Quantum Annealing for Flight Gate Assignment

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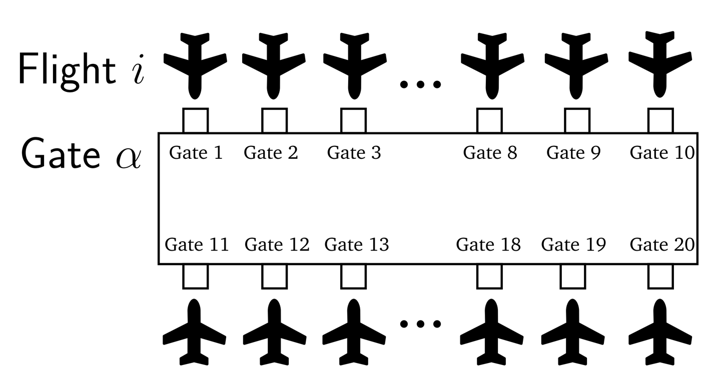
Abstract

We report on our efforts to solve a real world problem from airport planning with a D-Wave quantum annealer. The problem is related to the quadratic assignment problem which is known to be a hard combinatorial optimization problem. As problem instances, we use real data from a mid-size German airport. We present an analysis of the problem and compare quantum annealing solutions to classical approaches.

Problem Formulation

We investigate the problem of optimal assignment of flights to gates at airports [1]. The goal is to assign the flights in such a way, that the transit time for the passengers is minimal.



Problem variable:

$$x_{i\alpha} = \begin{cases} 1 & \text{if flight } i \text{ is assigned to gate } \alpha \\ 0 & \text{otherwise} \end{cases}$$

Cost Function Contribution

The cost function is the total transit time of all passengers.

1. Transit time of arriving passengers

$$T_{\text{Arrival}} = \sum_{i\alpha} n_i^a t_\alpha^a x_{i\alpha}$$

where
- t_α^a is the time it takes to get from gate α to the baggage claim
- n_i^a is the number of arriving passengers from flight i

2. Transit time of departing passengers

$$T_{\text{Departure}} = \sum_{i\alpha} n_i^d t_\alpha^d x_{i\alpha}$$

where
- t_α^d is the time it takes to get from check-in to gate α
- n_i^d is the number of departing passengers with flight i

3. Transit time of transferring passengers

$$T_{\text{Transfer}} = \sum_{ij\alpha\beta} n_{ij} t_{\alpha\beta} x_{i\alpha} x_{j\beta}$$

where
- $t_{\alpha\beta}$ is the time it takes to get from gate α to gate β
- n_{ij} is the number passengers transferring from flight i to flight j and vice versa

The total cost function reads

$$T_{\text{Total}} = T_{\text{Arrival}} + T_{\text{Departure}} + T_{\text{Transfer}} \quad (1)$$

Constraints

1. A single flight i can only be assigned to exactly one gate:

$$\forall i: \quad \sum_\alpha x_{i\alpha} = 1 \quad (2)$$

2. Flights with overlapping times (plus a buffer) must not be assigned to the same gate

$$\forall \alpha,\ \forall (i,j) \in F: \quad x_{i\alpha} \cdot x_{j\alpha} = 0 \quad (3)$$

with

$$F = \left\{ (i,j) \mid (t_i^{in} - t_j^{out} < t^{\text{buf}}) \wedge (t_j^{in} - t_i^{out} < t^{\text{buf}}) \right\}$$

Problem Instances

In order to investigate real world problems, we used the flight schedules and passenger flow of one single day at a mid-sized German airport. From this data, we extracted the problem parameters n_i^a, n_i^d and n_{ij} which are related to the passenger flow. The times from and to the gates t_α^a, t_α^d as well as in between the gates $t_{\alpha\beta}$ were extracted as mean values from agent based simulations [2]. Figure 1 shows the total problem instance which consists of 293 flights and 97 gates.

