps, with the test particle images being the same ones as in Figure 4.9. They show clear maxima where multiple test maps all included a neighbor candidate, and relatively uniform, lower values elsewhere. The location of the maxima within each map is different due to a change in the velocity vector across the image.

This criterion works well for areas with relatively even seeding, but is problematic in images with a wide range of seeding densities across the image. The particle image density usually only varies greatly in the vertical axis (depending on the area in which the seeding area has accumulated most). As the analysis focuses on a certain vertical area with a given shock wave strength, and the shock wave strength varies greatly in the vertical axis as well, this is not a significant issue. Otherwise, it might be necessary to run the algorithm with a lower number of similar particle images for the lower, less densely seeded area and with a higher number for the upper, more dense area in the image - or implement an automatic correction for the local particle image density.

Because the experiment is run with ice particles of varying sizes, and the sizes are not always identical within a single image, a slight variation in the velocity vector between different traces can be expected even in the same area of the image. Additionally, the exact vector is likely to vary within a given search area (and the maximum may be skewed slightly) due to an area of high seeding density on one side of the particle image compared to the other. To reduce the sensitivity to such effects, which are likely to only affect the weight map maximum by very few pixels, an option for weight map greyscale dilation was added ${ }^{3}$. Greyscale dilation is a

[^7]morphology operation which determines the new pixel value by taking the maximum from the given image within a certain area ${ }^{4}$ around the pixel position. It therefore effectively increases the size of any maxima without increasing the overall maximum value. Here, it is implemented with a circular structuring element of a user-defined radius $r_{\mathrm{d}}$. This radius should be chosen to roughly match the expected variation in the velocity vector within half the search area.
(a)
(b)


Figure 4.10: Weight maps from a PTV measurement image. As before, the resolution of the maps is $16 \mathrm{px} \times 16 \mathrm{px}$, they each include the sum of 64 nearby neighbor maps with a search aspect ratio for the closest neighbors of 4.0. Search area and $\sigma$ settings are identical to those in Figure 4.9. All maps were scaled such that 1.0 represents the highest single pixel value in the respective map. No dilation was applied.
a) Weight maps without histogram equalization applied.
b) Weight maps with histogram equalization applied (see Section 4.4.3).

## Image Border Treatment

In finding the $n$ nearest neighbors, an additional criterion is applied in order to avoid using neighbor maps that sample areas beyond the image boundaries. Such maps can only contain particle images in the area that is within the image boundaries, and as a result may not contain the actual correct partner particle image even if it would exist outside of the image. Moreover, due to the normalization of the weight maps, maps at the boundaries would falsely apply a high total weight value to some other point within the image boundaries which is now the maximum. This is not a big problem, as it mostly affects the very right edge of the image - far away from the shock wave which is most interesting here. However, it is also quite simply avoided by sampling the $n$ nearest neighbors only after removing all neighbors from the $k$-d tree that are so close to the border that their maps would extend beyond it. If there was a strong velocity gradient in this area, this would reduce the validity of the weight maps that are being generated. However, the area is furthest away from the shock wave and has the smallest velocity gradients of the entire image. Therefore, the effect of sampling neighbor maps from particle images slightly further left than the particle images to which they end up being applied is acceptable.

[^8]
### 4.4.3 Total Weight

The total weight is a property of a neighbor candidate particle image. In other words, the same neighbor candidate has a different total weight value if evaluated from a different test particle image. As such, the total weight must be calculated for each neighbor candidate of each test particle image, where test particle images are all detected particle images in the image and neighbor candidates are all particle images within the search area of the given test particle image.

## Total Weight Calculation

The total weight of a given particle image pairing is the result of a multiplication of the intensity weight and the square of the map weight, divided by the maximum achievable weight based on the $\sigma_{\mathrm{I}}$ that was chosen by the user, i.e.

$$
\begin{equation*}
W_{\mathrm{T}}=\frac{W_{\mathrm{I}}}{f_{\mathrm{pdf}}\left(x=0, \mu=0, \sigma=\sigma_{\mathrm{I}}\right)} \frac{W_{\mathrm{M}}{ }^{2}}{1.0} . \tag{4.5}
\end{equation*}
$$

A normalization for the map weights is not necessary as weight maps are already normalized. Taking the square of the map weight reduces the background level in these weight maps, which results from dense particle image regions with many "false" neighbor candidates. This makes it easier to set good threshold values.

The map weight is calculated by mapping the position of the neighbor candidate particle image onto the weight map of the test particle image and retrieving the weight map value at this position. A two-dimensional linear interpolation is used to find the weight value at subpixel positions of the weight map. ${ }^{5}$

The density of particle images in the image affects how bright the weight maps are outside of their maximum. The more particle images are in the image, the smaller the relative contribution from the actual respective neighbor in each neighbor map becomes. As such, the lower threshold for the total weight below which connections are discarded must be set by the user until a reasonable amount of connections is found for the entire image. To help with this, an optional histogram equalization step was implemented as a final step in the creation of weight maps. This adjusts the brightness values of all pixels in the weight map such that there is an equal amount of pixels assigned to each brightness level, resulting in a "flat" histogram. As a result, for particle images of equal intensity, a set minimum weight corresponds directly with a percentage of the area of the weight map to be included in the neighbor search. This should counteract a brightness distribution in the weight maps that is too flat for high density areas or images.

Histogram equalization does indeed result in a number of resulting traces which is about proportional to the density of particle images in a region. The quality of the filtering appears to decrease, however, if the density becomes too high, resulting in the loss of some legitimate

[^9]traces for weight threshold levels that work well on lower density areas. Figure 4.10 gives an indication of a different difficulty: The equalization removes the strong contrast between the local maximum and the surrounding area. As a consequence, the choice of the correct minimum map weight becomes much more sensitive, even though an equally good value can likely still be found. As a result of these problems, this equalization is not performed by default. This is also in part due to the fact that test have shown the regular method to be sufficiently robust when subjected to slight changes in density, with the angle filtering getting rid of additional invalid traces.

### 4.4.4 Trace Creation

Traces are assembled by going through all particle images and finding potential neighbors that have a total weight $W_{\mathrm{T}}$ value higher than the user-provided total weight threshold $W_{\mathrm{T}, \min }$. Whenever this is the case, the test particle image and neighbor particle image have their indices listed in a pair array, along with their total weight and absolute angle (with respect to the image coordinate system). From this list of single connections, traces are then chained together iteratively. This is done by taking each connections' final particle image index and finding other connections in the list that begin with this particle image index. Each such match is entered into a new list of traces which is one connection longer than the previous one. After the maximum number of iterations is reached, the result is a list of traces of equal length.

### 4.4.5 Angle Filtering

The goal of this step is to discard traces with kinks or doglegs at any point that are implausible. While a constant small angle between all connections of a trace, resulting in a curve overall, is possible as the flow follows the curvature of the airfoil and is also disturbed by the shock wave, what is much less plausible is a kink in one direction followed directly by a kink in the opposite direction. In order to filter out traces that show this characteristic, an angle $\beta$ representing the overall non-straightness of each trace is calculated to be

$$
\begin{equation*}
\beta=\max _{i=0, \ldots, n_{\mathrm{c}, \mathrm{t}}-1}\left\{\left|\alpha_{\{i, i+1\}}-\frac{1}{4} \alpha_{\{i-1, i\}}-\frac{1}{4} \alpha_{\{i+1, i+2\}}\right|\right\} . \tag{4.6}
\end{equation*}
$$

Here,,$n_{\mathrm{c}, \mathrm{t}}$ is the number of connections within a trace ( 7 for our experiment) and $\alpha_{\{i, i+1\}}$ is the angle between connection $i$ and its following connection to the right, $i+1$. For values of $\alpha$ at the beginning and end of a trace that cannot be computed due to the lack of a previous or next connection, the respective angle is set to zero. The result is a $50 \%$ weighting between the angle between two connections ( $\alpha_{\{i, i-1\}}$ ) and the derivative of this angle along the trace.

Traces are removed from the list if their angle $\beta$ exceeds a user-defined value $\beta_{\max }$. It is worth noting that, to some extent, decreasing the minimum total weight and lowering the $\beta_{\text {max }}$ angle in the right proportion results in very similar results in the end, and the user can choose which of these two filtering mechanisms should be applied more rigorously. Limiting in this is a
very large amount of initial, unfiltered traces in the case of a very low minimum total weight, as the number of final traces grows with a factor of approximately $n_{\mathrm{c}, \mathrm{p}}{ }^{n_{\mathrm{c}, \mathrm{t}}}$ where $n_{\mathrm{c}, \mathrm{p}}$ is the average number of valid connections per particle image and $n_{c, t}$ is the number of connections in a trace ( 7 for full-length traces in this experiment). Increasing $n_{c, p}$ beyond a value of $\sim 2$ is therefore problematic as it increases the overall runtime significantly. To give an example: With 8000 detected particle images, an average of 1.8 valid connections per particle image results in $\sim 5 \times 10^{5}$ traces which have to be filtered individually. With 2.5 average valid connections, this number already increases to $\sim 5 \times 10^{6}$.

### 4.4.6 Collision Filtering

Up to this point in the algorithm, some traces may still overlap. For example, two traces may be identical in all but a single point at their beginning or their last points. If this is the case, not all traces at this stage can represent actual particle paths - which should ideally be corrected by finding better thresholds that only return actual traces. However, if this is not possible, the result can still be improved by choosing the best of a given set of traces in all cases where a particle image is part of more than a single trace.

This is done by finding all such points, comparing the trace weights of all competing traces and discarding all but the trace with the highest value. Here, the trace weight is simply the sum of the total weights of all connections that make up the given trace. As such, even if multiple traces are compared at multiple collision points, the one with the highest weight overall is guaranteed to be preserved.

### 4.4.7 PIV-Like Analysis of Weight Maps

The weight maps represent a probability distribution of the local velocity vector at the position of each particle image. As a secondary usage of this information (beyond the trace assembly), a velocity vector field can be interpolated throughout the image.

Specifically, the three-point Gaussian center determination method can be applied to all weight maps to get a value for the position of their highest peak with subpixel accuracy. Each of the resulting vectors is assigned to an image position halfway along the vector (i.e. in the center between the particle image and the weight map maximum). Using an interpolation method ${ }^{6}$, a flow field covering almost the entire original image is obtained. The coverage area is only limited by the area of the image that contains detected particle neighbor candidates.

An example for the result of this analysis is shown in Figure 4.11. Figure A. 12 and the following figures in the Appendix show the same results as shown here for a second example case with, presumably, larger ice particle diameters and therefore a less pronounced shock. Information outside of the bounds of the detected particle images is extrapolated here using the nearest neighbor method, as described in Section 4.4.2.

[^10]

Figure 4.11: Absolute velocity in the NFT camera image area: Visualization of the PIV-like analysis of the weight map data in the image space.


Figure 4.12: Horizontal velocity profiles in the image area from the PIV-like analysis of the NFT camera image shown in Figure 4.11. Values of ${ }_{\mathrm{i}} \hat{x}_{2}$ are vertical locations in image coordinates, where 0.0 corresponds to the vertical center and 1.0 corresponds to the bottom edge of the image.

This functionality, although not directly related to the main PTV measurements, provides two major benefits:

## Weight Map Validation

First, it visualizes the results of the weight map calculation in a simple manner. With a new set of measurement images, this makes it much easier to find good parameters for the number of neighbors and the aspect ratio in the neighbor search, as well as for the different neighbor map options. For a good set of parameters, the method produces the expected representation of the local flow vector throughout the image. Otherwise, the visualization of the PIV-like data shows outliers and artifacts in areas where the peak position in the weight maps does not correspond with the actual velocity vector.

## Shock Wave Position Determination

As shown in Figure 4.12 (see Figure A. 13 for a different case), the PIV result gives a usable, if not highly precise, velocity profiles along horizontal cuts through the image area. The Figure only shows cuts from the center of the image downwards as the shock wave is significantly weakened in the upper half. This is a consequence of the increased distance from the airfoil model further up in the flow.

The shock wave coincides with the horizontal minimum of the local change in velocity, i.e. the minimum of the derivative in the horizontal image axis direction. This derivative is calculated after filling in the missing outer areas of the image using a nearest-neighbor fill method. Otherwise, a later application of image filtering would cause problems in areas with missing velocity information. The fill method is constant and does therefore not introduce any additional gradients. As a result, it does not negatively affect the determination of the shock wave location.

The velocity results from this method are effectively subject to a convolution with the particle displacement between two pulses. This is the case because all velocity information is integrated over the distance between two corresponding particle images. As a result, an ideal shock wave would correspond to a linear drop in velocity, with a region of near-constant velocity before and after the shock wave. The area after the shock wave will still decrease in velocity somewhat due to the slip velocity of the particles which is the subject of this analysis, and due to the shape of the airfoil. Additionally, the velocity information is not resolved very well due to the low particle image density that is (intentionally) present in the images.

For these reasons, taking the minimum of the derivative in the horizontal direction is not advisable as even in an ideal reproduction, the outcome could be any position of the linear slope. Instead, to both smooth the data somewhat and achieve a distinct single minimum at the position of the shock wave, a Gaussian blur filter is applied to the image. The filter is used with a standard deviation of half the length of the search vector in both axes. While the horizontal
application of the filter is more essential, the vertical component smoothes out shifts in the shock wave position due to the very low seeding density and resulting undersampling of the shock wave. For a continued drop in velocity after the shock, the shock wave position that is retrieved using this method will err towards the right. The smoothed derivative flow field is shown in Figures A. 10 and 4.13 (Figures A. 14 and A. 15 for a different case).


Figure 4.13: Horizontal profiles of the absolute velocity horizontal derivative in the image area from the PIV-like analysis. Corresponds to the image shown in Figure A.10.

Taking the minimum of this smoothed horizontal derivative for every vertical position in the image gives the line shown in Figure A.11. The result suggests that this method is suitable for determining the position of the shock wave to within a few pixels, which is sufficient for determining whether a trace connection is located ahead of or behind the shock wave.

### 4.5 Precise Distance Determination

As discussed in Chapter 3, the problem at hand is unusual compared to most existing cases in PTV and PIV in that the particles do not resemble small, rotationally symmetrical and mostly diffraction-limited Gaussian or Airy disk-like intensity profiles. Instead, they occasionally show multiple maxima with significant space between them, presumably due to their irregular shapes. They are also much larger than the diffraction limit of the given optical setup. Several examples for such non-radially symmetrical large particle images are shown in Figure 4.14.

While regular PTV approaches to subpixel particle location determination discard particles that are not sufficiently close to being circular in shape (one such approach is described in Shortis et al., 1994, p. 5), this is not possible here. After all, the specific purpose of this experiment is to understand the behavior of ice particles of different sizes and, potentially, different shapes.


Figure 4.14: Assortment of particle images showing multiple local maxima or shapes which are not radially symmetrical. Images have been inverted, normalized to their maximum brightness and extracted from the main image using a Gaussian tapered mask.

The usual approach would be to fit a well-defined intensity distribution, such as a Gaussian PDF, an Airy disk or a spline of some order to the existing particle images and determine individual centers from the best fit. Various such methods are summarized in Section 3.1. Based on the center locations, the difference in position is then calculated to be the difference in center coordinates for an arbitrary pair of particle images.

The number of particle images of which positions need to be compared here is very low-only particle images from filtered traces require a high precision distance measurement. Therefore, the computational time that can be invested in each connection is much larger than even in regular PTV, and the particle image pairs that require a comparison of their location are individually known at the time of the analysis (i.e. it is known which particle image has to be compared to which other particle image). As a result, it is possible to not calculate absolute centers and compare them later, but to instead compare them in pairs while ignoring their absolute position in the image.

### 4.5.1 Interpolated Intensity Least Squares Shift

Specifically, the first algorithm uses the intensity distribution of one particle image in a given particle image pair and matches it to the intensity distribution of the second one. Doing this for all connections in the few traces that were detected allows the time that is spent on a single comparison to be around two orders of magnitude greater than in a global center determination
for several thousand particle images.
The particle images are normalized to have a total sum of intensities of 1 across all pixels in order to enable a good possible fit. This is important because the laser pulses in the test section are not always equal in intensity, and the particle images can therefore not be expected to be equal in brightness.

In order to perform the particle image comparison at a subpixel level, it is necessary to interpolate the pixel values of one of the images (enabling the determination of intensity values at arbitrary positions). Then, it is possible to determine a cost function or objective function, as shown in Section 4.5.5. It represents the sum of the squared differences of the intensities at each pixel position for a given shift in position between the two particle images.

This method will subsequently be referred to as the interpolated intensity least squares shift (IILSS) algorithm. Additional details regarding the implementation are given in Sections 4.5.3 and 4.5.5.

## Particle Image Cutout Method

The particle image region that was identified using the thresholding method as part of the particle image identification is used as the basis to isolate a given particle image from the entire image. The challenge then is to remove any nearby particle images without creating any new intensity features that affect the center determination negatively, and without causing strong overshoots in the spline interpolation. These requirements rule out a simple multiplication with the binary region that would cause a sudden drop to a value of 0 .

Instead, a convolution of the particle image region with a two-dimensional Gaussian PDF with a peak value of 1 is capped to a maximum value of 1 . It is then multiplied with the particle image. As a result, the binary image region is included fully, and pixels outside of the region are tapered to a multiplication of 0 across a user-defined radius. The default $\sigma$ for the PDF is set to 0.75 px , resulting in a relatively steep but not entirely sharp cutoff. Figure 4.15 shows, in the second and third column, examples for the isolation of particle images from the full image using this technique.

## Challenges at High Background Intensities

Some of the PTV images contain not only significant noise due to unwanted background seeding but also an increased black level. This base intensity is not constant across the image, but variations are dominant at scales much larger than the size of a particle image such that, within a single particle image, the level can be regarded as near-constant with a certain noise standard deviation. The overall background intensity is not directly related to the noise standard deviation, as there is a component in this background intensity which is effectively homogeneous fog with low-frequency local variation, while other components consist of fine seeding that can be approximated with a noise standard deviation.


Figure 4.15: Examples for particle image masking and derivatives. The colorbar shown on the right applies to derivatives only and is scaled symmetrically to match the respective minima and maxima.
(PI: particle image)
a) Original image section intensity.
b) Image intensity after multiplication with mask.
b) Mask (with Gaussian taper).
b) Horizontal derivative.
b) Vertical derivative.

The adaptive thresholding technique is not negatively affected by this and will still produce useful particle image regions. However, the cutout method described above will start to dominate the intensity profile of the resulting particle image with increasing background intensities. As a result, the particle image distance determination method increasingly compares the particle image regions rather than the particle image intensities within the region, which can be expected to significantly reduce the accuracy of the results.

This is illustrated for an imaginary intensity distribution in Figure 4.16: A slightly different intensity distribution between the two particle images away from the peak plays a major role because it changes the mask significantly. While the mask is insignificant without the background intensity, it now becomes dominant.

For an entirely constant background intensity level, it would be possible to simply subtract this level from the particle image value. As discussed in Section 3.2.1, this causes an error that is significant for small particle images, but would probably be acceptable for larger ones. However, for any gradient in the background intensity level, this does not solve the problem. Therefore,


Figure 4.16: Illustration of the influence of a high background intensity on the particle distance determination using the IILSS method. The dark area in the upper row shows the respective particle image intensity distribution, the lighter areas show the intensity distribution after masking.
an alternative method using derivatives was implemented, which is described in the following section.

### 4.5.2 Interpolated Derivative Least Squares Shift

The derivatives of the intensity of a particle image are not affected by any background intensity level. Therefore, taking the derivatives in both image axes and performing the offset optimization on these rather than the intensities avoids any problems with an increased black level. The derivatives can be calculated from a spline interpolation with identical order to the one described in Section 4.5.3.

The mask for isolating the particle image can not be applied to the intensity here - this would cause additional derivatives and the benefit of this method would be lost. Instead, the mask is multiplied with the derivative images.

The intensity difference between two images per pixel is inherently weighted by the intensity of each pixel, i.e. a bright pixel has a larger potential influence on the objective function. This property is not as pronounced when using derivatives, effectively giving a stronger influence to low intensity values. To compensate for that, the derivatives are multiplied with the intensity function.

This method will subsequently be referred to as the interpolated derivative least squares shift (IDLSS) algorithm.

### 4.5.3 Particle Image Interpolation

Because an image represents the integral of the intensity over discrete pixel regions around the pixel location, this interpolation serves the purpose of finding values for the integral over any assumed pixel region around a location in-between existing pixel locations. In other words, the result of the interpolation is not the underlying intensity distribution, but rather the continuous approximation of the convolution of this intensity distribution with a two-dimensional rectangular function representing a square pixel. Consequently, the resulting interpolation function does not need to be integrated, but only sampled at discrete points in a grid. This grid will always be a shifted version of the pixel grid of the original image.

Common interpolation methods for images, listed in order of their complexity, are the nearest-neighbor interpolation, bilinear, bicubic and higher degree polynomial spline methods as well as various windows sinc approximation kernels (most commonly using Lanczos windows). Figure 4.17 demonstrates spline interpolation results for different degrees of polynomials on a one-dimensional pixel intensity data set. Because a linear spline interpolation shows large errors


Figure 4.17: Demonstration of different spline interpolation polynomial orders, on an example of the one-dimensional interpolation of pixel intensities.
and is not continuous in the first derivative, it is not suitable for the purpose at hand. Meijering et al. published an extensive analysis of the errors introduced by these and more methods under geometric transformations (rotation and, most relevant for this case, translation) on medical images. They found that, "regardless of image modality [...] or type of transformation (rotation or translation), spline interpolation offers the best trade-off between accuracy and computational cost" (Meijering et al., 2001, p. 125). In their subpixel translation experiment (pp. 119-120), cubic $(k=3)$ spline interpolation shows larger errors than higher degrees $(k=5,7,9)$, but the differences for degrees greater than 5 are usually small. A similar analysis was performed by Thévenaz et al. (2000) on white noise and synthetic images, with a similarly superior performance of B-splines with increasing degrees, and much better performance compared to the sinc function
approximation. Here, o-Moms functions were also tested and show slightly better performance than third degree B-splines at a similar execution time, but higher orders were not tested and fifth-degree B-splines still perform slightly better in both tests. The o-Moms functions also have the disadvantage compared to B-Splines of not offering continuous derivatives. Therefore, the highest degree spline interpolation will be used that is fast enough to be acceptable in the optimization, and that is available in SciPy.

The upper limit that is supported by the regular grid B-spline interpolation class offered by $\mathrm{SciPy}^{7}$ is $k=5$. In an experiment over 28 images, measuring the time to compute the objective function with different degrees of spline interpolation shows noticeable, but not dramatic, differences in runtime. The results are summarized in Table 4.1. The test was performed on the

| Spline polynomial degree |  | $k=1$ | $k=3$ | $k=5$ |
| :--- | :---: | :---: | :---: | :---: |
| Total time for $2.8 \times 10^{6}$ evaluations | $[\mathrm{s}]$ | 117.23 | 142.29 | 181.09 |
| Average time per evaluation | $[\mu \mathrm{s}]$ | 41.869 | 50.818 | 64.675 |

Table 4.1: Runtime for evaluating spline interpolated particle images for different degrees of polynomials. The test was performed using 28 different particle images (i.e. 4 traces) with, for each particle image, 100000 evaluations of the objective function, each time at all necessary subpixel positions.
computer that will be used for the analysis, which uses a Intel Yorkfield family processor running at 2.83 GHz . Implementing a higher degree method would not be worth the small possible benefit in precision. Due to the acceptable differences in the runtime, spline interpolation with degree $k=5$ is chosen.

For now, the implementation is not parallelized as the runtime is short enough to not be a major problem. However, parallelization would likely be quite effective, i.e. scaling across many processors would be very good.

### 4.5.4 Smoothing

In order to see how it would affect the accuracy of the results, the option to smooth the particle images was added. This is performed as part of the B-spline interpolation mentioned above, which offers a smoothing factor limiting the squared error sum across all pixels as a result of smoothing. In order to normalize the smoothing factor $\varphi_{\text {interpolation }}$ used by the interpolation function for smoothing ${ }^{8}$, a user-given smoothing factor $\varphi$ is multiplied with a value that describes the squared sum of the difference from the particle image mean, i.e.

$$
\begin{equation*}
\varphi_{\text {interpolation }}=\varphi \cdot \sum_{\mathrm{p}}\left(I_{\mathrm{p}}-\overline{I_{\forall \mathrm{p}}}\right)^{2} \tag{4.7}
\end{equation*}
$$

[^11]As such, a smoothing factor of $100 \%$ allows the image to be flattened completely, and reasonable values are two orders of magnitude smaller. By default, however, this is disabled.

The methods with smoothing enabled are referred to as interpolated smoothed intensity least squares shift (ISILSS) and interpolated smoothed derivative least squares shift (ISDLSS), respectively.

### 4.5.5 Optimization Problem

The problem statement for the global bound optimization problem can be written as:

$$
\begin{array}{ll}
\text { Minimize } & f_{\text {obj. }}(\mathbf{x}) \\
\text { subject to } & \mathbf{x}^{\mathrm{L}} \leq \mathbf{x} \leq \mathbf{x}^{\mathrm{U}} \tag{4.9}
\end{array}
$$

where (in the case of the IILSS method)

$$
\begin{align*}
\mathbf{x} & =\left\{\begin{array}{l}
x_{1} \\
x_{2}
\end{array}\right\} \text { and }  \tag{4.10}\\
f_{\text {obj., IILSS }}(\mathbf{x}) & =\sum_{\mathrm{p}=0}^{n_{\mathrm{p}, \mathrm{I}_{\mathrm{a}}}}\left[I_{\mathrm{a}}\left(\left\{\begin{array}{l}
x_{\mathrm{p}, 1} \\
x_{\mathrm{p}, 2}
\end{array}\right\}\right)-\mathrm{I}_{\mathrm{b}}\left(\left\{\begin{array}{l}
x_{\mathrm{p}, 1} \\
x_{\mathrm{p}, 2}
\end{array}\right\}+\mathbf{x}\right)\right]^{2} . \tag{4.11}
\end{align*}
$$

Here, $f_{\text {obj. }}$. () is the objective function, $\mathbf{x}$ is the design variable, which is the vector of the horizontal $\left(x_{1}\right)$ and vertical $\left(x_{2}\right)$ displacement. Additionally, $n_{p, \mathrm{I}_{\mathrm{a}}}$ are the number of pixels in $\mathrm{I}_{\mathrm{a}}$ and $x_{\mathrm{p}, i}$ are the discrete pixel coordinate positions of all pixels p of the first particle image. $\mathrm{I}_{\mathrm{a}}$ is the first particle image and $I_{b}$ is the second particle image $\left(I_{a}\right.$ are intensities at discrete pixel locations, $I_{b}$ at arbitrary positions). Because $I_{a}$ only offers discrete intensities within the image boundaries, it needs to be enlarged such that it is at least as big as the non-zero area of $I_{b}$ plus the bound region of $\mathbf{x}$. This is possible because the previous center determination already offers a first estimate of the relative location of the two images relative to each other to within at most one or two pixels, such that the lower and upper bounds $\mathbf{x}^{L}$ and $\mathbf{x}^{U}$ are small when $\mathbf{x}=\mathbf{0}$ is taken to be the existing estimate of the center displacement. Figure 4.18 shows examples for objective function results across horizontal and vertical shifts in the particle image position, for seven pairs of particle images in a trace (see Figure 4.19 for the corresponding particle images).

For the IDLSS method, the images $I_{a, b}$ are simply replaced with one of the derivatives of each of the particle images. The objective function results for the two derivatives are then added together, such that

$$
f_{\text {obj., IDLSS }}(\mathbf{x})=\sum_{i \in\{1,2\}}\left[\sum_{\mathrm{p}=0}^{n_{\mathrm{p}, \mathrm{I}_{\mathrm{a}}}}\left[\frac{\partial \mathrm{I}_{\mathrm{a}}}{\partial x_{i}}\left(\left\{\begin{array}{l}
x_{\mathrm{p}, 1}  \tag{4.12}\\
x_{\mathrm{p}, 2}
\end{array}\right\}\right)-\frac{\partial \mathrm{I}_{\mathrm{b}}}{\partial x_{i}}\left(\left\{\begin{array}{l}
x_{\mathrm{p}, 1} \\
x_{\mathrm{p}, 2}
\end{array}\right\}+\mathbf{x}\right)\right]^{2}\right]
$$

This is a bivariate multi-modal function and can therefore have multiple local minima, but $f_{\text {obj. }}(\mathbf{x})$ is continuous throughout the bounds of the problem. The first derivatives of the


Figure 4.18: Objective function results for successive particle images in a trace, by relative particle image position. The centers of the images represent the relative position as determined by the centroid method. The largest shift shown is $\pm 4.0 \mathrm{px}$, i.e. the value at the left image border represents a shift of the second image by 4.0 px towards the left. Objective function results are scaled to the respective maximum for each plot.
objective function,

$$
\begin{equation*}
\frac{\partial}{\partial \mathbf{x}} f_{\text {obj., IILSS }}(\mathbf{x}) \tag{4.13}
\end{equation*}
$$

can be obtained from the derivatives of the B-spline interpolation as

$$
\begin{align*}
\frac{\partial}{\partial \mathbf{x}} f_{\text {obj., IILSS }}(\mathbf{x}) & =\frac{\partial}{\partial \mathbf{x}} \sum_{\mathrm{p}=0}^{n_{\mathrm{p}, \mathrm{I}_{\mathrm{a}}}}\left[\mathrm{I}_{\mathrm{a}}-\mathrm{I}_{\mathrm{b}}(\mathbf{x})\right]^{2}  \tag{4.14}\\
& =\sum_{\mathrm{p}=0}^{n_{\mathrm{p}, \mathrm{I}_{\mathrm{a}}}} \frac{\partial\left[\mathrm{I}_{\mathrm{a}}-\mathrm{I}_{\mathrm{b}}(\mathbf{x})\right]^{2}}{\partial \mathbf{x}}  \tag{4.15}\\
& =-2 \sum_{\mathrm{p}=0}^{n_{\mathrm{p}, \mathrm{I}_{\mathrm{a}}}}\left[\mathrm{I}_{\mathrm{a}}-\mathrm{I}_{\mathrm{b}}(\mathbf{x})\right] \cdot \frac{\partial \mathrm{I}_{\mathrm{b}}(\mathbf{x})}{\partial \mathbf{x}} . \tag{4.16}
\end{align*}
$$

The result for the IDLSS method can be found in the same manner.
For this reason, both global/non-smooth zero-order methods such as Nelder-Mead Simplex (Nelder and Mead, 1965), Simulated Annealing (SA; Kirkpatrick et al., 1983) or Dividing Rectangles (DIRECT; Jones et al., 1993) can be applied, as well as a multi-start approach with a first- or second-order optimization method such as Sequential Quadratic Programming (SQP; Wilson, 1963; Han, 1976; Powell, 1969). SQP approximates the local shape of the objective function with a quadratic function based on objective function values and first derivatives. The fifth-degree splines that describe the inter-pixel regime of the second particle image are approximated relatively well by this approach. This therefore requires very few iterations (approximately 10 on average) to successfully meet an abort criterion for the change in the objective function between iterations. Some global/non-smooth methods reduce the likelihood of getting trapped in local minima during optimization compared to simple gradient-based methods, but they still do not necessarily find a global minimum among several local minima before having sampled a large number of design variable combinations.

Solving the problem benefits from a relatively limited range of the design variables $\mathbf{x}$ of only a few pixels: The particle images are limited in size and the centers are already known to within
a small area. Additionally, the computation of the result of the objective function is very fast in the given problem (see Table 4.1), so sampling a large part of the design variable space is not computationally expensive. Some of the global/non-smooth methods on the other hand add a lot of computational overhead compared to simply trying a much larger set of $\mathbf{s}$ and are therefore not very useful in this specific case.

