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Measurement Techniques

Statistical analysis of contrail lifetimes from a satellite perspective

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Abstract

The study of the lifetimes of contrails from a satellite perspective benefits from the extended coverage and close temporal monitoring. However, the initial stages of the contrail development are not observed from a satellite platform due to the sub-pixel size of the forming cloud. The final stages may be unobserved as well when the contrails get spatially diluted and when they lose contrast with their background. In this paper we apply a Weibull distribution model to describe the survival rate of contrails during their observed life-span and adjust its two defining parameters (λ and k) to fit a dataset of over 2300 contrails. Using the Weibull distribution, it is possible to estimate the expected further lifetime of the contrails after satellite observation ceased. Depending on the actually observed lifetime, the expected extension can range from about 1 to 4 h, but the overall mean of this duration is about 1.3 h. The time elapsed between contrail formation and first satellite observation is estimated from the initial width distribution of the contrails. Under the assumption of a 5 km/h spreading rate, the average age of contrails at the time of their first satellite detection is 1.5 ± 0.4 h. Using a Monte Carlo simulation, we are able to compute the cumulative distribution of the complete (i.e. initial spreading, tracking, and after-tracking periods) lifetime of persistent contrails. This complete lifetime has a mean value of 3.7 ± 2.8 h. The Weibull distribution ($k < 1$) shows that the probability to survive increases with contrail age, which corresponds to the actual decay rate of contrails in nature. Additionally, we analyse lifetime differences between daytime and nighttime contrails. We find that nighttime contrails have slightly shorter lifetimes than daytime contrails. Although this difference is statistically significant, it remains to be shown whether this has important physical consequences.

Keywords: statistical methods, process memory, contrails, satellite data

1 Introduction

In order to determine the impact of contrails on the climate system, it is necessary to observe their evolution during their entire lifetime. Geostationary satellites constitute an excellent tool because of their coverage, spatial resolution and temporal resolution. With their help it is possible to follow closely how different properties associated with contrails evolve in time. However, the limited spatial resolution of the satellite causes one inherent problem: how long does the contrail live before it is observed on the satellite image. Currently the only way to answer the question is by direct observation of a contrail and a subsequent positive detection on a satellite image. There have been several case studies in this manner such as the works by Duda et al. (2004) and Atlas et al. (2006).

Using satellite imagery, tracking algorithms can be developed that allow us to identify large numbers of individual contrails and closely study the evolution of their properties and statistically describe their behaviour. Two examples are the Automatic Contrail Tracking Algorithm (ACTA) by Vázquez-Navarro et al. (2010), and ForTRACC (forecast and tracking the evolution of cloud clusters) from Vila et al. (2008).

Observations suggest that contrails in suitable environments can be persistent and cover the sky during several hours before vanishing. However, an analysis of the contrail lifetimes is not complete if the elapsed time between formation and observation on satellite is unknown. Furthermore, a contrail does not necessarily cease to exist after it is no longer seen on the satellite image; it might be too optically thin or might have too low contrast with its background. Contrail lifetimes are thus longer than the satellite data suggest. It is important to estimate the unobserved fraction of contrail lifetimes. Without such information it is not possible to correctly quantify the effect of contrails on the climate system. We have used a mathematical approach based on a Weibull distribution to describe the observed part of contrail lifetimes. This distribution is a generalised form of the exponential distribution and with a negative exponent it describes processes whose chance to proceed a further unit of time *increases* with time. We show that this is a natural property of longliving contrails (or contrail cirrus). The Weibull distribution allows furthermore to compute the expected additional lifetime of a contrail after its observation ceases. Unfortunately, it seems there is no analogous statistical method for the determination of the

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time elapsed between contrail formation and its detection by the satellite, that is, for the pre-detection time. Instead, we use the initial width information, which is part of the ACTA data as well.

We have applied the analysis to a large dataset of contrails tracked by ACTA over the heavily flown areas of the North-Atlantic and Europe. The Rapid Scan Service of the Meteosat Second Generation satellite enables ACTA to track contrails in 5 minute intervals. The dataset used in this work is described in Vázquez–Navarro et al. (2015); it spans a full year and comprises over 2300 contrails. The longest lived contrail successfully tracked by ACTA lasted 18 hours.

