Early-Late Discriminator Performance of CDMA: Limitations of the Spectral Separation Coefficient

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***Abstract*—Performance of global navigation satellite system (GNSS) receivers in the presence of intra- and intersystem interference needs to be carefully modeled and monitored for future navigation signal design or for performance assessment of safety-critical applications. Conventional performance models are based on the well-known spectral separation coefficient or related parameters, and rely on the false assumption that GNSS signals are wide-sense stationary, circularly-symmetric Gaussian random processes. We propose a new accurate non-Gaussian performance model for early-late discriminator performance, taking into account the signals' wide-sense cyclostationary property and their non-circularity. The model does not rely on the assumption of many power-balanced interfering satellites, and reliably predicts the shape of the code-phase error distribution, including its tails. We verify the proposed model with numerical simulations, considering all civil GPS/Galileo signals transmitted in the L1/E1 band.**

I. Introduction

Global navigation satellite systems (GNSS) rely on asynchronous code-division multiple access (CDMA) to allow simultaneous broadcast of multiple signals in a common frequency band. Unlike orthogonal multiple access strategies, asynchronous CDMA involves a controlled level of multiple access interference (MAI) received by the user. This leads to a slight degradation of the estimation and detection of key raw observables, such as code-phase, carrier-phase and navigation data. While the effect of MAI is more subtle than perturbances such as multipath propagation and ionospheric effects, it is a form of nominal, always present nuisance much like noise [1, Sec. 4.2]. As such, MAI is an important contributor to a navigation service's nominal performance, and should be considered accurately during the design of navigation signals [2] and the dimensioning and spectrum coordination of navigation systems [3], [4]. A conservative performance analysis is also of interest for integrity assessment of safety-critical services such as ground-based augmentation systems. [5]

The most common approach to model intra- and intersystem MAI is to approximate each interfering signal as a complex-valued random process that is non-white, wide-sense stationary (WSS), and circularly-symmetric Gaussian (CSG). Such a WSS/CSG random process is fully characterized by its power spectral density (PSD) as a function of frequency. For this model, the signal-to-interference ratio (SIR) observed at the prompt correlator output of a desired signal was shown to be the product of three factors: the ratio of desired to interfering power (prior to correlation), the predetection integration time, and the inverse of the spectral separation coefficient (SSC) [6]. The SSC is easily calculated as the inner product of the normalized PSDs of desired and interfering signal. It is maximized for the case of matched-spectrum interference (self-SSC). With the SIR as a single figure of merit, the SSC is used today to ensure radio frequency compatibility (RFC) in satellite navigation, following ITU-R M1831 [3], [7]. Usually, perfectly aperiodic spreading codes are assumed for the pseudorandom-noise (PRN) sequences, so that the smooth PSD resulting from the pulse shape can be used; however, for low data rates, the periodicity of the PRN should be taken into account, resulting in a PSD with fine features. [3], [8]–[10].

The SSC in its basic form is not suited to assess RFC in terms of code-phase estimation accuracy, except if the interference is not only WSS/CSG but also white within the considered bandwidth. For instance, for a signal with binary phase-shift keying modulation, non-white interference with power concentrated at the band edges may lead to acceptable SIR, but to unfavorable code-phase variance. A more accurate analysis is obtained if, rather than the prompt correlator SIR, the joint early and late correlator statistics are taken into account: [6], [8] provide expressions for the code-tracking variance for arbitrary correlator spacing, as well as for infinitesimally small correlator spacing (code-tracking SSC).

The main problem of using the above methods for GNSS MAI assessment is that they rely on the WSS/CSG assumption, even though not a single one of today's GNSS signals actually matches this model. (A counter-example would be IS-95-type CDMA signals with quaternary phase-shift keying, aperiodic codes, and low time-bandwidth product [10], [11].) The application of the WSS/CSG model for MAI assessment in GNSS is often supported by the intuitive argument that for many asynchronous satellites with random relative code- and carrier-phases, and with approximately the same power, the Gaussian approximation is valid due to the central limit

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theorem [3], [6], [7]. The WSS model may be justified by the fact that spreading codes are very long or have additional secondary code. However, it is unclear just how long the codes should be, or how many satellites there should be, in order for these approximations to be valid. Also, the WSS/CSG model is not at all suited for performance analysis in near-far scenarios [12]. For a thorough analysis of MAI, the wide-sense cyclostationarity and non-circularity of the GNSS signals should be taken into account, as was recently proposed in [10].