For these reasons, the method of starting a SQP minimization from every point in a small grid of positions a few pixels around the expected center was chosen. The size of the grid, as well as the number of divisions per pixel, can be configured by the user. As an example, a radius of around 3 px around the previously estimated centers and a division of $\sim 2 \mathrm{px}^{-1}$ should be sufficient for almost any combination of particle images and only requires 169 computations of the objective function (i.e. around 11 ms on the previously mentioned reference computer). From these minimizations, the $\mathbf{x}$ of the smallest resulting value of the objective function is taken to be the global minimum.

Figure 4.19 shows a comparison, for the same trace for which the objective function results were given in Figure 4.18, of the relative centers as determined by the centroid and IILSS method. It is important to note here that the absolute position of the markers in either image is not significant and does not need to align with the intensity maxima-only the alignment between the marker in the top image relative to the one in the bottom image is significant. This trace has particularly challenging particle images, with only some showing secondary maxima and the overall particle images generally varying quite strongly. While this trace should likely not be used in the analysis, it illustrates the differences in the two algorithms well. In case IV, for example, the IILSS method aligns the single peak from the previous particle image with the larger peak on the following particle image, while the centroid method chooses a point in-between the two peaks.


Figure 4.19: Centroid and IILSS relative positions visualized for the particle images in a trace. Particle images have been scaled such that their respective maxima are shown as white pixels. Images on top show the centroid center (green marker) only, which is used as the basis for both the centroid (green marker) and the IILSS (red marker) offset in the images below.

### 4.6 Calculation of Slip Velocities

The slip velocities of the particles are determined as the difference between the PTV particle velocity and the PIV flow field velocity.

For a given trace, the PTV velocity is calculated for each connection based on the absolute difference in position between the two particle images. This pixel distance is multiplied with the reproduction scale in order to find the absolute distance in meters. It is then divided by the illumination pulse separation time to find the velocity.

### 4.6.1 Pulse Delay Fluctuation Correction

The delay between the illumination pulses can fluctuate by a factor of approximately $10^{-2}$. This deviation is small enough to not significantly affect trace finding, but it does impact the slip velocity calculation. The difference in the resulting particle image distances corresponds to about $3 \mathrm{~m} \mathrm{~s}^{-1}$ in the calculated slip velocity.

Input fields were added to the GUI that give the user the option to enter small deviations in the pulse timing in order to correct for this. The deviations are only applied in the slip velocity calculation and are not considered in previous analysis steps.

### 4.6.2 PIV Velocity Field Determination and Sampling

In order to calculate the PIV velocity field, a user-selected PIV data file is imported which contains velocity vectors (in image coordinates) at discrete positions in the image. This file represents the output of an external PIV analysis application. It comes in a Tecplot spacedelimited ASCII format with columns for vector positions and velocities. For the area between the given discrete PIV values, linear interpolation is used in order to achieve a continuous flow field description. For the image area outside of the value array (close to the edges of the image where the PIV algorithm was not able to determine a velocity), a nearest neighbor algorithm is used for extrapolation. This reduces the accuracy of the velocity determination in this area. However, as the shock wave is located in the left half of the image and only the very outer borders are extrapolated, no useful traces are likely to be affected.

For the absolute velocity, which is subtracted from the PTV connection velocity in order to find the slip velocity, the PIV velocity field is sampled at the center of the given connection. Because the PIV velocity field is already a representation of a $10 \mu$ s integration, just like the PTV connection velocity, this returns the PTV-equivalent PIV velocity.

### 4.7 Particle Response Fitting Modifications

The C++ code of the particleresponse application was also adjusted slightly to make it compilable with Linux using the GNU Compiler Collection (GCC).

Beyond that, the only change that was made was the removal of an input parameter for the velocity change across the shock wave. ${ }^{9}$ Initially, the idea was to find traces that would start more or less exactly at or in front of the shock wave and fit the slip velocities starting with this initial velocity difference. In this case, a single slip velocity could be used across all traces for a given image, assuming a shock that is equally strong across the entire image. However, this was deemed impractical as the shock wave is not sharply resolved in PIV measurements, meaning that the region shortly after the shock wave still show major gradients. Additionally, finding traces that would cross the shock wave or start exactly at the shock wave increased the difficulties with the refractive effects of the shock wave, and was generally unlikely. The strength of the shock wave also approached zero towards the top of the NFT camera images, making a single shock wave strength for all traces impractical to use.

The slip velocity equations described in Section 1.7.2 do not consider the position of the actual shock wave but only some initial and decreasing velocity difference between a given particle and the surrounding flow. This means that a trace starting at a later point is still equally usable for the aerodynamic diameter determination. To nevertheless obtain the local initial slip velocity for each trace, $u_{\mathrm{s}}$ was included as a variable in the least squares fit.

Despite these changes, it is desirable to find traces for the analysis which start reasonably early behind the shock wave. This does produce the highest possible difference in velocity between the fluid and the particle, and as a result also the highest accuracy.

[^12]
## 5

## Calibration and Validation

The PTV and PIV measurements require calibration in the sense that pixel distances in the respective images need to be translated to real-world distances and velocities, which is done via an image reproduction scale. Whether this scale is constant across the image or requires local resolution (due to distortion and/or refractive effects) has to be determined as part of this process.

Additionally, the new methods introduced into the analysis require validation. The goal is a quantitative measurement of the quality of the results in absolute terms, and in relation with the previous analysis method where possible.

These two topics contain some overlap in their methodology and experiments, which is why they have been merged into this chapter and will be discussed in the following sections.

### 5.1 Particle Trace Matching Quality

Because the actual traces are not known in any NFT camera image with realistic seeding, evaluating the success rate of the trace matching algorithm is difficult. The first indication is of course whether the traces look plausible, which is the case here, but this is not satisfactory for a validation. There are however some measures which can give an indication for how many false positives are contained in the resulting traces.

It is worth noting that any image can contain particles flying in parallel to each other at such a distance that they overlap for some of their illumination points, which can result in collisions and in traces of extended length. Therefore, realistically, the criterion in either of the following sections should not be a complete lack of deviations from the expected value but rather a very small number of outliers.

The following figures show results for $r_{\mathrm{d}}=1 \mathrm{px}$ only-figures of $r_{\mathrm{d}}=0 \mathrm{px}$ and $r_{\mathrm{d}}=2 \mathrm{px}$ are available in Appendix A.5.2.

### 5.1.1 Collisions

One indication for the quality of the traces that are detected by the algorithm is the percentage of collisions that occur. If the trace finding algorithm up to the point of collision filtering only returns true traces, very few or none of these should collide, whereas the inclusion of false positives returns numerous collisions. Figure 5.1 shows the percentage of traces that survived collision filtering in a parameter study across different values of $\beta_{\max }$ and $W_{\mathrm{T}, \mathrm{min}}$. Trace ratios


Figure 5.1: Percentage of traces that survive collision-filtering, under variations of $\beta_{\max }$ and $W_{\mathrm{T}, \mathrm{min}}$. The dark blue area highlights cases in which not a single trace remained, in which case the percentage is undefined. Grey dots highlight parameter combinations that were sampled, intermediate data shows the result of linear interpolation. See Figure A. 17 for different values of $r_{\mathrm{d}}$.
of close to $100 \%$ indicate a low number of false positives. For reference, Figure 5.2 shows the number of traces that were found in the same parameter ranges and with the same settings.

### 5.1.2 Extended Length Traces

Another indication can be taken from the number of traces that are found when the trace length is set to be greater than the actual number of illumination pulses that were recorded in the given image. Every trace that is detected with such a parameter is known to be either a false positive or to represent more than one particle at some of the illuminations. Figure 5.3 shows an example for a trace which clearly contains a second particle in the first three illuminations, and would therefore be allowed to extend beyond the given maximum length without representing


Figure 5.2: Number of traces found, under variations of $\beta_{\max }$ and $W_{\mathrm{T}, \text { min }}$. Grey dots highlight parameter combinations that were sampled, intermediate data shows the result of linear interpolation. See Figure A. 18 for different values of $r_{\mathrm{d}}$.


Figure 5.3: Particle images detected as single particles on a single trace. The first three images likely show a second particle on a very similar path that was probably illuminated at earlier points as well. Images have been inverted and isolated from the main image using a gaussian tapered mask.
a false positive. Ideally, if this occurs frequently, the particle identification threshold should be increased in order to recognize two separate particles in this instance. However, images in Figure 4.14 show that some real particles are almost indistinguishable from single particle images that sometimes show multiple local maxima. For this reason, a perfect distinction between multiple particles and oddly shaped single particles is likely not achievable.

Any set of parameters used for the final evaluation should produce very few or none of these $\left(n_{\mathrm{c}, \mathrm{t}}+1\right)$ length traces. Figure 5.4 shows the number of traces found for different trace lengths, starting with the correct number of connections of $n_{c, t}=7$.


Figure 5.4: Number of traces found for a higher number of pulses than actually captured $\left(n_{c, t}=8\right.$, i.e. extended by one), under variation of $\beta_{\max }$ and $W_{\mathrm{T}, \min }$. Grey dots highlight parameter combinations that were sampled, intermediate data shows the result of linear interpolation. See Figure A. 19 for different values of $r_{d}$.

### 5.1.3 Conclusion

This analysis shows that with $\sigma_{\mathrm{I}}=1.0$ and $\sigma_{\mathrm{I}}=2.0$, with $\beta_{\max }$ smaller than $1.35^{\circ}$ and $W_{\mathrm{T}, \min }$ greater than $74 \%$, it is possible to assume that the vast majority of detected traces represents real particle traces. This is not universally applicable for all measurement images, but shows that good parameters can be found. The application does contain the tools to easily perform this study again for different input data. The results indicate that the algorithm does at least perform well in this case, while the old trace assembly method lacked the fine control to even easily conduct such a study.

### 5.2 Preliminary Image Reproduction Scale Calibration

In order to convert the pixel displacements that are determined in the software to real-world distances and finally velocities, the reproduction scale needs to be determined. This was done before the actual test runs, with the wind tunnel test section moved into the open position. A grid target was placed in the test section such that it covered the entire field of view of the NFT camera and most of the relevant field of view of the WFT camera. It was positioned into the line of the PTV illumination laser sheet using a three-axis construction laser for alignment. Due to safety concerns, the InnoLas SpitLight 1000 laser could not be active for this purpose even on its lowest power setting. The reference for the alignment was the exit lens of the laser beam into the test section. As this lens is a few millimeters wide and the beam could exit at a slight angle, this limits the precision of the placement of the target to within about 10 mm .

The resulting images from the two PTV cameras are shown in Figure 5.5. The distortion in the lens of the WFT camera mentioned in Section 3.2.2 is quite noticeable on this target. From


Figure 5.5: Image reproduction scale calibration images. Black and white levels have been adjusted for better contrast.
a) narrow field tracking camera.
b) wide field tracking camera. The approximate field of view of the NFT camera is shown in red.
the 5 mm grid on the target, a image reproduction scale of

$$
\begin{align*}
s_{\mathrm{NF}, \text { horizontal }} & =\frac{8 \times 5 \times 10^{3} \mu \mathrm{~m}}{\sqrt{1894 \mathrm{px}^{2}+17 \mathrm{px}^{2}}} & & =21.12 \mathrm{\mu m} \mathrm{px}^{-1}  \tag{5.1}\\
s_{\mathrm{NF}, \text { vertical }} & =\frac{8 \times 5 \times 10^{3} \mu \mathrm{~m}}{\sqrt{16 \mathrm{px}^{2}+1896 \mathrm{px}^{2}}} & & =21.10 \mathrm{\mu m} \mathrm{px}^{-1}  \tag{5.2}\\
s_{\mathrm{NF}} & =21.11 \mathrm{\mu m} \mathrm{px}^{-1} \pm \sim 0.02 \mathrm{\mu m} \mathrm{px}^{-1} & & \tag{5.3}
\end{align*}
$$

was found for the NFT camera, and a scale of

$$
\begin{align*}
s_{\mathrm{WF}, \text { horizontal }} & =\frac{33 \times 5 \times 10^{3} \mu \mathrm{~m}}{\sqrt{1960.5 \mathrm{px}^{2}+13 \mathrm{px}^{2}}} & & =84.16 \mu \mathrm{mpx}^{-1}  \tag{5.4}\\
s_{\mathrm{WF}, \text { vertical }} & =\frac{16 \times 5 \times 10^{3} \mathrm{\mu m}}{\sqrt{9 \mathrm{px}^{2}+947 \mathrm{px}^{2}}} & & =84.47 \mathrm{~mm} \mathrm{px}^{-1}  \tag{5.5}\\
s_{\mathrm{WF}} & =84.32 \mathrm{mpx}^{-1} \pm \sim 0.3 \mathrm{pmpx}^{-1} & & \tag{5.6}
\end{align*}
$$

was found for the WFT camera. The uncertainty given above applies only to the measurement itself, based on how well the line location can be determined on the target. For the WFT camera, the distortion also causes a difference between the horizontal and vertical measurement and therefore increases the uncertainty. However, the actual accuracy of these values is mostly limited by the significant inaccuracy of the alignment of the target to the laser sheet plane - they may be inaccurate by about $\pm 5 \%$ of the result.

The image from the WFT camera also shows the $\frac{x_{\mathrm{a}}}{c}$ markings in steps of $10 \%$ on the airfoil model, giving a reference for the position of the field of view relative to the profile. This was verified additionally by setting the construction laser to the trailing edge of the airfoil model, both vertically and horizontally, and measuring the distance to the a/d marking on the grid target. The vertical distance here was 36.0 mm , the horizontal distance was 90.0 mm . The lower image border of the NFT camera was found to be located 54 mm above the airfoil model trailing edge. The accuracy of these measurements is only given to within about 0.5 mm , mostly due to the difficulty of working under the very limited accessibility of this part of the test section.

This target was also used to set the initial focus of the two cameras. Corrections are possible using two adjustment motors that move the camera mounting positions relative to the lens, without opening the camera box or even entering the wind tunnel area. By this method, the focus was later fine tuned with actual seeding in the test section and under realistic pressure and illumination conditions.

### 5.3 Displacement Experiment

It is desirable to test the distance determination accuracy with a setup which resembles the actual test section scenario as closely as possible. Additionally, a better calibration for the image reproduction scale of the pixel distances in the NFT camera image plane is necessary-specifically one that is not subject to inaccuracies in the placement of any target into the laser sheet plane.

Furthermore, values for the image reproduction scale and distance determination accuracy that are spatially resolved across the image of the NFT camera are helpful to narrow down any possible distortion as well as errors caused by the PSF in the outer areas of the image.

The following setup and results describe an attempt to achieve all three aforementioned goals.

### 5.3.1 Setup

To achieve all of this, a custom test rig was assembled to be placed around the airfoil model in the test section. In order to get the most realistic representation of the distance from the camera, the target is illuminated by the InnoLas SpitLight 1000 laser system. In order to be able to use the full workflow of the software that was developed, the target consists of small round plastic particles $20 \mu \mathrm{~m}$ in size. These should resemble the actual ice particles in their optical behavior. In order to avoid any optical interference from adhesives or any other mounting material, the particles were lightly pressed between two sheets of glass using Kapton tape. This resulted in some reflections of the particles between the sheets of glass. However, these reflections should be either out of focus or not illuminated and should therefore not disturb the measurement. In order to have particles positioned at all depths from the camera, such that the visibility would only depend on the focus and the illumination (as in the real experiment) and not on the placement of the target, the sheets of glass were mounted at an angle of $\epsilon=23.1^{\circ}$ from the laser beam axis (rotated counter-clockwise looking down).

Moving this target at sufficiently precise intervals proved very difficult. It was therefore decided to use a linear motor to move the target and to record the imprecise movement separately using a distance gauge. For this purpose, a Mahr MarCator 1075R was used (see Table 5.1 for specifications).

| Type designation | MarCator $\mathbf{1 0 7 5 R}$ |
| :--- | ---: |
| Subtype | no. 4336020 |
| Manufacturer | Mahr GmbH |
| Measurement range | 12.5 mm |
| Resolution | $5 \mu \mathrm{~m}$ |
| Error limit | $15 \mu \mathrm{~m}$ |
| Measuring force | $0.5-1 \mathrm{~N}$ |

Table 5.1: Specifications for the Mahr MarCator 1075R digital indicator (Mahr GmbH, 2017).
Figure 5.6 shows the target displacement setup both outside of the tunnel and in place. Both the linear motor (with the target attached) as well as the distance gauge were mounted on a bridge constructed across the airfoil model. The "bridge" itself was constructed using the X95 aluminium optical rail system (Newport Corporation, 2011) in order to provide the highest possible stiffness. The alignment of the movement axis of the linear motor with the axis of the


Figure 5.6: Displacement experiment setup.
a) The full setup outside of the test section.
b) Detail of the target area inside of the test section.
c) The full setup in the test section, looking against the direction of flow. The airfoil model is visible directly below the target with its orange TSP paint.
laser beam was achieved using a construction laser. Here, the precision of the spanwise position was not critical.

## Procedure

Because the digital indicator is only capable of measuring displacements of up to 12.5 mm and the field of view of the NFT camera spanned about 42 mm , the setup had to be used in intervals. For recording the final images, the entire "bridge" was moved such that the target was located at different fifths of the NFT camera's horizontal field of view.

The motor was then used to displace the target to five positions, in approximately equally spaced increments, by a total length of about 12 mm towards the right. At each interval, the digital indicator reading was recorded. These images were later combined by taking the maximum across all five images. Each such combined, resulting image is referred to as a stack.

## Geometric Target Distortion

In order to achieve a somewhat even laser illumination on the particles, the angle on the target sheets was necessary. However, this creates a distortion of the position of each particle compared to a sensor-parallel target. This is illustrated in Figure 5.7.


Figure 5.7: Illustration of the geometric distortion of the particle image distances due to the rotation of the target. The red distances represent the actual particle displacement. Note that for Equations 5.7-5.11, the shift of the target is equal to the radius $r$. Sizes, angles and distances are not drawn to scale.

As a result, any connections starting a few tens or hundreds of pixels further to the left or right than a different one will be subject to a slightly different angle difference, and as a result,
will show a slightly different connection length $\delta_{i} \hat{x}_{1}$. Specifically,

$$
\begin{align*}
\delta_{\mathrm{i}} \hat{x}_{1} & ={ }_{\mathrm{i}} \hat{x}_{1,0}-{ }_{\mathrm{i}} \hat{x}_{1, \alpha}  \tag{5.7}\\
& ={ }_{\mathrm{i}} \hat{r}\left(\cos \left(\epsilon^{\prime}+\alpha\right)-\cos \left(\epsilon^{\prime}\right)\right), \tag{5.8}
\end{align*}
$$

with

$$
\begin{align*}
\epsilon^{\prime} & =90^{\circ}-\epsilon=66.9^{\circ}  \tag{5.9}\\
\alpha & =\arctan \left(\frac{w_{\text {sensor }} \Delta_{\mathrm{i}} \hat{x}_{1}}{2 l_{\mathrm{f}}}\right)  \tag{5.10}\\
{ }_{\mathrm{i}} \hat{r} & =\cos \left(\alpha+\epsilon^{\prime}\right) \Delta_{\mathrm{i}} \hat{x}_{1} \tag{5.11}
\end{align*}
$$

Here, ${ }_{i} \hat{x}_{1,0}$ is the observed distance from the (arbitrary) axis of rotation of the target if the target was parallel to the sensor plane (assuming constant $x_{1}$ positions of the particles, i.e. sheared rather than rotated), ${ }_{i} \hat{x}_{1, \alpha}$ is the actual relative position for an angle $\alpha$ away from the optical axis. Additionally, ${ }_{i} \hat{r}$ is the actual distance of the particle from the axis of rotation of the target.

Given that the illuminated range of the particle target in the $x_{1}$ direction is approximately $\Delta_{\mathrm{i}} \hat{x}_{1}=350 \mathrm{px}$ wide, there is a viewing angle difference of $\delta \alpha \approx 1.73^{\circ}$ between the starting particles of a given connection. This gives a value of $\delta_{\mathrm{i}} \hat{x}_{1}$ of $\sim-3.4 \times 10^{-3}$ or $\delta_{\mathrm{i}} x_{1} \approx-3.48 \mathrm{px}$. Clearly, this deviation is not acceptable if the assumption will later be that the displacement of all particles will be equal. If the expectation is to measure an accuracy down to about $1 \times 10^{-2} \mathrm{px}$, only particles within 18 px to one side of an arbitrary vertical axis on the target can be used. Therefore, a window of this width is extracted from the particle images, making sure that the strips show the same particle images in all 5 images respectively for each stack. This also serves the purpose of removing out-of-focus particle images and most internal reflections in the target setup.

Furthermore, it should be noted that the two panels of glass between which the particles are located will cause some shift due to refraction. This shift should be similar for all particles in a given position, and very small in its magnitude. As such, the windowing that is performed to compensate for the geometric distortion of the angled target is sufficient to minimize the impact of this error as well.

### 5.3.2 Resulting Images and Processing

## Unmodified Images

The result of this setup are images that resemble, in some aspects, seeding with particle traces containing four connections each. An example image is shown in Figure 5.8a. The bright blocks in the upper area are part of the mounting setup for the glass plates.

The fact that only the lower part of this image contains useful particle images is not a problem, as the actual measurement images only contain a strong shock in the lower half of the image as well. For this reason, the upper half (minus a 100 px buffer area for the weight map generation)


Figure 5.8: Example for a stacked displacement experiment image, based on five imgaes from the NFT camera.
a) Full image stack of 5 illuminations, without windowing. Only the area within the blue outline is used for the analysis.
b) Image as used in the analysis, windowed to 18 px wide sections of the original images before stacking.
c) Detail of a $256 \times 128 \mathrm{px}$ area in the lower half of (a), showing some of the particle image shapes in detail.
was excluded, such that the analysis was performed on $2048 \mathrm{px} \times 1124 \mathrm{px}$ images.
The particle images, as can be seen in Figure 5.8b, are roughly Gaussian due to the spherical shape and diffuse reflectivity of the particles themselves. As such, the strengths of the improved distance determination methods cannot fully be tested here, but they can be expected to perform at least equally to the centroid and Gaussian methods.

## Filtering

The four displacement distances in the experiment are far from equal due to a lack of fine control over the position. In this aspect, the test is therefore a bad representation of an actual PTV image from the experiment. This is problematic for the creation of weight maps, and as a result, for the trace assembly in general. The trace matching algorithm still produces valid results here, though likely with a few false positives.

The latter can be filtered out based on the knowledge that all traces should resemble each other to within a small error (though this does affect the usefulness of the standard deviation numbers below). The deviation of a single connection distance from the mean distance of the given connection across all traces in a given stack was used for the filtering. So, for example, the second connection in a given trace should not deviate further than a set distance from the mean across the second connection distances of all traces in the stack. A limit of 1.0 px was chosen as the filter criterion for removing clearly invalid traces in the analysis of the results. This filter is applied to the results of all methods, choosing only connections which pass the test in every case, in order to use identical traces across methods. For the vast majority of particle images, none of the methods produce an error for the center offset greater than that amount.

## Noise Simulation

These calibration images contain no background seeding and, as a result, almost no noise compared to the level that is present in regular measurement images. As described in Chapter 3, noise is expected to be among the major error sources for the distance determination. An estimate of the influence is possible by adding noise to the images.

Because the background seeding is sometimes persistent even after several wind tunnel runs without seeding, images are available that contain only background seeding with no bright particles at all. For the purpose of adding noise to the calibration images, an image was chosen which had a similar noise standard deviation as the measurement image analyzed in Figure 3.2 on page 51 . The image was then scaled in its intensity by $-8 \%$ for an even better match of the standard deviations. Figure 5.9 shows the standard deviation histograms of the images with seeding as well as the reference noise image without seeding.

In order to not create new saturated particle images through the addition of the relatively high noise level, the original stack (i.e. the signal) was not added directly to the reference noise image. Instead, the signal was scaled by the difference between the local noise pixel brightness
and the saturation level. For example, if the local noise reference pixel has a value of 0 , the signal would simply be added, whereas for a noise pixel value of 0.5 , it would be divided by 2 before being added.

One of the resulting stacks with the added noise is shown in Figure 5.10.


Figure 5.9: Local standard deviation histograms for images from NFT camera with and witout recent seeding. Both histograms are normalized such that the area under the histogram equals 1.
a) Local standard deviation histogram for a set of 5 images with seeding ( $3 \mathrm{px} \times 3 \mathrm{px}$ area) from the NFT camera.
b) Local standard deviation histogram for the noise reference image without recent seeding ( $7 \mathrm{px} \times 7 \mathrm{px}$ area) from the NFT camera.


Figure 5.10: Displacement experiment image stack with added reference noise image.

### 5.3.3 Results

Unless stated otherwise, the following results were obtained using the IILSS method described in Section 4.5. The symbol ${ }_{i} \hat{d}_{c}$ represents the distance, in image coordinates, within a given connection, whereas $d_{c}$ indicates the absolute real-world distance between two particle illuminations. A comparison of the results with the previous (centroid) distance determination method follows at the end of this section.

## Distance Error

Figure 5.11 shows the analyzed connection length deviation from measured displacement distance, in px. The horizontal axis corresponds to the distance of the given trace from the image center, which potentially shows any influences of the image position on the error. In order to convert real-world distances to pixel distances in this case, the median analyzed connection length in each connection across the different traces $\widetilde{{ }_{\mathrm{i}} d_{\mathrm{c}}}$ was used here to calculate a single image reproduction scale for each separate connection in a given stack. These four different image reproduction scales per stack were then compared to the measured distances as a basis for the conversion. For comparison of the error distribution, Figure 5.12 offers histograms of this error (i.e. the vertical axis in the previous figure) for the different methods. Given that there is a clear resemblance with a Gaussian distribution, calculating standard deviations is suitable to characterize the error of each method. Standard deviation statistics for the stacks and connections are also presented individually in Table A. 12.

Any error in the real-world displacement measurement using the digital indicator does not affect the results because the standard deviation is only calculated from the deviations within a given stack and connection, where the results should, ideally, be identical.


- Connection 1 ॰ Connection 2 - Connection 3 - Connection 4
- Stack no. 1 - Stack no. 2 - Stack no. 3 - Stack no. 4 - Stack no. 5

Figure 5.11: Median deviation of measured connection length. Conversion from pixels to $\mu \mathrm{m}$ was performed using the median length of the given connection across all traces.

## Reproduction Scale

As an alternative way of interpreting the data without applying any general reproduction scales, Figure 5.13 shows the image reproduction scale of every single trace connection, i.e. the measured real-world distance divided by the analyzed connection length. This is shown only for the IILSS method, but results differ very little for other methods. The vertical offset between different stacks and connections is likely the result of measurement errors from the digital indicator or of refraction in the particle holder glass. In the case of one very strong deviation (for example connection 1 in stack no. 2), it is likely attributable to some shift in the setup. The corresponding mean and standard deviation values for the stacks and connections are given in Table A. 13 .

The exact length of each individual real-world displacement is only known to within the measurement accuracy of the digital indicator, which is $15 \mu \mathrm{~m}$ and corresponds to about 0.74 px on the NFT camera. Some of the measurement error can be expected to average out across the different measurements, which were all taken across approximately 200 px (giving a worst case error of $\sim 0.4 \%$ ). Therefore, this analysis is more accurate for determining the image reproduction scale than what was described in Section 5.2.

It is important to not bias the determination of the reproduction scale based on the choice of vertical windowing that is applied. Therefore, for this purpose, the windowing is not applied and the full displacement images are analyzed. Taking the median value of the resulting image reproduction scale gives a value of $s_{\mathrm{NF}} \approx 20.34 \mathrm{~mm} \mathrm{px}^{-1}$ (for the IILSS method). The median value in Table A. 13 was derived from the windowed images and is therefore slightly different. The reproduction scale is within a range of $\pm 0.01 \mathrm{\mu m} \mathrm{px}^{-1}$ for all tested distance determination methods.


Figure 5.12: Histograms of analyzed connection length deviation from measured displacement distance.
Values in brackets show the standard deviation of the error for a given distance determination method in pixels.
a) Methods without smoothing.
b) Methods with $2.5 \%$ smoothing. Centroid method included for reference only.


- Connection 1 ॰ Connection 2 - Connection 3 จ Connection 4
- Stack no. 1 - Stack no. 2 - Stack no. 3 - Stack no. 4 - Stack no. 5

Figure 5.13: Measured displacement divided by analyzed connection length. This is equivalent to the image reproduction scale for each connection in each trace.

## Lens Distortion

In order to estimate the total distortion for the lens on the NFT camera, the data from all connections in all stacks was statistically combined. This entails disregarding the variability in the absolute values while only looking at the changes within the given connection-and then observing how these are affected by increasing distance from the center of the field of view outwards. The result is fitted to a function approximating the lens distortion. Again, this is shown only for the IILSS method.

Due to the low signal to noise ratio in the data, anything but a simple radially symmetrical distortion function would overfit the data. Zernike (1934) has developed a widely used mathematical model to describe lens distortion components using a series of polynomials. These are ordered by angular frequencies $m$ and radial orders $n$ (Lakshminarayanan and Fleck, 2011). For radial symmetry, $m$ equals 0 , giving $n=0,2,4$ as the lowest order distortions. This corresponds to the following deviation terms $R_{n}^{m}(r)$ :
$m=0, n=0$ :

$$
\begin{equation*}
R_{0}^{0}(r)=1 \tag{5.12}
\end{equation*}
$$

$m=0, n=2$ :

$$
\begin{equation*}
R_{2}^{0}(r)=2{ }_{\mathrm{i}} \hat{r}^{2}-1 \tag{5.13}
\end{equation*}
$$

$m=0, n=4:$

$$
\begin{equation*}
R_{4}^{0}(r)=6{ }_{\mathrm{i}} \hat{r}^{4}-6{ }_{\mathrm{i}} \hat{r}^{2}+1 \tag{5.14}
\end{equation*}
$$

Here, ${ }_{i} \hat{r}$ is the radius from the center of the image. For curve fits to the individual connection
data in each stack, the function

$$
\begin{equation*}
D_{2}\left(r, C_{0}^{0}, C_{2}^{0}\right)=C_{0}^{0} R_{0}^{0}+C_{2}^{0} R_{2}^{0} \tag{5.15}
\end{equation*}
$$

was used, where $C_{n}^{m}$ are the coefficients or amplitudes of the respective polynomial component. The $C_{4}^{0} R_{4}^{0}$ component was skipped as it already showed indications of overfitting the existing data. The least squares fits of this function to each connection and stack are shown as dotted lines in Figure 5.13.

For all particles on the target, the respective connection distances are equal, except for the small differences that are caused by the finite depth of the illumination sheet (resulting in slightly different image reproduction scales) and other possible geometric misalignments. In Figure 5.14, $D_{2}\left({ }_{\mathrm{i}} \hat{x}_{1, \text { node }}, C_{n}^{0} \forall n\right)$ was subtracted from the error data in order to align the different stacks and connections vertically. Here, ${ }_{\mathrm{i}} \hat{x}_{1, \text { node }}$ is the median of the absolute ${ }_{\mathrm{i}} \hat{x}_{1}$ positions of all measured points. The underlying assumption is that the curve fit across the entire data set is likely the most accurate at $x_{\text {node }}$. As a result, only the change in reproduction ratio across the field of view for each connection is shown, with potential measurement errors from the digital indicator being disregarded. The dark line in Figure 5.14 represents the $D_{2}$ least squares curve fit across the entire data set, showing essentially zero distortion.

A correction of the original data allows for an update of the standard deviations that were given in Table A.12, giving the the results shown in Figure A. 23 and Table A.14.


- Stack no. 1 - Stack no. 2 - Stack no. 3 - Stack no. 4 - Stack no. 5

Figure 5.14: Relative change across the field of view of the camera of the measured displacement divided by analyzed connection length. This is equivalent to the change in image reproduction scale for each connection in each trace.
a) Data with subtracted absolute offset, including $D_{2}$-fit through the entire dataset. The thick black line represents the overall bests fit, which is almost exactly flat.
b) Data corrected using the overall fit shown in (a). Median and $1 \sigma$ are shown.

