

## How to Send a Signal to Fixed Ground Antennas from a Non-Geostationary Satellite

Dominik Quantius

DLR German Aerospace Center, Institute of Space Systems, System Analysis Space Segment, Robert-Hooke-Str. 7, 28359 Bremen, Germany, [dominik.quantius@dlr.de](mailto:dominik.quantius@dlr.de)

### Abstract

There are orbits which have quasi-stationary periods twice per revolution, as seen from a specific position on Earth. They could serve as a broadcast alternative for the geostationary orbit and since such orbits are inclined, in addition they cover the Polar Regions. The conditions for those special cases and the dependency on orbit radius and inclination are elaborated in this paper. Exemplarily an application for a global information service will be introduced in combination with the conditions for a repeating ground track using just one single satellite.

**Keywords:** Zig zag; Earth orbit; sharp edged repeating ground track; GEO; satellite; super GSO.

### Nomenclature

$\omega_{ground}$ : Ground velocity

$\omega_{Earth}$ : Earth rotation

$T_{sid}$ : Period of one Earth rotation (sidereal Time)

$\mu$ : Gravitational parameter

$r$ : Orbit radius

$i_{temporary\ stationary}$ : Orbit inclination meeting temporary stationary conditions

$r_{temporary\ stationary}$ : Orbit radius meeting temporary stationary conditions

$T_{orbit}$ : Orbit period

$r_{repeating\ gt}$ : Orbit radius meeting repeating ground track conditions

### 1. Introduction

For most of us sinus curved ground tracks of satellite orbits such as the one of the International Space Station (see Fig. 1) look familiar. But there exist also ground track patterns that are more like zig zag with sharp edges (c.t. Fig. 2). The class of orbits that create the latter are elaborated in the following by deriving the needed conditions and dependencies on orbit inclination and radius and by describing their meaning in terms of coverage and communication.



Fig. 1. ISS ground track.

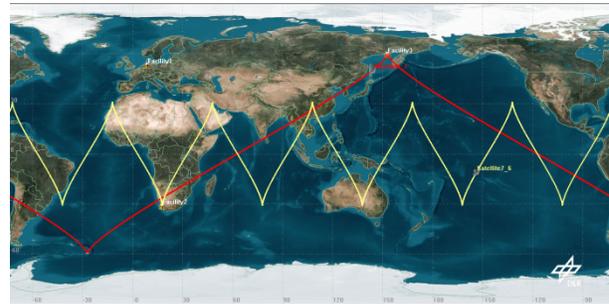


Fig. 2. Ground track of temporary stationary orbits.

### 2. Theory and calculation

A sharp edge on the ground track of a satellite means that at the edge the satellite has the same velocity over ground as the Earth's rotation velocity at the related longitude. Since the longitude is given by the orbit's inclination  $i$ , the ground velocity  $\omega_{ground}$  can be calculated with the Earth rotation  $\omega_{Earth} = \frac{2\pi}{T_{sid}} = 7.2921 \cdot 10^{-5} \frac{1}{s}$  ( $T_{sid} = 86164$  s) by:

$$\omega_{ground} = \omega_{Earth} \cos i \quad (1)$$

The orbit velocity is given with the standard gravitational parameter  $\mu = 3.986 \cdot 10^{14} \frac{m^3}{s^2}$  and the orbit radius  $r$  by:

$$\omega_{orbit} = \sqrt{\frac{\mu}{r^3}} \quad (2)$$

Equalising (1) and (2) results in:

$$i_{temporary\ stationary}(r) = \arccos\left(\frac{\sqrt{\frac{\mu}{r^3}}}{\omega_{Earth}}\right) \quad (3a)$$

or

$$r_{temporary\ stationary}(i) = \sqrt[3]{\frac{\mu}{(\omega_{Earth} \cos i)^2}} \quad (3b)$$

Solving equation (3b) for an orbit inclination range of 0° to 70° creates the blue curve given in Fig. 3. It can be seen that all orbits fulfilling the condition are retrograde orbits with an altitude above the geostationary orbit.

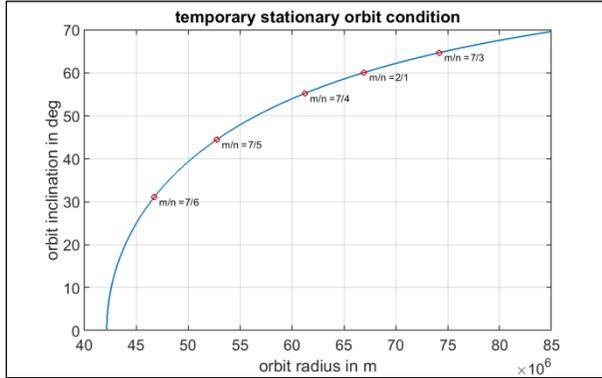


Fig. 3. Orbit inclination over orbit radius meeting temporary stationary orbit conditions (marks in addition fulfil repeating ground track conditions).