In this work, we propose a novel analytical methodology to determine the non-Gaussian probability density function (pdf) of code-phase estimation error due to MAI *per satellite*. Rather than a WSS/CSG model, we use the wide-sense cyclostationary/non-circular (WSCS/NC) signal model from [10], taking into account primary (chips) and secondary (bits) modulation of all interplexed signal components. Monte Carlo simulations with the civil L1/E1 signals in space as defined in [13]–[15] serve as a means of verification. The proposed model is found to be already accurate for a single interfering satellite, which obviates the need to invoke the central limit theorem for many asynchronous and power-balanced interferers. Instead, the exact error pdf for two or more randomly asynchronous interferers can be obtained by convolution of the individual non-Gaussian pdfs provided in this work. The proposed model is also perfectly suitable for error analysis in near-far problems. One important result is that the classic WSS/CSG model generally tends to underestimate the tails of the code error pdf, both for legacy and modernized signals.

II. System Model

Consider the following complex baseband received signal

$$y(t) = x_1(t;\boldsymbol{\theta}_1) + x_2(t;\boldsymbol{\theta}_2) \tag{1}$$

given as the superposition of signals received from two satellites $k = 1, 2$. From the kth satellite, L_k components are received as a composite signal $x_k(t;\boldsymbol{\theta}_k)$, where the synchronization parameter vector $\boldsymbol{\theta}_k = [\tau_k, \nu_k, \phi_k]^T$ includes delay (code-phase) τ_k, Doppler shift ν_k and carrier-phase ϕ_k. Each composite signal is modeled as

$$x_k(t;\boldsymbol{\theta}_k) = e^{\mathrm{j}\phi_k} e^{\mathrm{j}2\pi\nu_k t} \sum_{\ell=1}^{L_k} e^{\mathrm{j}\alpha_{k,\ell}} \sqrt{C_{k,\ell}},\, s_{k,\ell}(t-\tau_k) \tag{2}$$

where $C_{k,\ell}$ and $\alpha_{k,\ell}$ denote the (k,ℓ)th component's power and modulation angle, respectively. For the (k,ℓ)th navigation signal, we adopt the generic WSCS/NC model from [10]

$$s_{k,\ell}(t) = \sum_{m=-\infty}^{\infty} b_{k,\ell}^{(m)} q_{k,\ell}(t - mT_{k,\ell}) \tag{3}$$

with a random binary sequence $b_{k,\ell}^{(m)} \in \{-1,+1\}$, and a waveform $q_{k,\ell}(t)$ with double-sided bandwidth W. The waveform is normalized to $\int_{-\infty}^{\infty} |q_{k,\ell}(f)|^2\,\mathrm{d}t = T_{k,\ell}$. We define its Fourier transform as $Q_{k,\ell}(f) = \int_{-\infty}^{\infty} q_{k,\ell}(t) e^{-\mathrm{j}2\pi f t}\,\mathrm{d}t$, which is strictly zero for frequencies outside the interval $[-W/2, W/2]$.

Let the desired signal component have index $(k,\ell) = (1,1)$ (we do not consider joint tracking of multiple components). We consider a coherent early-late delay-locked loop (DLL) operating on the received signal $z(t)$ with Doppler- and carrier-wipeoff, i.e., $z(t) \triangleq e^{-\mathrm{j}2\pi\nu_1 t} e^{-\mathrm{j}(\phi_1+\alpha_{1,1})} y(t)$. For a predetection integration time T_{int}, we define the prompt local replica

$$\hat{s}_{1,1}(t) = \sum_{n=1}^{N} b_{1,1}^{(n)} \hat{q}_{1,1}(t - \hat{\tau}_1 - nT_{1,1}) \tag{4}$$

with $N = \lceil T_{\mathrm{int}}/T_{1,1} \rceil$, and the delay estimate $\hat{\tau}_1$ coming from acquisition or an earlier tracking epoch. Note that the replica waveform $\hat{q}_{1,1}(t)$ may be mismatched with respect to the received waveform $q_{1,1}(t)$. By definition, the replica is normalized $N\int_{-\infty}^{\infty} |\hat{q}_{1,1}(t)|^2\,\mathrm{d}f = T_{\mathrm{int}}$, and has Fourier transform $\hat{Q}_{1,1}(f) \triangleq \int_{-\infty}^{\infty} \hat{q}_{1,1}(t) e^{-\mathrm{j}2\pi f t}\,\mathrm{d}t$.