## Influence of Noise

For the images with added noise (and some background intensity), Table A. 30 shows the standard deviation results equivalent to those in Table A.12. Additionally, Figure 5.15 shows histograms of the errors for the different methods (equivalent to Figure 5.12 without noise). The results with corrected distortion are skipped here because the benefit is questionable and the comparison is equally valid using the uncorrected data. The increase in standard deviation compared to the images without noise in the overall mean is $4.6 \%$ for the IILSS algorithm and $4.5 \%$ for the centroid method. These changes are identical between the two methods within the precision of this analysis. The increase in the error is underestimated somewhat as a consequence of an effective upper limit in the error for each single data point at 1.0 px . A visual comparison of the histograms does not show very different distributions either, indicating a small overall effect of the noise here. The noise error can be analyzed more easily in the more controlled synthetic validation described in Section 5.4.

## Peak Locking

A number of potential sources of error, most notably sampling errors, lead to peak locking as described in Section 3.2.1.

A good indicator for peak locking can be found by generating a linear least squares fit through the errors of all individual connections when plotted across

$$
\begin{equation*}
{ }_{\mathrm{i}} d_{\mathrm{c}, \mathrm{dd}}-\left\lfloor{ }_{\mathrm{i}} d_{\mathrm{c}, \mathrm{dd}}\right\rceil=\left(\left\|\Delta\left({ }_{\mathrm{i}} \mathrm{x}\right)\right\|+0.5\right) \bmod (1)-0.5 . \tag{5.16}
\end{equation*}
$$

This is shown in Figure 5.16. The methodology for obtaining the errors is the same that was used for Figure A.12. Here, it is clear especially from the test with added noise that the grey value offset does appear to cause a slight peak locking effect. The strongest slope can be observed in the Gaussian and IILSS methods for the image with added noise. For the IDLSS method, the effect is weaker but still present. Without noise, only the Gaussian method shows noticeable peak locking. As expected for this error, there is a negative correlation between the distance from the integer value and the displacement error. The exception here are some weak positive slopes, which cannot be explained through peak locking and are likely within the margin of error of this test. Overall, even the most significant effects are an order of magnitude smaller than the overall error.

## Sensitivity to the Objective Function Result

The result of the objective function in the distance determination minimization is stored along with each connection. It gives an indication for how well the algorithm was able to match the given particles with each other. This should depend on how similar the particle images actually are. For example, in cases where one of the particle images was illuminated slightly differently or affected by the speckle pattern of the laser, the objective function is larger. When plotting


Figure 5.15: Histograms of analyzed connection length deviation from measured displacement distance, for the images with added reference noise. Conversion from pixels to $\mu \mathrm{m}$ was performed using the median length of the given connection across all traces. Values in brackets show the standard deviation of the error for a given distance determination method in pixels.
a) Methods without smoothing.
b) Methods with $2.5 \%$ smoothing. Centroid method included for reference.
(a)


| Centroid | Gaussian | IILSS | IDLSS | ISILSS | ISDLSS |
| :---: | :---: | :---: | :---: | :---: | :---: |
| (0.0024) | (-0.0803) | (0.0093) | -(0.0161) | (0.0214) | (0.0046) |

(b)


| Centroid | Gaussian | IILSS | IDLSS | ISILSS | ISDLSS |
| :---: | :---: | :---: | :---: | :---: | :---: |
| (0.0007) | (-0.0765) | (-0.0569) | (-0.0300) | (-0.0490) | (0.0036) |

Figure 5.16: Least squares fit through connection errors (on the vertical axis) over the distance from an integer interval. This highlights any prevalence of errors in the direction of whole integer interval distances in the displacement experiment dataset. Values in brackets show the respective slope of the line fit.
a) Original stacks.
b) Stacks with added noise.
the position error over the objective function result, one can expect a correlation between high objective function values and high errors. This relationship for the IILSS and IDLSS methods is shown in Figure 5.17.

Generally, lower objective function results do show lower error standard deviation and mean values. In all cases, the lowest objective function results correlate with an error standard deviation of approximately 0.1 px , which is a significant improvement over the error of the entire set of connections. This increasing error trend at low results does not continue for higher objective function results, though the much smaller sample sizes make these measurements less useful.

These results indicate that it might be possible to improve the overall error by filtering out traces with high objective function values in their connections. This, however, is not trivial as a suitable threshold is generally not known for a given measurement - absolute objective function results will be different for different particle image sizes, shapes and noise levels.

While the objective function distribution of the IDLSS method is not affected significantly by the added noise and base intensity, objective function values of the IILSS method decrease significantly with the added noise floor. This can be explained by some base intensity which is common to all particle images, such that the absolute differences between particle images become smaller.

### 5.3.4 Conclusions

## Lens Distortion

There is a large variance in the distortion fits for different stacks and connections. Additionally, the total standard deviation is not reduced by applying the distortion correction to the results. This indicates that any distortion is well within the measurement error of the analysis. Consequently, applying any correction to the main measurements appears unnecessary until or unless the measurement error can be improved further. For this reason, the NFT camera images will be treated as being free of any distortion.

## Lens Aberrations

Almost all lens aberrations are more severe in their effect outside of the center of the image. As a result, they tend to increase the result standard deviation with increasing distance from $\left\|_{i} \hat{x}\right\|=0$. The fact that the standard deviation is relatively constant from the center of the image to the outer areas (see the point distribution in, for example, Figure 5.11) indicates that these effects are at least not dominant in the overall error.
(a)

(b)

(c)

(d)


Figure 5.17: Connection distance errors by the objective function results from the distance determination minimization. Circles show the standard deviation of the given set of connections. Cross markers show the mean error for the given bin. Connection numbers contained in each bin are given on top.
a) IILSS method. b) IILSS method with added noise.
c) IDLSS method.
d) IDLSS method with added noise.

## Image Reproduction Scale

A value of $s_{\mathrm{NF}} \approx 20.34 \mathrm{~mm} \mathrm{px}^{-1}$ was found for the image reproduction scale, with an expected accuracy of approximately $\pm 0.05 \mu \mathrm{mpx}^{-1}$. In all cases where image determined velocities are given, this value was used as a scaling factor.

## Noise

The results in Section 5.3.3 indicate that the noise level does not dominate the location determination error. This is important as it was found to be among the dominant error sources in Chapter 3. The largest increase in error due to noise is observable in the Centroid method, where the standard deviation is increased by approximately $22 \%$. The new methods, i.e. IILSS and IDLSS, show about $3 \%$ and $3.5 \%$ of noise deterioration respectively, with almost identical differences for the corresponding results with $2.5 \%$ of smoothing enabled.

## Peak Locking

Peak locking does not appear to contribute significantly to the overall error, even though it is present in some methods to varying degrees.

## Comparison of Distance Determination Methods

In all cases, the new IILSS method performs slightly but not substantially better than the old centroid method, with improvements of the standard deviation of up to $19 \%$ only in the case with noise. All methods perform significantly better than the Gaussian method, despite the symmetrical shape of the particle images - likely mostly due to their large sizes. The IDLSS method actually performs worse than the IILSS and Centroid variants without noise. Even with added noise, where an improvement over IILSS was expected, it performs worse than this method and marginally better than the Centroid method.

Overall, the IILSS method therefore shows the best performance. A low amount of added smoothing is benefitial within the margin of error of these standard deviation numbers, and overall appears not to be worth the increased complexity in the method and additional consideration for the user regarding in the choice of a suitable smoothing parameter.

## Overall Precision

Due to the high seeding density on the target and the tendency of the granular particles to accumulate in certain areas, multiple particle images are often recognized as a single one. This can be seen when looking at the individual particle cutouts from the IILSS algorithm. These aggregates would not necessarily pose a problem for either of the algorithms in the case of perfect reproduction and illumination. However, the separation of the particle images is occasionally
inconsistent throughout a trace, resulting in potentially large increases in the position error. Similar effects can be observed in real seeding test images, but at much lower rates.

Consequently, the absolute error here is not fully representative for the error in the main experiment. It is noteworthy and a valuable result that among the error sources that were predicted to dominate (noise, lens aberrations and sampling errors), the magnitude of the first two has been found to not be dominant for the new distance determination methods. An error with a standard deviation around 0.2 px remains, and cannot be assigned to any specific phenomenon based on this analysis. The reason for this error may lie in differences in the illumination depending on the target location due to the uneven light sheet from the laser and potentially due to speckle effects. Some slack in the orientation of the target as it is being moved may also cause some rotation, which would result in additional distance errors.

In the most realistic scenario, i.e. with added noise, the IILSS algorithm achieves an accuracy in this test of about 0.2 px standard deviation (with or without $2.5 \%$ of smoothing).

### 5.4 Synthetic Image Validation

In order to understand the quality of the proposed methods in a fully controlled environment with known precise connection distances, synthetic PTV images were created. These allow for a test of the analysis pipeline including particle identification, trace finding and particle distance analysis.

### 5.4.1 Image Generation

The goal in creating the images was to match the actual PTV images from the experiment as closely as possible, but with entirely synthetically generated and placed particle images such that their positions relative to each other are known exactly.

A description of the velocity field that was generated for these tests can be found in Appendix A.4. The field resembles the one found in the experiment and contains some randomization for the trace distances.

## Randomized Particle Images

Eight particle images are being generated for each trace. Initially, these are based on images with five times the final resolution, so that they can later be spline-interpolated and scaled down to the correct subpixel position. This serves the purpose of making sure that the higher resolution source particle images contain additional detail compared to what can be shown in the final result. As a result of the additional resolution and the spline interpolation, the positioning of the particle images at subpixel positions is realistic in that it does not exhibit any biases towards certain alignments with the pixel grid.

The final subpixel position is calculated as described above. Particle traces are started at a random position within an image that extends 1024 px horizontally and 128 px vertically around the actual output image, such that traces can start before the left edge and end beyond the right edge. If this was not the case, the particle density would decrease towards the edges of the image, and finding correct traces would be easier for the algorithm.

The particle image generation algorithm has three main goals: First, it should create both single-maxima and multiple-maxima particle images with one algorithm. Second, the resulting particle images should vary in size and intensity. Third, the particle images it creates should have realistic amounts of variation and consistency within a given trace. All mentions of "random" variables below are results of NumPy and Python pseudorandom number generators, and thus not truly random.

To generate the actual intensity distributions of the particle images, several small kernels are generated to simulate the non-symmetrical non-Gaussian nature of the real PTV particle images. These small kernels are created as noisy Gaussian peaks with a width of $\sigma_{\mathrm{k}}$. The images are slightly noisy in the full resolution version as a result of an under-sampled Monte-Carlo-like buildup of the pixel intensities. Specifically, random coordinates are picked from a Gaussian distribution and the pixel value at that coordinate is then increased. This has only a slight randomness effect on the downsampled final particle images (see Figure 5.18).

Values of $\sigma_{\mathrm{k}}$ are themselves randomly generated for each kernel, using the generator function $f_{\text {variation }}\left(x_{\mathrm{a}}, x_{\mathrm{b}}, x_{\mathrm{c}}\right)$ with $x_{\mathrm{a}}=0.6 \mathrm{px}, x_{\mathrm{b}}=0.4 \mathrm{px}$ and $x_{\mathrm{c}}=3.0$. This function is defined to be

$$
\begin{equation*}
f_{\text {variation }}\left(x_{\mathrm{a}}, x_{\mathrm{b}}, x_{\mathrm{c}}\right)=s_{\mathrm{k}} \min \left(\max \left(\frac{x_{\mathrm{a}}}{\left|r_{f_{\mathrm{pdf}}\left(\mu=1.0, \sigma=x_{\mathrm{b}}\right)}\right|}, \frac{x_{\mathrm{a}}}{x_{\mathrm{c}}}\right), x_{\mathrm{a}} x_{\mathrm{c}}\right) \tag{5.17}
\end{equation*}
$$

where

$$
\begin{equation*}
s_{\mathrm{k}}=5 \tag{5.18}
\end{equation*}
$$

Here, $r_{f_{\text {pdf }}(\mu, \sigma)}$ is a random sample from a Gaussian normal PDF with standard deviation $\sigma$ and center $\mu$, and $s_{\mathrm{k}}$ is the scaling factor by which the image is higher in resolution relative to its eventual mapping onto the image. The parameter $x_{\mathrm{a}}$ controls the most common output value, $x_{\mathrm{b}}$ the variation of the values and $x_{c}$ limits the upper and lower extreme values to avoid very narrow or wide distributions. A total of 8 of these kernels are then placed onto the particle image, with the placement also being randomized. The final standard deviation for this placement is found using $f_{\text {variation }}\left(x_{\mathrm{a}}, x_{\mathrm{b}}, x_{\mathrm{c}}\right)$ again, this time with $x_{\mathrm{a}}=0.6 \mathrm{px}, x_{\mathrm{a}}=0.6 \mathrm{px}$ and $x_{\mathrm{c}}=4.0$. Each kernel has a random overall brightness, where any value between 0.5 and 1.0 is equally likely.

This last value, the overall brightness per kernel, is the only value which is randomly picked even between the different particle images within a given trace. All other randomly chosen values are only picked once for the entire trace. As a result, the overall shape between the different particle images in a trace is mostly consistent, as the kernels are placed in the same position and have the same width. However, just as different reflection points may change in intensity over the path of a particle, the different sub-maxima can vary in intensity. Such
changes in the intensity of the local maxima in particle images can frequently be observed in the real images. As a result of this, no perfect alignment can be expected any more from any distance determination algorithm (as is the case for the actual experiment).

The set of eight particle images is finally scaled to a peak brightness where any value between 2048 and 65344 is equally likely. This prevents the generation of any saturated particle images, as these would later be discarded in the analysis. This peak brightness here is the brightest maximum in the set, such that the relative brightness between the particle images remains unchanged.

The various randomness values were chosen, mostly through trial and error, in order to make the particle images look similar to those in the real images. Figure 5.18 shows some examples for the resulting particle images both in the full as well as in the reduced resolution.


Figure 5.18: Synthetic randomized particle images, before and after downsampling to the final resolution. All images in two consecutive lines belong to the same trace. For each trace, the versions before downsampling are shown on top and the versions in the final image resolution are shown below. Greyscale values are inverted for easier observation of low intensity values. Only the central $12 \times 12 \mathrm{px}$ areas of the full $16 \times 16 \mathrm{px}$ images are shown.

## Gaussian Particle Images

As a reference for a near-ideal seeding scenario, images with purely Gaussian peaks were also created. The method of generation is similar to the one above, but with no randomization of the position of the kernels or their standard deviation. As a result, all particle images are of a Gaussian PDF-like shape with a standard deviation of 1.5 px . An example for the resulting images is shown in Figure 5.19.


Figure 5.19: Synthetic Gaussian particle images, before and after downsampling to the final resolution. All images belong to the same trace, with the versions before downsampling are shown on top and the versions in the final image resolution are shown below. Greyscale values are inverted for easier observation of low intensity values. Only the central $12 \times 12 \mathrm{px}$ areas of the full $16 \times 16 \mathrm{px}$ images are shown.

## Image Assembly

For the actual placement of these particle images onto the test images, they will be shifted by subpixel amounts, resulting in different actual low-resolution renderings than shown here. Particle images are placed on the main image using a maximum filter. As a result, overlapping particle images do not create pixel intensities higher than the highest intensity in either of them.

The final image is written into a 16 -bit integer .tiff file, using the same encoding in which the PTV camera images are stored. Of the traces that are being generated per image, only approximately one quarter are fully shown within the image area.

## Noise

The synthetic image generation script saves one file with only the particle images added, and no additional noise. In addition, a second image is saved which contains noise such that

$$
\begin{equation*}
I_{\mathrm{p}, \mathrm{n}}=I_{\mathrm{p}} \frac{I_{\mathrm{sat} .}-5 \sigma_{\mathrm{n}}}{I_{\mathrm{sat}}}+\left(\mathrm{G}\left(\sigma_{\mathrm{G}}\right) * r_{f_{\mathrm{pdf}}\left(\mu=4 \sigma_{\mathrm{n}}, \sigma=\sigma_{\mathrm{n}}\right)}\right), \tag{5.19}
\end{equation*}
$$

where

$$
\begin{align*}
& \sigma_{\mathrm{n}}=\frac{10^{2.8}}{\varsigma} \text {, and }  \tag{5.20}\\
& \varsigma=\sqrt{\sum_{i=-\infty}^{\infty} \sum_{j=-\infty}^{\infty}\left(\mathrm{G}_{i, j}\left(\sigma_{\mathrm{G}}\right)\right)^{2}} \quad \text { (cf., for example, Rubin and Weisberg, 1974) }  \tag{5.21}\\
& =\sqrt{\sum_{i=-\infty}^{\infty} \sum_{j=-\infty}^{\infty}\left(\frac{1}{2 \pi \sigma_{\mathrm{G}}{ }^{2}} \mathrm{e}^{\frac{-\left(i^{2}+j^{2}\right)}{2 \sigma_{\mathrm{G}}^{2}}}\right)^{2} \approx 0.6599 .} \tag{5.22}
\end{align*}
$$

Here, $I_{\mathrm{p}, \mathrm{n}}$ is the local pixel intensity with added noise, $I_{\mathrm{p}}$ is the original pixel intensity, $I_{\mathrm{sat}}=$ 65344 is the saturation intensity of the image format and $\sigma_{\mathrm{n}}$ is the chosen noise standard deviation. The base intensity is increased by $5 \sigma_{\mathrm{n}}$ in order to avoid a large amount of negative pixel values as a result of the noise. Any few remaining negative pixels are set to 0 . This also has an effect which is similar to the background intensity in the measurements, and is therefore not unrealistic.

The noise component is blurred using a Gaussian kernel $\mathrm{G}\left(\sigma_{\mathrm{G}}\right)$ with standard deviation
$\sigma_{\mathrm{G}}=0.5 \mathrm{px}$. This more realistically represents noise that is partly caused by small particles or droplets in the flow, as lower frequencies dominate in this scenario due to the limits of the optical system and the finite size of the particles. The standard deviation of the noise prior to applying blur, $\sigma_{\mathrm{n}}$, is increased by the factor $\frac{1}{\varsigma}$ in order to compensate for the fact that the Gaussian blur decreases the noise standard deviation by $\varsigma$. The standard deviation of the resulting noise, $10^{2.8}$, is a common background noise level in the experiment as determined from measurement images.

### 5.4.2 Analysis Methodology

The synthetic images are processed using the analysis tool in the same way that a real measurement image would be treated. Because the velocities and particle sizes simulated in the synthetic image are consistent with those observed in the actual measurements, almost all settings can be kept identical. The particle detection threshold was adjusted in order to avoid the detection of nonexistent particles, but kept constant across all synthetically generated images, with or without noise. Conventionally, this adjustment should be performed for any image, based on the noise level.

As part of the image generation, an output array is written which contains the true distances for all traces that are entirely within the bounds of the image (i.e. all 8 illuminations are visible in the synthetic output image). The comparison of this true position array with the measured array from the distance determination allows for precise error calculations of the connection distances, but not of the absolute positions. The latter are not clearly defined, and not of any significant interest.

Measured and true traces are considered to be the same if their starting particle image positions are within 3 px of each other. For subsequent particle images in such a trace, a deviation of 2 px is used to determine if an incorrect connection was made along the way. Here, the first particle image distance is subtracted in order to correct for a non-centered position detection which is consistent along the trace-hence the lower allowed deviation distance. In the latter case, any incorrect connection is not considered within the overall connection distance error. This decision is made for all methods together based on the centroid data only, in order to use the same traces throughout.

The effect of smoothing was not determined here as the SciPy smoothing implementation showed instability issues on very few particle images. ${ }^{1}$

[^13]
### 5.4.3 Results

## Trace Detection Ratio and False Positives

A comparison of the traces that have been placed onto the synthetic image with the ones that have actually been found allows for further validation of the trace detection algorithm. Figure 5.20 shows the false positive rate among the detected traces, as well as the percentage of existing traces that were found by the algorithm. The exact number of true traces and particle images in each image is somewhat random as a result of the generation method of these images. As a result, they differ between the Gaussian peaks particle image cases and the randomized cases. For the images with and without noise, the particle images and locations are exactly the same - the noise is added to the existing synthetic test image.

The false positive rate stays below $10 \%$ in all cases with reasonably dense seeding. The problem in low seeding densities, where up to $20 \%$ false traces are detected, is likely a lack of weight map data. For very high seeding densities, the rate increases slightly. The Gaussian peak particle images are much less susceptible to false positive trace detections than the randomized multi-peak particle images. A likely reason for this is the detection of secondary peaks in the randomized particle images as separate particle images, or simply the reduced centroid location accuracy. Added noise has no strong effect on the false positive rate.

The detection rate decreases significantly at high seeding densities, which is expected as particle images are much more likely to be very close to each other. At low seeding densities, the detection rate is significantly lower for the randomized images (and almost constant), where the Gaussian particle image detection rate continues to increase with lower densities. This is likely a result of some particle images being discarded due to being too large or too small, which explains the constant ceiling at around $80 \%$. For very high seeding densities, the detection rate for the Gaussian particle images, curiously, is worse than for the randomized ones. The randomization may help distribute the weight map peaks slightly, increasing both the detection rate as well as the false positive rate. This could likely be emulated using the weight map dilation parameter in the Gaussian peak case - though a low false positive rate is more important here than a high detection rate. Again, added noise has no strong effect here.

The number of traces that lose the correct path along the way, also shown in Figure 5.20, is generally lower than the false positive rate and stays below $10 \%$ in all randomized particle image scenarios and below $2 \%$ in all Gaussian particle image scenarios. It increases with seeding density, which brings possible connection candidate particle images closer together.

From these results, it is evident that, with these settings, the trace detection rate above about 5000 particle images decreases enough that not many additional traces are found. Below about 1500 particle images, trace finding suffers from a lack of particle images for generating accurate weight maps. Thus, for the best trace finding results, the threshold for the particle image detection should usually be set such that the number of particle images that are detected stays somewhere in the region of 1500-5000.

## Connection Distance Error

Figure 5.21 shows the connection distance error across all connections in the respective images. The results from the different seeding densities have been combined here. In general, lower seeding densities produce slightly lower errors than high seeding densities. The combination of all data is a conservative representation of the real scenario, giving the most weight to high seeding density images. The overall trends between different methods are very similar between different seeding density results.

As expected, the Gaussian particle image distance errors are much lower than the randomized ones. In the case of the IILSS method, almost no connections show an error larger than 0.02 px in the Gaussian case - for the randomized one, this level is about one order of magnitude larger. The effect on the overall standard deviation is lower, likely as a result of few outliers in the Gaussian case that have a strong effect on the $\sigma$ values there. It is interesting that despite near-perfect Gaussian peaks and no added noise, the Gaussian method is by far the least accurate on Gaussian peaks. This may be a result of the convolution over the pixel area, which deforms the Gaussian peaks. A correction for this effect may be possible, but it is not likely that it would make the method competitive in the randomized case. Another difficulty for the Gaussian method is the large size of the particle images (see Figure 5.19), resulting in small central gradients.

Across almost all methods and cases, added noise increases the error by about $0.03-0.04 \mathrm{px}$ in standard deviation. Exceptions are, in the case of the randomized particle images, the Gaussian method (which improves with added noise according to the standard deviation, though the central peak of the distribution is still lower) and the IDLSS method (which only shows an error increase by about 0.015 px ).

In the noise-free cases, the IILSS method outperforms all other methods by a significant margin, both in standard deviation and in the distribution of errors. Thanks to its reduced sensitivity to the increased noise floor, the IDLSS method outperforms the IILSS method in the noisy randomized case. The centroid method, while outperformed by the two least squares methods, shows lower standard deviations and fewer outliers than the Gaussian method across all tests. As such, it is the better of the two fast and universally applicable methods, confirming it as a good default choice for trace finding.

## Comparison With the Results of the Displacement Calibration

The errors without (see Figure 5.12) and with added noise (see Figure 5.15) measured in the displacement experiment should be comparable to the ones determined here (Figure 5.21). The particle images in the displacement case were mostly not as irregular as in the randomized synthetic case (and not as irregular as those in the measurements), but more so than the synthetic Gaussian particle images - though some were indeed quite irregular in their shapes. Yet, the errors found in the displacement experiment are slightly larger than those found in the
synthetic case with randomized images.
One possible reason for this are changes in the illumination between the different target positions in the displacement experiment. This would mean that the displacement experiment results are more indicative of the actual error that can be expected in the measurements. Another factor may be slight rotations of the target as a result of it being moved, which would create differences in connection length between the uppermost and lowermost traces. Finally, the image sensor may not be equally sensitive across all pixels, and the optical system may distort particle images in different ways at different positions. Theoretically, it is even possible that particles may have shifted slightly on the target between the exposures as a result of the motor vibration. As most of these factors cannot be determined or ruled out without a more controlled experiment, it is best to use the synthetic image results to evaluate the quality of the different methods, while conservatively assuming that the overall error is in the range of what was found using the displacement experiment.


Figure 5.20: Trace matching quality results in the synthetic image validation. Images with increasing seeding density are shown on the horizontal axis, where $n_{\mathrm{t}, \mathrm{true}}$ is the number of drawn traces fully within the image. Numbers for $n_{\mathrm{p}}$ on top show the total number of particle images drawn per image, including partially visible traces. For the detected true traces, $n_{\mathrm{t}, \mathrm{a}}$ is the number of detected traces that match a true trace, and $n_{\mathrm{t}, \mathrm{b}}$ is the number of true traces $n_{\mathrm{t}, \text { true }}$. For the detected false positives, $n_{\mathrm{t}, \mathrm{a}}$ is the number of detected traces not found among the true traces, and $n_{\mathrm{t}, \mathrm{b}}$ is the number of detected traces that match a true trace.
I) Noise-free images.
II) Images with added noise.
a) Randomized multi-peak particle images.
b) Single Gaussian peak particle images.

$\square$ Centroid ( $\sigma=0.230$ )
------ Gaussian $(\sigma=0.496)$
)........ IILSS $(\sigma=0.152)$
$\stackrel{\square}{\square-\square} \operatorname{IDLSS}(\sigma=0.175)$

$\square$ Centroid ( $\sigma=0.273$ )
------- Gaussian $(\sigma=0.430)$
.......... IILSS $(\sigma=0.177)$
$\stackrel{-\because .!}{\square} \operatorname{IDLSS}(\sigma=0.189)$

$\square$ Centroid ( $\sigma=0.154$ )
-------- Gaussian $(\sigma=0.192)$
$\cdots \operatorname{IILSS}(\sigma=0.135)$
$\stackrel{-}{-\cdots} \operatorname{IDLSS}(\sigma=0.154)$

$\square$ Centroid ( $\sigma=0.186$ )
------- Gaussian $(\sigma=0.219)$
$\ldots \operatorname{IILSS}(\sigma=0.151)$
$\xrightarrow{-\cdots} \operatorname{IDLSS}(\sigma=0.173)$

Figure 5.21: Connection distance error in the synthetic image validation. In the legends, $\sigma$, values in brackets are standard deviations of the error, in px. Values in the top left corners indicate the number of outlier connections not shown in the histogram, for each method, in the same order in which the methods are listed in the legend.
I) Noise-free images.
II) Images with added noise.
a) Randomized multi-peak particle images.
b) Single Gaussian peak particle images. Note the difference in axis scaling.

## 6

## Application of the Method

This chapter focuses on applying the algorithms described in Chapter 4 to images from the experimental setup outlined in Chapter 2. The goal is to find any problem in the practical application of the tool that did not become apparent during validation and prior testing. Close inspection of the intermediate and final results should also give further insight into the accuracy of the entire analysis.

### 6.1 PIV Flow Velocity Field

In order to determine slip velocities for any of the measurements, it is first necessary to determine the background flow field from very low diameter seeding through PIV analysis, as established in Section 4.6. This analysis was not part of this work but rather provided by R. Konrath. Figure 6.1 shows the PIV data after interpolation, derived from a single NFT camera image taken during a standard Scenario C wind tunnel run without large particle seeding. For a better impression of the horizontal gradients, Figure 6.2 contains the velocity profiles at four different image heights from the center of the image downwards.

### 6.1.1 Secondary Velocity Gradients

Here, the shock wave is visible, but the gradient of the velocity after the shock wave is still very significant in comparison, unlike the ideal case drawn in Figure 1.8 and assumed for the fitted slip velocity model. This is due to the shape of the airfoil, which by itself creates a negative velocity gradient as it tapers towards the trailing edge. As a result, it becomes more difficult to isolate the amount that the slip velocity of a particle is decreasing over time, as both the PIV as well as the PTV velocity show a negative gradient. Even without any shock, a small slip velocity would remain solely due to the shape of the airfoil.


Figure 6.1: Horizontal and vertical PIV velocities for $M=0.76, R=9 \times 10^{6}$ and AOA of $2.0^{\circ}$. Red diagonal crosshairs indicate the locations of the PIV-provided velocity vectors that were interpolated, $u_{1,2}$ are positive in positive directions of ${ }_{\mathbf{i}} \hat{x}_{1,2}$.


Figure 6.2: Horizontal PIV velocities for $M=0.76, R=9 \times 10^{6}$ and AOA of $2.0^{\circ}$, at multiple image heights. Values of ${ }_{\mathrm{i}} \hat{x}_{2}$ are vertical locations in image coordinates, where 0.0 corresponds to the vertical center and 1.0 corresponds to the bottom edge of the image.

### 6.1.2 Resolution

The shock itself is not resolved as an immediate drop in velocity due to a number of effects causing a loss of detail in the PIV analysis. Among these are the inherent resolution limit of PIV due to the finite correlation window size (resulting in a $91 \times 95$ velocity vector array across the $2048 \times 2048 \mathrm{px}$ image area), as well as the relatively large pulse separation time of $10 \mu \mathrm{~s}$. Additionally, the resolution is limited here by the finite particle size, resulting in some relaxation time even for the PIV seeding. Finally, the optical effects of test gas refraction may cause additional loss of detail in the area immediately surrounding the shock wave.

The PTV results share most of these effects, but not the correlation window size. As a result, it is possible that the PTV analysis result for small particles resolve the shock better than is possible in the PIV method. A comparison of Figures 6.2 (PIV flow field) and 4.12 (PTV-derived flow field) illustrates this difference in resolution.

### 6.2 Pulse Delay Fluctuations

In the sequencer (for pulses 1, 3, 5 and 7) and lasers (for the even-numbered illumination pulses), the delays were set to create delays of $10 \mu$ in-between all eight pulses. However, results show that this time delay was likely not kept exactly. Initial examinations of the PTV flow velocities showed per-connection velocity deviation patterns that were consistent across all traces within a given image, regardless of the position in the image. More precisely, deviations of approximately $0.15 \mu \mathrm{~s}$ (absolute difference between shorter and longer time delays) for every other connection could be seen in the traces of some, but not all, wind tunnel runs. These were visible as staggered/zigzag PTV velocity results, as shown in Figure 6.3 -an example for the
velocity across the connections within a trace, with unadjusted pulse delay timings.


Figure 6.3: Example for PTV connection velocities without pulse delay fluctuation adjustments applied (including the comparison with PIV velocities).

As described in Section 4.6.1, a correction for this variability was implemented in the calculation of slip velocities. The only plausible explanation for the phenomenon is a fluctuation in the laser pulse delay. The exact delays had only been measured to within a few percent during the actual experiment using an oscilloscope, and these measurements had not been recorded. As a result, no primary data could be used to correct for this effect as part of the analysis. Instead, the delay offsets needed to be determined

For all cases shown here, the pulse timing fluctuations were corrected by adjusting all 4 odd connection time steps by +150 ns. The 3 intermediate connection time steps were not modified. Whether the odd delays were too long or the even delays too short, or a combination of both effects occurred, cannot be determined after the fact due to a lack of recorded pulse timing data. The difference in absolute velocity has a negligible impact on the results, which is why the correction was chosen to match the distances that the PIV analysis appears to have converged to.

In the generation of the trace velocity plots shown below, the corrected time delays are considered. For this reason, they show a difference of $60.6 \mu \mathrm{~s}$ between the first and last connection center, rather than an even $60 \mu \mathrm{~s}$ as expected with constant delays of $10 \mu \mathrm{~s}$.