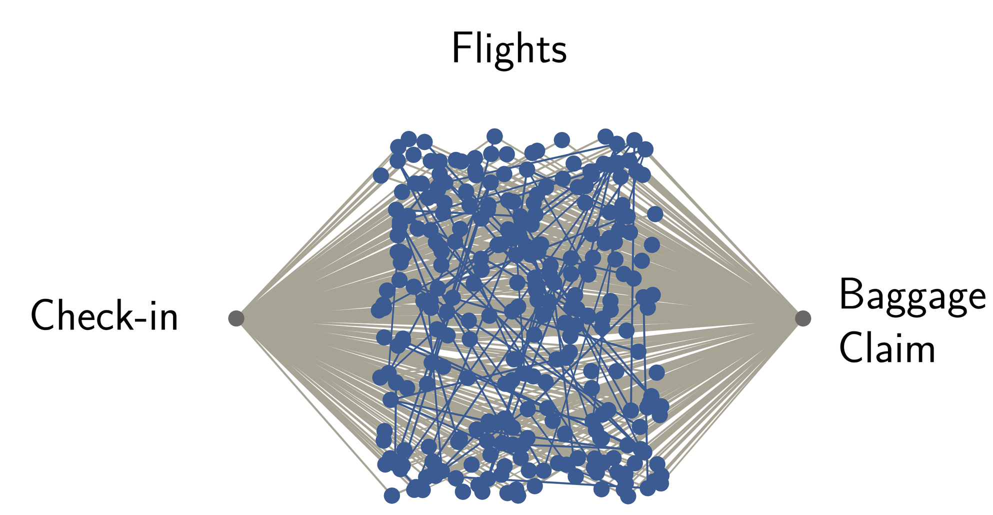


Figure 1: Total problem instance represented as a graph. The flights are represented as blue nodes and the blue edges represent passengers from one flight to another (n_{ij}). The gray nodes represent check-in and baggage claim. The gray edges represent passengers from check-in to the gates (n_i^a) and from the gates to baggage claim (n_i^d). The edge thickness represents the number of passengers.

Instance Subdivision

- Create smaller instances by restricting the time frame to durations of 20 minutes
- Use these smaller instances for testing on the quantum annealer

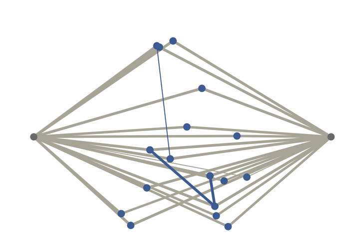


Figure 2: Example of an instance extracted from flights arriving within a 20 minute time frame.

Mapping to QUBO

- The cost function (1) is already a quadratic unconstrained binary optimization problem (QUBO)

$$Q_c = \frac{1}{f_{\text{A}}} T_{\text{Arrival}} + \frac{1}{f_{\text{D}}} T_{\text{Departure}} + \frac{1}{f_{\text{T}}} T_{\text{Transfer}}$$

  where f_{A}, f_{D} and f_{T} are normalization factors.
- The constraint (2) can be formulated as a penalty term

$$Q_u = \sum_i \left(\sum_\alpha x_{i\alpha} - 1 \right)^2$$

- The penalty term for the constraint (3) reads

$$Q_t = \sum_\alpha \sum_{(i,j)\in F} x_{i\alpha} x_{j\alpha}$$

- The total QUBO reads

$$Q_{\text{total}} = Q_c + \lambda_u Q_u + \lambda_t Q_t$$

  where we introduced the penalty weights λ_u and λ_t

Choice of the Penalty Weights

The penalty weights λ_u and λ_t have to be sufficiently large to ensure $Q_u = Q_t = 0$ in the solution.