To determine the unknown offset delay $\delta_1 \triangleq \hat{\tau}_1 - \tau_1$, the early and late correlator outputs are generated as

$$\begin{bmatrix} \mathcal{E}_1 \\ \mathcal{L}_1 \end{bmatrix} = \frac{1}{T_{\mathrm{int}}} \int_{-\infty}^{\infty} \begin{bmatrix} \hat{s}_{1,1}(t - \hat{\tau}_1 + d/2) \\ \hat{s}_{1,1}(t - \hat{\tau}_1 - d/2) \end{bmatrix}^* z(t)\,\mathrm{d}t \tag{5}$$

with double-sided correlator spacing $d > 0$. Finally, the offset delay estimate is obtained as

$$\hat{\delta}_1 = \frac{\mathrm{Re}\{\mathcal{E}_1 - \mathcal{L}_1\}}{K_1(d)\sqrt{C_{1,1}}}. \tag{6}$$

with the constant

$$K_1(d) = \frac{\int_{-\frac{W}{2}}^{\frac{W}{2}} 4\pi f\, \hat{Q}_{1,1}^*(f) Q_{1,1}(f) \sin(\pi f d)\,\mathrm{d}f}{\int_{-\frac{W}{2}}^{\frac{W}{2}} |\hat{Q}_{1,1}(f)|^2\,\mathrm{d}f}. \tag{7}$$

III. Spreading Code Models

The generic WSCS/NC model (3) can be used with or without consideration of the exact PRN codes. While this makes no difference for the analytical derivations, the former requires considerably more processing effort to evaluate the model numerically. In the following, we introduce the periodic code (PC) model with the actual deterministic PRNs, and the aperiodic code (AC) model with perfectly random primary code. Note that there are also other possible models: for instance, an elegant random but periodic model is used in [9].

A. Periodic code (PC) model

We model $b_{k,\ell}^{(m)}$ for all k,ℓ as independent sequences of independent and identically distributed (i.i.d.) elements, taking values $\{-1,+1\}$ with equal probability. The elements represent data, pseudo-data or secondary code bits used on the (k,ℓ)th signal component, while $1/T_{k,\ell}$ is the respective secondary modulation rate (e.g. 50 Hz for L1 C/A). The primary spreading code and the chip pulse shape are incorporated jointly in the purely deterministic spreading waveform $q_{k,\ell}(t)$, which is recurring every $T_{k,\ell}$ seconds. This waveform is modeled exactly as specified in the interface control documents (ICDs) [13]–[15], but ideally band-limited to the receiver bandwidth W. Such spreading waveforms have fine spectral

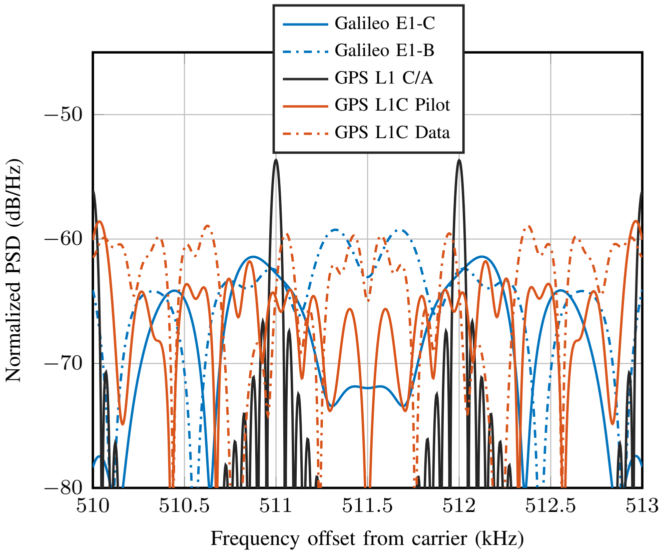

Fig. 1. Fine PSD features of Galileo and GPS civil signal components in the L1/E1 band. Exemplary PRNs and frequency range.

features on the order of Kilohertz resulting from the PRN, and are shown for an exemplary range of frequencies and PRNs in Fig. 1.

The local replica waveform $\hat{q}_{1,1}(t)$ is modeled likewise, but may be mismatched with respect to $q_{1,1}(t)$, to allow for the following relevant special cases:

- For L1 C/A, one may wish to consider only one (or more) length-1023 segments of the length-20460 primary code, if $T_{\text{int}} < T_{1,1} = 20\,\text{ms}$.
- For L1C Pilot or E1 OS, the receiver may use a BOC(1,1) reference pulse shape only, instead of using the respective MBOC modulation.

While the PC model is very accurate, it is rarely used for assessment of MAI due to its complexity (even though it should be). We shall see later in which scenarios its use is essential for an accurate analysis.