### 6.3 Problematic Trace Scenarios

A look at scenarios in which no useful particle diameter can be obtained helps to understand the limitations of the analysis method, and potentially the experiment as a whole. Apart from traces that are clearly false positive detections (which are usually quite easy to spot in the type of visualization shown below), three interesting classes of difficulty in obtaining a diameter fit are discussed here.

### 6.3.1 Minimal Slip Traces

Even when setting the particle image identification threshold relatively high, such that mostly larger particles are expected to be detected, many trace velocity curves resemble the corresponding PIV velocities very closely. Figure 6.4 shows the PTV and PIV velocities for a number of particles that show almost identical velocity curves between PTV and PIV-in two of the cases with a significant offset in absolute velocity. As the slip velocities represent the difference


Figure 6.4: PTV connection velocities in comparison with PIV velocities of several traces with minimal initial slip velocities.
between these two tracks, those are essentially constant here and no useful result can be obtained from a fit to the expected relaxation curve (determined diameters are often orders of magnitude too large for these cases).

These traces occur if the trace starts too far behind the shock wave or for particles that are likely very small. Any of these two reasons, or a combination of them, results in a slip velocity that has already declined to a values close to zero at the beginning of the trace. As the PIV field was also generated from particles of a finite size, there is necessarily a lower limit in particle size where even a trace starting directly behind the shock wave shows no significant slip velocity.

### 6.3.2 High Slip Low Gradient Traces

There are cases where traces start with a significant amount of slip, but this slip is maintained almost without change throughout the entire time of observation. Figure 6.5 shows examples
for such traces. The corresponding traces in the image show relatively bright and therefore,


Figure 6.5: PTV connection velocities in comparison with PIV velocities of several traces with significant slip velocities but minimal change in slip velocity from beginning to end.
presumably, large particles that are located reasonably close to the shock wave. The change in slip velocity from beginning to end is within the combined margin of error for the PTV and PIV measurements, especially given that those are taken from different wind tunnel runs with some variance in the exact flow conditions. As a result, determining a diameter here is not possible with reasonable precision. The difference in slip velocity change between a relatively large particle (for example around $100 \mu \mathrm{~m}$ ) and an incredibly large particle which cannot occur in this experiment is simply too small.

### 6.3.3 Negative Slip Velocity Traces

Some traces that look very promising otherwise exhibit small negative slip velocity values at one or more points which prevent a solution using the fitting algorithm.

The examples in Figure 6.6 show very small negative slip velocities towards the end of a trace, which tends to occur towards the end of the relaxation period. Here, the particle is already very close to the fluid velocity, such that a small error or a small change in velocity between the PIV and PTV image can easily cause a negative slip velocity.

The examples in Figure 6.7 on the other hand show a PTV velocity which is consistently lower than the PIV velocity even at the beginning of the trace. The PTV velocity is also almost entirely constant across the entire trace in both cases. This is frequently seen among very large


Figure 6.6: PTV connection velocities in comparison with PIV velocities of several traces with small negative slip velocities but a generally decreasing trend.


Figure 6.7: PTV connection velocities in comparison with PIV velocities of several traces with initially negative slip velocities.
and bright particle image traces. An explanation for this is that these particles had never fully accelerated to the velocity ahead of the shock in the first place and are therefore crossing the shock at an already much lower velocity than the fine PIV seeding. Although they are not accessible to any particle response fitting, these traces still indicate the presence of very larger ice particles in these images.

### 6.4 Usable Traces

For traces starting reasonably close to the shock and representing particles that are not too big or too small, the method works as expected and a usable fit for the particle diameter can be achieved. Several examples from the experiment are shown in Figure 6.8, along with the particle diameter that was determined in each case. These results show mostly plausible particle diameters that are somewhat lower than expected. Because results too close to the shock wave interfere with the wide shock wave area in the PIV data, initial slip velocities in
these examples are low, between 5 and $20 \mathrm{~m} \mathrm{~s}^{-1}$. Under these circumstances, particles with determined diameters greater than $10 \mu \mathrm{~m}$ already show relatively small relaxation gradients, i.e. relatively horizontal slip velocity curves. Particles that would be significantly larger would therefore quickly hit the resolution limit and require larger initial slip velocities to yield a result.

Considering the results of Section 3.3 regarding the fitting error for a given position error, it is likely that these particle diameters are among the successful fits at least partly because other diameters would not converge or produce plausible results. Specifically, Figure 3.8 III a indicates that for a position error of around 0.2 px and an initial slip velocity of around $10 \mathrm{~m} \mathrm{~s}^{-1}$, these particle diameters are the only ones that can be found at all. As such, a higher initial slip velocity would be necessary in order to make definitive statements about the diameter distribution of the ice particles in this experiment.


Figure 6.8: PTV connection velocities in comparison with PIV velocities of a total of 6 traces with decreasing slip velocities. Above, respectively, the comparison of the velocities from PTV and PIV. Below, both the resulting slip velocities as well as the fitted relaxation curve (with the corresponding particle diameter $D_{\mathrm{p}}$ ).

## 7

## Summary and Conclusions

### 7.1 Summary

An experiment was conducted in the cryogenic Ludwieg tube type wind tunnel (KRG) of the DNW with the aim of observing the extent of laminar flow across a natural laminar airfoil under the influence of ice particles in the flow. In this thesis, data from PTV and PIV measurements was combined in order to find the velocity difference (or slip velocity) between the ice particles and the flow surrounding them. This was done in a region behind the recompression shock wave on the top surface of the airfoil model. Here, the relaxation time of particles can be observed as they decelerate after encountering the sudden drop in flow velocity. Fitting the time series of particle slip velocities to a model for the motion of a spherical particle across a shock wave, particle diameters can be obtained.

The ice particle seeding presents several challenges for PTV analysis, namely a wide range of particle image sizes and shapes that are not ideal for exact position determination, as well as relatively high background noise and intensity levels from unwanted seeding components. Additionally, it was necessary to capture eight laser pulse illuminations within a single camera image in order to obtain a velocity trace to which a relaxation curve can be fitted. This makes it challenging to find the particle images that belong to the same particle, especially in dense seeding conditions.

An application was developed for the entire process of analyzing the PTV images from the given experiment. For this, existing software components for finding particle images and for fitting a slip velocity response to a particle diameter were reused. All settings and adjustments necessary for this were combined in a single GUI. Both the GUI as well as all new algorithms were written in the Python programming language.

An entirely new algorithm was developed for finding particle image traces in any single recorded PTV image from an initial estimate of the flow field across the image. This estimate is generated from the previously determined approximate particle image positions under the
assumption that, for any given region in the measurement image, most particle images will have a partner image in the direction of flow from a subsequent illumination pulse. This flow estimate is stored as a probability distribution of the flow vector at any particle image location, rather than as a flow field, thereby allowing for more than one solution to coexist in regions of high velocity gradients such as a shock wave. The flow field estimate takes into account how similar different particle images are in their intensity, as two images of the same particle are likely to share similar absolute intensities. Combining the flow field probabilities with the differences in intensity into a total pairing weight, traces are assembled from the most likely particle partners. Then, these are filtered to remove traces with large angle deviations as well as traces that share particle images with other traces. The accuracy of this method was validated using synthetic PTV images, with results indicating that it is sufficiently functional and recognizes very few false positives. Using the method on recorded PTV images confirmed that the trace finding method works very well for the particle image densities occurring in this experiment.

In order to improve the accuracy of the PTV particle image distance determination, a corrector algorithm was developed. It uses the existing centroid particle positions and the previously determined trace information and was developed specifically to produce good particle distance estimates for large, non-symmetrical and non-identical particle images. For each pair of particle images for which the distance needs to be determined, this algorithm creates a custom correlation kernel from one of the images and performs a number of SQP minimizations in order to find the position of the smallest squared intensity difference between the two images. This minimization works with a fifth-order spline-interpolated intensity distribution and therefore returns distances with subpixel accuracy - errors of less than 0.02 px were achieved for Gaussian peaks in the vast majority of cases.

Validation of the particle distance determination method using synthetic PTV images with both ideal Gaussian peaks as well as intentionally randomized particle images showed a superior performance of this method compared to the centroid method, which still showed better results than Gaussian peak fitting. Especially in the case of randomized particle images in a noisy image, the error distribution and standard deviation of the newly developed method outperformed the other methods significantly. Validation using specifically captured test images of a movable target with known constant displacements for all particles showed less distinct benefits of the method compared to the centroid method. This validation was performed in the tunnel test section using the same hardware, resulting in nearly identical optical conditions and illumination characteristics. Seeding particles however differed significantly in their appearance from the ice particles from the actual experiment.

A derivative of the correlation method for determining distances was developed that uses horizontal and vertical intensity derivatives rather than absolute intensity values for the least squares minimization. This was done with the expectation of achieving superior results under conditions of high and non-constant background intensities. The synthetic image validation confirmed that this method showed a smaller loss of precision as a result of the added background
intensity and noise, but the overall precision was always found to be lower than the main method.
An initial error estimate concluded that, for the given experimental setup, it should be possible to achieve an accuracy of approximately $10^{-1} \mathrm{px}$ for the particle positions,. Limiting error influences were determined to likely be sampling errors, noise from background seeding as well as lens aberrations. Both validation methods found that, for particle images of the order of tens of pixels, the actual error is around or below 0.2 px (standard deviation), matching the initial estimate quite well.

A first-order estimate of refractive effects of the density difference across the shock wave was performed as well. The results indicate that PTV or PIV analysis of the areas in front of the shock wave, from the camera positioned behind the shock wave, produces position errors of several pixels. For this reason, the velocity in front of the shock wave can not be determined accurately from these images. The particle slip velocity response fitting was therefore changed to include the initial slip velocity as a fitting parameter. This enables obtaining particle diameters without initial slip velocity information.

The application of the entire method on some of the recorded images produced usable diameter results while also highlighting challenges and possible improvements to the analysis and experimental setup (see Section 7.2). A numerical analysis of the particle diameter fitting error was performed beforehand, observing changes as a function of the initial slip velocity, particle diameter, as well as the position error of the data to which the fit was applied. Here, it was found that for initial slip velocities of around $10 \mathrm{~m} \mathrm{~s}^{-1}$ and position errors of around 0.2 px , only a narrow range of particle diameters could successfully be fitted. The valid results from the measurements match these particle diameters. As such, the range of diameters that can be measured is very limited for this set of recorded images.

### 7.2 Conclusions

With the tools in place to quickly analyze the PTV images and visualize the results, several difficulties have become apparent. These prevent the results of the recorded wind tunnel measurements to yield definitive conclusions regarding the influence of ice particles on the laminar boundary layer of an airfoil. This section mainly describes suggested improvements to the experimental setup and analysis methodology as far as they can be specified from the observations made as part of this thesis.

Chiefly among those is the ice particle seeding from the IPG not being monodisperse and being difficult to control, which by itself prevents any conclusions regarding the critical Stokes numbers and particle diameters. This is not within the scope of this thesis, but is a prerequisite for achieving the broader goal of better understanding the transition effects of ice particles on a laminar airfoil in a repeatable and controlled environment. As already mentioned by Girnth (2017), a heated airfoil would be very helpful for the repeatability of wind tunnel runs and for making the practice of keeping the airfoil free of particles and ice a less fastidious one.

Regarding the capture of PTV images, a field of view that is centered much closer to the upper airfoil surface than was the case here would offer the benefit of a stronger shock wave across the entire vertical image area. This could be achieved even with the current placement of camera boxes using a lens position that is shifted down relative to the camera sensor. As an added benefit, the surrounding flow would match any numerical simulation more closely and also deviate less from the measurements taken at the pressure taps on the airfoil. As a result, the flow field could more easily be treated as a known quantity rather than one that needs to be measured using the PIV method.

In order to further improve the PTV measurements, the pulse delay fluctuations need to be measured to within around $10-20 \mathrm{~ns}$, or even better, controlled to this precision. While any small fluctuations can be corrected for in the PTV analysis, the PIV solution relies on equal delays throughout: The PIV method will otherwise always contain two competing solutions from two different pulse lengths. A simple way to avoid the problem here without any improvement in the sequencer timing uncertainty would be to only use a single double pulse, or double pulses with much longer delays in-between, whenever PIV images are captured.

A better PIV flow field could be determined with more usable measurement images available. Rather than evaluating a single image, results should ideally be averaged across at least a few dozen images. Alternatively, an improved correlation between the PTV and PIV solutions may be achievable by using the PTV measurement images, with large particle images removed, as the basis for the respective PIV field, such that the PIV solution originates from the identical run and even time within the run as the PTV traces. This might be possible due to the (otherwise not desirable) background seeding that is present in most of the PTV images as a side-effect of the ice particle generation method.

Regarding the problem that the flow velocity is not constant behind the shock wave, and the solution will therefore not match the solution of the BBO equation for the idealized shock wave, better results may be achievable using a more complex flow assumption. Specifically, the entire flow around the airfoil, including but not limited to the shock wave, could be used for the solution, and finally for the particle diameter fit. The flow assumption here could be based on the simulated flow around the airfoil, fitted to the PIV solution in the NFT camera observation window. At the expense of a much more complex fitting model, this would significantly improve the accuracy of the results as well as the diameter range for which any results could be obtained, including the case of very large particles never fully accelerating to the velocity in front of the shock wave.

As for the accuracy of the PTV results themselves, additional improvements in the distance determination method appear to be difficult to achieve. Given that the trace finding algorithm works very well now, increasing the pulse separation times beyond $10 \mu \mathrm{~m}$ is likely feasible. Especially for big particles where the accuracy is currently the most limited as a result of very large particle images, longer connection distances would directly lead to more precise velocities.

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## Bibliography

Airy, George Biddell (1835). "On the diffraction of an object-glass with circular aperture." In: Transactions of the Cambridge Philosophical Society 5, pp. 283-291.

Basler AG (2012). Basler acA2040-180km Camera Specification - Measurement protocol using the EMVA Standard 1288. BD000594. Version 03. URL: https://www.baslerweb.com/pp-1489067453/ media/downloads/documents/emva_data/BD00059403_Basler_acA2040-180km_EMVA_Standard_1288.pdf.

Basset, Alfred Barnard (1888). A treatise on hydrodynamics: with numerous examples. Vol. 2. Deighton, Bell and Company.

Baughcum, Steven L, Stephen C Henderson, and Terrance G Tritz (1996). Scheduled civil aircraft emission inventories for 1976 and 1984: Database development and analysis. NASA technical report NASA-CR-4722.

Baughcum, Steven L, Donald J Sutkus, and Stephen C Henderson (1998). Year 2015 aircraft emission scenario for scheduled air traffic. NASA technical report NASA/CR-1998-207638.

Berghof, Ralf, A Schmitt, C Eyers, K Haag, J Middel, M Hepting, A Grübler, and R Hancox (2005). CONSAVE 2050 final technical report. Tech. rep. DLR.

Boussinesq, Joseph (1885). "Sur la résistance qu'oppose un liquide indéfini en repos." In: Comptes rendus de l'Académie des Sciences 100, pp. 935-937.

Brown, Phillip P and Desmond F Lawler (2003). "Sphere drag and settling velocity revisited." In: Journal of environmental engineering 129.3, pp. 222-231.

Cao, Shuyang and Yukio Tamura (2009). "Reynolds number dependence of the velocity shear effects on flow around a circular cylinder." In: Proceedings of the 5th European $\mathcal{B}$ African Conference on Wind Engineering, EACWE, Florence, Italy, pp. 1-12.

CCMOSIS BVBA (2015). CMV4000 v3 Datasheet. URL: http:// www.cmosis.com / ?ACT = $52 \& k e y=Z 3 B N d l h y S G Y x N G d a a G 91 e W R I N z J p U n F x K 24 v N W 83 Q 1 d i Y W p s S H B 3 M k Z D M n p Y b 050 a W h q V m Z K e m d v Q n g$ wZmhVWjJMWW5TdUJKdzRvamlTR2cyYXNMSUE9PQ==.

Cheng, M, DS Whyte, and J Lou (2007). "Numerical simulation of flow around a square cylinder in uniform-shear flow." In: Journal of Fluids and Structures 23.2, pp. 207-226.

Clarke, Timothy A (1995). "A frame grabber related error in subpixel target location." In: The Photogrammetric Record 15.86, pp. 315-322.

Clarke, Timothy A, M A R Cooper, and J G Fryer (1993). "An estimator for the random error in subpixel target location and its use in the bundle adjustment." In: Optical 3-D measurements techniques II, Herbert Wichmann Verlag, Karlsruhe, pp. 161-168.
Clift, R., J.R. Grace, and M.E. Weber (1978). Bubbles, Drops, and Particles. Academic Press. ISBN: 9780121769505.

CMOSIS BVBA (2017). CMV4000 Area Scan Sensors. URL: http://www.cmosis.com/assets/generate-single-pdfs.php?products=cmv4000.
Coherent, Inc (2013a). Flare Analog OEM Lasers Operator's Manual. Part No. 1260793 Rev. AA. Unpublished.

Coherent, Inc (2013b). Flare Passively Q-Switched DPSS Laser. MC-009-13-1M0113. Unpublished.

Costantini, Marco, Uwe Fey, Ulrich Henne, and Christian Klein (2015). "Nonadiabatic surface effects on transition measurements using temperature-sensitive paints." In: AIAA Journal.

Cowen, EA and SG Monismith (1997). "A hybrid digital particle tracking velocimetry technique." In: Experiments in fluids 22.3, pp. 199-211.

Dandy, David S and Harry A Dwyer (1990). "A sphere in shear flow at finite Reynolds number: effect of shear on particle lift, drag, and heat transfer." In: Journal of Fluid Mechanics 216, pp. 381-410.

Davis, Richard E, Dal V Maddalon, and Richard D Wagner (1987). Performance of laminar-flow leading-edge test articles in cloud encounters. NASA technical report. NASA Langley Research Center. URL: https://ntrs.nasa.gov/search.jsp?R=19900003195.

Davis, Richard E, Dal V Maddalon, Richard D Wagner, David F Fisher, and Ronald Young (1989). Evaluation of cloud detection instruments and performance of laminar-flow leading-edge test articles during NASA leading-edge fight-test program. NASA technical report. NASA Langley Research Center. URL: https://ntrs.nasa.gov/search.jsp?R=19910014886.
Delbeke, Jos and Peter Vis (2016). EU climate policy explained. previously published by Routledge (2015). URL: https://ec.europa.eu/clima/sites/clima/files/eu_climate_policy_explained_en.pdf.

Deutsches Zentrum für Luft- und Raumfahrt e. V. (2013). LuFoV-1. Technologievorhaben LDAinOp. Vorhabenbeschreibung des DLR. Unpublished.
DNW German Dutch Wind Tunnels (n.d.). The Cryogenic Ludwieg-Tube Göttingen (KRG). Fact sheet. Business Unit GUK.
Drela, Mark (2007). "A User's Guide to MSES 3.05." In: Massachusetts Institute of Technology (MIT), Cambridge. URL: http://web.mit.edu/drela/Public/web/mses/.

Dryden, Hugh L (1953). "Review of published data on the effect of roughness on transition from laminar to turbulent flow." In: J. Aeronaut. Sci 20.7, pp. 477-482.

Edmund Optics Inc. (2017). TECHSPEC 8mm FL f/4, Blue Series M12 $\mu$-Video ${ }^{\text {tm }}$ Imaging Lens. URL: https://www.edmundoptics.com/imaging-lenses/micro-video-lenses/8mm-fl-f4-blue-series-m12-mu-videotrade-imaging-lens/ (visited on 2017-12-14).
Egami, Yasuhiro, Uwe Fey, and Jürgen Quest (2007). "Development of New Two-Component TSP for Cryogenic Testing." In: AIAA Aerospace Sciences Meeting and Exhibit, pp. 8-11.

Eyers, CJ, P Norman, J Middel, M Plohr, S Michot, K Atkinson, and RA Christou (2004). "AERO2k global aviation emissions inventories for 2002 and 2025." In: DLR.

Fey, Uwe, Yasuhiro Egami, and Rolf H Engler (2006). "High Reynolds number transition detection by means of temperature sensitive paint." In: AIAA Paper 514, p. 2006.

Fey, Uwe, Yasuhiro Egami, and Christian Klein (2007). "Temperature-sensitive paint application in cryogenic wind tunnels: Transition detection at high reynolds numbers and influence of the technique on measured aerodynamic coefficients." In: Instrumentation in Aerospace Simulation Facilities, 2007. ICIASF 2007. 22nd International Congress on. IEEE, pp. 1-17.

Gardner, RM, JK Adams, T Cook, LG Larson, RS Falk, E Fleuit, W Förtsch, M Lecht, David S Lee, MV Leech, et al. (1998). "ANCAT/EC2 aircraft emissions inventories for 1991/1992 and 2015." In: Final report. Produced by the ECAC/ANCAT and EC working group. European civil aviation conference.

Girnth, Martina Anna (2017). "Investigation of the transition on a laminar profile impacted by ice particles in the incoming flow." Master's thesis. Georg-August-Universität Göttingen.

Gregory, NT and WS Walker (1956). The effect on transition of isolated surface excrescences in the boundary layer. HM Stationery Office.

Gui, L and ST Wereley (2002). "A correlation-based continuous window-shift technique to reduce the peak-locking effect in digital PIV image evaluation." In: Experiments in Fluids 32.4, pp. 506-517.

Hall, Gordon R (1964). On the mechanics of transition produced by particles passing through an initially laminar boundary layer and the estimated effect on the LFC performance of the X-21 aircraft. Tech. rep. Northrop Corporation. URL: https://ntrs.nasa.gov/search.jsp?R=19790071148.

Hama, Francis R (1957). "An efficient tripping device." In: Journal of the aeronautical sciences 24.3, pp. 236-237.

Han, Shih-Ping (1976). "Superlinearly convergent variable metric algorithms for general nonlinear programming problems." In: Mathematical Programming 11.1, pp. 263-282. Doi: 10.1007/ BF01580395.

Hess, Cecil F and Drew L'Esperance (2009). "Droplet imaging velocimeter and sizer: a twodimensional technique to measure droplet size." In: Experiments in fluids 47.1, pp. 171182.

Hölzer, Andreas (2007). "Bestimmung des Widerstandes, Auftriebs und Drehmoments und Simulation der Bewegung nichtsphärischer Partikel in laminaren und turbulenten Strömungen mit dem Lattice-Boltzmann-Verfahren." PhD thesis. Martin-Luther-Universität Halle-Wittenberg.

Iijima, Y, Yasuhiro Egami, A Nishizawa, K Asai, Uwe Fey, and Rolf H Engler (2003). "Optimization of temperature-sensitive paint formulation for large-scale cryogenic wind tunnels." In: Instrumentation in Aerospace Simulation Facilities, 2003. ICIASF'03. 20th International Congress on. IEEE, pp. 70-76.

Infinity Photo-Optical Company (2011). Model K2/SC ${ }^{T M}$ Long-Distance Microscope System. Data sheet. URL: http://www.infinity-usa.com/downloads/OEM_Handbook/Model-K2-SC.pdf.

InnoLas Laser GmbH (2013). SpitLight 1000 Nd:YAG Laser System: User Manual. Revision 2.0. Unpublished.

IPCC working group I (2007). Climate change 2007: The Physical Science Basis. Contribution of Working Group I to the Fourth Assessment Report of the Intergovernmental Panel on Climate Change. Ed. by Susan Solomon, D. Qin, M. Manning, Z. Chen, M. Marquis, K. B. Averyt, M. Tignor, and H. L. Miller. Vol. 4. Cambridge University Press.

IPCC working group III (2007). Climate change 2007: Mitigation. Contribution of Working Group III to the Fourth Assessment Report of the Intergovernmental Panel on Climate Change. Ed. by Bert Metz, O. R. Davidson, P. R. Bosch, R. Dave, and L. A. Meyer. Vol. 4. Cambridge University Press.

Jasperson, William H, Gregory D Nastrom, Richard E Davis, and James D Holdeman (1985). "Variability of cloudiness at airline cruise altitudes from GASP measurements." In: Journal of climate and applied meteorology 24.1, pp. 74-82.

Johnson, CB (1972). "Circular aperture diffraction limited MTF: approximate expressions." In: Applied Optics 11.8, pp. 1875-1876.

Jones, Donald R, Cary D Perttunen, and Bruce E Stuckman (1993). "Lipschitzian optimization without the Lipschitz constant." In: Journal of Optimization Theory and Applications 79.1, pp. 157-181.

Kaskas, A (1964). "Berechnung der stationären und instationären Bewegung von Kugeln in ruhenden und strömenden Medien." Diploma thesis. Technische Universität Berlin.

Kaups, Kalle and Tuncer Cebeci (1977). "Compressible laminar boundary layers with suction on swept and tapered wings." In: Journal of Aircraft 14.7, pp. 661-667.

Kim, Brian Y, Gregg G Fleming, Joosung J Lee, Ian A Waitz, John-Paul Clarke, Sathya Balasubramanian, Andrew Malwitz, Kelly Klima, Maryalice Locke, Curtis A Holsclaw, et al. (2007). "System for assessing Aviation's Global Emissions (SAGE), Part 1: Model description
and inventory results." In: Transportation Research Part D: Transport and Environment 12.5, pp. 325-346.

Kirkpatrick, Scott, C Daniel Gelatt, Mario P Vecchi, et al. (1983). "Optimization by simulated annealing." In: science 220.4598 , pp. 671-680.

Klebanoff, PS, GB Schubauer, and KD Tidstrom (1955). "Measurements of the effect of 2dimensional and 3-dimensional roughness elements on boundary-layer transition." In: Journal of the Aeronautical Sciences 22.11, pp. 803-804.

Konrath, Robert (2014). "Definition of test parameters." In: Technical kickoff meeting "Ice Crystal Effects" of Lufo V-1 project LDAinOp. (2014-04-17). Unpublished. Göttingen.

Konrath, Robert (2015). "Status of ice crystal test at DNW-KRG." In: Progress Meeting "Ice Crystal Effects" of Lufo V-1 project LDAinOp. (2015-12-03). Unpublished. Göttingen.

Kosin, Rüdiger E (1965). "Laminar flow control by suction as applied to the X-21A airplane." In: Journal of Aircraft 2.5.

Kurose, Ryoichi and Satoru Komori (1999). "Drag and lift forces on a rotating sphere in a linear shear flow." In: Journal of Fluid Mechanics 384, pp. 183-206.

Lakshminarayanan, Vasudevan and Andre Fleck (2011). "Zernike polynomials: a guide." In: Journal of Modern Optics 58.7, pp. 545-561.

Lawson, R Paul, Brad Baker, Bryan Pilson, and Qixu Mo (2006). "In situ observations of the microphysical properties of wave, cirrus, and anvil clouds. Part II: Cirrus clouds." In: Journal of the atmospheric sciences 63.12, pp. 3186-3203. D OI: 10.1175/JAS3803.1. eprint: https://doi.org/10.1175/JAS3803.1. URL: https://doi.org/10.1175/JAS3803.1.

Lee, David S, David W Fahey, Piers M Forster, Peter J Newton, Ron CN Wit, Ling L Lim, Bethan Owen, and Robert Sausen (2009). "Aviation and global climate change in the 21st century." In: Atmospheric Environment 43.22, pp. 3520-3537.

Lee, David S, Giovanni Pitari, Volker Grewe, K Gierens, JE Penner, A Petzold, MJ Prather, U Schumann, A Bais, T Berntsen, et al. (2010). "Transport impacts on atmosphere and climate: Aviation." In: Atmospheric Environment 44.37, pp. 4678-4734.

Lee, Sungsu and James M Wilczak (2000). "The effects of shear flow on the unsteady wakes behind a sphere at moderate Reynolds numbers." In: Fluid Dynamics Research 27.1, pp. 1-22.

Legendre, Dominique and Jacques Magnaudet (1998). "The lift force on a spherical bubble in a viscous linear shear flow." In: Journal of Fluid Mechanics 368, pp. 81-126.

Liao, Shi-Jun (2002). "An analytic approximation of the drag coefficient for the viscous flow past a sphere." In: International Journal of Non-Linear Mechanics 37.1, pp. 1-18.

Liebling, Michael, Thierry Blu, and Michael Unser (2003). "Fresnelets: new multiresolution wavelet bases for digital holography." In: IEEE Transactions on image processing 12.1, pp. 2943.

Liepmann, Hans W and Gertrude H Fila (1947). Investigations of effects of surface temperature and single roughness elements on boundary-layer transition. NASA technical report. NASA, Washington DC.

Magnus, Gustav (1853). "Über die Abweichung der Geschosse, und: Über eine auffallende Erscheinung bei rotirenden Körpern." In: Annalen der Physik 164.1, pp. 1-29.

Mahr GmbH (2017). Digital Indicator MarCator 1075R. 3759777. Operating Instructions. URL: https://www.mahr.com/scripts/relocateFile.php?ContentID=19483\&NodeID=20678\&FileID=15047\&ContentDataID= 57308\&save=0\&isBlog $=0$.
Marxen, M, PE Sullivan, MR Loewen, and B Jähne (2000). "Comparison of Gaussian particle center estimators and the achievable measurement density for particle tracking velocimetry." In: Experiments in Fluids 29.2, pp. 145-153.

Meijering, Erik HW, Wiro J Niessen, and Max A Viergever (2001). "Quantitative evaluation of convolution-based methods for medical image interpolation." In: Medical Image Analysis 5.2, pp. 111-126.
Mikheev, AV and VM Zubtsov (2008). "Enhanced particle-tracking velocimetry (EPTV) with a combined two-component pair-matching algorithm." In: Measurement Science and Technology 19.8, p. 085401.

Molerus, Otto (2013). Fluid-Feststoff-Strömungen: Strömungsverhalten feststoffbeladener Fluide und kohäsiver Schüttgüter. Springer-Verlag.

Nastrom, Gregory D, James D Holdeman, and Richard E Davis (1981). Cloud-encounter and particle-concentration variabilities from GASP data. NASA technical report. NASA Langley Research Center.

Nayfeh, Ali H (1988). "Stability of compressible boundary layers." In: NASA Conference Publication 3020 1, Part 2, pp. 629-689. URL: https://ntrs.nasa.gov/archive/nasa/casi.ntrs.nasa.gov/ 1989001582.pdf.

Nelder, John A and Roger Mead (1965). "A simplex method for function minimization." In: The Computer Journal 7.4, pp. 308-313.

Newport Corporation (2011). X95 Structural Rails and Carriers. BR-101103-EN. Brochure. URL: https://www.newport.com/medias/sys_master/images/images/h6f/hf4/8797305634846/X95-Brochure.pdf.

Nikolas, Vogt (2009). "Numerische Simulation partikelbeladener Gasströmungen mit der Euler-Lagrange-Methode." Master's thesis. Georg-August-Universität Göttingen.

Nobach, Holger (2004). "Accuracy of sub-pixel interpolation in PIV and PTV image processing." In: Technical University of Darmstadt, Darmstadt, Germany, pp. 114-130.

Nobach, Holger, Nils Damaschke, and Cameron Tropea (2005). "High-precision sub-pixel interpolation in particle image velocimetry image processing." In: Experiments in Fluids 39.2, pp. 299-304.

Oseen, C W (1927). Neure Methoden und Ergebnisse in der Hydrodynamik. Akademische Verlagsgesellschaft.

Owen, B and David S Lee (2006). "Allocation of international aviation emissions from scheduled air traffic-future cases, 2005 to 2050." In: Centre for Air Transport and the Environment. Manchester Metropolitan University, United Kingdom.

PCO AG (2009). pixelfly high performance digital 12 bit CCD camera system. URL: https: //www.pco.de/fileadmin/user_upload/db/products/datasheet/pixelfly_20090505_02.pdf.

Peck, Edson R and Baij Nath Khanna (1966). "Dispersion of nitrogen." In: Journal of the Optical Society of America 56.8, pp. 1059-1063.

Penner, Joyce E (1999). Aviation and the global atmosphere: a special report of the Intergovernmental Panel on Climate Change. Cambridge University Press.