- Solve QUBO exactly with classical solver
- Box shaped *validity boundary*
- Need to choose penalty weights “above“ validity boundary

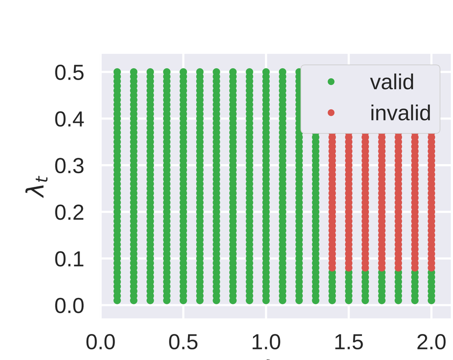


Figure 3: Example for a validity boundary of an instance extracted from flights arriving within a 20 minute time frame.

Quantum Annealing

Embedding

- Use D-Wave embedding algorithm on a D-Wave 2000Q

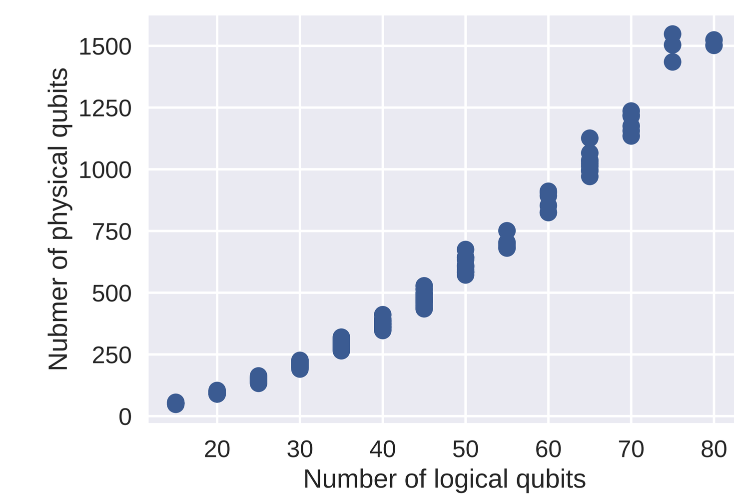


Figure 4: Number of logical and physical qubits for instances extracted from flights arriving within a 20 minute time frame and embedded on a D-Wave 2000Q

- Instances with up to approximately 100 logical variables are embeddable

Time to Solution

- 10000 annealing runs
- No gauges
- 3 different embeddings
- Expected time to solution with 99% success probability

$$T_{99} = \frac{\ln(1-0.99)}{\ln(1-p)} T_{\text{Anneal}}$$

  where p is the success probability
- Time to solution increases for larger problems. We conjecture, that this is related to the increased precision requirements for larger instances (cf. [3]).

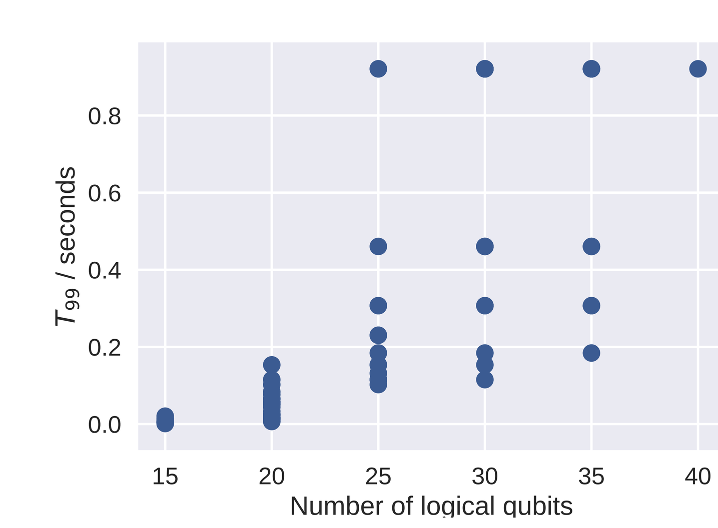


Figure 5: Time to solution with 99 % with probability T_{99} for QUBO instances extracted from flights arriving within a 20 minute time frame in dependence of the number of logical qubits.

Acknowledgment

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