B. Aperiodic code (AC) model

We model $b_{k,\ell}^{(m)}$ for all k, ℓ as independent sequences of i.i.d. elements, taking values $\{-1,+1\}$ with equal probability. The elements represent the primary code chips. The primary spreading code is modeled as purely random and aperiodic, and absorbs the randomness of second-order modulation. The waveform $q_{k,\ell}(t)$ consists only of the chip pulse shape specified in the ICDs (ideally band-limited to W), and repeats every $T_{k,\ell}$ seconds, $1/T_{k,\ell}$ being the chipping rate. The local replica $\hat{q}_{1,1}(t)$ may be a mismatched pulse (relevant for L1C Pilot or E1 OS).

This considerably simpler model is actually not describing the navigation signal statistics as specified in the ICDs, and does not show the fine spectrum features from Fig. 1. However, it is reasonably accurate in many cases.

IV. Theoretical Code-Phase Estimation Performance

To evaluate code-phase estimation performance in the presence of MAI, we must find the error pdf $p_{\hat{\delta}_1}(\delta)$. For that, we consider the joint MAI from all components $s_{2,\ell}(t)$, $\ell = 1, \ldots, L_2$, while we neglect interchip-/intersymbol-interference from $s_{1,1}(t)$, as well as cross-talk from $s_{1,\ell}(t)$ for $\ell \neq 1$. For simplicity, we further assume that the true delay offset is $\delta_1 = 0$.

The WSS/CSG model naturally leads to a circular Gaussian distribution of the correlator outputs $(\mathcal{E}_1, \mathcal{L}_1)$ [6], hence to a Gaussian distributed $\hat{\delta}_1 \sim \mathcal{N}(0, \Sigma_S)$ with some variance Σ_S. However, this is not generally true for the WSCS/NC model. In the following, we argue under which conditions a Gaussian distribution can be assumed, and how this distribution can be transformed for the general case.

A. Conditional Gaussian approximation (CGA)

Correlator outputs of the form (5), with an energy-limited replica and a power-limited input signal, can be shown to converge in distribution to a *conditionally* Gaussian random variable for large N, if the only randomness comes from $b_{k,\ell}^{(m)}$ [11]. Since $\hat{\delta}_1$ is a widely linear combination of $\mathcal{E}_1$ and $\mathcal{L}_1$, it is also conditonally Gaussian distributed, which we denote by

$$\hat{\delta}_1 | \boldsymbol{\theta} \sim \mathcal{N}\big(0, \Sigma_C(\boldsymbol{\theta})\big). \tag{8}$$

Let the corresponding conditional pdf be given by $p_{\hat{\delta}_1|\boldsymbol{\theta}}(\delta|\boldsymbol{\theta})$. The conditional variance $\Sigma_C(\boldsymbol{\theta})$ for the WSCS/NC model in a delay/Doppler channel was first derived in [10]. If there is more than one component ($L_2 > 1$), the conditional variances simply add, since the binary sequences $b_{2,\ell}^{(m)}$ are conditionally independent over ℓ.

Note that except for the sequences $b_{k,\ell}^{(m)}$, all parameters, including the synchronization parameters $\boldsymbol{\theta} \triangleq [\boldsymbol{\theta}_1^T, \boldsymbol{\theta}_2^T]^T$, must be in the conditioning. In particular, the frequent assumption of uniformly random relative delay $\Delta\tau \triangleq \tau_2 - \tau_1$ and relative carrier-phase $\Delta\phi \triangleq \phi_2 - \phi_1$ will destroy the Gaussian shape of the distribution of $(\mathcal{E}_1, \mathcal{L}_1)$ and $\hat{\delta}_1$. Only when many power-balanced interferers are present, they will eventually assume a Gaussian shape again. Assuming moderate to large N is a much milder assumption than having many transmitters, and we will see that in practice, even short predetection integration times are sufficient for our purpose.

While this conditional Gaussian approximation (CGA) is very accurate, its dependency on the instantaneous values of the parameters $\Delta\tau, \Delta\phi$ makes it impractical to assess MAI on a larger scale. These parameters should actually be considered as asynchronous, hence uniformly random. In the remainder of this section, they are removed from the conditioning via expectation (standard Gaussian approximation) or marginalization (improved Gaussian approximation).

If one chooses not to use the assumption of randomly asynchronous satellites, the CGA can be evaluated point-by-point for a complete constellation scenario [10].