Perkins, Porter and Ulf R C Gustafsson (1975). "An automated atmospheric sampling system operating on 747 airliners." In: International Conference on Environmental Sensing and Assessment, sponsored by the IEEE. Vol. TM X-71790. NASA. URL: https://ntrs.nasa.gov/archive/ nasa/casi.ntrs.nasa.gov/19750023546.pdf.

Powell, M J D (1969). "A method for nonlinear constraints in minimization problems." In: Optimization. Ed. by R Fletcher. New York: Academic Press, pp. 283-298.

Rayleigh, Lord (1879). "XXXI. Investigations in optics, with special reference to the spectroscope." In: Philosophical Magazine 8.49, pp. 261-274. D OI: 10.1080/14786447908639684.

Rosemann, Henning (1997). "The Cryogenic Ludwieg-tube Tunnel at Gottingen." In: Special Course on Advances in Cryogenic Wind Tunnel Technology. AGARD Report 812. (1996-05-20/1996-05-24). Cologne: NATO Advisory Group for Aerospace Research and Development (AGARD), pp. 8-01-8-13.

Rosemann, Henning, E Stanewsky, and G Hefer (1995). "The Cryogenic Ludwieg-Tube of DLR and its new adaptive wall test section." In: Fluid Dynamics Conference, p. 2198.

Rubin, Donald B and Sanford Weisberg (1974). "The variance of a linear combination of independent estimators using estimated weights." In: ETS Research Report Series 1974.2.

Saffman, PGT (1965). "The lift on a small sphere in a slow shear flow." In: Journal of Fluid Mechanics 22.2, pp. 385-400.

Sausen, Robert and Ulrich Schumann (2000). "Estimates of the climate response to aircraft CO 2 and NO x emissions scenarios." In: Climatic Change 44.1, pp. 27-58.
Scarano, Fulvio and Michel L Riethmuller (2000). "Advances in iterative multigrid PIV image processing." In: Experiments in Fluids 29, S051-S060.
Schiller, L (1932). Handbook of Experimental Physics. Vol. IV.4. Leipzig, pp. 189-192.
Schneider Kreuznach (11/20/2008). Apo-Xenoplan 2.8/50 - Ruggedized. Version 2.0. Data sheet. Jos. Schneider Optische Werke GmbH. URL: http://www.schneiderkreuznach.com/fileadmin/user_ upload/bu_industrial_solutions/industrieoptik/22mm_Lenses/4_8_Megapixel_Anti_Shading/Apo-Xenoplan_2.850_ruggedized.pdf.

Schneider Kreuznach (2009). Schneider Kreuznach C-Mount Compact Lenses. Brochure. Jos. Schneider Optische Werke GmbH. URL: https://www.schneideroptics.com/pdfs/industrial/compact_ lenses_brochure.pdf.

Schneider Kreuznach (06/17/2013). Cinegon 1.8/16 - Ruggedized. Version 3.0. Data sheet. Jos. Schneider Optische Werke GmbH. URL: http://www.schneiderkreuznach.com/fileadmin/user_upload/bu_ industrial_solutions/industrieoptik/16mm_Lenses/Compact_Lenses/Cinegon_1.8-16_ruggedized.pdf.
Sellmeier, Wilhelm (1871). "Zur Erklärung der abnormen Farbenfolge im Spectrum einiger Substanzen." In: Annalen der Physik und Chemie 219.6, pp. 272-282.
Shortis, Mark R, Timothy A Clarke, and Tim Short (1994). "A comparison of some techniques for the subpixel location of discrete target images." In: Proceedings of SPIE. Vol. 2350, pp. 239-250.
Smith, AMO (1959). "The smallest height of roughness capable of affecting boundary-layer transition." In: Journal of the Aerospace Sciences 26.4, pp. 229-245.

Sony Corporation (2002). ICX285AL - Diagonal 11 mm (Type 2/3) Progressive Scan CCD Image Sensor with Square Pixel for B/W Cameras. E00Y42A27. URL: https://www.bnl.gov/atf/ docs/ChipSet_ICX285AL.pdf.

Stokes, George Gabriel (1851). "For creeping flow around an object of arbitray shape." In: Transactions of the Cambridge Philosophical Society 9.8.
Sutkus Jr, Donald J, Steven L Baughcum, and Douglas P DuBois (2001). Scheduled civil aircraft emission inventories for 1999: database development and analysis. NASA technical report NASA/CR-2001-211216.

Tani, Itiro (1961). "Effect of two-dimensional and isolated roughness on laminar flow." In: Boundary Layer and Flow Control 2, pp. 637-656.
Thévenaz, Philippe, Thierry Blu, and Michael Unser (2000). "Image interpolation and resampling." In: Handbook of Medical Imaging, Processing and Analysis 1.1, pp. 393-420.

Thorlabs, Inc. (2017). DET10A/M Si Biased Detector. Revision H. User Guide. URL: https://www. thorlabs.com/drawings/95e991c8b89e4112-667525CF-D6F5-382F-EFFA58FEC9A47AB1/DET10A_M-Manual.pdf.

Van de Hulst, HC and RT Wang (1991). "Glare points." In: Applied Optics 30.33, pp. 4755-4763.
Wagner, Richard D, DV Maddalon, and DF Fisher (1990). "Laminar flow control leading-edge systems in simulated airline service." In: Journal of aircraft 27.3, pp. 239-244.

Westerweel, Jerry (1993). "Digital particle image velocimetry - theory and application." PhD thesis. Technische Universiteit Delft.

Westerweel, Jerry (1997). "Fundamentals of digital particle image velocimetry." In: Measurement Science and Technology 8.12, p. 1379.

Westerweel, Jerry (2000). "Theoretical analysis of the measurement precision in particle image velocimetry." In: Experiments in Fluids 29, S003-S012.

Willert, Christian (1996). "The fully digital evaluation of photographic PIV recordings." In: Applied Scientific Research 56.2, pp. 79-102.

Wilson, Robert B (1963). "A simplicial algorithm for concave programming." PhD thesis. Harvard University.

Wischnewski, Berndt (2017). Calculation of thermodynamic state variables of nitrogen. URL: http://www.peacesoftware.de/einigewerte/stickstoff_e.html (visited on 2017-08-22).

Zernike, F (1934). "Diffraction theory of the knife-edge test and its improved form, the phasecontrast method." In: Monthly Notices of the Royal Astronomical Society 94, pp. 377-384.

## Appendix

## A. 1 Additional Technical Measurement Setup Descriptions

The following sections describe the technical details of measurement techniques used in this experiment that are not of very high importance to the methodology and results discussed in this thesis.

## A.1.1 Temperature Sensitive Paint

Illumination and cameras for the TSP measurements are located on either side of the test section within camera boxes which also contain cameras for the PTV and PIV measurements. These boxes are placed within the airfoil mounting system and rotate with the airfoil when the AOA is changed, thereby making sure that the angle of observation does not change relative to the airfoil. Figure A. 1 shows photos of the two camera boxes before installation.


Figure A.1: Camera boxes containing cameras, lenses and illumination for TSP, PTV and PIV measurements as well as an overview over the current state of the wind tunnel and airfoil model. Both boxes contain five installations which are listed below.
a) Camera box installed on the left side of the tunnel (in the direction of flow). Installations from left to right: TSP LED, WFT camera, main TSP camera, NFT camera, TSP LED.
b) Camera box installed on the right side of the tunnel (in the direction of flow). Installations from left to right: TSP LED, Backup TSP wide angle camera, main TSP-camera, localized airfoil model illumination (unused), TSP LED.

The TSP cameras are located in the very center of each camera box in order to get the highest possible viewpoint looking down at the airfoil model. One LED is placed on each outer corner of each camera box. Each of them has an excitation center wavelength of $\lambda=453 \mathrm{~nm}$. The photos in Figure A. 2 show the test section under LEDs illumination. These LEDs are equipped with bandpass filters with a range of $430-490 \mathrm{~nm}$. The placement as far away from the lens as possible was chosen in order to achieve even illumination with minimal reflections from the LEDs into the TSP camera on the opposite side. This setup of two cameras and four LEDs is


Figure A.2: Test section and airfoil model under TSP illumination (very strong blue color). The image on the left shows the flange of the opened test section area. In the image on the right, the camera box windows are visible as bright areas on each side above the airfoil model mounts.
used to then reconstruct an unwrapped top view of the model surface that is distortion-free.
The 8 mm focal length wide angle lenses (see Table A. 2 for specifications) are equipped with long-pass spectral filters with a cut-on wavelength of $\lambda=590 \mathrm{~nm}$ in order to block any LED light.

## A.1.2 Holographic Shadowgraphy

On the illumination side, a FLARE Analog OEM laser system (see Table A. 3 for the specifications) is used as the coherent light source. As shown in Figure A.3, the beam is first polarized using a prism type polarizing beam splitter with the secondary exit covered with light-absorbing material. Then, the beam is sent through a collimating lens ( $l_{\mathrm{f}}=15 \mathrm{~mm}$ ) and a pinhole aperture,


Figure A.3: Optical setup for holographic shadowgraphy in the test section. Light which is diffracted by the ice particles is illustrated in red. The diagram does not show correct proportions. The much more complex optical setup inside of the microscope objective is represented by a single ideal lens.
positioned 15 mm behind the lens, with a $20 \mu \mathrm{~m}$ diameter. Together, these work as a a spatial filter. Two additional lenses (at distances of 80 mm and 205 mm behind the pinhole aperture) control the size of the beam before collimating it before it enters the test section on the port side. The entire optical system is set up on a 30 mm cage system. Figure A. 4 shows the illumination setup for this measurement system.

| Type designation | pco.pixelfly qe <br> PCO AG |
| :--- | ---: |
| Manufacturer | serial LVDS <br> (via 8P8C connector, |
| Interface type | shielded Ethernet cable, PCI) |
|  | 12 bits $/$ pixel |
| Output format | C-mount |
| Lens mount | $5 \mu \mathrm{~s}-65 \mathrm{~s}$ |
| Exposure time | ICX285AL |
| Sensor | Sony Corporation |
| Sensor Manufacturer | CCD |
| Sensor type | none (greyscale) |
| Color filter array | effective: $1392 \times 1040$ pixels |
| Resolution | active: $1360 \times 1024$ pixels |
| Pixel dimensions | $6.45 \mu \mathrm{~m} \times 6.45 \mu \mathrm{~m}$ |
| Chip size | $10.2 \mathrm{~mm} \times 8.3 \mathrm{~mm}(H \times V)$ |
| Shutter type | electronic, global |
| Maximum frame rate | $12 \mathrm{fps}($ without binning) |
| Full well capacity | $18000 \mathrm{e}^{-}$ |
| Dynamic range | $\sim 69.5 \mathrm{~dB}$ |
| Quantum efficiency | $62 \%$ at $\lambda=500 \mathrm{~nm}$ |

Table A.1: Specifications for the pxo.pixelfly qe camera and ICX285AL sensor used in TSP measurements, data from PCO AG (2009) and Sony Corporation (2002).

On the measurement side, an Infinity K2/S $C F-1 / B$ long-distance microscope objective (see Table A.4) is used to achieve very high magnification at a relatively long distance from the focal plane. The microscope focus is set to the center of the flow. As a result, the non-diffracted light from the laser is not in focus and acts as a homogeneous background for the holographic interference pattern. The image is recorded using a Basler acA2040-180km camera (see Table 2.2 on page 32 ).

Figure A. 5 shows an example image from the measurement system with particle seeding and high velocity flow in the test section.

## Challenges

Holographic shadowgraphy requires a very clean optical path, as anything outside of the plane of focus can still greatly affect the resulting image. This setup requires the measurements to be performed though a high-speed fluid which shows significant variations in density and therefore in refractive index. As a result, wavefronts are not uniform and oscillate at high frequencies.

Additionally, two windows on each side of the test section can potentially collect dust or

| Type designation | TECHSPEC 8 mm FL f/4 Blue Series M12 $\mu$-Video ${ }^{\text {TM }}$ |
| :---: | :---: |
| Manufacturer | Edmund Optics |
| Focal length | 8 mm |
| Relative aperture | 1/4 |
| Image Circle | $\sim 6 \mathrm{~mm}$ ( $1 / 3$ in type) |
| Lens mount | S-Mount (M12 $\times 0.5$ ) |
| Scheimpflug angle | $7.4^{\circ}$ |
| Lens tilt angle | $16^{\circ}$ |
| Magnification range | $0.053 \times-0.032 \times$ |
| Working distance | 150 mm - 250 mm (up to 400 mm ) |

Table A.2: Specifications for the TECHSPEC 8 mm FL f/4 Blue Series M12 $\mu$-Video ${ }^{T M}$ lens (Edmund Optics Inc., 2017) used for TSP measurements.
condensation both on the inside as well as on the outside. Significant coverage of the window surfaces, especially with droplets of any size, randomizes the wavefronts enough to prevent any useful analysis of the results. Condensation on the outside was prevented by pointing a stream of hot air towards the windows on both sides of the test section. On the inside, limiting certain parameters of the IPG was usually enough to prevent the slow accumulation of condensation. As a result, this measurement technique only ended up working somewhat reliably with $40 \mu \mathrm{~m}$ target size particle seeding.

## Calibration

Using the USAF 1951 Resolution test chart (see Appendix A.1.3), the focal plane was first set to the center of the test section (within about $\pm 2 \mathrm{~mm}$. Then, a dot grid target (dot size $125 \mu \mathrm{~m}$, dot spacing $250 \mu \mathrm{~m}$ ) was used to determine the magnification of the Infinity $K 2 / S C F-1 / B$ at this distance. The result showed a magnification factor of $1.51 \times$ or $3.65 \mathrm{\mu m} \mathrm{px}^{-1}$.

Finally, the depth scale after analysis of the holographic interference pattern was calibrated by traversing the USAF 1951 Resolution test chart along the optical axis and taking images at intervals of $250 \mu \mathrm{~m}$.


Figure A.4: Photo of the illumination side of the holographic shadowgraphy setup. The frozen water on some wind tunnel parts are visible, as well as the clear glass window which is being heated with warm air.

| Type designation | FLARE Analog OEM |
| :---: | :---: |
| Model | Flare 532-40-100 laser system (Analog Controller) |
| Manufacturer | Coherent, Inc. |
| Laser type | diode-pumped Nd:YAG |
| Classification | Class 4 |
| Wavelength | $\lambda=1064 \mathrm{~nm}$ <br> (frequency doubled to 532 nm ) |
| Pulse control | Analog external trigger (TTL level) |
| Energy per pulse | $434 \mu \mathrm{~J} \pm 8 \mathrm{\mu J}{ }^{\mathrm{i}}$ |
| Pulse frequency | up to $100 \mathrm{~Hz}^{\text {i }}$ |
| Pulse width (FWHM) | $1.9 \mathrm{~ns}^{\text {i }}$ |
| Divergence | $<0.7 \mathrm{mrad}^{\text {i }}$ |
| Beam diameter | $0.5-0.8 \mathrm{~mm}^{\text {i }}$ |
| Dimensions | $143 \times 61 \times 31 \mathrm{~mm}$ (without power supply/pulse generator) |

${ }^{\text {i }}$ Values taken from the laser test sheet for the specific individual laser that was used (performed in March 2015).

Table A.3: Specifications for the Coherent FLARE Analog OEM laser system (Coherent, Inc, 2013b; Coherent, Inc, 2013a) used in holographic shadowgraphy measurements.

| Type designation | $\begin{array}{r}\text { Infinity K2/S CF-1/B } \\ \text { Manufacturer }\end{array}$ | Infinity Photo-Optical Company |
| :--- | ---: | ---: |$]$| Working distance | 222 mm | $\sim 297 \mathrm{~mm}$ |
| :--- | :---: | ---: |
| Focal length | $\sim 310 \mathrm{~mm}$ | $\sim 16 \mathrm{~mm}$ |
| Image Circle | C-Mount, Nikon F-Mount |  |
| Interface | $1.4 \times$ | $0.71 \times$ |
| Magnification | $3.9 \mu \mathrm{~m}$ | $7.3 \mu \mathrm{~m}$ |
| Resolution | $70 \mu \mathrm{~m}$ | $260 \mu \mathrm{~m}$ |
| DOF | 0.086 mm | 0.045 mm |
| NA |  |  |

Table A.4: Specifications for the Infinity $K 2 / S C F-1 / B$ long-distance microscope (Infinity PhotoOptical Company, 2011) used in holographic shadowgraphy measurements.
DOF: depth of field, NA: numerical aperture.


Figure A.5: Image from the holographic shadowgraphy measurement system. Taken with $40 \mu \mathrm{~m}$ target size ice particle seeding in the test section.

## A.1.3 USAF 1951 Resolution Test Chart

The standardized US Air Force (USAF) resolution test chart shown in Figure A. 6 was used for different calibration purposes during the experiment. Table A. 5 shows the corresponding line widths. All lines have an aspect ratio of $5: 1$, such that all elements of three lines are square and have overall dimensions of five line widths in both directions.


Figure A.6: 1951 USAF resolution test chart, conforming to the MIL-STD-150A standard. Groups are numbered using a bold font weight, elements are numbered using the regular font weight.

| Element | Group Number |  |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | -2 | -1 | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
| 1 | 2000.00 | 1000.00 | 500.00 | 250.00 | 125.00 | 62.50 | 31.25 | 15.63 | 7.81 | 3.91 |
| 2 | 1781.80 | 890.90 | 445.45 | 222.72 | 111.36 | 55.68 | 27.84 | 13.92 | 6.96 | 3.48 |
| 3 | 1587.40 | 793.70 | 396.85 | 198.43 | 99.21 | 49.61 | 24.80 | 12.40 | 6.20 | 3.10 |
| 4 | 1414.21 | 707.11 | 353.55 | 176.78 | 88.39 | 44.19 | 22.10 | 11.05 | 5.52 | 2.76 |
| 5 | 1259.92 | 629.96 | 314.98 | 157.49 | 78.75 | 39.37 | 19.69 | 9.84 | 4.92 | 2.46 |
| 6 | 1122.46 | 561.23 | 280.62 | 140.31 | 70.15 | 35.08 | 17.54 | 8.77 | 4.38 | 2.19 |

Table A.5: Line widths of the 1951 USAF resolution test chart. All sizes given in $\mu \mathrm{m}$.

## A. 2 Testing Sequence and Practices

The following sections describe the sequence of measurements and events both across a whole day of testing as well as within a single wind tunnel run. The structure of a measurement day is important because the results of a run can be affected by events on the previous measurement day or even further back in time. On the other hand, the exact sequence during a run is significant for what the temporal resolution of different measurements is, how the images from a run are selected and how correlation between different runs is limited.

## A.2.1 Testing Day

The tunnel, even if it had been cooled down to $\sim 180 \mathrm{~K}$ on the previous day, will usually be at a temperature of around $270-290 \mathrm{~K}$ in the early morning because it is not actively cooled during the night and this set of experiments was conducted in the months of June and July, with outside temperatures usually around $20 \pm 10^{\circ} \mathrm{C}$. To avoid condensation, however, the tunnel will always remain closed if any runs are planned for the following day, retaining a dry nitrogen atmosphere inside. Opening the tunnel, for example in order to manually clean the airfoil model, prevents cryogenic runs for at least two days. This is therefore avoided unless it is absolutely necessary.

## Warm Runs

At the beginning of the day, the tunnel will usually contain some remnants of the seeding of the previous day. Especially material which may have frozen in certain areas the day before and may not have caused any issues will likely have melted due to the increase in temperature, with most of it accumulating at the bottom of the storage tube. It is important to get rid of this material while the tunnel is still warm in order to avoid accelerating any large clusters of ice through the test section and potentially damaging the model, the laser optics or the tunnel itself. Small droplets on the airfoil model itself are also easier to aerodynamically clean off while the tunnel is still warm as liquid droplets are easier to blow away than frozen ones and evaporation is much fast at higher temperatures.

For these reasons, between two and eight wind tunnel runs will be performed every morning without any measurements and with warm nitrogen gas. A crucial function of these is the removal of any remnants of seeding from the airfoil model. Unfortunately, the TSP on the airfoil model does not emit enough light to evaluate the point of transition at temperatures above about 230 K . Therefore, running the measurement equipment during warm runs would not yield any feedback regarding the contamination state of the model. Consequently, the number of warm runs that are performed is based purely on past experience and an estimate of how contaminated the test section is likely to be from the previous day of testing.

## Tunnel Cooldown

After the warm runs have been completed, the tunnel is cooled down using liquid nitrogen. At a rate of approximately $20 \mathrm{~K} \mathrm{~h}^{-1}$ initially (above 250 K ) and $\approx 10 \mathrm{Kh}^{-1}$ towards the end (below 200 K ), the tunnel is brought to its target temperature for the day. During the cooling, an occasional run is performed in order to verify that the testing equipment works, to see how much remaining seeding is in the flow coming from the storage tube, and to test the evenness of the pulse brightness from the PTV lasers, adjusting those as necessary. As soon as the temperature is low enough for the TSP to respond, these runs also show how many turbulent wedges are present on the upper surface, indicating the amount of contamination still remaining.

## Baseline Runs

As soon as the target temperature is reached, additional runs are performed with all measurements running in order to establish a baseline set of results for a clean model and a flow with negligible seeding. If seeding was used on the previous day, even these PTV images will usually show a low single-digit number of particles in the PTV/PIV images. However, the density of these particles is not nearly high enough to cause any significant change in the time-averaged point of transition. An exception might be the case of such a particle getting stuck close to the stagnation point of the airfoil model, in which case it will add an additional turbulent wedge to the TSP image. Tis can be ignored as long as some spanwise section remains free of turbulent wedges.

## Ice Particle Generation

In order to introduce seeding into the test section during the steady state flow period, the IPG is activated. This is usually done directly in advance of a run, but in rare instances will also be done during the actual run.

The seeding material is gathered directly ahead of the gate valve and therefore at the very front of the storage tube. Leaving it there would push it through the test section before a steady state flow is established. The goal, then, is to place the seeding so far back in the storage tube that it travels through the test section precisely when the steady state flow is achieved. This is achieved by activating the circulation blower of the tunnel for a certain amount of time (usually between 10 and 30 s ) and at a given rpm. Pushing the seeding material further back into the tube creates a thinner seeding during the test and potentially not enough material in the relevant interval. Additionally, it causes more material to stay in the storage tube throughout the run, and potentially even following runs, such that it becomes hard to remove it again and achieve runs that are entirely free from seeding material.

A combined cleaning and baseline run is performed before each subsequent seeding at least once, though those will usually be less clean than the first baseline run of the day. In order to show a clear effect of the ice particles on the position of the transition point on the airfoil, it is
therefore necessary to show that the boundary layer was less turbulent both before and after the respective run with the full amount of seeding.

## A.2.2 Single Run Sequence

Each run is entirely controlled by a sequencer in the wind tunnel room which is connected to all but the TSP cameras and all lasers through BNC connector cables. A script contains all timings and instructions for the experimental run, including some variables which can easily be changed in-between runs. This script is compiled into an explicit linear sequence and then transmitted to the hardware sequencer. Figure A. 7 illustrates a typical sequence of events that is performed during a single laser period (this refers to the InnoLas SpitLight 1000 lasers) of $\sim 1 / 15 \mathrm{~s}$ (actually $66666 \mu \mathrm{~s}$ ). For the cameras, the length of the signal determines the exposure time. The pulsed lasers on the other hand emit light for a much shorter period of time than their respective $q$-switch signals - for them, durations are given in the specification tables if available. At different times, different parts of this core sequence can be active or inactive.

The idle sequence is used when no measurement is performed but lasers need to be pumped continuously to keep them stable and ready for a measurement. In this sequence, only the channels shown in Figure A. 8 are active. With inactive $q$-switches, the lasers are not firing but still being pumped with flashlamps.

When a wind tunnel run is performed, the sequencer switches more channels to the active state as shown in Figure A.9. It shows that the holographic shadowgraphy is running for 1 s before and after the main measurement period, with all other measurements starting to record even earlier. This serves the purpose of recording the state of the wind tunnel both before there is any motion in the test section and after the flow has mostly come to a halt. When this sequence is finished, the program returns to the idle sequence immediately.

Previous test campaigns had shown problems in reproducing the shock wave position between different runs. This spoils any attempts of using the PIV data from one run as a point of comparison for the PTV measurement taken during a different run. In order to improve the repeatability of the timing between the KRG and the entire measurement sequence, an additional relay was added which would suppress the trigger signal for the wind tunnel until the sequencer also sends a signal. This signal is represented by the "KRG trigger" channel in the figures shown in this chapter. With this improvement in place, results still show slight variation in the shock wave position between runs with otherwise very similar parameters.


Figure A.7: Core sequence for a single SpitLight period. Arrows highlight the beginning and end of each transmission period for better visibility of short periods.
a) Detailed view of the first $300 \mu \mathrm{~s}$ of the sequence
b) Detailed view of the first $3000 \mu \mathrm{~s}$ of the sequence
c) Overview over the entire $1 / 15 \mathrm{~s}$ period


Figure A.8: Active sequencer channels for the idle sequence. Within each active channel, the sequence from Figure A. 7 is repeated every $1 / 15$ s.


Figure A.9: Active sequencer channels for the measurement run sequence. Within each channel, the sequence from Figure A. 7 is repeated every $1 / 15$ s. The main measurement period is shown to highlight the 10 laser periods (or 0.67 s ) in which the main analysis is done, i.e. somewhere within this time, a steady state high-speed flow is expected to occur.

## A. 3 Calculation of the Refractive Effects of the Shock Wave

For this simulation, it was assumed that the distortion of the image due to the constant refractive index of the gas behind the shock wave is corrected, and only the difference in gas density across the shock wave causes an error.

The exact refractive index of nitrogen is available from Peck and Khanna (1966, p. 1061) at a temperature of $0^{\circ} \mathrm{C}$ and $p_{\text {st. }}=101325 \mathrm{~Pa}$ in the form

$$
\begin{equation*}
10^{8}\left(n_{\mathrm{r}, \mathrm{~N}_{2}}-1\right)=6855.200+\frac{3243157.0}{144-\tilde{\nu}^{2}} \tag{A.1}
\end{equation*}
$$

where $n$ is the refractive index and $\tilde{\nu}$ is the spectroscopic wavenumber in $\mu^{-1}$ (number of wavelengths per unit distance). This can be rewritten, more closely resembling standard Sellmeier coefficients, as

$$
\begin{equation*}
n_{\mathrm{r}, \mathrm{~N}_{2}}=1+6855.200 \times 10^{-8}+\frac{\frac{3243157.0 \times 10^{-8}}{144} \lambda^{2}}{\lambda^{2}-\frac{1}{144 \times 10^{12}}} \tag{A.2}
\end{equation*}
$$

where $\lambda$ is the wavelength of the refracted light. For a wavelength of $\lambda=532 \mathrm{~nm}$, this gives

$$
\begin{equation*}
n_{\mathrm{r}, \mathrm{~N}_{2}, \mathrm{st} ., 532 \mathrm{~nm}}=1.000293771 \tag{A.3}
\end{equation*}
$$

Wischnewski (2017) gives the required properties of nitrogen for the following cases (see Section 2.5.2):

$$
\begin{array}{rll}
\text { Standard } & T_{\text {st. }}=0^{\circ} \mathrm{C}, & p_{\text {st. }}=101325 \mathrm{~Pa} \\
\text { Undisturbed flow } & T_{\infty}=164.2 \mathrm{~K}, & p_{\infty}=185.9 \mathrm{kPa} \tag{A.5}
\end{array}
$$

namely

$$
\begin{array}{ll}
c_{p, \mathrm{~N}_{2}, \text { st. }}=1.04103 \times 10^{3} \mathrm{~J} \mathrm{~kg}^{-1} \mathrm{~K}^{-1} & \left(T_{\text {st. }}, p_{\text {st. }}\right) \\
c_{v, \mathrm{~N}_{2}, \text { st. }}=0.742904 \times 10^{3} \mathrm{~J} \mathrm{~kg}^{-1} \mathrm{~K}^{-1} & \left(T_{\text {st. }}, p_{\text {st. }}\right) \\
c_{p, \mathrm{~N}_{2}, \infty}=1.053386 \times 10^{3} \mathrm{~J} \mathrm{~kg}^{-1} \mathrm{~K}^{-1} & \left(T_{\infty}, p_{\infty}\right) \\
c_{v, \mathrm{~N}_{2}, \infty}=0.745073 \times 10^{3} \mathrm{~J} \mathrm{~kg}^{-1} \mathrm{~K}^{-1} & \left(T_{\infty}, p_{\infty}\right) \tag{A.9}
\end{array}
$$

Here, $c_{p}$ is the specific isobaric heat capacity and $c_{v}$ is the specific isochoric heat capacity. These values lead to

$$
\begin{align*}
R_{\mathrm{st.}} & =c_{p, \mathrm{~N}_{2}, \mathrm{st} .}-c_{v, \mathrm{~N}_{2}, \mathrm{st} .} & & =298.126 \mathrm{~J} \mathrm{~kg}^{-1} \mathrm{~K}^{-1}  \tag{A.10}\\
R_{\infty} & =c_{p, \mathrm{~N}_{2}, \infty}-c_{v, \mathrm{~N}_{2}, \infty} & & =308.313 \mathrm{~J} \mathrm{~kg}^{-1} \mathrm{~K}^{-1}  \tag{A.11}\\
\gamma_{\mathrm{st} .} & =c_{p, \mathrm{~N}_{2}, \mathrm{st} .} / c_{v, \mathrm{~N}_{2}, \mathrm{st} .} & & =1.40130  \tag{A.12}\\
\gamma_{\infty} & =c_{p, \mathrm{~N}_{2}, \infty} / c_{v, \mathrm{~N}_{2}, \infty} & & =1.41380 \tag{A.13}
\end{align*}
$$

where $R$ is the specific gas constant and $\gamma$ is the adiabatic index. The density of nitrogen for both cases $\left(\rho_{\mathrm{N}_{2}, \text { st. }}\right.$ and $\left.\rho_{\mathrm{N}_{2}, \infty}\right)$ are therefore

$$
\begin{equation*}
\rho_{\mathrm{N}_{2}, \text { st. }}=\frac{p_{\text {st. }}}{R_{\text {st. }} * T_{\text {st. }}}=1.2443 \mathrm{~kg} \mathrm{~m}^{-3}, \text { and } \rho_{\mathrm{N}_{2}, \infty}=\frac{p_{\infty}}{R_{\infty} * T_{\infty}}=3.6728 \mathrm{~kg} \mathrm{~m}^{-3} \tag{A.14}
\end{equation*}
$$

For a Mach number of $\mathrm{M}_{\infty}=0.76$, the velocity of the undisturbed flow $u_{\infty}$ follows as

$$
\begin{equation*}
u_{\infty}=\mathrm{M}_{\infty} \sqrt{\gamma_{\infty} R_{\infty} T_{\infty}}=203.325 \mathrm{~m} \mathrm{~s}^{-1} . \tag{A.15}
\end{equation*}
$$

The pressure coefficients before and after the shock wave for the upper airfoil surface, $c_{p, \mathrm{I}}$ and $c_{p, \text { II }}$, can be taken from a simulation of the LV2F airfoil model using MSES (Drela, 2007). The simulation used parameters of $\mathrm{M}_{\infty}=0.76, \mathrm{Re}=10 \times 10^{6}$, and $\mathrm{AOA}=2.0^{\circ}$ (see Figure 2.9). The pressure coefficients were taken from the pressure coefficient minimum of the upper surface

$$
\begin{equation*}
c_{p, \mathrm{I}}=-1.08245 \quad \text { at } \frac{x_{\mathrm{a}}}{c}=60.58 \% \tag{A.16}
\end{equation*}
$$

and about $5 \%$ further back with

$$
\begin{equation*}
c_{p, \mathrm{II}}=-0.531076 \quad \text { at } \frac{x_{\mathrm{a}}}{c}=65.22 \% . \tag{A.17}
\end{equation*}
$$

Here, value I represents the supersonic flow ahead of the shock wave, while value II represents the subsonic flow behind the shock wave.