If the sharp edges of the ground track of such temporary stationary orbits shall appear at the same location on Earth after a specific timeframe, in addition they have to fulfil the criteria of a repeating ground track. That means that an integral multiple  $n$  of the satellite's orbit period  $T_{orbit}$  must be equal to an integral multiple  $m$  of the Earth's rotation period  $T_{sid}$ :

$$T_{orbit} = \frac{m}{n} \cdot T_{sid} = 2\pi \sqrt{\frac{r^3}{\mu}}; \quad m, n \in \mathbb{Z}$$

That means that in addition to equation (3b) also the following must count for the satellite's orbit radius:

$$r_{repeating\ gt} = \sqrt[3]{\mu \left( \frac{m}{n} \cdot \frac{T_{sid}}{2\pi} \right)^2} \quad (4)$$

The results of equation (4) in combination with equation (3a) are given in Table 1 for  $m$  and  $n$  going from 1 to 9. Values for  $m = 7$  mean, that the ground track will repeat after seven days; for  $m = 2$  every second day the satellite will appear over the same location. The latter two cases are represented in Fig. 3 by red circles and their orbits are depicted in Fig. 4 in the Earth inertial reference frame.

Table 1. Orbit parameters meeting conditions of temporary stationary orbits with repeating ground track.

m	n	radius (m)	inclination (deg)
2	1	66931376	60,0
3	2	55250633	48,2
4	3	51078199	41,4
5	2	77666981	66,4
5	3	59271000	53,1
5	4	48927132	36,9
6	5	47613552	33,6
7	3	74175579	64,6
7	4	61230591	55,2
7	5	52766924	44,4
7	6	46727687	31,0
8	3	81081587	68,0
8	5	57679711	51,3
8	7	46089753	29,0
9	4	72398811	63,6
9	5	62391404	56,3
9	7	49854695	38,9
9	8	45608393	27,3

### 3. Implications for the communication

Strictly speaking, the above derived orbits are stationary only for one moment: At the very edge of the ground track pattern. Analysing the duration in which the satellites position at the edge stays within a cone of one degree seen from a fixed ground antenna is about 75 min. for the  $m/n = 2/1$  case and 83 min. for the  $m/n = 7/6$  case. That proves that a quasi-stationary condition is met for significantly more than an hour. Since the antenna does not need to be located exactly below the discussed quasi-stationary edge, but has to point towards it only, a 100% coverage of the globe can be achieved by just a single satellite, as visualised in yellow and red in Fig. 2 and Fig. 4., including the poles. Such a mission could serve as an independent global information service e.g. operated by the United Nations available under use of just simple antenna equipment. A constellation on these orbits could have even bigger impacts on communication and data distribution on Earth.

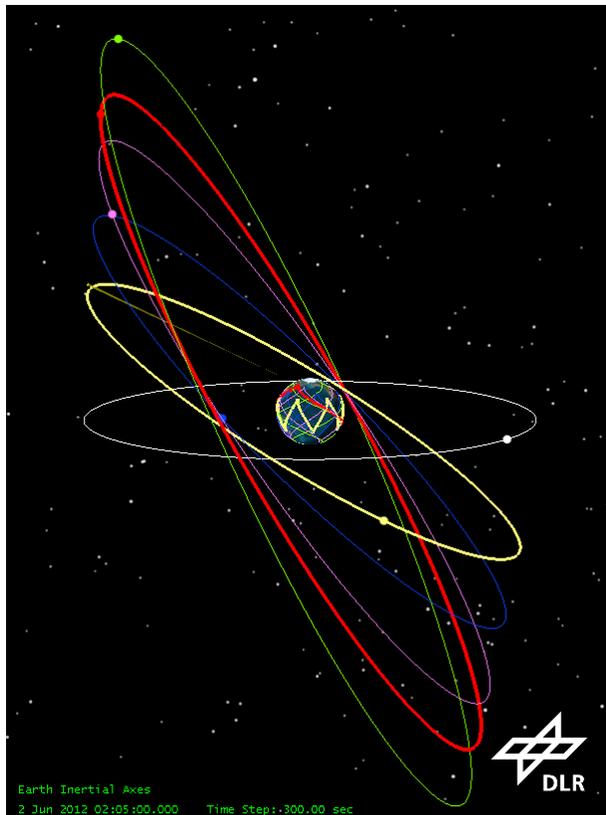


Fig. 4. Visualisation of temporary stationary orbits with repeating ground track patterns in the Earth inertial reference system.

#### 4. Discussion

The calculations above represent two body point mass solutions only. In further studies the influence of perturbations on the orbit stability has to be analysed. Also the applicability of receiving a signal just every second day or even just once per week for over one hour has to be further discussed. Nevertheless it is a group of orbits which should not be neglected, also in terms of end-of-life regulations for geostationary satellites.

#### 5. Conclusions

The examples presented above are introduced to explain the concept of quasi-stationary orbit types with and without a repeating ground track. Choosing the right conditions in terms of orbit radius and inclination there exist orbit types which appear for more than an hour stationary within a tolerance of plus/minus  $0.5^\circ$  from ground. This behaviour is hereby being numbered and visualised for inclinations up to  $70^\circ$ .