B. Standard Gaussian approximation (SGA) — SSC-like approach

The standard Gaussian approximation (SGA) [10], [11], [16] relies on the assumption that

$$\hat{\delta}_1 \sim \mathcal{N}\big(0, \mathrm{E}\left[\Sigma_C(\boldsymbol{\theta})\right]\big) \tag{9}$$

where the expectation is taken with respect to uniformly distributed $\Delta\phi \in [-\pi, \pi)$ and $\Delta\tau \in [-T_m, T_m)$, with $T_m = T_{1,1} + \max_\ell \{T_{2,\ell}\}$. This radical step simplifies the complexity of the performance model considerably, but completely neglects that the accuracy of code-phase estimation is actually a function of $\Delta\tau, \Delta\phi$. Interestingly, $\mathrm{E}[\Sigma_C(\boldsymbol{\theta})] = \Sigma_S$, i.e., the SGA is equivalent to the performance model that is obtained for the WSS/CSG approach [10], [11]. For instance, if $\mathrm{E}[\Sigma_C(\boldsymbol{\theta})]$ is calculated for very small correlator spacing, the result is simply a scaled version of the code-tracking SSC from [6].

C. Improved Gaussian Approximation (IGA)

By the law of total probability, we can obtain the error pdf for randomly uniform relative carrier-phase and relative delay by the marginalization

$$p_{\hat{\delta}_1}(\delta) = \frac{1}{2\pi}\frac{1}{2T_m}\int_{-\pi}^{\pi}\int_{-T_m}^{T_m} p_{\hat{\delta}_1|\boldsymbol{\theta}}(\delta|\boldsymbol{\theta})\, \mathrm{d}\Delta\tau\, \mathrm{d}\Delta\phi. \tag{10}$$

While the resulting pdf needs by no means be Gaussian, we refer to it as improved Gaussian approximation (IGA) since the required conditional pdf is based on a Gaussian approximation. The terminology (CGA, SGA, IGA,...) has been used for similar performance models assessing bit-error-rate in CDMA communications [11], [17], [18].

Note that the both SGA and IGA in the presented form are still conditional with respect to the Doppler frequencies ν_1, ν_2, which are not usually assumed uniformly random distributed.

V. Numerical Results

For the GPS/Galileo composite signal, we consider L1 C/A and L1C, or E1 OS, respectively, as transmitted at $1575.42\,\mathrm{MHz}$. Since we never consider more than two satellites at a time, we choose one representative PRN for each satellite, and generate the primary codes and pulse shapes exactly as defined in the respective ICD [13]–[15]. Bits or secondary code are generated randomly at the respective secondary modulation rate (L1 C/A: 50 Hz, L1C: 100 Hz, E1 OS: 250 Hz). The relative delay $\Delta\tau$ and relative carrier-phase $\Delta\phi$ are also random, while the relative Doppler $\Delta\nu = \nu_2 - \nu_1$ is fixed at the given value. Various combinations of pre-correlation bandwidth W, early-late spacing d and predetection integration time T_{int} are considered.

With notations like C/A←L1, we indicate that a C/A user experiences interference from the whole interplexed L1 signal, i.e., the composite signal of L1 C/A, L1C Pilot and L1C Data. For Galileo, the composite signal consists of E1-B and E1-C. Note that we assume a fixed relative power between the individual components of a composite signal. For instance, L1 C/A is 1 dB stronger than L1C Pilot and L1C Data combined,

TABLE I
Fixed power ratios between interplexed signal components (arbitrary power constants C_1 and C_2)

Signal	L1 C/A	L1C Data	L1C Pilot	E1-B	E1-C
$C_{1,\ell}$	$1.26\,C_1$	$0.25\,C_1$	$0.75\,C_1$	$0.50\,C_1$	$0.50\,C_1$
$C_{2,\ell}$	$1.26\,C_2$	$0.25\,C_2$	$0.75\,C_2$	$0.50\,C_2$	$0.50\,C_2$