Based on the definition of the pressure coefficient,

$$
\begin{equation*}
c_{p}=\frac{p-p_{\infty}}{1 / 2 \rho u_{\infty}{ }^{2}}, \tag{A.18}
\end{equation*}
$$

the pressures ahead of the shock wave and behind the shock wave can be derived to be

$$
\begin{align*}
p_{\mathrm{I}} & =c_{p, \mathrm{I}}\left(1 / 2 \rho_{\mathrm{N}_{2}, \infty} u_{\infty}^{2}\right)+p_{\infty}=103758 \mathrm{~Pa} \text { and }  \tag{A.19}\\
p_{\mathrm{II}} & =c_{p, \mathrm{II}}\left(1 / 2 \rho_{\mathrm{N}_{2}, \infty} u_{\infty}^{2}\right)+p_{\infty}=145617 \mathrm{~Pa} . \tag{A.20}
\end{align*}
$$

Assuming a non-oblique shock wave, it follows that

$$
\begin{align*}
\frac{p_{\mathrm{II}}}{p_{\mathrm{I}}} & =\frac{2 \gamma_{\infty} \mathrm{M}_{\mathrm{I}}^{2}-\left(\gamma_{\infty}-1\right)}{\gamma_{\infty}+1}  \tag{A.21}\\
& =1+\frac{2 \gamma_{\infty}}{\gamma_{\infty}+1}\left(\mathrm{M}_{\mathrm{I}}^{2}-1\right) . \tag{A.22}
\end{align*}
$$

From this, the Mach number ahead of the shock wave can be derived to be

$$
\begin{equation*}
\mathrm{M}_{\mathrm{I}}=\sqrt{\left(\frac{p_{\mathrm{II}}}{p_{\mathrm{I}}}-1\right)\left(\frac{\gamma_{\infty}+1}{2 \gamma_{\infty}}\right)+1}=1.159 . \tag{A.23}
\end{equation*}
$$

Just for reference, the Mach number behind the shock wave is

$$
\begin{equation*}
\mathrm{M}_{\mathrm{II}}=\frac{1+\frac{\gamma_{\infty}-1}{2} \mathrm{M}_{\mathrm{I}}^{2}}{\gamma_{\infty} \mathrm{M}_{\mathrm{I}}^{2}-\frac{\gamma_{\infty}-1}{2}}=0.869 \tag{A.24}
\end{equation*}
$$

The density ahead of the shock wave, assuming an adiabatic process, is

$$
\begin{equation*}
\rho_{\mathrm{I}}=\frac{p_{\mathrm{I}}}{R_{\infty} T_{\mathrm{I}}}=2.43112 \mathrm{~kg} \mathrm{~m}^{-3} \tag{A.25}
\end{equation*}
$$

using the temperature ahead of the shock wave, $T_{\mathrm{I}}$, from

$$
\begin{equation*}
T_{\mathrm{I}}=T_{\infty}\left(\frac{p_{\infty}}{p_{\mathrm{I}}}\right)^{\frac{1-\gamma_{\infty}}{\gamma_{\infty}}}=138.43 \mathrm{~K} \tag{A.26}
\end{equation*}
$$

Via the ratio of the densities,

$$
\begin{equation*}
\frac{\rho_{\mathrm{II}}}{\rho_{\mathrm{I}}}=\frac{\left(\gamma_{\infty}+1\right) \mathrm{M}_{\mathrm{I}}^{2}}{2+\left(\gamma_{\infty}-1\right) \mathrm{M}_{\mathrm{I}}^{2}}=1.26945 \tag{A.27}
\end{equation*}
$$

one arrives at the density of the gas behind the shock wave,

$$
\begin{equation*}
\rho_{\mathrm{II}}=\rho_{\mathrm{I}} * \frac{\rho_{\mathrm{II}}}{\rho_{\mathrm{I}}}=3.08617 \mathrm{~kg} \mathrm{~m}^{-3} \tag{A.28}
\end{equation*}
$$

From these two densities, we can use the density dependency equation 3.16 and the value from equation A. 3 to arrive at

$$
\begin{array}{ll}
n_{\mathrm{r}, \mathrm{I}}=\left(n_{\mathrm{r}, \mathrm{~N}_{2}, \mathrm{st} ., 532 \mathrm{~nm}}-1\right) \frac{\rho_{\mathrm{I}}}{\rho_{\mathrm{N}_{2}, \mathrm{st}}}+1 & =1.00057398 \text { and } \\
n_{\mathrm{r}, \mathrm{II}}=\left(n_{\mathrm{r}, \mathrm{~N}_{2}, \mathrm{st} ., 532 \mathrm{~nm}}-1\right) \frac{\rho_{\mathrm{II}}}{\rho_{\mathrm{N}_{2}, \mathrm{st} .}}+1 & =1.00072864 \tag{A.30}
\end{array}
$$

For the geometric analysis of the problem, the horizontal angle from the center of the image, $\alpha_{\text {test }}$, is defined to be

$$
\begin{equation*}
\alpha_{\mathrm{test}}=\arctan \left(\frac{w_{\text {sensor }{ }_{\mathrm{i}}} \hat{x}}{2 l_{\mathrm{f}}}\right) \tag{A.31}
\end{equation*}
$$

with

$$
\begin{align*}
w_{\text {sensor }} & =5.50 \mu \mathrm{~m} \times 2048 \mathrm{px}=11.264 \mathrm{~mm}  \tag{A.32}\\
-1.0 & \leq{ }_{\mathrm{i}} \hat{x} \leq 1.0 \text { and }  \tag{A.33}\\
l_{\mathrm{f}} & =62.315 \mathrm{~mm} \tag{A.34}
\end{align*}
$$

Where $w_{\text {sensor }}$ is the width of the sensor, ${ }_{\mathrm{i}} \hat{x}$ is the test variable representing the (horizontal) particle position in the final image, and $l_{\mathrm{f}}$ is the focal length of the NFT camera lens. In addition to the test variable ${ }_{i} \hat{x}$, the position of the shock wave ${ }_{i} \hat{x}_{s, 1}$ is also kept variable for now as it can change its position slightly between different runs.

The camera is positioned behind the shock wave. Particles ahead of the shock wave will thus be displaced compared to their actual position. Therefore, the refraction happens from $n_{\mathrm{r}, \mathrm{II}}$ into $n_{\mathrm{r}, \mathrm{I}}$, which is why arabic indices are used for the angles below, instead of the roman indices used for the gas properties. If

$$
\begin{equation*}
\sin \left(\theta_{1}\right) \frac{n_{\mathrm{r}, \mathrm{II}}}{n_{\mathrm{r}, \mathrm{I}}}>1.0 \tag{A.35}
\end{equation*}
$$

there is a total internal reflection at the shock wave and the refracted angle changes from

$$
\begin{equation*}
\theta_{2}=\arcsin \left(\sin \left(\theta_{1}\right) \frac{n_{\mathrm{r}, \mathrm{II}}}{n_{\mathrm{r}, \mathrm{I}}}\right) \tag{A.36}
\end{equation*}
$$

to

$$
\begin{equation*}
\theta_{2}=\pi-\theta_{1} \tag{A.37}
\end{equation*}
$$

In these equations, $\theta_{1}=\frac{\pi}{2}-\alpha_{\text {test }}$ is the angle before refraction, and $\theta_{2}=\frac{\pi}{2}-\alpha_{\text {refracted }}$ is the angle after refraction, in both cases relative to the shock wave surface. To convert back into a camera-centric coordinate system, $\alpha_{\text {refracted }}$ is used for the angle after refraction.

Using simple trigonometry, the perceived and actual position of a particle are

$$
\begin{align*}
{ }_{\mathrm{i}} \hat{x}_{\text {perceived }} & =\tan \left(\alpha_{\text {test }}\right) z_{\text {obs. }}  \tag{A.38}\\
{ }_{\mathrm{i}} \hat{x}_{\text {actual }} & =\tan \left(\alpha_{\text {refracted }}\right) z_{1}+\tan \left(\alpha_{\text {test }}\right) z_{2} \tag{A.39}
\end{align*}
$$

with

$$
\begin{align*}
z_{2} & =\min \left\{\frac{\mathrm{i} \hat{x}_{\mathrm{s}, 1} x_{\mathrm{FOV}}}{\tan \left(\alpha_{\mathrm{test}}\right)} ; z_{\mathrm{obs} .}\right\}  \tag{A.40}\\
z_{1} & =z_{\text {obs. }}-z_{2}  \tag{A.41}\\
x_{\mathrm{FOV}} & =\left(\frac{w_{\text {sensor }}}{2 l_{\mathrm{f}}}\right) z_{\mathrm{obs}} \tag{A.42}
\end{align*}
$$

where

$$
\begin{equation*}
z_{\text {obs. }}=230 \mathrm{~mm} \text { (approximate) } \tag{А.43}
\end{equation*}
$$

is the depth from the NPP of the lens to the focal plane,
$z_{1}$ is the depth from the NPP to the shock wave,
$z_{2}$ is the depth from the shock wave to the focal plane, and $x_{\text {FOV }}$ is the horizontal absolute field of view (FOV) in the focal plane.

## A. 4 Synthetic Image Validation Velocity Field

A synthetically generated velocity field is the basis for the offset determination of the particle images. The image scale for the conversion from real-world to pixel distances is $20.34 \mathrm{~mm} \mathrm{px}^{-1}$.

A velocity before the shock of $245 \mathrm{~m} \mathrm{~s}^{-1}$ is used, with a drop of $50 \mathrm{~ms}^{-1}$ at the shock. The position of the shock is set to

$$
\begin{equation*}
{ }_{\mathrm{i}} x_{\mathrm{s}, 1}=512 \mathrm{px}-\frac{25}{262144}\left({ }_{\mathrm{i}} x_{2}-2048 \mathrm{px}\right)^{2}, \tag{A.44}
\end{equation*}
$$

with ${ }_{i} x_{1}$ and ${ }_{i} x_{2}$ measured in pixels from the top left corner. This is an approximation of the slightly curved shape of the shock that can be observed in the PIV analysis images.

The velocity of the particles after the shock is based the differential equation for the particle slip velocity (see Equation 1.8). Additionally, the definition of $\mathrm{R}_{D_{\mathrm{p}}}$ (see Equation 1.1) is used. Regarding the drag coefficient $c_{\mathrm{d}}$ of a sphere, the approximation by Kaskas (1964) (see Equation 1.10) is not easily applicable as a solution for $u(x)$ is not reasonably possible even when using a symbolic computing environment.

For this reason, a simpler approximation for the drag coefficient was used here. The actual particle diameters that were achieved during the wind tunnel experiments are mostly expected to be between $20 \mu \mathrm{~m}$ and $40 \mu \mathrm{~m}$, and the maximum slip velocity directly after the shock is expected to be at around $50 \mathrm{~m} \mathrm{~s}^{-1}$ at most (see the largest differences in velocity in, for example, Figure 4.11). With a value of the kinematic viscosity of $\nu_{\infty} \approx 2.97 \times 10^{-6} \mathrm{~m} \mathrm{~s}^{-2}$ (see Section 2.5.2), the particle Reynolds numbers should be at and below $R_{D_{\mathrm{p}}=20 \mu \mathrm{~m}}=336$ and $\mathrm{R}_{D_{\mathrm{p}}=40 \mu \mathrm{~m}}=673$. Unfortunately, neither the Stokes approximation of the sphere drag (Equation 1.9), nor a constant approximation of $c_{\mathrm{d}}=0.4$ often used for high Reynolds numbers, is particularly suitable in this Reynolds number regime (see Figure 1.9). Nevertheless, the constant approximation is used for this slip velocity model for the sake of simplicity and lack of an easily accessible better option. A particle size of $D_{\mathrm{p}}=20 \mu \mathrm{~m}$ is assumed.

Given Equation 1.8 and $c_{\mathrm{d}}=0.4$, the slip velocity equation can be rearranged into

$$
\begin{equation*}
\frac{\dot{u}_{\mathrm{s}}(t)}{u_{\mathrm{s}}(t)^{2}}=-\frac{3}{10} \frac{\rho_{\mathrm{f}}}{\rho_{\mathrm{p}} D_{\mathrm{p}}} \tag{A.45}
\end{equation*}
$$

Integration over $t$ gives

$$
\begin{equation*}
-\frac{1}{u_{\mathrm{s}}(t)}=-\frac{3}{10} \frac{\rho_{\mathrm{f}}}{\rho_{\mathrm{p}} D_{\mathrm{p}}}+c_{\mathrm{i}, 1} \tag{A.46}
\end{equation*}
$$

and with $c_{\mathrm{i}, 1}$ set such that $u_{\mathrm{s}}(t=0)=\Delta u_{\mathrm{s}}$ follows

$$
\begin{align*}
-\frac{1}{u_{\mathrm{s}}(t)} & =-\frac{3}{10} \frac{\rho_{\mathrm{f}}}{\rho_{\mathrm{p}} D_{\mathrm{p}}}-\frac{1}{\Delta u_{\mathrm{s}}}, \text { or }  \tag{A.47}\\
u_{\mathrm{s}}(t) & =\frac{10 \rho_{\mathrm{p}} D_{\mathrm{p}} \Delta u_{\mathrm{s}}}{3 t \Delta u_{\mathrm{s}} \rho_{\mathrm{f}}+10 D_{\mathrm{p}} \rho_{\mathrm{p}}} \tag{A.48}
\end{align*}
$$

Furhter integration over $t$ results in

$$
\begin{equation*}
x(t)=\frac{10}{3} \frac{D_{\mathrm{p}} \rho_{\mathrm{p}} \ln \left(3 t \Delta u_{\mathrm{s}} \rho_{\mathrm{f}}+10 D_{\mathrm{p}} \rho_{\mathrm{p}}\right)}{\rho_{\mathrm{f}}}-c_{\mathrm{i}, 2} \tag{A.49}
\end{equation*}
$$

and, with $c_{\mathrm{i}, 2}$ adjusted such that $\Delta x_{\mathrm{s}}(t=0)=0$,

$$
\begin{align*}
\Delta x_{\mathrm{s}}(t) & =\frac{10}{3} \frac{D_{\mathrm{p}} \rho_{\mathrm{p}} \ln \left(3 t \Delta u_{\mathrm{s}} \rho_{\mathrm{f}}+10 D_{\mathrm{p}} \rho_{\mathrm{p}}\right)}{\rho_{\mathrm{f}}}-\frac{10}{3} \frac{D_{\mathrm{p}} \rho_{\mathrm{p}} \ln \left(10 D_{\mathrm{p}} \rho_{\mathrm{p}}\right)}{\rho_{\mathrm{f}}}  \tag{A.50}\\
& =\frac{10}{3} \frac{D_{\mathrm{p}} \rho_{\mathrm{p}}}{\rho_{\mathrm{f}}} \ln \left(\frac{1}{10} \frac{3 t \Delta u_{\mathrm{s}} \rho_{\mathrm{f}}+10 D_{\mathrm{p}} \rho_{\mathrm{p}}}{D_{\mathrm{p}} \rho_{\mathrm{p}}}\right) \tag{A.51}
\end{align*}
$$

Solving Equation A. 51 for $t$ and substituting the result for $t$ in Equation A. 48 finally gives the slip velocity as a function of the position from the origin,

$$
\begin{equation*}
u_{\mathrm{s}}\left(\Delta x_{\mathrm{s}}\right)=\Delta u_{\mathrm{s}} \mathrm{e}^{-\frac{3}{10} \frac{\Delta x_{\mathrm{s}} \rho_{\mathrm{f}}}{D_{\mathrm{p}} \rho_{\mathrm{p}}}} . \tag{A.52}
\end{equation*}
$$

This can be used as a velocity predictor at a given image position for any particle, where $\Delta x_{\mathrm{s}}$ is the distance from the particle to the position where the initial slip velocity $\Delta u_{\mathrm{s}}$ was given. For the purposes of this simulation, the shock simply produces an immediate velocity drop of $\Delta u_{\mathrm{s}}=50 \mathrm{~m} \mathrm{~s}^{-1}$. The densities are set to $\rho_{\mathrm{p}}=900 \mathrm{~kg} \mathrm{~m}^{-3}$ and $\rho_{\mathrm{f}}=3.826 \mathrm{~kg} \mathrm{~m}^{-3}$.

A slight vertical velocity component was added, in a linear horizontal gradient from a slight upwards velocity at the very left to a slight downward velocity on the very right (downstream). A similar gradient is observable in the measurement due to the curvature of the airfoil.

For each individual connection, a random displacement is added in order to simulate minor variations in the flow field. A Gaussian standard deviation of 0.5 px horizontally, and 0.1 px vertically, is used for this. Final particle image positions, after randomization, are recorded in a file to simplify calculating the error in the analysis.

## A. 5 Additional Results and Figures

## A.5.1 PIV-Like Analysis of Weight Maps

## Case A

Shown here are additional visualizations for the case A from Section 4.4.7.


Figure A.10: Horizontal derivative of the velocity field from the PIV-like analysis, case A, with Gaussian blur applied.


Figure A.11: Position of the shock wave (green line) on top of the original PIV-like velocity result, case A.

## Case B

In this section, all figures shown in Section 4.4.7 as well as Section A.5.1 are repeated for a different recorded PTV image (case B). Both were taken at very similar flow conditions, but with different seeding properties.


Figure A.12: Absolute velocity in the NFT camera image area: Visualization of the PIV-like analysis of the weight map data, case B in the image space.


Figure A.13: Horizontal velocity profiles in the image area from the PIV-like analysis of the NFT camera image, case B, shown in Figure A.12. Values of ${ }_{\mathrm{i}} \hat{x}_{2}$ are vertical locations in image coordinates, where 0.0 corresponds to the vertical center and 1.0 corresponds to the bottom edge of the image.


Figure A.14: Horizontal derivative of the velocity field from the PIV-like analysis, case B, with Gaussian blur applied.


Figure A.15: Horizontal profiles of the absolute velocity horizontal derivative in the image area from the PIV-like analysis, case B. Corresponds to the image shown in Figure A.14.


Figure A.16: Position of the shock wave (green line) on top of the original PIV-like velocity result, case B.

## A.5.2 Particle Trace Matching Quality

Shown here are collision test results and the total number of traces found for $8\left(n_{\mathrm{c}, \mathrm{t}}=7\right)$ and 9 $\left(n_{\mathrm{c}, \mathrm{t}}=8\right)$ illuminations, as discussed in Section 5.1. These figures cover values of the dilation radius of $r_{\mathrm{d}}=0 \mathrm{px}$ and $r_{\mathrm{d}}=2 \mathrm{px}$.


Figure A.17: Percentage of traces that survive collision-filtering, under variations of $\beta_{\max }$ and $W_{\mathrm{T}, \min }\left(r_{\mathrm{d}}=0 \mathrm{px}, 2 \mathrm{px}\right)$. The dark blue area highlights cases in which not a single trace remained, in which case the percentage is undefined.


Figure A.18: Number of traces found, under variations of $\beta_{\max }$ and $W_{\mathrm{T}, \min }\left(r_{\mathrm{d}}=0 \mathrm{px}, 2 \mathrm{px}\right)$.
(a)

$$
\begin{aligned}
r_{\mathrm{d}} & =0 \mathrm{px} \\
n_{\mathrm{c}, \mathrm{t}} & =8 \\
\sigma_{\mathrm{I}} & =2.0
\end{aligned}
$$


(b)


Figure A.19: Number of traces found for a higher number of pulses than actually captured ( $n_{\mathrm{c}, \mathrm{t}}=8$, i.e. extended by one), under variation of $\beta_{\max }$ and $W_{\mathrm{T}, \min }$ ( $\left.r_{\mathrm{d}}=0 \mathrm{px}, 2 \mathrm{px}\right)$.

## A.5.3 Displacement Experiment

The following tables show the statistical information on the individual connections of all image stacks used for the displacement calibration. In all of the tables, the connection and stack mean values are calculated with the respective connections and stacks in the table equally weighted, regardless of the number of traces each value is based on. The bottom right value shows the mean across all values in the respective table.

For each method, the connection length deviation scatter plot is also shown as a visual indication of the distribution of the errors across the stacks and across the distance from the center of the image.

## Images Without Added Noise

## Centroid Method



Figure A.20: Centroid method: Median deviation of measured connection length.

Centroid standard deviation (across traces)

| Connection no. <br> Units | $\mathbf{1}$ <br> $[\mathrm{px}]$ | $\mathbf{2}$ <br> $[\mathrm{px}]$ | $\mathbf{3}$ <br> $[\mathrm{px}]$ | $\mathbf{4}$ <br> $[\mathrm{px}]$ | Stack mean <br> $[\mathrm{px}]$ |
| :--- | ---: | :--- | ---: | ---: | ---: |
| Stack no. $\mathbf{1}$ | 0.1877 | 0.1642 | 0.1912 | 0.1674 | 0.1776 |
| Stack no. 2 | 0.2160 | 0.1412 | 0.1659 | $\mathrm{n} / \mathrm{a}$ | 0.1744 |
| Stack no. 3 | 0.2036 | 0.1712 | 0.2109 | 0.1763 | 0.1905 |
| Stack no. 4 | 0.1783 | 0.1705 | 0.1634 | 0.2103 | 0.1806 |
| Stack no. 5 | 0.2580 | 0.3082 | 0.2063 | 0.2631 | 0.2589 |
| Connection mean | 0.2087 | 0.1911 | 0.1875 | 0.2042 | 0.1976 |

Entire dataset: Median: 0.0042 px Mean: 0.0011 px Standard deviation: 0.1977 px
Table A.6: Centroid method: Median deviation of measured connection length.

Centroid standard deviation (across traces)

| Connection no. | $\mathbf{1}$ | $\mathbf{2}$ | $\mathbf{3}$ | $\mathbf{4}$ | Stack mean |
| :--- | :---: | :---: | :---: | :---: | ---: |
| Units | $\left[\frac{\mathrm{\mu m}}{\mathrm{px}}\right]$ | $\left[\frac{\mu \mathrm{m}}{\mathrm{px}}\right]$ | $\left[\frac{\mu \mathrm{m}}{\mathrm{px}}\right]$ | $\left[\frac{\mathrm{\mu m}}{\mathrm{px}}\right]$ | $\left[\frac{\mathrm{m}}{\mathrm{px}}\right]$ |
| Stack no. 1 | 0.0287 | 0.0226 | 0.0228 | 0.0234 | 0.0244 |
| Stack no. 2 | 0.0289 | 0.0196 | 0.0217 | $\mathrm{n} / \mathrm{a}$ | 0.0234 |
| Stack no. 3 | 0.0251 | 0.0280 | 0.0291 | 0.0214 | 0.0259 |
| Stack no. 4 | 0.0254 | 0.0209 | 0.0259 | 0.0255 | 0.0244 |
| Stack no. 5 | 0.0372 | 0.0381 | 0.0283 | 0.0366 | 0.0350 |
| Connection mean | 0.0291 | 0.0258 | 0.0256 | 0.0267 | 0.0268 |

Median (across traces)

| Connection no. | $\mathbf{1}$ | $\mathbf{2}$ | $\mathbf{3}$ | $\mathbf{4}$ | Stack mean |
| :--- | :---: | :---: | :---: | :---: | ---: |
| Units | $\left[\frac{\mathrm{mm}}{\mathrm{px}}\right]$ | $\left[\frac{\mathrm{\mu m}}{\mathrm{px}}\right]$ | $\left[\frac{\mathrm{\mu m}}{\mathrm{px}}\right]$ | $\left[\frac{\mathrm{mm}}{\mathrm{px}}\right]$ | $\left[\frac{\mathrm{\mu m}}{\mathrm{px}}\right]$ |
| Stack no. $\mathbf{1}$ | 20.2091 | 20.3043 | 20.3776 | 20.3785 | 20.3174 |
| Stack no. 2 | 19.7451 | 20.3806 | 20.4935 | $\mathrm{n} / \mathrm{a}$ | 20.2064 |
| Stack no. 3 | 20.1570 | 20.3680 | 20.4027 | 20.3589 | 20.3217 |
| Stack no. 4 | 20.1883 | 20.4311 | 20.4358 | 20.3432 | 20.3496 |
| Stack no. 5 | 20.2062 | 20.5503 | 20.3882 | 20.4638 | 20.4021 |
| Connection mean | 20.1011 | 20.4069 | 20.4196 | 20.3861 | 20.3254 |

Entire dataset: Median: $20.3653 \mu_{\mathrm{mpx}} \mathrm{p}^{-1} \quad$ Mean: $20.3115 \mu \mathrm{mpx}^{-1}$
Standard deviation: $0.1796 \mu \mathrm{mpx}^{-1}$
Table A.7: Centroid method: Measured displacement divided by analyzed connection length. This is equivalent to the image reproduction scale for each connection in each trace.

## Centroid standard deviation (across traces)

| Connection no. Units | $\begin{aligned} & \mathbf{1} \\ & {[\mathrm{px}]} \end{aligned}$ | $\begin{aligned} & \mathbf{2} \\ & {[\mathrm{px}]} \end{aligned}$ | $\begin{aligned} & \mathbf{3} \\ & {[\mathrm{px}]} \end{aligned}$ | $\begin{aligned} & \mathbf{4} \\ & {[\mathrm{px}]} \end{aligned}$ | Stack mean [px] |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Stack no. 1 | 0.1849 | 0.1655 | 0.1927 | 0.1692 | 0.1781 |
| Stack no. 2 | 0.2155 | 0.1417 | 0.1662 | n/a | 0.1744 |
| Stack no. 3 | 0.2067 | 0.1710 | 0.2123 | 0.1783 | 0.1921 |
| Stack no. 4 | 0.1777 | 0.1711 | 0.1628 | 0.2102 | 0.1805 |
| Stack no. 5 | 0.2606 | 0.3130 | 0.2020 | 0.2627 | 0.2596 |
| Connection mean | 0.2091 | 0.1925 | 0.1872 | 0.2051 | 0.1981 |

Entire dataset: Median: -0.0001 px Mean: 0.0002 px
Standard deviation: 0.2028 px
Table A.8: Centroid method: Median deviation of measured connection length, with distortion correction.

## Gaussian Method


$\Delta$ Connection $1 \Delta$ Connection $2 \Delta$ Connection 3 - Connection 4

- Stack no. 1 - Stack no. 2 - Stack no. 3 - Stack no. 4 - Stack no. 5

Figure A.21: Gaussian method: Median deviation of measured connection length.

Gaussian standard deviation (across traces)

| Connection no. Units | $\begin{aligned} & \mathbf{1} \\ & {[\mathrm{px}]} \end{aligned}$ | $\begin{aligned} & \mathbf{2} \\ & {[\mathrm{px}]} \end{aligned}$ | $\begin{aligned} & \mathbf{3} \\ & {[\mathrm{px}]} \end{aligned}$ | $\begin{aligned} & \mathbf{4} \\ & {[\mathrm{px}]} \end{aligned}$ | Stack mean [px] |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Stack no. 1 | 0.3687 | 0.3896 | 0.4393 | 0.3238 | 0.3804 |
| Stack no. 2 | 0.4577 | 0.4822 | 0.4145 | n/a | 0.4515 |
| Stack no. 3 | 0.3882 | 0.3341 | 0.4197 | 0.3794 | 0.3804 |
| Stack no. 4 | 0.4958 | 0.3629 | 0.3969 | 0.4205 | 0.4190 |
| Stack no. 5 | 0.4103 | 0.3629 | 0.3354 | 0.3004 | 0.3523 |
| Connection mean | 0.4241 | 0.3864 | 0.4011 | 0.3560 | 0.3938 |

Entire dataset: Median: 0.0006 px Mean: 0.0302 px Standard deviation: 0.4094 px
Table A.9: Gaussian method: Median deviation of measured connection length.

Gaussian standard deviation (across traces)

| Connection no. | $\mathbf{1}$ | $\mathbf{2}$ | $\mathbf{3}$ | $\mathbf{4}$ | Stack mean |
| :--- | :---: | :---: | :---: | :---: | ---: |
| Units | $\left[\frac{\mathrm{\mu m}}{\mathrm{px}}\right]$ | $\left[\frac{\mu \mathrm{m}}{\mathrm{px}}\right]$ | $\left[\frac{\mathrm{\mu m}}{\mathrm{px}}\right]$ | $\left[\frac{\mathrm{\mu m}}{\mathrm{px}}\right]$ | $\left[\frac{\mathrm{m}}{\mathrm{px}}\right]$ |
| Stack no. 1 | 0.0566 | 0.0536 | 0.0526 | 0.0453 | 0.0520 |
| Stack no. 2 | 0.0613 | 0.0671 | 0.0542 | $\mathrm{n} / \mathrm{a}$ | 0.0608 |
| Stack no. 3 | 0.0479 | 0.0546 | 0.0581 | 0.0460 | 0.0516 |
| Stack no. 4 | 0.0707 | 0.0445 | 0.0632 | 0.0509 | 0.0573 |
| Stack no. 5 | 0.0593 | 0.0449 | 0.0459 | 0.0417 | 0.0479 |
| Connection mean | 0.0591 | 0.0529 | 0.0548 | 0.0460 | 0.0536 |

Median (across traces)

| Connection no. | $\mathbf{1}$ | $\mathbf{2}$ | $\mathbf{3}$ | $\mathbf{4}$ | Stack mean |
| :--- | :---: | :---: | :---: | :---: | ---: |
| Units | $\left[\frac{\mathrm{\mu m}}{\mathrm{px}}\right]$ | $\left[\frac{\mathrm{\mu m}}{\mathrm{px}}\right]$ | $\left[\frac{\mathrm{\mu m}}{\mathrm{px}}\right]$ | $\left[\frac{\mathrm{\mu m}}{\mathrm{px}}\right]$ | $\left[\frac{\mathrm{m}}{\mathrm{px}}\right]$ |
| Stack no. 1 | 20.2023 | 20.2961 | 20.3886 | 20.3629 | 20.3125 |
| Stack no. 2 | 19.7550 | 20.3885 | 20.4876 | $\mathrm{n} / \mathrm{a}$ | 20.2103 |
| Stack no. 3 | 20.1794 | 20.3475 | 20.3949 | 20.3558 | 20.3194 |
| Stack no. 4 | 20.1804 | 20.4433 | 20.4093 | 20.3484 | 20.3454 |
| Stack no. 5 | 20.1948 | 20.5705 | 20.3703 | 20.4723 | 20.4019 |
| Connection mean | 20.1024 | 20.4092 | 20.4101 | 20.3848 | 20.3236 |

Entire dataset: Median: $20.3536 \mu \mathrm{px}^{-1} \quad$ Mean: $20.3153 \mu \mathrm{mpx}{ }^{-1}$
Standard deviation: $0.1853 \mu \mathrm{mpx}^{-1}$
Table A.10: Gaussian method: Measured displacement divided by analyzed connection length. This is equivalent to the image reproduction scale for each connection in each trace.

## Gaussian standard deviation (across traces)

| Connection no. Units | $\begin{aligned} & \mathbf{1} \\ & {[\mathrm{px}]} \end{aligned}$ | $\begin{aligned} & \mathbf{2} \\ & {[\mathrm{px}]} \end{aligned}$ | $\begin{aligned} & \mathbf{3} \\ & {[\mathrm{px}]} \end{aligned}$ | $\begin{aligned} & \mathbf{4} \\ & {[\mathrm{px}]} \end{aligned}$ | Stack mean [px] |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Stack no. 1 | 0.3683 | 0.3898 | 0.4388 | 0.3234 | 0.3801 |
| Stack no. 2 | 0.4575 | 0.4825 | 0.4147 | n/a | 0.4516 |
| Stack no. 3 | 0.3889 | 0.3339 | 0.4199 | 0.3800 | 0.3807 |
| Stack no. 4 | 0.4953 | 0.3630 | 0.3967 | 0.4206 | 0.4189 |
| Stack no. 5 | 0.4111 | 0.3636 | 0.3349 | 0.3007 | 0.3526 |
| Connection mean | 0.4242 | 0.3866 | 0.4010 | 0.3562 | 0.3939 |

Entire dataset: Median: -0.0397 px Mean: -0.0028 px Standard deviation: 0.4134 px
Table A.11: Gaussian method: Median deviation of measured connection length, with distortion correction.

