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Fluid Love numbers with the matrix propagator method with an application to GJ436b

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Abstract

Investigations of the interior structure of exoplanets mostly rely on their measured radius and mass. Large observational errors and the intrinsic non-uniqueness of mass-radius relationships limit the inferences that can be made. The expected future determination of the fluid Love number k_2, which is sensitive to the interior structure, provides an additional constraint. We illustrate here an analytical method to calculate the fluid Love numbers based on the matrix propagator method, and apply it to the object GJ436b. We show that a precise measurement of its k_2 might partially solve some of the current degeneracies present in mass-radius relationships.

1. Introduction

The ever-blooming catalogue of exoplanets counts more than 3700 confirmed members (e.g., https://exoplanetarchive.ipac.caltech.edu). For about 10% of these objects a reliable mass M and radius R have been measured and the resulting bulk density provides a first handle on the interior structure—e.g., rocky vs. gaseous. However, a given M-R pair can be compatible with different interior structures as in the case of GJ436b [1]. The fluid Love number k_2, which depends on the concentration of mass in the interior similarly to the moment of inertia, can provide an additional constraint. Its determination can be obtained through photometry and/or measurement of the orbital precession. With improving instruments and continuous observations, the number of exoplanets for which M, R, and k_2 will be available will steadily increase.

2. Fluid Love numbers with the matrix propagator method

The calculation of the Love numbers can be obtained from the solution of the equation (e.g., [2])

$$T''_{\mathrm{n}}(r)+\frac{2}{r}T'_{\mathrm{n}}+\left[\frac{4\pi G\rho'(r)}{V'(r)}-\frac{n(n+1)}{r^2}\right]T_n(r)=0, \tag{1}$$

where r, V, and ρ are the radial coordinate, the gravitational potential, and the density of the unperturbed body (i.e., spherically symmetric), respectively. A prime indicates derivation with respect to r, and $V'(r) = -g(r)$ the gravitational acceleration. The function T has the dimensions of a potential and describes the total change in potential of the planet under a perturbing potential (e.g., tidal potential, rotational potential). Mixed boundary conditions are applied at the center ($r = 0$), where T must be finite, and at at the surface ($r = R_{\mathrm{P}}$), where its derivative must be continuous. It can be shown (e.g., [2]), that the fluid Love numbers k_n and h_n can be obtained from

$$k_{\mathrm{n}} = \frac{T_{\mathrm{n}}(R_{\mathrm{P}})}{R_{\mathrm{P}}g(R_{\mathrm{P}})}-1, \quad h_{\mathrm{n}} = k_{\mathrm{n}}+1 \tag{2}$$

where the subscript n indicates the degree in the harmonic expansion of the perturbing potential.

The solution of equation (1) with mixed boundary conditions can be obtained with the shooting method (e.g., [4]). Alternatively, by discretising the internal density profile of a given interior model, the term dependent on the radial density derivative in equation (1) can be dropped and the equation reduces to an Euler-Cauchy equation, which we solve with the matrix propagator method (e.g., [5]). We recast the second-order differential equation as a system of two first-order equations in the non-dimensional quantities $\mathbf{y}_1$ and $\mathbf{y}_2$, defined as

$$T = \frac{GM}{R}\mathbf{y}_1, \tag{3}$$

$$P = \frac{dT}{dr} = \frac{GM}{R^2}\left(\frac{R}{r}\right)\mathbf{y}_2. \tag{4}$$

By adopting power solutions for the $\mathbf{y}$'s and imposing continuity of T and P across internal boundaries–i.e., at each density jump–it is possible to obtain the solution for $\mathbf{y}$ at the surface and, through equations (2) and (3), to determine the value of k_{n}.

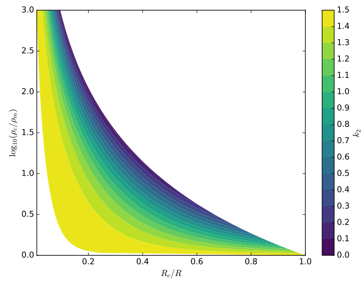

Figure 1: For a two constant-density layer model, fluid Love number k_2 (colors) as a function of the radius of the core and the density contrast $\rho_\mathrm{c}/\rho_\mathrm{m}$.

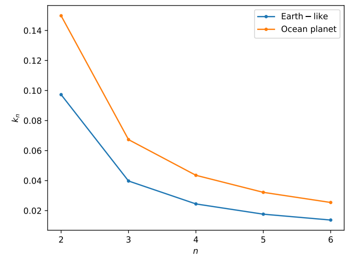

Figure 2: Value of k_n as a function of n for two different interior models of GJ436b, an Earth-like model and an Ocean-planet model (described in [1]).

3. Results

3.1. Interior structure and $\mathrm{k_2}$

We use a two constant-density layer planet model to illustrate the relation between the Love number k_2 and mass concentration in the planet. The parameters of the model are the core density ρ_c, the mantle density ρ_m, the radius of the core R_C, and the radius R. Figure 1 shows how k_2 varies as a function of the radius of the core and the density contrast between core and mantle. In the case of equal density in the core and mantle, or of a very small core, k_2 reaches the value of 1.5, characteristics of a homogeneous body. For the opposite case of a small high-density core, k_2 approaches the value of 0, which corresponds to a point-mass core surrounded by a massless envelope. Intermediate values indicate various concentration of mass in the interior.

3.2. Application to GJ436b

As an application we choose the planet GJ436b, a roughly 21 Earth masses and 4.3 Earth radii planet. We apply the matrix propagator method to two possible interior models [1], an Earth-like model with a very dense metallic core surrounded by a silicate mantle, and a water rich model with a silicate core and a water mantle. Both models have an outer hydrogen/helium envelope and are compatible with the measured mass and radius. We digitised the density profiles published in [1] and computed the fluid Love number k_n for n = 2,3,4,5,6. The results are shown in Figure 2. The Earth-like model, characterised by a dense core, which corresponds to a more concentrated density profile, has a smaller value for the k_n considered.

4. Summary and Conclusions

We developed a python package, which we intend to freely release, to compute the fluid Love numbers k_n and h_n for a given interior structure model using the matrix propagator method. The accurate knowledge of M, R, and k_n cannot remove the degeneracy present in interior structure models comprising more than two layers [3], but may allow to broadly discriminate the interior model classes, e.g., Earth-like versus Ocean-planet for the case GJ436b as illustrated in Figure 2.

Acknowledgements

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