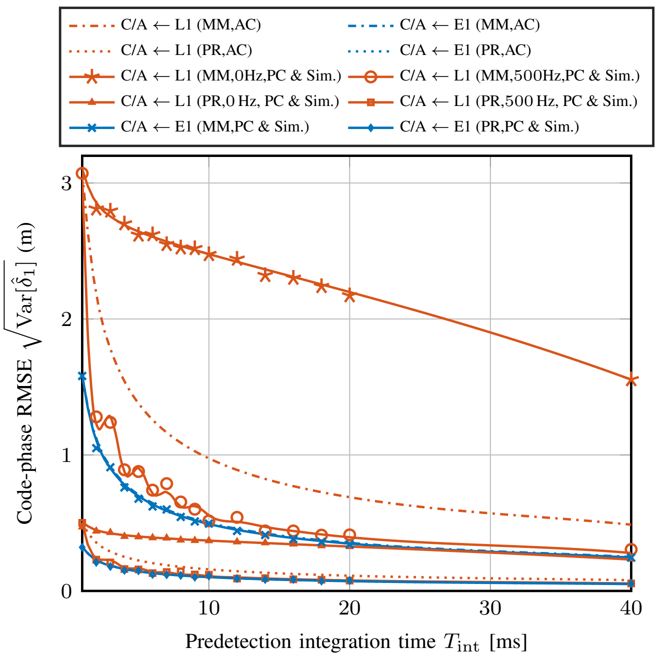

Fig. 2. Code-phase RMSE experienced by an L1 C/A user versus predetection integration time. Configuration for mass-market (MM) receiver: $W = 2.046\,\mathrm{MHz}$, $d = 1.0$ C/A chips; professional (PR) receiver: $W = 20.46\,\mathrm{MHz}$, $d = 0.1$ C/A chips.

and L1C Pilot and Data have a relative power ratio of 3:1. The components on E1 OS have equal power. For an overview, refer to Table I. Note that for presentation of all results, we choose arbitrarily $C_1 = C_2$, but all code-phase errors scale with $\sqrt{C_2/C_1}$.

A. Code-phase root-mean-squared error (RMSE)

First, we consider the root-mean-squared error (RMSE) as a figure of merit, regardless of the actual shape of the error pdf $p_{\hat{\delta}_1}(\delta)$ or its approximations. Note that SGA and IGA pdf have identical second moments, and will therefore always yield the identical RMSE — the two approximations differ only in the shape of their pdf. Only the choice of the spreading code model (PC or AC) will influence the predicted results.

The RMSE experienced by an L1 C/A user as a function of predetection integration time is shown in Fig. 2. In case the interference is from a GPS satellite, the alignment or misalignment of the spectral lines of replica and interfering C/A-code has considerable implications on performance: if the replica includes several 1 ms segments, coherent averaging will not lead to the expected decorrelation over time, but

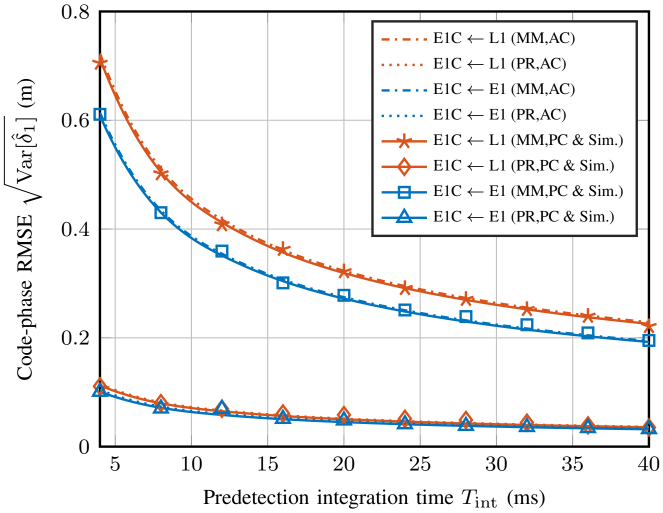

Fig. 3. Code-phase RMSE experienced by an E1-C user versus predetection integration time. Configuration for mass-market (MM) receiver: $W = 4.092\,\mathrm{MHz}$, $d = 0.5$ C/A chips; professional (PR) receiver: $W = 20.46\,\mathrm{MHz}$, $d = 0.1$ C/A chips.

may in fact be much better ($\Delta\nu = 500\,\mathrm{Hz}$) or much worse ($\Delta\nu = 0\,\mathrm{Hz}$) than predicted by an AC model. While the PC model correctly predicts this behaviour (the RMSE predicted by SGA and IGA are equivalent), the AC model seems to produce an average over all possible relative Doppler shifts. Note that the performance of snapshot code-phase estimation with $T_{\mathrm{int}} = 1\,\mathrm{ms}$ is not affected by this phenomenon. No such Doppler-dependency could be observed for interference on or from E1 OS (cf. Fig 3). We conclude that the AC model is sufficiently accurate in terms of RMSE, unless C/A-code self-interference needs to be modeled, which is in agreement with [9]. However, the PC model may become relevant for the design and RFC assessment of (quasi-)pilot signals with short primary code and slow (or no) secondary code.

B. Code-phase error distribution

We show histograms for some of the simulations performed in the previous section, along with their respective SGA/IGA for the PC and AC spreading code model. Note that the simplest form, i.e., SGA in combination with AC, is equivalent to the frequently-used SSC-type models as presented in [3], [6].