## IILSS Method



- Connection $1 \Delta$ Connection $2 \Delta$ Connection 3 - Connection 4
- Stack no. 1 - Stack no. 2 - Stack no. 3 - Stack no. 4 - Stack no. 5

Figure A.22: IILSS method: Median deviation of measured connection length.

IILSS standard deviation (across traces)

| Connection no. Units | $\begin{aligned} & \mathbf{1} \\ & {[\mathrm{px}]} \end{aligned}$ | $\begin{aligned} & \mathbf{2} \\ & {[\mathrm{px}]} \end{aligned}$ | $\begin{aligned} & \mathbf{3} \\ & {[\mathrm{px}]} \end{aligned}$ | $\begin{aligned} & \mathbf{4} \\ & {[\mathrm{px}]} \end{aligned}$ | Stack mean [px] |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Stack no. 1 | 0.1646 | 0.1560 | 0.1959 | 0.1637 | 0.1701 |
| Stack no. 2 | 0.1325 | 0.1300 | 0.1757 | n/a | 0.1461 |
| Stack no. 3 | 0.2040 | 0.1424 | 0.1920 | 0.1708 | 0.1773 |
| Stack no. 4 | 0.2182 | 0.1939 | 0.1684 | 0.2557 | 0.2091 |
| Stack no. 5 | 0.2586 | 0.3316 | 0.2585 | 0.2304 | 0.2698 |
| Connection mean | 0.1956 | 0.1908 | 0.1981 | 0.2052 | 0.1970 |

Entire dataset: Median: 0.0041 px Mean: 0.0131 px Standard deviation: 0.1970 px
Table A.12: IILSS method: Median deviation of measured connection length.

IILSS standard deviation (across traces)

| Connection no. | $\mathbf{1}$ | $\mathbf{2}$ | $\mathbf{3}$ | $\mathbf{4}$ | Stack mean |
| :--- | :---: | :---: | :---: | :---: | ---: |
| Units | $\left[\frac{\mu \mathrm{m}}{\mathrm{px}}\right]$ | $\left[\frac{\mu \mathrm{m}}{\mathrm{px}}\right]$ | $\left[\frac{\mu \mathrm{m}}{\mathrm{px}}\right]$ | $\left[\frac{\mu \mathrm{m}}{\mathrm{px}}\right]$ | $\left[\frac{\mathrm{m}}{\mathrm{px}}\right]$ |
| Stack no. $\mathbf{1}$ | 0.0252 | 0.0215 | 0.0234 | 0.0229 | 0.0233 |
| Stack no. 2 | 0.0177 | 0.0181 | 0.0230 | $\mathrm{n} / \mathrm{a}$ | 0.0196 |
| Stack no. 3 | 0.0252 | 0.0233 | 0.0265 | 0.0207 | 0.0239 |
| Stack no. 4 | 0.0312 | 0.0237 | 0.0268 | 0.0310 | 0.0282 |
| Stack no. 5 | 0.0373 | 0.0410 | 0.0354 | 0.0321 | 0.0364 |
| Connection mean | 0.0273 | 0.0255 | 0.0270 | 0.0267 | 0.0266 |

Median (across traces)

| Connection no. | $\mathbf{1}$ | $\mathbf{2}$ | $\mathbf{3}$ | $\mathbf{4}$ | Stack mean |
| :--- | :---: | :---: | :---: | :---: | ---: |
| Units | $\left[\frac{\mathrm{m}}{\mathrm{px}}\right]$ | $\left[\frac{\mathrm{\mu m}}{\mathrm{px}}\right]$ | $\left[\frac{\mathrm{\mu m}}{\mathrm{px}}\right]$ | $\left[\frac{\mathrm{\mu m}}{\mathrm{px}}\right]$ | $\left[\frac{\mathrm{\mu m}}{\mathrm{px}}\right]$ |
| Stack no. 1 | 20.2032 | 20.3117 | 20.3806 | 20.3861 | 20.3204 |
| Stack no. 2 | 19.7511 | 20.3770 | 20.4940 | $\mathrm{n} / \mathrm{a}$ | 20.2074 |
| Stack no. 3 | 20.1533 | 20.3665 | 20.4062 | 20.3636 | 20.3224 |
| Stack no. 4 | 20.1863 | 20.4313 | 20.4404 | 20.3365 | 20.3486 |
| Stack no. 5 | 20.1987 | 20.5451 | 20.3851 | 20.4513 | 20.3950 |
| Connection mean | 20.0985 | 20.4063 | 20.4213 | 20.3844 | 20.3246 |

Entire dataset: Median: $20.3687 \mu_{\mathrm{px}^{-1}} \quad$ Mean: $20.3134 \mu_{\mathrm{p}}^{\mathrm{px}}{ }^{-1}$
Standard deviation: $0.1810 \mu \mathrm{mpx}{ }^{-1}$
Table A.13: IILSS method: Measured displacement divided by analyzed connection length. This is equivalent to the image reproduction scale for each connection in each trace.

IILSS standard deviation (across traces)

| Connection no. <br> Units | $\mathbf{1}$ <br> $[\mathrm{px}]$ | $\mathbf{2}$ <br> $[\mathrm{px}]$ | $\mathbf{3}$ <br> $[\mathrm{px}]$ | $\mathbf{4}$ <br> $[\mathrm{px}]$ | Stack mean <br> $[\mathrm{px}]$ |
| :--- | ---: | :--- | ---: | ---: | ---: |
| Stack no. 1 | 0.1649 | 0.1559 | 0.1958 | 0.1636 | 0.1701 |
| Stack no. 2 | 0.1327 | 0.1300 | 0.1757 | $\mathrm{n} / \mathrm{a}$ | 0.1461 |
| Stack no. 3 | 0.2038 | 0.1425 | 0.1917 | 0.1708 | 0.1772 |
| Stack no. 4 | 0.2183 | 0.1940 | 0.1685 | 0.2557 | 0.2091 |
| Stack no. 5 | 0.2582 | 0.3312 | 0.2586 | 0.2304 | 0.2696 |
| Connection mean | 0.1956 | 0.1907 | 0.1981 | 0.2051 | 0.1970 |

Entire dataset: Median: -0.0111 px Mean: 0.0001 px Standard deviation: 0.1988 px
Table A.14: IILSS method: Median deviation of measured connection length, with distortion correction.


- Connection $1 \Delta$ Connection $2 \triangleleft$ Connection 3 - Connection 4
- Stack no. 1 - Stack no. 2 - Stack no. 3 • Stack no. 4 - Stack no. 5

Figure A.23: Median deviation of measured connection length. Low order symmetrical lens distortions were corrected.
Conversion from pixels to $\mu \mathrm{m}$ was performed using the results of the distortion curve fitting in conjunction with the fitted reproduction scale ratios for each stack and connection.


Figure A.24: IDLSS method: Median deviation of measured connection length.

IDLSS standard deviation (across traces)

| Connection no. <br> Units | $\mathbf{1}$ <br> $[\mathrm{px}]$ | $\mathbf{2}$ <br> $[\mathrm{px}]$ | $\mathbf{3}$ <br> $[\mathrm{px}]$ | $\mathbf{4}$ <br> $[\mathrm{px}]$ | Stack mean <br> $[\mathrm{px}]$ |
| :--- | ---: | :---: | :---: | :---: | ---: |
| Stack no. 1 | 0.1866 | 0.1875 | 0.2228 | 0.1839 | 0.1952 |
| Stack no. 2 | 0.1457 | 0.1493 | 0.2219 | $\mathrm{n} / \mathrm{a}$ | 0.1723 |
| Stack no. 3 | 0.2412 | 0.1586 | 0.2107 | 0.1788 | 0.1973 |
| Stack no. 4 | 0.2605 | 0.2101 | 0.2233 | 0.2897 | 0.2459 |
| Stack no. 5 | 0.2666 | 0.3521 | 0.3087 | 0.2498 | 0.2943 |
| Connection mean | 0.2201 | 0.2115 | 0.2375 | 0.2255 | 0.2236 |

Entire dataset: Median: 0.0035 px Mean: 0.0113 px
Standard deviation: 0.2230 px
Table A.15: IDLSS method: Median deviation of measured connection length.

IDLSS standard deviation (across traces)

| Connection no. | $\mathbf{1}$ | $\mathbf{2}$ | $\mathbf{3}$ | $\mathbf{4}$ | Stack mean |
| :--- | :---: | :---: | :---: | :---: | ---: |
| Units | $\left[\frac{\mathrm{\mu m}}{\mathrm{px}}\right]$ | $\left[\frac{\mathrm{\mu m}}{\mathrm{px}}\right]$ | $\left[\frac{\mathrm{\mu m}}{\mathrm{px}}\right]$ | $\left[\frac{\mathrm{mm}}{\mathrm{px}}\right]$ | $\left[\frac{\mathrm{mm}}{\mathrm{px}}\right]$ |
| Stack no. $\mathbf{1}$ | 0.0286 | 0.0259 | 0.0266 | 0.0257 | 0.0267 |
| Stack no. 2 | 0.0195 | 0.0207 | 0.0291 | $\mathrm{n} / \mathrm{a}$ | 0.0231 |
| Stack no. 3 | 0.0297 | 0.0259 | 0.0291 | 0.0217 | 0.0266 |
| Stack no. 4 | 0.0372 | 0.0257 | 0.0355 | 0.0351 | 0.0334 |
| Stack no. 5 | 0.0384 | 0.0436 | 0.0422 | 0.0347 | 0.0397 |
| Connection mean | 0.0307 | 0.0284 | 0.0325 | 0.0293 | 0.0303 |

Median (across traces)

| Connection no. | $\mathbf{1}$ | $\mathbf{2}$ | $\mathbf{3}$ | $\mathbf{4}$ | Stack mean |
| :--- | :---: | :---: | :---: | :---: | ---: |
| Units | $\left[\frac{\mu \mathrm{m}}{\mathrm{px}}\right]$ | $\left[\frac{\mu \mathrm{m}}{\mathrm{px}}\right]$ | $\left[\frac{\mathrm{mm}}{\mathrm{px}}\right]$ | $\left[\frac{\mu \mathrm{m}}{\mathrm{px}}\right]$ | $\left[\frac{\mu \mathrm{m}}{\mathrm{px}}\right]$ |
| Stack no. 1 | 20.2008 | 20.3166 | 20.3856 | 20.3886 | 20.3229 |
| Stack no. 2 | 19.7495 | 20.3791 | 20.4951 | $\mathrm{n} / \mathrm{a}$ | 20.2079 |
| Stack no. 3 | 20.1545 | 20.3737 | 20.4118 | 20.3600 | 20.3250 |
| Stack no. 4 | 20.1876 | 20.4348 | 20.4482 | 20.3443 | 20.3537 |
| Stack no. 5 | 20.1895 | 20.5558 | 20.3881 | 20.4533 | 20.3967 |
| Connection mean | 20.0964 | 20.4120 | 20.4257 | 20.3866 | 20.3272 |

Entire dataset: Median: $20.3689 \mu_{\mathrm{mpx}^{-1}} \quad$ Mean: $20.3147 \mu \mathrm{mpx}{ }^{-1}$
Standard deviation: $0.1822 \mu \mathrm{mpx}{ }^{-1}$
Table A.16: IDLSS method: Measured displacement divided by analyzed connection length.
This is equivalent to the image reproduction scale for each connection in each trace.

IDLSS standard deviation (across traces)

| Connection no. | $\mathbf{1}$ <br> $[\mathrm{px}]$ | $\mathbf{2}$ <br> $[\mathrm{px}]$ | $\mathbf{3}$ <br> $[\mathrm{px}]$ | $\mathbf{4}$ <br> $[\mathrm{px}]$ | Stack mean <br> $[\mathrm{px}]$ |
| :--- | :---: | :---: | :---: | ---: | ---: |
| Units | 0.1865 | 0.1875 | 0.2228 | 0.1839 | 0.1952 |
| Stack no. 1 | 0.1457 | 0.1493 | 0.2220 | $\mathrm{n} / \mathrm{a}$ | 0.1723 |
| Stack no. 2 | 0.2413 | 0.1586 | 0.2108 | 0.1789 | 0.1974 |
| Stack no. 3 | 0.2605 | 0.2100 | 0.2233 | 0.2897 | 0.2459 |
| Stack no. 4 | 0.2667 | 0.3523 | 0.3086 | 0.2498 | 0.2943 |
| Stack no. 5 | 0.2201 | 0.2115 | 0.2375 | 0.2256 | 0.2236 |
| Connection mean | Mean: -0.0002 px |  |  |  |  |
| Entire dataset: | Median: -0.0018 px | Mandard deviation: 0.2256 px |  |  |  |

Table A.17: IDLSS method: Median deviation of measured connection length, with distortion correction.

## ISILSS Method



Figure A.25: ISILSS method: Median deviation of measured connection length.

ISILSS standard deviation (across traces)

| Connection no. <br> Units | $\mathbf{1}$ <br> $[\mathrm{px}]$ | $\mathbf{2}$ <br> $[\mathrm{px}]$ | $\mathbf{3}$ <br> $[\mathrm{px}]$ | $\mathbf{4}$ <br> $[\mathrm{px}]$ | Stack mean <br> $[\mathrm{px}]$ |
| :--- | ---: | :--- | ---: | ---: | ---: |
| Stack no. 1 | 0.1696 | 0.1549 | 0.1995 | 0.1479 | 0.1680 |
| Stack no. 2 | 0.1512 | 0.1464 | 0.1756 | $\mathrm{n} / \mathrm{a}$ | 0.1577 |
| Stack no. 3 | 0.1958 | 0.1433 | 0.2061 | 0.1659 | 0.1778 |
| Stack no. 4 | 0.2159 | 0.1930 | 0.1614 | 0.2395 | 0.2024 |
| Stack no. 5 | 0.2509 | 0.3093 | 0.2504 | 0.2063 | 0.2542 |
| Connection mean | 0.1967 | 0.1894 | 0.1986 | 0.1899 | 0.1938 |

Entire dataset: Median: 0.0009 px Mean: 0.0090 px Standard deviation: 0.1936 px
Table A.18: ISILSS method: Median deviation of measured connection length.

ISILSS standard deviation (across traces)

| Connection no. | $\mathbf{1}$ | $\mathbf{2}$ | $\mathbf{3}$ | $\mathbf{4}$ | Stack mean |
| :--- | :---: | :---: | :---: | :---: | ---: |
| Units | $\left[\frac{\mathrm{\mu m}}{\mathrm{px}}\right]$ | $\left[\frac{\mathrm{\mu m}}{\mathrm{px}}\right]$ | $\left[\frac{\mathrm{\mu m}}{\mathrm{px}}\right]$ | $\left[\frac{\mathrm{\mu m}}{\mathrm{px}}\right]$ | $\left[\frac{\mathrm{\mu m}}{\mathrm{px}}\right]$ |
| Stack no. $\mathbf{1}$ | 0.0260 | 0.0214 | 0.0238 | 0.0207 | 0.0230 |
| Stack no. 2 | 0.0202 | 0.0203 | 0.0230 | $\mathrm{n} / \mathrm{a}$ | 0.0212 |
| Stack no. 3 | 0.0241 | 0.0234 | 0.0285 | 0.0201 | 0.0240 |
| Stack no. 4 | 0.0308 | 0.0236 | 0.0257 | 0.0290 | 0.0273 |
| Stack no. 5 | 0.0362 | 0.0382 | 0.0343 | 0.0287 | 0.0343 |
| Connection mean | 0.0275 | 0.0254 | 0.0270 | 0.0246 | 0.0262 |

Median (across traces)

| Connection no. | $\mathbf{1}$ | $\mathbf{2}$ | $\mathbf{3}$ | $\mathbf{4}$ | Stack mean |
| :--- | :---: | :---: | :---: | :---: | ---: |
| Units | $\left[\frac{\mu \mathrm{m}}{\mathrm{px}}\right]$ | $\left[\frac{\mathrm{\mu m}}{\mathrm{px}}\right]$ | $\left[\frac{\mathrm{\mu m}}{\mathrm{px}}\right]$ | $\left[\frac{\mathrm{\mu m}}{\mathrm{px}}\right]$ | $\left[\frac{\mu \mathrm{m}}{\mathrm{px}}\right]$ |
| Stack no. $\mathbf{1}$ | 20.2038 | 20.3113 | 20.3822 | 20.3860 | 20.3208 |
| Stack no. 2 | 19.7507 | 20.3724 | 20.4889 | $\mathrm{n} / \mathrm{a}$ | 20.2040 |
| Stack no. 3 | 20.1610 | 20.3635 | 20.4045 | 20.3645 | 20.3234 |
| Stack no. 4 | 20.1880 | 20.4268 | 20.4456 | 20.3364 | 20.3492 |
| Stack no. 5 | 20.1968 | 20.5513 | 20.3856 | 20.4480 | 20.3954 |
| Connection mean | 20.1001 | 20.4051 | 20.4214 | 20.3837 | 20.3246 |

Entire dataset: Median: $20.3664 \mu_{\mathrm{mpx}^{-1}} \quad$ Mean: $20.3128 \mu \mathrm{mpx}{ }^{-1}$
Standard deviation: $0.1804 \mu \mathrm{mpx}{ }^{-1}$
Table A.19: ISILSS method: Measured displacement divided by analyzed connection length. This is equivalent to the image reproduction scale for each connection in each trace.

ISILSS standard deviation (across traces)

| Connection no. Units | $\begin{aligned} & \mathbf{1} \\ & {[\mathrm{px}]} \end{aligned}$ | $\begin{aligned} & \mathbf{2} \\ & {[\mathrm{px}]} \end{aligned}$ | $\begin{aligned} & \mathbf{3} \\ & {[\mathrm{px}]} \end{aligned}$ | $\begin{aligned} & 4 \\ & {[\mathrm{px}]} \end{aligned}$ | Stack mean [px] |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Stack no. 1 | 0.1702 | 0.1548 | 0.1993 | 0.1478 | 0.1680 |
| Stack no. 2 | 0.1514 | 0.1463 | 0.1754 | n/a | 0.1577 |
| Stack no. 3 | 0.1954 | 0.1433 | 0.2056 | 0.1657 | 0.1775 |
| Stack no. 4 | 0.2161 | 0.1930 | 0.1616 | 0.2395 | 0.2026 |
| Stack no. 5 | 0.2503 | 0.3084 | 0.2507 | 0.2062 | 0.2539 |
| Connection mean | 0.1967 | 0.1892 | 0.1985 | 0.1898 | 0.1937 |
| Entire dataset: M | Median: -0.0125 px Mean: 0.0000 px Standard deviation: 0.1972 px |  |  |  |  |

Table A.20: ISILSS method: Median deviation of measured connection length, with distortion correction.

## ISDLSS Method



Figure A.26: ISDLSS method: Median deviation of measured connection length.

## ISDLSS standard deviation (across traces)

| Connection no. <br> Units | $\mathbf{1}$ <br> $[\mathrm{px}]$ | $\mathbf{2}$ <br> $[\mathrm{px}]$ | $\mathbf{3}$ <br> $[\mathrm{px}]$ | $\mathbf{4}$ <br> $[\mathrm{px}]$ | Stack mean <br> $[\mathrm{px}]$ |
| :--- | ---: | ---: | ---: | ---: | ---: |
| Stack no. 1 | 0.1901 | 0.1746 | 0.2377 | 0.1680 | 0.1926 |
| Stack no. 2 | 0.2378 | 0.1680 | 0.2045 | $\mathrm{n} / \mathrm{a}$ | 0.2034 |
| Stack no. 3 | 0.2096 | 0.1689 | 0.2296 | 0.2034 | 0.2029 |
| Stack no. 4 | 0.2502 | 0.2129 | 0.1922 | 0.2646 | 0.2300 |
| Stack no. 5 | 0.2844 | 0.3595 | 0.2842 | 0.2328 | 0.2902 |
| Connection mean | 0.2344 | 0.2168 | 0.2296 | 0.2172 | 0.2249 |

Entire dataset: Median: 0.0008 px Mean: 0.0167 px Standard deviation: 0.2247 px
Table A.21: ISDLSS method: Median deviation of measured connection length.

ISDLSS standard deviation (across traces)

| Connection no. | $\mathbf{1}$ | $\mathbf{2}$ | $\mathbf{3}$ | $\mathbf{4}$ | Stack mean |
| :--- | :---: | :---: | :---: | :---: | ---: |
| Units | $\left[\frac{\mathrm{\mu m}}{\mathrm{px}}\right]$ | $\left[\frac{\mathrm{\mu m}}{\mathrm{px}}\right]$ | $\left[\frac{\mathrm{\mu m}}{\mathrm{px}}\right]$ | $\left[\frac{\mathrm{mm}}{\mathrm{px}}\right]$ | $\left[\frac{\mathrm{mm}}{\mathrm{px}}\right]$ |
| Stack no. $\mathbf{1}$ | 0.0291 | 0.0241 | 0.0284 | 0.0235 | 0.0263 |
| Stack no. 2 | 0.0317 | 0.0233 | 0.0268 | $\mathrm{n} / \mathrm{a}$ | 0.0273 |
| Stack no. 3 | 0.0259 | 0.0276 | 0.0317 | 0.0247 | 0.0275 |
| Stack no. 4 | 0.0357 | 0.0261 | 0.0306 | 0.0320 | 0.0311 |
| Stack no. 5 | 0.0410 | 0.0445 | 0.0389 | 0.0324 | 0.0392 |
| Connection mean | 0.0327 | 0.0291 | 0.0313 | 0.0281 | 0.0304 |

Median (across traces)

| Connection no. | $\mathbf{1}$ | $\mathbf{2}$ | $\mathbf{3}$ | $\mathbf{4}$ | Stack mean |
| :--- | :---: | :---: | :---: | :---: | ---: |
| Units | $\left[\frac{\mu \mathrm{m}}{\mathrm{px}}\right]$ | $\left[\frac{\mu \mathrm{m}}{\mathrm{px}}\right]$ | $\left[\frac{\mathrm{mm}}{\mathrm{px}}\right]$ | $\left[\frac{\mu \mathrm{m}}{\mathrm{px}}\right]$ | $\left[\frac{\mu \mathrm{m}}{\mathrm{px}}\right]$ |
| Stack no. 1 | 20.2013 | 20.3117 | 20.3851 | 20.3848 | 20.3207 |
| Stack no. 2 | 19.7482 | 20.3736 | 20.4908 | $\mathrm{n} / \mathrm{a}$ | 20.2042 |
| Stack no. 3 | 20.1600 | 20.3619 | 20.4068 | 20.3635 | 20.3231 |
| Stack no. 4 | 20.1836 | 20.4309 | 20.4447 | 20.3377 | 20.3492 |
| Stack no. 5 | 20.1918 | 20.5546 | 20.3854 | 20.4525 | 20.3961 |
| Connection mean | 20.0970 | 20.4065 | 20.4226 | 20.3846 | 20.3247 |

Entire dataset: Median: $20.3671 \mu_{\mathrm{mpx}^{-1}} \quad$ Mean: $20.3134 \mu_{\mathrm{mpx}}{ }^{-1}$
Standard deviation: $0.1820 \mu \mathrm{mpx}{ }^{-1}$
Table A.22: ISDLSS method: Measured displacement divided by analyzed connection length. This is equivalent to the image reproduction scale for each connection in each trace.

ISDLSS standard deviation (across traces)

| Connection no. <br> Units | $\mathbf{1}$ <br> $[\mathrm{px}]$ | $\mathbf{2}$ <br> $[\mathrm{px}]$ | $\mathbf{3}$ <br> $[\mathrm{px}]$ | $\mathbf{4}$ <br> $[\mathrm{px}]$ | Stack mean <br> $[\mathrm{px}]$ |
| :--- | ---: | :---: | ---: | ---: | ---: |
| Stack no. 1 | 0.1933 | 0.1742 | 0.2368 | 0.1677 | 0.1930 |
| Stack no. 2 | 0.2382 | 0.1674 | 0.2038 | $\mathrm{n} / \mathrm{a}$ | 0.2031 |
| Stack no. 3 | 0.2081 | 0.1695 | 0.2268 | 0.2029 | 0.2018 |
| Stack no. 4 | 0.2514 | 0.2131 | 0.1936 | 0.2648 | 0.2308 |
| Stack no. 5 | 0.2810 | 0.3542 | 0.2852 | 0.2321 | 0.2881 |
| Connection mean | 0.2344 | 0.2157 | 0.2292 | 0.2169 | 0.2244 |
| Entire dataset: | Median: -0.0134 px | Mean: 0.0000 px |  |  |  |
|  | Standard deviation: 0.2287 px |  |  |  |  |

Table A.23: ISDLSS method: Median deviation of measured connection length, with distortion correction.

## Images With Reference Noise

## Centroid Method



- Connection $1 \Delta$ Connection 2 - Connection 3 - Connection 4
- Stack no. 1 - Stack no. 2 - Stack no. 3 • Stack no. 4 - Stack no. 5

Figure A.27: Centroid method with reference noise: Median deviation of measured connection length.

Centroid standard deviation (across traces)

| Connection no. Units | $\begin{aligned} & \mathbf{1} \\ & {[\mathrm{px}]} \end{aligned}$ | $\begin{aligned} & \mathbf{2} \\ & {[\mathrm{px}]} \end{aligned}$ | $\begin{aligned} & \mathbf{3} \\ & {[\mathrm{px}]} \end{aligned}$ | $\begin{aligned} & \mathbf{4} \\ & {[\mathrm{px}]} \end{aligned}$ | Stack mean [px] |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Stack no. 1 | 0.2070 | 0.1943 | 0.2421 | 0.2077 | 0.2128 |
| Stack no. 2 | 0.2523 | 0.1477 | 0.2716 | n/a | 0.2239 |
| Stack no. 3 | 0.2527 | 0.2175 | 0.2149 | 0.2338 | 0.2297 |
| Stack no. 4 | 0.2163 | 0.2794 | 0.2258 | 0.2762 | 0.2494 |
| Stack no. 5 | 0.3198 | 0.2971 | 0.2336 | 0.2688 | 0.2798 |
| Connection mean | 0.2496 | 0.2272 | 0.2376 | 0.2466 | 0.2399 |

Entire dataset: Median: 0.0041 px Mean: 0.0034 px Standard deviation: 0.2416 px

Table A.24: Centroid method with reference noise: Median deviation of measured connection length.

Centroid standard deviation (across traces)

| Connection no. | $\mathbf{1}$ | $\mathbf{2}$ | $\mathbf{3}$ | $\mathbf{4}$ | Stack mean |
| :--- | :---: | :---: | :---: | :---: | ---: |
| Units | $\left[\frac{\mathrm{m}}{\mathrm{px}}\right]$ | $\left[\frac{\mathrm{\mu m}}{\mathrm{px}}\right]$ | $\left[\frac{\mathrm{\mu m}}{\mathrm{px}}\right]$ | $\left[\frac{\mathrm{\mu m}}{\mathrm{px}}\right]$ | $\left[\frac{\mathrm{\mu m}}{\mathrm{px}}\right]$ |
| Stack no. $\mathbf{1}$ | 0.0317 | 0.0268 | 0.0289 | 0.0290 | 0.0291 |
| Stack no. 2 | 0.0338 | 0.0205 | 0.0355 | $\mathrm{n} / \mathrm{a}$ | 0.0299 |
| Stack no. 3 | 0.0312 | 0.0355 | 0.0297 | 0.0283 | 0.0312 |
| Stack no. 4 | 0.0308 | 0.0342 | 0.0359 | 0.0335 | 0.0336 |
| Stack no. 5 | 0.0461 | 0.0366 | 0.0320 | 0.0374 | 0.0380 |
| Connection mean | 0.0347 | 0.0307 | 0.0324 | 0.0321 | 0.0325 |

Median (across traces)

| Connection no. | $\mathbf{1}$ | $\mathbf{2}$ | $\mathbf{3}$ | $\mathbf{4}$ | Stack mean |
| :--- | ---: | :---: | :---: | ---: | ---: |
| Units | $\left[\frac{\mu \mathrm{m}}{\mathrm{px}}\right]$ | $\left[\frac{\mu \mathrm{m}}{\mathrm{px}}\right]$ | $\left[\frac{\mu \mathrm{m}}{\mathrm{px}}\right]$ | $\left[\frac{\mu \mathrm{m}}{\mathrm{px}}\right]$ | $\left[\frac{\mu \mathrm{m}}{\mathrm{px}}\right]$ |
| Stack no. $\mathbf{1}$ | 20.2082 | 20.3036 | 20.3774 | 20.3825 | 20.3179 |
| Stack no. 2 | 19.7433 | 20.3734 | 20.5053 | $\mathrm{n} / \mathrm{a}$ | 20.2073 |
| Stack no. 3 | 20.1551 | 20.3659 | 20.4097 | 20.3579 | 20.3222 |
| Stack no. 4 | 20.1877 | 20.4405 | 20.4374 | 20.3501 | 20.3539 |
| Stack no. 5 | 20.2201 | 20.5407 | 20.3737 | 20.4590 | 20.3984 |
| Connection mean | 20.1029 | 20.4048 | 20.4207 | 20.3874 | 20.3259 |

Entire dataset: Median: $20.3670 \mu_{\mathrm{mpx}^{-1}} \quad$ Mean: $20.3163 \mu \mathrm{mpx}{ }^{-1}$
Standard deviation: $0.1740 \mu \mathrm{~m} \mathrm{px}{ }^{-1}$
Table A.25: Centroid method with reference noise: Measured displacement divided by analyzed connection length. This is equivalent to the image reproduction scale for each connection in each trace.

Centroid standard deviation (across traces)

| Connection no. Units | $\begin{aligned} & \mathbf{1} \\ & {[\mathrm{px}]} \end{aligned}$ | $\begin{aligned} & \mathbf{2} \\ & {[\mathrm{px}]} \end{aligned}$ | $\begin{aligned} & \mathbf{3} \\ & {[\mathrm{px}]} \end{aligned}$ | $\begin{aligned} & \mathbf{4} \\ & {[\mathrm{px}]} \end{aligned}$ | Stack mean [px] |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Stack no. 1 | 0.2092 | 0.1931 | 0.2406 | 0.2046 | 0.2119 |
| Stack no. 2 | 0.2526 | 0.1464 | 0.2721 | n/a | 0.2237 |
| Stack no. 3 | 0.2516 | 0.2174 | 0.2126 | 0.2313 | 0.2282 |
| Stack no. 4 | 0.2166 | 0.2757 | 0.2253 | 0.2747 | 0.2481 |
| Stack no. 5 | 0.3173 | 0.2980 | 0.2338 | 0.2695 | 0.2796 |
| Connection mean | 0.2494 | 0.2261 | 0.2369 | 0.2450 | 0.2391 |

Entire dataset: Median: 0.0083 px Mean: -0.0002 px Standard deviation: 0.2409 px

Table A.26: Centroid method with reference noise: Median deviation of measured connection length, with distortion correction.

## Gaussian Method



Figure A.28: Gaussian method with reference noise: Median deviation of measured connection length.

Gaussian standard deviation (across traces)

| Connection no. <br> Units | $\mathbf{1}$ <br> $[\mathrm{px}]$ | $\mathbf{2}$ <br> $[\mathrm{px}]$ | $\mathbf{3}$ <br> $[\mathrm{px}]$ | $\mathbf{4}$ <br> $[\mathrm{px}]$ | Stack mean <br> $[\mathrm{px}]$ |
| :--- | ---: | ---: | ---: | ---: | ---: |
| Stack no. 1 | 0.3836 | 0.4434 | 0.4561 | 0.3172 | 0.4001 |
| Stack no. 2 | 0.4741 | 0.4869 | 0.4045 | $\mathrm{n} / \mathrm{a}$ | 0.4552 |
| Stack no. 3 | 0.3915 | 0.3755 | 0.4658 | 0.3731 | 0.4015 |
| Stack no. 4 | 0.4642 | 0.4318 | 0.3869 | 0.4329 | 0.4290 |
| Stack no. 5 | 0.3273 | 0.3662 | 0.3328 | 0.2898 | 0.3290 |
| Connection mean | 0.4081 | 0.4208 | 0.4092 | 0.3532 | 0.4002 |

Entire dataset: Median: 0.0000 px Mean: 0.0286 px Standard deviation: 0.4132 px
Table A.27: Gaussian method with reference noise: Median deviation of measured connection length.