We present the mathematical analysis in Section 2 and apply it to the dataset in order to retrieve the elapsed time between contrail formation and first detection as well as the average survival time of a contrail after observation. A Monte Carlo method is used to determine the cumulative distribution of the total lifetime. In Section 3 we discuss the results and consider whether there are lifetime differences between daytime and nighttime contrails. Section 4 summarises the results.

2 Analysis

2.1 Distribution of observed lifetimes

The ACTA data set contains observational records of 2305 individual contrails, including their effects on the radiation field at the top of the atmosphere, their length and width. The individual contrails carry an identification number (ID) and are generally observed several times, that is, they appear in up to 225 subsequent 5-min scans of Meteosat-SEVIRI. For each contrail ID the maximum observation number is collected which equals its observation period in units of 5 min. We start with an analysis of the observed lifetimes. If the last observation time is t, one does not know whether the contrail actually ceased to exist between t and the next SEVIRI scan or whether it has merely grown too optically thin or otherwise too faint against the background scene to be still observable. This possibility is considered later in Section 2.3 where we make the reasonable assumption that the contrail "process" does not change at the end of the observation, or equivalently, that the observation does not cease *because* the contrail "process" undergoes a certain change that is not reflected in the lifetime distribution parameters to be derived in the present section. The initial part of contrail lifetime, between contrail formation and its first detection in satellite images will be treated in Section 2.2.

The ACTA observed lifetime data have roughly been fitted with exponential distributions in previous publications (e.g. Vázquez-Navarro et al. 2015) to get rough estimates of typical contrail lifetimes. But exponentials are not appropriate for thorough analyses because the assumption of memorylessness which is exclusively fulfilled for exponential decay processes (like radioactive

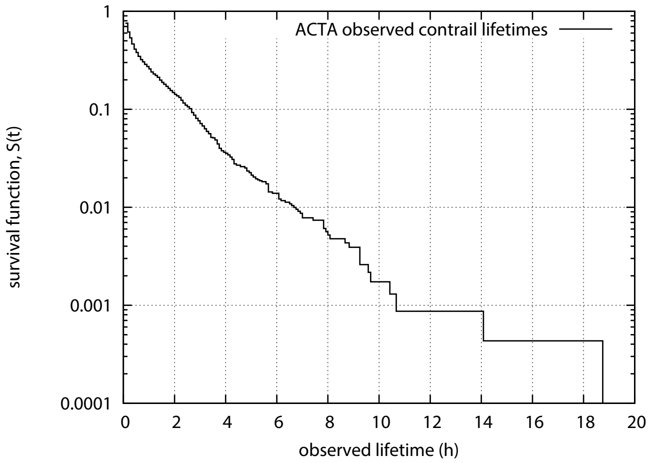

Figure 1: The distribution of contrail's observed lifetimes in the ACTA data base plotted as a survival function.

decay) is not justified for atmospheric phenomena. A better lifetime model is the Weibull-type decay process; it takes the memory of atmospheric processes into account and is more flexible. We shall use it in the following, beginning with the construction of an empirical survival function.

The survival function, $S(t)$, describes the fraction of objects (contrails) that are still existent (alive) at time t, in relation to the number of objects that existed initially, at $t = 0$. The data base has $n(0) = 2305$ contrails and in each 5 min time step the number of contrails decreases or stays constant. Let the number of contrails at time t be $n(t)$, then the empirical survival function is

$$S(t) = n(t)/n(0), \tag{2.1}$$

which is a non-increasing function with values in the real interval $[0, 1]$. The empirical survival function for the ACTA data is shown in Figure 1. One can see that the number of contrails decreases quickly at small observation times, but the decay rate gets smaller and smaller over time until the final contrail terminates after more than 18 h of observation. An exponential decay would lead to a straight line in such a (half-logarithmic) plot, and it is seen that such a model would be incorrect.