It becomes obvious from the results shown in Figs. 4-7 that the true error pdf does not, in most cases, resemble a Gaussian distribution. Interestingly, this is so for the L1 C/A user as well as for E1 self-interference (cf. Fig. 4). From Fig. 4b, we can observe that the true error pdf may decay at a much slower rate than a Gaussian pdf for large arguments, and that this is correctly modeled by the IGA. Why is the error pdf non-Gaussian even for self-interference of signals with very long primary code such as E1 OS? This is due to the fact that the spreading waveform has a relatively large time-bandwidth product $WT_{k,\ell}$, both for AC and PC assumptions. It is shown in [11] that the IGA remains a Gaussian pdf if $WT_{k,\ell} \leq 1$.

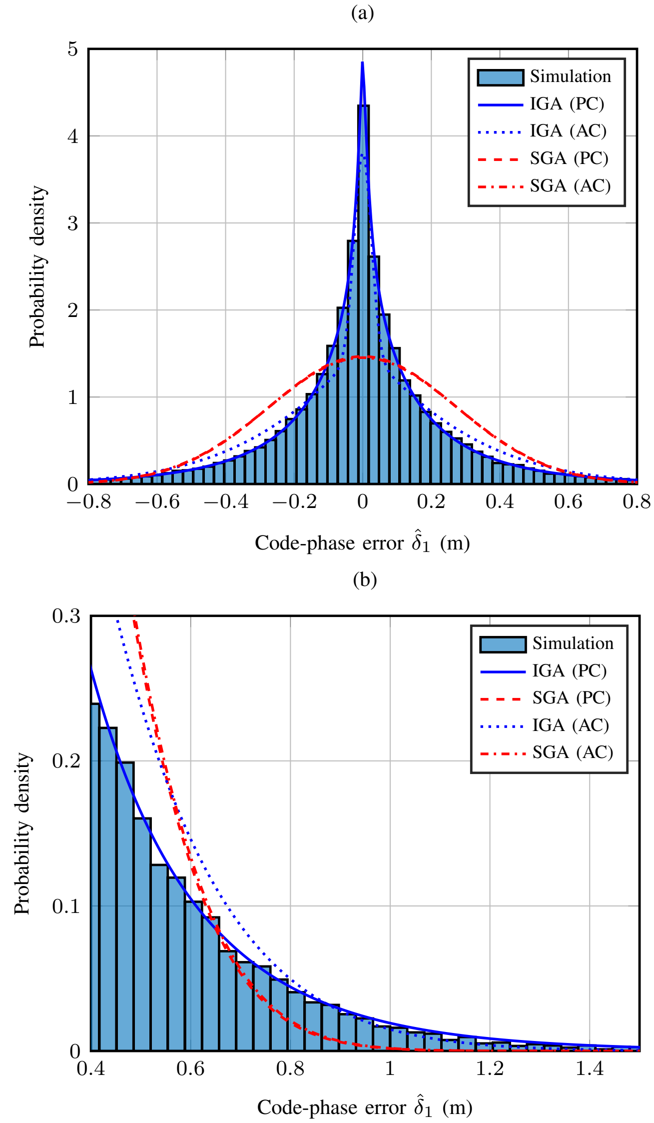

Fig. 4. Code-phase error distribution experienced by an E1-C user in the presence of interference from one satellite transmitting E1 OS. $T_{\mathrm{int}} = 20\,\mathrm{ms}$, $W = 4.092\,\mathrm{MHz}$, $d = 0.5$ C/A-chips. (a) Center (b) Tail.

We can conclude from the simulations that the computationally cheap AC model in combination with IGA is sufficiently accurate in most cases, except for mutual interference of C/A-like signals. For those scenarios, the AC model can heavily underestimate the RMSE of the distribution (cf. Fig 7); hence, considering the spreading codes' periodicity is essential.

VI. Conclusion

Considering that today's legacy and modernized GNSS signals can neither be considered WSS nor CSG, we proposed a new accurate performance model for early-late code-phase estimation in the presence of intra- or intersystem interference. The well-known SSC, code-tracking-SSC, and related methods rely on the WSS/CSG property, and are thus approximations that can only possibly be justified in the case of many