Gaussian standard deviation (across traces)

| Connection no. | $\mathbf{1}$ | $\mathbf{2}$ | $\mathbf{3}$ | $\mathbf{4}$ | Stack mean |
| :--- | :---: | :---: | :---: | :---: | ---: |
| Units | $\left[\frac{\mathrm{m}}{\mathrm{px}}\right]$ | $\left[\frac{\mathrm{\mu m}}{\mathrm{px}}\right]$ | $\left[\frac{\mathrm{\mu m}}{\mathrm{px}}\right]$ | $\left[\frac{\mathrm{\mu m}}{\mathrm{px}}\right]$ | $\left[\frac{\mathrm{\mu m}}{\mathrm{px}}\right]$ |
| Stack no. $\mathbf{1}$ | 0.0589 | 0.0610 | 0.0546 | 0.0444 | 0.0547 |
| Stack no. 2 | 0.0634 | 0.0677 | 0.0531 | $\mathrm{n} / \mathrm{a}$ | 0.0614 |
| Stack no. 3 | 0.0482 | 0.0614 | 0.0644 | 0.0453 | 0.0548 |
| Stack no. 4 | 0.0661 | 0.0529 | 0.0616 | 0.0524 | 0.0583 |
| Stack no. 5 | 0.0471 | 0.0453 | 0.0455 | 0.0402 | 0.0445 |
| Connection mean | 0.0568 | 0.0577 | 0.0558 | 0.0456 | 0.0544 |

Median (across traces)

| Connection no. | $\mathbf{1}$ | $\mathbf{2}$ | $\mathbf{3}$ | $\mathbf{4}$ | Stack mean |
| :--- | ---: | :---: | :---: | ---: | ---: |
| Units | $\left[\frac{\mu \mathrm{m}}{\mathrm{px}}\right]$ | $\left[\frac{\mu \mathrm{m}}{\mathrm{px}}\right]$ | $\left[\frac{\mu \mathrm{m}}{\mathrm{px}}\right]$ | $\left[\frac{\mu \mathrm{m}}{\mathrm{px}}\right]$ | $\left[\frac{\mu \mathrm{m}}{\mathrm{px}}\right]$ |
| Stack no. 1 | 20.1981 | 20.3042 | 20.3908 | 20.3679 | 20.3153 |
| Stack no. 2 | 19.7595 | 20.3539 | 20.4984 | $\mathrm{n} / \mathrm{a}$ | 20.2039 |
| Stack no. 3 | 20.1633 | 20.3710 | 20.3977 | 20.3564 | 20.3221 |
| Stack no. 4 | 20.1807 | 20.4383 | 20.4136 | 20.3566 | 20.3473 |
| Stack no. 5 | 20.2052 | 20.5684 | 20.3526 | 20.4703 | 20.3991 |
| Connection mean | 20.1014 | 20.4072 | 20.4106 | 20.3878 | 20.3235 |

Entire dataset: Median: $20.3572 \mu \mathrm{mpx}^{-1} \quad$ Mean: $20.3197 \mathrm{~mm} \mathrm{px}^{-1}$
Standard deviation: $0.1803 \mu \mathrm{mpx}{ }^{-1}$
Table A.28: Gaussian method with reference noise: Measured displacement divided by analyzed connection length. This is equivalent to the image reproduction scale for each connection in each trace.

Gaussian standard deviation (across traces)

| Connection no. <br> Units | $\mathbf{1}$ <br> $[\mathrm{px}]$ | $\mathbf{2}$ <br> $[\mathrm{px}]$ | $\mathbf{3}$ <br> $[\mathrm{px}]$ | $\mathbf{4}$ <br> $[\mathrm{px}]$ | Stack mean <br> $[\mathrm{px}]$ |
| :--- | ---: | ---: | ---: | ---: | ---: |
| Stack no. 1 | 0.3843 | 0.4434 | 0.4566 | 0.3177 | 0.4005 |
| Stack no. 2 | 0.4742 | 0.4865 | 0.4048 | $\mathrm{n} / \mathrm{a}$ | 0.4552 |
| Stack no. 3 | 0.3902 | 0.3758 | 0.4659 | 0.3720 | 0.4010 |
| Stack no. 4 | 0.4643 | 0.4318 | 0.3873 | 0.4326 | 0.4290 |
| Stack no. 5 | 0.3265 | 0.3660 | 0.3334 | 0.2895 | 0.3288 |
| Connection mean | 0.4079 | 0.4207 | 0.4096 | 0.3529 | 0.4001 |

Entire dataset: Median: -0.0318 px Mean: -0.0018 px Standard deviation: 0.4151 px

Table A.29: Gaussian method with reference noise: Median deviation of measured connection length, with distortion correction.

## IILSS Method



Figure A.29: IILSS method with reference noise: Median deviation of measured connection length.

IILSS standard deviation (across traces)

| Connection no. <br> Units | $\mathbf{1}$ <br> $[\mathrm{px}]$ | $\mathbf{2}$ <br> $[\mathrm{px}]$ | $\mathbf{3}$ <br> $[\mathrm{px}]$ | $\mathbf{4}$ <br> $[\mathrm{px}]$ | Stack mean <br> $[\mathrm{px}]$ |
| :--- | ---: | :---: | :---: | ---: | ---: |
| Stack no. $\mathbf{1}$ | 0.1948 | 0.1537 | 0.1809 | 0.1714 | 0.1752 |
| Stack no. 2 | 0.1848 | 0.1328 | 0.2060 | $\mathrm{n} / \mathrm{a}$ | 0.1745 |
| Stack no. 3 | 0.1932 | 0.1523 | 0.1881 | 0.2005 | 0.1835 |
| Stack no. 4 | 0.1965 | 0.2782 | 0.1603 | 0.2508 | 0.2214 |
| Stack no. 5 | 0.2881 | 0.2714 | 0.2241 | 0.1969 | 0.2451 |
| Connection mean | 0.2115 | 0.1977 | 0.1919 | 0.2049 | 0.2013 |

Entire dataset: Median: 0.0028 px Mean: -0.0035 px Standard deviation: 0.2026 px

Table A.30: IILSS method with reference noise: Median deviation of measured connection length.

IILSS standard deviation (across traces)

| Connection no. | $\mathbf{1}$ | $\mathbf{2}$ | $\mathbf{3}$ | $\mathbf{4}$ | Stack mean |
| :--- | :---: | :---: | :---: | :---: | ---: |
| Units | $\left[\frac{\mu \mathrm{m}}{\mathrm{px}}\right]$ | $\left[\frac{\mathrm{\mu m}}{\mathrm{px}}\right]$ | $\left[\frac{\mu \mathrm{m}}{\mathrm{px}}\right]$ | $\left[\frac{\mathrm{m}}{\mathrm{px}}\right]$ | $\left[\frac{\mathrm{\mu m}}{\mathrm{px}}\right]$ |
| Stack no. 1 | 0.0298 | 0.0212 | 0.0216 | 0.0240 | 0.0241 |
| Stack no. 2 | 0.0247 | 0.0184 | 0.0270 | $\mathrm{n} / \mathrm{a}$ | 0.0234 |
| Stack no. 3 | 0.0238 | 0.0249 | 0.0260 | 0.0243 | 0.0247 |
| Stack no. 4 | 0.0280 | 0.0340 | 0.0255 | 0.0304 | 0.0295 |
| Stack no. 5 | 0.0416 | 0.0335 | 0.0307 | 0.0274 | 0.0333 |
| Connection mean | 0.0296 | 0.0264 | 0.0261 | 0.0265 | 0.0272 |

Median (across traces)

| Connection no. | $\mathbf{1}$ | $\mathbf{2}$ | $\mathbf{3}$ | $\mathbf{4}$ | Stack mean |
| :--- | :---: | :---: | :---: | :---: | ---: |
| Units | $\left[\frac{\mu \mathrm{m}}{\mathrm{px}}\right]$ | $\left[\frac{\mathrm{pm}}{\mathrm{px}}\right]$ | $\left[\frac{\mu \mathrm{m}}{\mathrm{px}}\right]$ | $\left[\frac{\mu \mathrm{m}}{\mathrm{px}}\right]$ | $\left[\frac{\mu \mathrm{m}}{\mathrm{px}}\right]$ |
| Stack no. 1 | 20.2045 | 20.3066 | 20.3832 | 20.3881 | 20.3206 |
| Stack no. 2 | 19.7475 | 20.3737 | 20.4948 | $\mathrm{n} / \mathrm{a}$ | 20.2053 |
| Stack no. 3 | 20.1521 | 20.3660 | 20.4103 | 20.3609 | 20.3223 |
| Stack no. 4 | 20.1836 | 20.4458 | 20.4443 | 20.3571 | 20.3577 |
| Stack no. 5 | 20.2136 | 20.5406 | 20.3688 | 20.4534 | 20.3941 |
| Connection mean | 20.1002 | 20.4065 | 20.4203 | 20.3899 | 20.3260 |

Entire dataset: Median: $20.3681 \mu_{\mathrm{mpx}^{-1}} \quad$ Mean: $20.3167 \mathrm{\mu m} \mathrm{px}^{-1}$
Standard deviation: $0.1744 \mu_{\mathrm{mpx}^{-1}}$
Table A.31: IILSS method with reference noise: Measured displacement divided by analyzed connection length. This is equivalent to the image reproduction scale for each connection in each trace.

IILSS standard deviation (across traces)

| Connection no. <br> Units | $\mathbf{1}$ <br> $[\mathrm{px}]$ | $\mathbf{2}$ <br> $[\mathrm{px}]$ | $\mathbf{3}$ <br> $[\mathrm{px}]$ | $\mathbf{4}$ <br> $[\mathrm{px}]$ | Stack mean <br> $[\mathrm{px}]$ |
| :--- | ---: | :---: | :---: | ---: | ---: |
| Stack no. $\mathbf{1}$ | 0.1992 | 0.1522 | 0.1786 | 0.1673 | 0.1743 |
| Stack no. 2 | 0.1855 | 0.1304 | 0.2067 | $\mathrm{n} / \mathrm{a}$ | 0.1742 |
| Stack no. 3 | 0.1893 | 0.1527 | 0.1858 | 0.1962 | 0.1810 |
| Stack no. 4 | 0.1971 | 0.2720 | 0.1632 | 0.2502 | 0.2206 |
| Stack no. 5 | 0.2837 | 0.2725 | 0.2249 | 0.1965 | 0.2444 |
| Connection mean | 0.2110 | 0.1960 | 0.1918 | 0.2025 | 0.2002 |

Entire dataset: Median: 0.0142 px Mean: -0.0004 px Standard deviation: 0.2009 px

Table A.32: IILSS method with reference noise: Median deviation of measured connection length, with distortion correction.

## IDLSS Method



Figure A.30: IDLSS method with reference noise: Median deviation of measured connection length.

IDLSS standard deviation (across traces)

| Connection no. <br> Units | $\mathbf{1}$ <br> $[\mathrm{px}]$ | $\mathbf{2}$ <br> $[\mathrm{px}]$ | $\mathbf{3}$ <br> $[\mathrm{px}]$ | $\mathbf{4}$ <br> $[\mathrm{px}]$ | Stack mean <br> $[\mathrm{px}]$ |
| :--- | ---: | :---: | ---: | ---: | ---: |
| Stack no. $\mathbf{1}$ | 0.2244 | 0.1941 | 0.2116 | 0.2192 | 0.2123 |
| Stack no. 2 | 0.1329 | 0.1488 | 0.2426 | $\mathrm{n} / \mathrm{a}$ | 0.1748 |
| Stack no. 3 | 0.2438 | 0.1691 | 0.2194 | 0.2196 | 0.2130 |
| Stack no. 4 | 0.2222 | 0.2686 | 0.2184 | 0.2925 | 0.2504 |
| Stack no. 5 | 0.2652 | 0.3320 | 0.3236 | 0.2254 | 0.2865 |
| Connection mean | 0.2177 | 0.2225 | 0.2431 | 0.2392 | 0.2302 |

Entire dataset: Median: 0.0030 px Mean: 0.0055 px Standard deviation: 0.2313 px
Table A.33: IDLSS method with reference noise: Median deviation of measured connection length.

IDLSS standard deviation (across traces)

| Connection no. | $\mathbf{1}$ | $\mathbf{2}$ | $\mathbf{3}$ | $\mathbf{4}$ | Stack mean |
| :--- | :---: | :---: | :---: | :---: | ---: |
| Units | $\left[\frac{\mathrm{m}}{\mathrm{px}}\right]$ | $\left[\frac{\mathrm{\mu m}}{\mathrm{px}}\right]$ | $\left[\frac{\mathrm{\mu m}}{\mathrm{px}}\right]$ | $\left[\frac{\mathrm{\mu m}}{\mathrm{px}}\right]$ | $\left[\frac{\mathrm{\mu m}}{\mathrm{px}}\right]$ |
| Stack no. $\mathbf{1}$ | 0.0344 | 0.0268 | 0.0252 | 0.0306 | 0.0293 |
| Stack no. 2 | 0.0178 | 0.0207 | 0.0318 | $\mathrm{n} / \mathrm{a}$ | 0.0234 |
| Stack no. 3 | 0.0301 | 0.0276 | 0.0303 | 0.0266 | 0.0286 |
| Stack no. 4 | 0.0317 | 0.0329 | 0.0347 | 0.0354 | 0.0337 |
| Stack no. 5 | 0.0383 | 0.0411 | 0.0443 | 0.0313 | 0.0387 |
| Connection mean | 0.0304 | 0.0298 | 0.0333 | 0.0310 | 0.0311 |

Median (across traces)

| Connection no. | $\mathbf{1}$ | $\mathbf{2}$ | $\mathbf{3}$ | $\mathbf{4}$ | Stack mean |
| :--- | ---: | :---: | :---: | ---: | ---: |
| Units | $\left[\frac{\mu \mathrm{m}}{\mathrm{px}}\right]$ | $\left[\frac{\mu \mathrm{m}}{\mathrm{px}}\right]$ | $\left[\frac{\mu \mathrm{m}}{\mathrm{px}}\right]$ | $\left[\frac{\mu \mathrm{m}}{\mathrm{px}}\right]$ | $\left[\frac{\mu \mathrm{m}}{\mathrm{px}}\right]$ |
| Stack no. $\mathbf{1}$ | 20.1993 | 20.3165 | 20.3886 | 20.3919 | 20.3241 |
| Stack no. 2 | 19.7460 | 20.3809 | 20.4907 | $\mathrm{n} / \mathrm{a}$ | 20.2058 |
| Stack no. 3 | 20.1532 | 20.3729 | 20.4089 | 20.3581 | 20.3233 |
| Stack no. 4 | 20.1890 | 20.4430 | 20.4364 | 20.3608 | 20.3573 |
| Stack no. 5 | 20.2038 | 20.5446 | 20.3711 | 20.4539 | 20.3934 |
| Connection mean | 20.0982 | 20.4116 | 20.4191 | 20.3912 | 20.3268 |

Entire dataset: Median: $20.3693 \mu_{\mathrm{mpx}^{-1}} \quad$ Mean: $20.3183 \mu \mathrm{mpx}^{-1}$
Standard deviation: $0.1751 \mu \mathrm{mpx}^{-1}$
Table A.34: IDLSS method with reference noise: Measured displacement divided by analyzed connection length. This is equivalent to the image reproduction scale for each connection in each trace.

IDLSS standard deviation (across traces)

| Connection no. Units | $\begin{aligned} & \mathbf{1} \\ & {[\mathrm{px}]} \end{aligned}$ | $\begin{aligned} & \mathbf{2} \\ & {[\mathrm{px}]} \end{aligned}$ | $\begin{aligned} & \mathbf{3} \\ & {[\mathrm{px}]} \end{aligned}$ | $\begin{aligned} & \mathbf{4} \\ & {[\mathrm{px}]} \end{aligned}$ | Stack mean [px] |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Stack no. 1 | 0.2261 | 0.1942 | 0.2110 | 0.2180 | 0.2123 |
| Stack no. 2 | 0.1344 | 0.1484 | 0.2426 | $\mathrm{n} / \mathrm{a}$ | 0.1751 |
| Stack no. 3 | 0.2421 | 0.1693 | 0.2187 | 0.2180 | 0.2120 |
| Stack no. 4 | 0.2224 | 0.2663 | 0.2194 | 0.2927 | 0.2502 |
| Stack no. 5 | 0.2639 | 0.3314 | 0.3240 | 0.2251 | 0.2861 |
| Connection mean | 0.2178 | 0.2219 | 0.2431 | 0.2384 | 0.2299 |

Entire dataset: Median: 0.0021 px Mean: -0.0003 px Standard deviation: 0.2332 px

Table A.35: IDLSS method with reference noise: Median deviation of measured connection length, with distortion correction.

## ISILSS Method



Figure A.31: ISILSS method with reference noise: Median deviation of measured connection length.

ISILSS standard deviation (across traces)

| Connection no. <br> Units | $\mathbf{1}$ <br> $[\mathrm{px}]$ | $\mathbf{2}$ <br> $[\mathrm{px}]$ | $\mathbf{3}$ <br> $[\mathrm{px}]$ | $\mathbf{4}$ <br> $[\mathrm{px}]$ | Stack mean <br> $[\mathrm{px}]$ |
| :--- | ---: | ---: | ---: | ---: | ---: |
| Stack no. 1 | 0.1847 | 0.1524 | 0.1828 | 0.1650 | 0.1712 |
| Stack no. 2 | 0.1863 | 0.1365 | 0.2104 | $\mathrm{n} / \mathrm{a}$ | 0.1778 |
| Stack no. 3 | 0.1928 | 0.1536 | 0.1879 | 0.1949 | 0.1823 |
| Stack no. 4 | 0.1985 | 0.2724 | 0.1598 | 0.2461 | 0.2192 |
| Stack no. 5 | 0.2885 | 0.2758 | 0.2125 | 0.1905 | 0.2418 |
| Connection mean | 0.2102 | 0.1981 | 0.1907 | 0.1992 | 0.1996 |

Entire dataset: Median: 0.0015 px Mean: -0.0007 px Standard deviation: 0.2003 px
Table A.36: ISILSS method with reference noise: Median deviation of measured connection length.

ISILSS standard deviation (across traces)

| Connection no. | $\mathbf{1}$ | $\mathbf{2}$ | $\mathbf{3}$ | $\mathbf{4}$ | Stack mean |
| :--- | :---: | :---: | :---: | :---: | ---: |
| Units | $\left[\frac{\mathrm{m}}{\mathrm{px}}\right]$ | $\left[\frac{\mathrm{\mu m}}{\mathrm{px}}\right]$ | $\left[\frac{\mathrm{\mu m}}{\mathrm{px}}\right]$ | $\left[\frac{\mathrm{\mu m}}{\mathrm{px}}\right]$ | $\left[\frac{\mathrm{\mu m}}{\mathrm{px}}\right]$ |
| Stack no. $\mathbf{1}$ | 0.0283 | 0.0210 | 0.0218 | 0.0231 | 0.0236 |
| Stack no. 2 | 0.0249 | 0.0190 | 0.0276 | $\mathrm{n} / \mathrm{a}$ | 0.0238 |
| Stack no. 3 | 0.0238 | 0.0251 | 0.0260 | 0.0236 | 0.0246 |
| Stack no. 4 | 0.0283 | 0.0333 | 0.0254 | 0.0298 | 0.0292 |
| Stack no. 5 | 0.0416 | 0.0340 | 0.0291 | 0.0265 | 0.0328 |
| Connection mean | 0.0294 | 0.0265 | 0.0260 | 0.0257 | 0.0270 |

Median (across traces)

| Connection no. | $\mathbf{1}$ | $\mathbf{2}$ | $\mathbf{3}$ | $\mathbf{4}$ | Stack mean |
| :--- | ---: | :---: | :---: | ---: | ---: |
| Units | $\left[\frac{\mu \mathrm{m}}{\mathrm{px}}\right]$ | $\left[\frac{\mu \mathrm{m}}{\mathrm{px}}\right]$ | $\left[\frac{\mu \mathrm{m}}{\mathrm{px}}\right]$ | $\left[\frac{\mu \mathrm{m}}{\mathrm{px}}\right]$ | $\left[\frac{\mu \mathrm{m}}{\mathrm{px}}\right]$ |
| Stack no. 1 | 20.2029 | 20.3047 | 20.3847 | 20.3864 | 20.3197 |
| Stack no. 2 | 19.7443 | 20.3709 | 20.4929 | $\mathrm{n} / \mathrm{a}$ | 20.2027 |
| Stack no. 3 | 20.1523 | 20.3664 | 20.4062 | 20.3581 | 20.3208 |
| Stack no. 4 | 20.1821 | 20.4431 | 20.4438 | 20.3565 | 20.3564 |
| Stack no. 5 | 20.2152 | 20.5419 | 20.3675 | 20.4546 | 20.3948 |
| Connection mean | 20.0994 | 20.4054 | 20.4190 | 20.3889 | 20.3250 |

Entire dataset: Median: $20.3659 \mu_{\mathrm{mpx}^{-1}} \quad$ Mean: $20.3164 \mu_{\mathrm{mpx}}{ }^{-1}$
Standard deviation: $0.1742 \mu \mathrm{mpx}{ }^{-1}$
Table A.37: ISILSS method with reference noise: Measured displacement divided by analyzed connection length. This is equivalent to the image reproduction scale for each connection in each trace.

ISILSS standard deviation (across traces)

| Connection no. <br> Units | $\mathbf{1}$ <br> $[\mathrm{px}]$ | $\mathbf{2}$ <br> $[\mathrm{px}]$ | $\mathbf{3}$ <br> $[\mathrm{px}]$ | $\mathbf{4}$ <br> $[\mathrm{px}]$ | Stack mean <br> $[\mathrm{px}]$ |
| :--- | ---: | :---: | :---: | ---: | ---: |
| Stack no. $\mathbf{1}$ | 0.1896 | 0.1510 | 0.1803 | 0.1606 | 0.1704 |
| Stack no. 2 | 0.1871 | 0.1336 | 0.2111 | $\mathrm{n} / \mathrm{a}$ | 0.1773 |
| Stack no. 3 | 0.1890 | 0.1540 | 0.1854 | 0.1899 | 0.1796 |
| Stack no. 4 | 0.1994 | 0.2663 | 0.1622 | 0.2451 | 0.2182 |
| Stack no. 5 | 0.2839 | 0.2768 | 0.2133 | 0.1902 | 0.2410 |
| Connection mean | 0.2098 | 0.1963 | 0.1904 | 0.1965 | 0.1984 |

Entire dataset: Median: 0.0087 px Mean: -0.0003 px Standard deviation: 0.1991 px

Table A.38: ISILSS method with reference noise: Median deviation of measured connection length, with distortion correction.

## ISDLSS Method



Figure A.32: ISDLSS method with reference noise: Median deviation of measured connection length.

ISDLSS standard deviation (across traces)

| Connection no. <br> Units | $\mathbf{1}$ <br> $[\mathrm{px}]$ | $\mathbf{2}$ <br> $[\mathrm{px}]$ | $\mathbf{3}$ <br> $[\mathrm{px}]$ | $\mathbf{4}$ <br> $[\mathrm{px}]$ | Stack mean <br> $[\mathrm{px}]$ |
| :--- | ---: | :--- | ---: | ---: | ---: |
| Stack no. $\mathbf{1}$ | 0.2024 | 0.1715 | 0.2345 | 0.2834 | 0.2230 |
| Stack no. 2 | 0.1569 | 0.1591 | 0.2466 | $\mathrm{n} / \mathrm{a}$ | 0.1875 |
| Stack no. 3 | 0.2271 | 0.1797 | 0.2128 | 0.2318 | 0.2129 |
| Stack no. 4 | 0.2167 | 0.2592 | 0.1917 | 0.2947 | 0.2406 |
| Stack no. 5 | 0.3025 | 0.3365 | 0.2690 | 0.2224 | 0.2826 |
| Connection mean | 0.2211 | 0.2212 | 0.2309 | 0.2581 | 0.2315 |

Entire dataset: Median: 0.0030 px Mean: 0.0088 px Standard deviation: 0.2329 px
Table A.39: ISDLSS method with reference noise: Median deviation of measured connection length.

ISDLSS standard deviation (across traces)

| Connection no. | $\mathbf{1}$ | $\mathbf{2}$ | $\mathbf{3}$ | $\mathbf{4}$ | Stack mean |
| :--- | :---: | :---: | :---: | :---: | ---: |
| Units | $\left[\frac{\mathrm{m}}{\mathrm{px}}\right]$ | $\left[\frac{\mathrm{\mu m}}{\mathrm{px}}\right]$ | $\left[\frac{\mathrm{\mu m}}{\mathrm{px}}\right]$ | $\left[\frac{\mathrm{\mu m}}{\mathrm{px}}\right]$ | $\left[\frac{\mathrm{\mu m}}{\mathrm{px}}\right]$ |
| Stack no. $\mathbf{1}$ | 0.0310 | 0.0237 | 0.0280 | 0.0395 | 0.0305 |
| Stack no. 2 | 0.0210 | 0.0221 | 0.0323 | $\mathrm{n} / \mathrm{a}$ | 0.0251 |
| Stack no. 3 | 0.0280 | 0.0294 | 0.0294 | 0.0281 | 0.0287 |
| Stack no. 4 | 0.0309 | 0.0317 | 0.0304 | 0.0357 | 0.0322 |
| Stack no. 5 | 0.0437 | 0.0416 | 0.0368 | 0.0309 | 0.0383 |
| Connection mean | 0.0309 | 0.0297 | 0.0314 | 0.0336 | 0.0313 |

Median (across traces)

| Connection no. | $\mathbf{1}$ | $\mathbf{2}$ | $\mathbf{3}$ | $\mathbf{4}$ | Stack mean |
| :--- | ---: | :---: | :---: | ---: | ---: |
| Units | $\left[\frac{\mu \mathrm{m}}{\mathrm{px}}\right]$ | $\left[\frac{\mu \mathrm{m}}{\mathrm{px}}\right]$ | $\left[\frac{\mu \mathrm{m}}{\mathrm{px}}\right]$ | $\left[\frac{\mu \mathrm{m}}{\mathrm{px}}\right]$ | $\left[\frac{\mu \mathrm{m}}{\mathrm{px}}\right]$ |
| Stack no. $\mathbf{1}$ | 20.2001 | 20.3151 | 20.3886 | 20.3860 | 20.3225 |
| Stack no. 2 | 19.7509 | 20.3760 | 20.4921 | $\mathrm{n} / \mathrm{a}$ | 20.2063 |
| Stack no. 3 | 20.1552 | 20.3662 | 20.4075 | 20.3538 | 20.3207 |
| Stack no. 4 | 20.1820 | 20.4402 | 20.4386 | 20.3598 | 20.3552 |
| Stack no. 5 | 20.2008 | 20.5432 | 20.3744 | 20.4564 | 20.3937 |
| Connection mean | 20.0978 | 20.4081 | 20.4202 | 20.3890 | 20.3256 |

Entire dataset: Median: $20.3684 \mu_{m_{p x}}{ }^{-1} \quad$ Mean: $20.3182 \mu \mathrm{mpx}^{-1}$
Standard deviation: $0.1750 \mu \mathrm{~m} \mathrm{px}{ }^{-1}$
Table A.40: ISDLSS method with reference noise: Measured displacement divided by analyzed connection length. This is equivalent to the image reproduction scale for each connection in each trace.

ISDLSS standard deviation (across traces)

| Connection no. <br> Units | $\mathbf{1}$ <br> $[\mathrm{px}]$ | $\mathbf{2}$ <br> $[\mathrm{px}]$ | $\mathbf{3}$ <br> $[\mathrm{px}]$ | $\mathbf{4}$ <br> $[\mathrm{px}]$ | Stack mean <br> $[\mathrm{px}]$ |
| :--- | ---: | ---: | ---: | ---: | ---: |
| Stack no. 1 | 0.2039 | 0.1716 | 0.2342 | 0.2822 | 0.2230 |
| Stack no. 2 | 0.1577 | 0.1587 | 0.2466 | $\mathrm{n} / \mathrm{a}$ | 0.1877 |
| Stack no. 3 | 0.2259 | 0.1799 | 0.2121 | 0.2306 | 0.2121 |
| Stack no. 4 | 0.2169 | 0.2574 | 0.1924 | 0.2948 | 0.2404 |
| Stack no. 5 | 0.3016 | 0.3360 | 0.2695 | 0.2223 | 0.2824 |
| Connection mean | 0.2212 | 0.2207 | 0.2310 | 0.2575 | 0.2313 |

Entire dataset: Median: -0.0046 px Mean: -0.0001 px Standard deviation: 0.2351 px

Table A.41: ISDLSS method with reference noise: Median deviation of measured connection length, with distortion correction.


[^0]:    ${ }^{1}$ The LDAinOp project is supported by the German Federal Ministry for Economic Affairs and Energy (German: Bundesministerium für Wirtschaft und Energie) and carried out by the project lead, Airbus Operations GmbH, as well as four partners that include Airbus Group Innovations, Fraunhofer Gesellschaft, Lufthansa Technik GmbH and the German Aerospace Center (Deutsches Zentrum für Luft- und Raumfahrt e. V., DLR).

[^1]:    ${ }^{2}$ Mentioned studies include Tani, 1961; Klebanoff et al., 1955; Dryden, 1953; Gregory and Walker, 1956; Smith, 1959; Hama, 1957; Liepmann and Fila, 1947; Schiller, 1932.

[^2]:    ${ }^{1}$ Cryogenic Ludwieg tube Göttingen; German: Kryo-Rohrwindkanal Göttingen

[^3]:    ${ }^{2}$ Model: Thorlabs DET10A/M silicon detector, $0.8 \mathrm{~mm}^{2}$ active area, $200-1100 \mathrm{~nm}$ wavelength range (peak response at 730 nm ), 1 ns rise time (Thorlabs, Inc., 2017).

[^4]:    ${ }^{3}$ The specific Ruthenium complex is $\mathrm{Di}($ tripyridyl)ruthenium(II), often abbreviated to $\mathrm{Ru}(\operatorname{trpy})$.

[^5]:    ${ }^{1}$ The loss of contrast in this context refers to the reduction of the MTF from $100 \%$, i.e. $(1-\mathrm{T})$.

[^6]:    ${ }^{1}$ OpenCV is an open source library for computer vision applications. Its license is BSD-like.
    ${ }^{2} \mathbf{S c i P y}$ is an open source Python library containing modules that are useful for scientific applications. It is licensed under BSD-new.

[^7]:    ${ }^{3}$ The greyscale dilation was implemented using the SciPy function scipy.ndimage.grey_dilation().

[^8]:    ${ }^{4}$ The area is defined by a flat structuring element.

[^9]:    ${ }^{5}$ The linear interpolation uses the SciPy function scipy.interpolate.RectBivariateSpline(). A higher order interpolation can therefore easily be achieved by increasing the parameters kx and ky from the currently used value of 1 .

[^10]:    ${ }^{6}$ Here, scipy.interpolate.griddata() was used with linear interpolation between the value positions.

[^11]:    ${ }^{7}$ The class scipy.interpolate.RectBivariateSpline() was used without smoothing and with boundaries set to the image boundaries.
    ${ }^{8}$ See the documentation for scipy.interpolate.RectBivariateSpline(), parameter "s".

[^12]:    ${ }^{9}$ The implementation of this change was performed by the author of the particleresponse application, R. Konrath (DLR).

[^13]:    ${ }^{1}$ The corresponding bug appears to be fixed in the current release candidate of SciPy (version 1.0), but this version is not yet available for the platform on which this analysis was performed. Due to the Fortran implementation of these functions, which are called from Python code, building a workaround is not trivial. As earlier tests showed no significant benefit from smoothing, this was deemed acceptable.