A better model is the Weibull-model (Weibull 1951), which is a generalisation of the exponential one. Its survival function has the form

$$S(t) = \exp[-(\lambda t)^k], \tag{2.2}$$

with a generalised rate coefficient $\lambda > 0$ and an exponent $k > 0$. The exponential model has the special exponent $k = 1$, but in the case of contrails it turns out that $k < 1$. The two parameters can be estimated using a Weibull plot. It is easy to see that

$$\ln[-\ln(S)] = k \ln \lambda + k \ln t =: a + b \ln t. \tag{2.3}$$

Thus plotting $\ln[-\ln(S)]$ vs $\ln t$ should result in a straight line if the survival function, $S(t)$, is indeed of the Weibull-type. This is demonstrated in Figure 2.

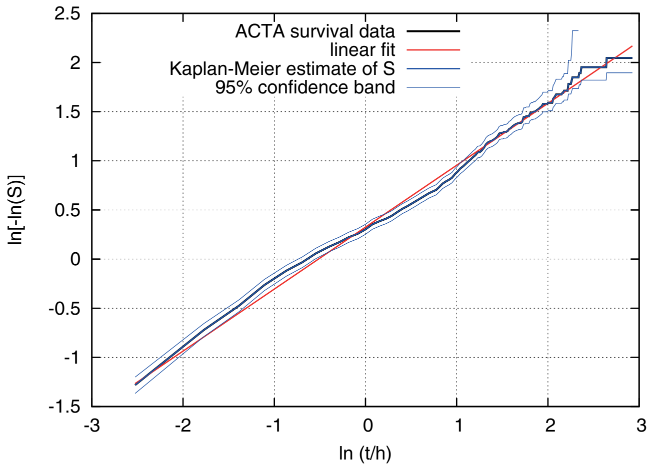

Figure 2: Weibull-plot of the observed contrail lifetimes (black), of the empirical survival function (thick blue, nearly congruent with the data curve) together with the corresponding 95 % confidence limits (thin blue). The red straight line is the best fit (in the chi-square sense) Weibull survival function from which the parameters can be determined.

The ACTA data are represented by the black line which is almost hidden behind the blue line, which in turn represents an empirical estimate of the survival function, the so-called Kaplan-Meier estimate (Kaplan and Meier 1958; Owen 2001). While this estimate itself is unnecessary (because it is essentially identical to the S taken directly as described above), it allows to compute a confidence band around the $S(t)$ estimate (thin blue lines, using Greenwood's formula, see Owen 2001). The confidence band is quite narrow in our case, only at the high end of the data range where the data become sparse, the confidence band gets broader. The red line represents a linear fit of $\ln[-\ln(S)]$ with parameters intercept $a = 0.325$ and slope $b = 0.630$. Note that the value of the intercept depends on the time unit on the abscissa, for which we choose 1 h. As the slope directly gives the exponent of the Weibull distribution, we get $k = 0.630$, that is, indeed a value below unity. This is characteristic for processes with ever increasing residual lifetime. This means, the probability that a contrail survives another 5 min increases with its age. In contrast, if the exponent were $k = 1$, the probability of surviving the next 5 min would be constant. This is not the case for contrails. Their tendency to survive the longer the older they are is a noteworthy property.

It can be seen that the red straight line in Figure 2 is occasionally outside the 95 % confidence band. This should neither be a problem nor a surprise. The Weibull decay model is just a simple but flexible model, not more. It cannot be derived from first principles or from contrail physics. It is a tool that condenses information on the individual atmospheric situations of 2305 contrails to two numbers, k and λ. It cannot be expected that this works perfectly. But we are confident that one can learn certain properties of contrails from such a condensation of information, for instance the property mentioned above, namely that the survival chance of a contrail during the next time unit increases with the contrail's age.

It remains to determine the parameter λ. This can be computed from the intercept value: $\lambda = e^{a/k} = 1.676\,\mathrm{h}^{-1}$. The mean observed lifetime according to the Weibull fit is given as

$$\bar{t} = \lambda^{-1}\,\Gamma\left(1 + \frac{1}{k}\right), \tag{2.4}$$

where $\Gamma(\cdot)$ is the gamma function. In our case we find $\bar{t} = 0.84$ h, while directly from the data we get 0.91 h, which is a satisfactory agreement. The variance is

$$\sigma_t^2 = \lambda^{-2}\left[\Gamma\left(1 + \frac{2}{k}\right) - \Gamma^2\left(1 + \frac{1}{k}\right)\right], \tag{2.5}$$

which is 1.96 h^2, implying a standard deviation of 1.40 h. Directly from the data we find 1.41 h. The standard deviation is considerably larger than the mean value, a consequence of the distribution being right-skewed. The skewness determined from the data is 3.65.