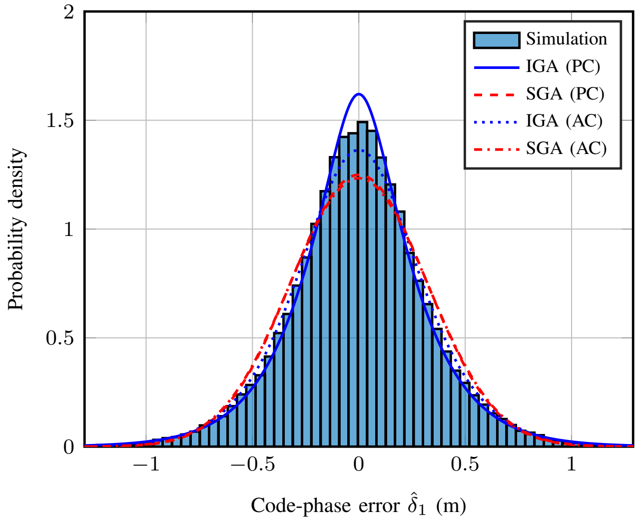

Fig. 5. Code-phase error distribution experienced by an E1-C user in the presence of interference from one satellite transmitting L1 C/A, L1C Data and L1C Pilot. $T_{\text{int}} = 20\,\text{ms}$, $W = 4.092\,\text{MHz}$, $d = 0.5$ C/A-chips.

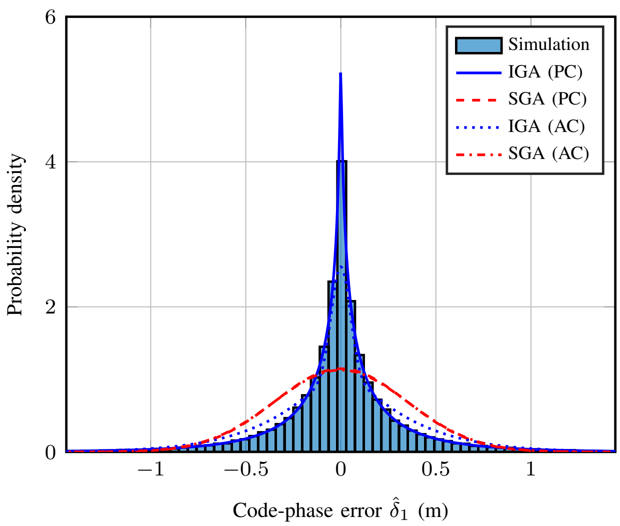

Fig. 6. Code-phase error distribution experienced by an L1 C/A user in the presence of interference from one satellite transmitting E1-B and E1-C. $T_{\text{int}} = 20\,\text{ms}$, $W = 2.046\,\text{MHz}$, $d = 1.0$ C/A-chips.

randomly asynchronous satellites with comparable powers. Whenever this is not the case (near-far scenarios, few in-view satellites), the proposed IGA provides an accurate analysis that reveals the actual shape of the code-phase error distribution rather than just the average accuracy. For signals whose PSD is nearly smooth (such as E1 OS or L1C), the IGA can be used in combination with the assumption of randomly aperiodic codes, which is computationally cheap. We verified the proposed method by means of numerical simulations, where signals were generated following the ICDs. While we considered only a single interferer, the proposed methodology can easily be used for scenarios with more than one asynchronous interferer as follows: parametrize the error pdfs for a set of relevant

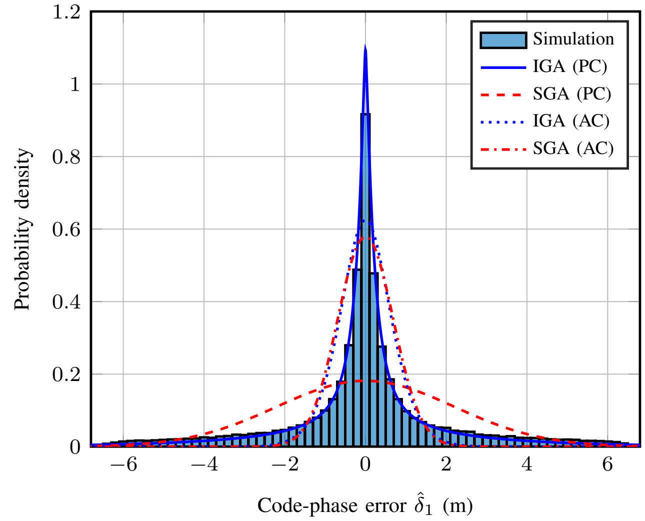

Fig. 7. Code-phase error distribution experienced by an L1 C/A user in the presence of interference from one satellite with relative Doppler shift $\Delta\nu = 0\,\text{Hz}$ transmitting L1 C/A, L1C Data and L1C Pilot. $T_{\text{int}} = 20\,\text{ms}$, $W = 2.046\,\text{MHz}$, $d = 1.0$ C/A-chips.

receiver configurations (and, if necessary, relative Doppler frequencies), and then obtain the aggregate error pdf by convolution of the individual pdfs. This will allow a more realistic RFC assessment of future navigation signals or error-overbounding for safety-critical applications.

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