2.2 Initial unobserved lifetime

As SEVIRI cannot observe the initial expansion of a contrail the initial lifetime is unknown. We can however estimate it from the initial width distribution using the rule of thumb that contrails expand with pedestrian speed (as the late Hermann Mannstein stated in a private communication), that is, at about 5 km/h. The initial width distribution is shown in Figure 3. Directly from the data we determine the cumulative distribution function (black curve). The shown probability density function of the widths is determined using a kernel density estimate (Epanechnikov kernel with bandwidth 300 m). This density looks quite symmetric and indeed a Gaussian fit with mean and standard deviation determined from the width data, 7.7 km and 2.2 km, respectively, looks quite appropriate for further analysis purposes. A plot of observed lifetimes against initial widths of the contrails does not display any kind of correlation, so we assume that contrail ages at the beginning of observation and the lifetimes under observation are independent variables. However, uncorrelatedness does not generally imply independence, so we check this further using the relation $\overline{WT} = \overline{W} \cdot \overline{T}$ between mean values of independent random variables, W and T. Indeed we find that this relation is valid for the ACTA data (the difference between the two sides of the equation is 0.5 %). Thus, initial widths of the contrails and the total durations of their observation seem independent quantities. Using Mannstein's rule of pedestrian speed (see also Freudenthaler et al. 1995), the mean age of the contrails when they start to appear in the ACTA data is 1.5 ± 0.4 h. The given standard deviation of 0.4 h represents only the variation of initial widths in the ACTA data set. The actual uncertainty of the initial spreading

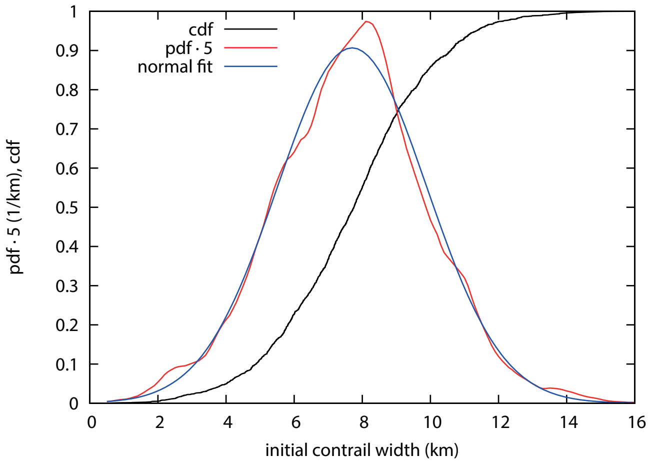

Figure 3: Contrail widths at the moment when they are first detected by the satellite. Shown are the empirical cumulative distribution function (black) and a density function (red, multiplied by 5 for better visibility) determined using a kernel density estimate. Most contrails are about 8 km wide initially, but there are also very narrow ones with a width of less than 1 km and very broad ones with a width exceeding 15 km. The blue line is a normal distribution (times 5) with mean 7.7 km and standard deviation 2.2 km. It can be used as an analytical model for the initial width distribution.

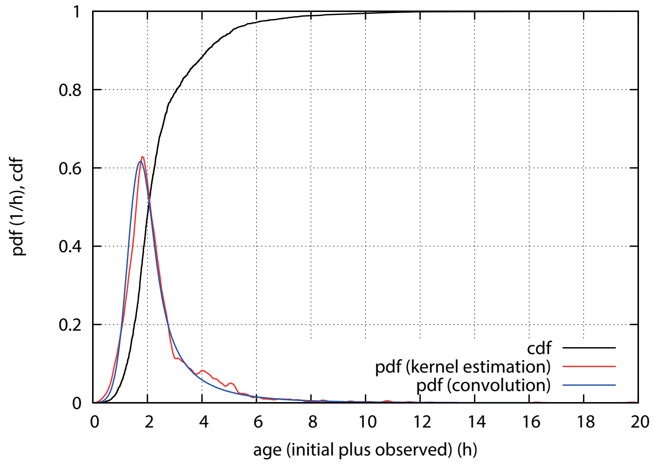

Figure 4: Cumulative distribution (black curve) of the initial plus observed lifetime of contrails in the ACTA data set estimated as the sum of the observed lifetime and the time it took to reach the respective initial width with an expansion rate of 5 km/h. The red line is the corresponding probability density computed from the data using a kernel density estimate. The similar blue line is the convolution of the Weibull distribution that describes the observed lifetimes and the normal distribution that describes the initial unobserved times (computed from the initial widths). The computed maximum is close to 20 h.

time is larger because contrails expand with different speeds. The mean age of a contrail when its observation commences is inversely proportional to the assumed spreading rate. That is, a relative uncertainty of p percent in the spreading rate leads to the same relative uncertainty of in the derived initial time.

The total lifetime before the end of observation of each contrail in the ACTA data can thus be estimated as the sum of the observed lifetime and the time it took for the contrail to reach its initial width with an expansion rate of 5 km/h. Figure 4 shows both the cumulative distribution (black) and the corresponding probability density (red curve, determined using an Epanechnikov kernel with bandwidth 0.1 h). These curves are computed from the data and are thus representative for the ACTA data set. However, if the duration of the initial unobserved expansion phase is independent from the observed lifetime one can get a more general result by determination of the probability density of the initial plus observed lifetime as a sum of these independent components. This is achieved as a convolution of the two probability densities fitted to the data above, the Weibull density for the observed lifetimes and the normal distribution for the initial expansion phase. This convolution has to be performed numerically since no simple analytical treatment for this case is known to us. As a convolution is equivalent to the product of the characteristic functions, we compute the Fourier transforms of the two pdfs, multiply them and Fourier transform back into the time domain. The result is shown as the blue curve in Figure 4 and it is quite similar to the curve obtained from the data directly. We conclude from this that the result is not too special for just the ACTA data and we expect that the pdf of the total lifetime before observation ends should be similar to the one obtained here when other observation systems were used.

According to Figure 4 most contrails that can be observed in satellite imagery reach an age of almost 2 h, 90 % have an age below 4 h until the observation ends, but some can reach very long lifetimes. The lifetime pdf is strongly right-skewed.

We note that the pdfs in Figure 4 can be fitted up to $t \approx 3$ h with a gamma distribution (shape parameter $k = 8$, rate parameter $\lambda = 4$), but the right tail of the gamma pdf falls off too fast (not shown). Plotting the survival function (one minus the cdf of Figure 4) on Weibull probability paper yields a curve (black) that consists of two straight “arms” with a slope change at $\ln(T/\mathrm{h}) \approx 0.7$, corresponding to about 2 h. This is shown in Figure 5. The survival behaviour of contrails younger than that age differs very much from the older ones, which is signified by the exponents of the Weibull distributions which are greater than unity for young contrails but lower than unity for old contrails. For young contrails up to about 2 h age the probability to survive the next 5 min thus *decreases* with age, while for older contrails the survival probability increases, as we have already seen above. This is clearly a peculiar behaviour that has so far neither been reported nor been explained. This peculiar behaviour is best shown using the hazard function

$$h(t) = -\frac{\mathrm{d}n}{n\,\mathrm{d}t} = -\frac{\mathrm{d}S}{S\,\mathrm{d}t}, \qquad (2.6)$$

see Figure 6. $n(t)$ is here the number of contrails that reach the age t. The hazard function thus describes the

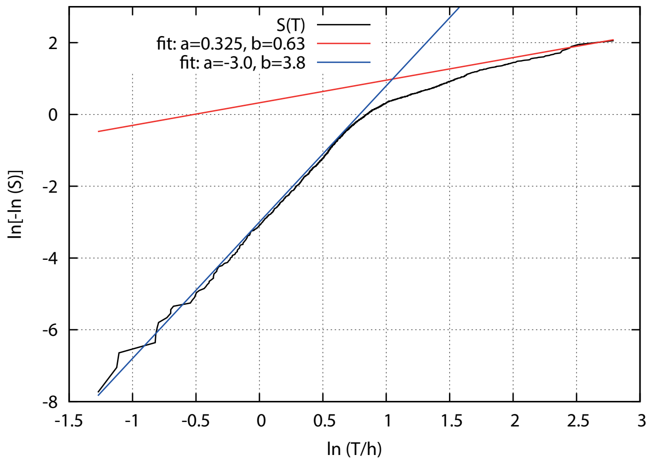

Figure 5: Weibull-plot of the estimated initial plus observed lifetimes, T (black). The slope of the line has a break at about 2 h. A pair of straight lines can be fitted at the two ends of the survival curve. The red line represents the same fit as used earlier for the observed lifetimes. The blue line fits the distribution of the short lifetimes up to the break point. It is noteworthy that in this T-regime the slope parameter exceeds unity which means that the survival behaviour of contrails of short ages is clearly different from that of large ages.

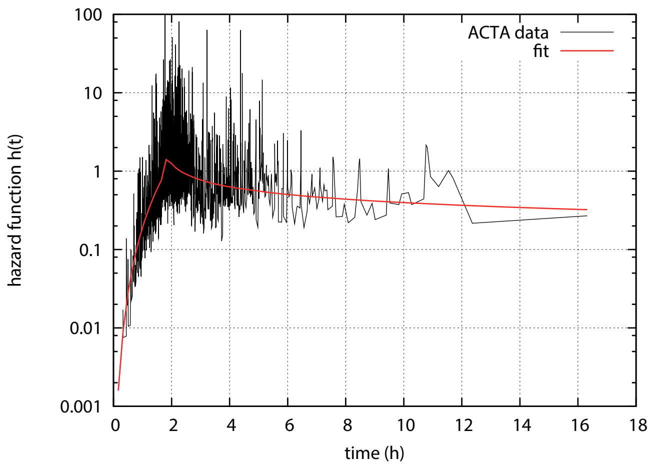

Figure 6: The hazard rate function for the initial plus observed contrail lifetimes in the ACTA data versus time. $h(t)$ computed directly from the data (thin black line) is very noisy due to taking the logarithmic derivative of noisy data. The red fit is constructed from the two straight line fits to the survival function in Figure 5.

current relative rate of contrail disappearance from the data set. Despite of the strong noise caused by taking the derivative of noisy data, Figure 6 shows clearly that the hazard rate increases within the first 2 hours or so, and then decreases. The red line in Figure 6 is a hazard rate function constructed from the two Weibull fits of Figure 5; it shows the peculiar behaviour of contrail lifetimes more clearly than the noisy data.

2.3 Mean extended residual life function

As already stated, a contrail either does cease to exist or it becomes unobservable for SEVIRI after its final record in the ACTA data. Both these cases can be treated as follows.

The probability that the actual lifetime, T, of an object exceeds a time τ is given by the survival function, $S(\tau)$. Thus the probability that the actual lifetime even exceeds $\tau + \delta$ is $S(\tau + \delta)$. With these probabilities one can compute the *conditional* probability that an object that existed at τ is still present at $\tau + \delta$. This conditional probability is

$$P\{T > \tau + \delta | T > \tau\} = \frac{S(\tau + \delta)}{S(\tau)}, \qquad (2.7)$$

which is to be read as $P\{\text{event in question} \,|\, \text{given condition}\}$. The corresponding conditional probability density, $f(\delta|\tau)$, is given as the negative derivative of $P\{T > \tau + \delta | T > \tau\}$:

$$f(\delta|\tau) = -\frac{\mathrm{d}}{\mathrm{d}\delta}\left(\frac{S(\tau + \delta)}{S(\tau)}\right). \qquad (2.8)$$

Note that this is a probability density for δ with τ being a parameter. Now,

$$\bar{\delta}(\tau) = \int_0^\infty \delta\, f(\delta|\tau)\,\mathrm{d}\delta. \qquad (2.9)$$

$\bar{\delta}(\tau)$ is the expected (i.e. mean) additional lifetime of all objects that have an actual age τ. Since it depends on the age, it is possible to compute an overall mean by integrating it using the age distribution density as weighting function:

$$\langle\bar{\delta}\rangle = \int_0^\infty \bar{\delta}(\tau)\, f(\tau)\,\mathrm{d}\tau, \qquad (2.10)$$

which does not anymore depend on age.

Now we apply this theoretical program to the residual lifetime distribution determined above, that is, the Weibull distribution with rate parameter $\lambda = 1.676\,\mathrm{h}^{-1}$ and shape parameter $k = 0.630$. Assuming that the statistics determined above are characteristics of the contrails' dispersion phase (i.e. of contrail evolution more than about 5 min after formation, when aircraft influence is no longer effective) the Weibull model with its two parameters is sufficient for the characterisation of the post-tracking part of the lifetime, which is part of the dispersion phase as well.

The survival function for a Weibull-type decay process has been given above in Eq. (2.2). The conditional probability for contrails of age τ that their lifetime is extended at least by δ is thus

$$P\{T > \tau + \delta | T > \tau\} = \frac{\exp\{-[\lambda(\tau + \delta)]^k\}}{\exp[-(\lambda\tau)^k]}. \qquad (2.11)$$

The conditional probability density for the extended lifetime δ, given an age τ, is the derivative of equation (2.11) times minus one, that is

$$f(\delta|\tau) = -\exp[(\lambda\tau)^k]\,\frac{d}{d\delta}\,\exp\{-[\lambda(\tau + \delta)]^k\}. \qquad (2.12)$$

It is not necessary to perform the derivative since we compute the conditional expectation of δ as the first moment of this conditional density.

$$\bar{\delta}(\tau) = -\exp[(\lambda\tau)^k] \int_0^\infty \delta \frac{d}{d\delta} \exp\{-[\lambda(\tau+\delta)]^k\}\,\mathrm{d}\delta. \tag{2.13}$$

The integral is computable via partial integration.

$$\int_0^\infty \delta \frac{d}{d\delta} \exp\{-[\lambda(\tau+\delta)]^k\}\,\mathrm{d}\delta = \left[\delta \exp\{-[\lambda(\tau+\delta)]^k\}\right]_0^\infty - \int_0^\infty \exp\{-[\lambda(\tau+\delta)]^k\}\,\mathrm{d}\delta. \tag{2.14}$$

The first rhs term is zero and the remaining integral (substitute t for $[\lambda(\tau+\delta)]^k$) is given by an incomplete gamma function ($\Gamma(\cdot,\cdot)$, Abramowitz and Stegun 1972, formula 6.5.3) such that the final expression for the desired conditional expectation is

$$\bar{\delta}(\tau) = \frac{\exp[(\lambda\tau)^k]}{k\lambda}\,\Gamma\left[\frac{1}{k}, (\lambda\tau)^k\right]. \tag{2.15}$$

This means, if a contrail has been observed for a period τ, one can expect that its lifetime is extended on average by $\bar{\delta}(\tau)$. Generally this expression depends on τ. It differs from the mean value of the Weibull density, but in the limit $\lim_{\tau\to 0} \bar{\delta}(\tau) = \bar{\tau}$.

As $\bar{\delta}(\tau)$ depends on age τ, it can be integrated further over τ with the Weibull density $f(\tau)$ as the weighting function.

$$\begin{aligned}\langle\bar{\delta}\rangle &= \int_0^\infty \frac{\exp[(\lambda\tau)^k]}{k\lambda}\,\Gamma\left[\frac{1}{k}, (\lambda\tau)^k\right] f(\tau)\,\mathrm{d}\tau \\ &= \int_0^\infty (\lambda\tau)^{k-1}\,\Gamma\left[\frac{1}{k}, (\lambda\tau)^k\right]\mathrm{d}\tau.\end{aligned} \tag{2.16}$$

The integral can be solved using the substitution $x = (\lambda\tau)^k$ which leads to

$$\begin{aligned}\langle\bar{\delta}\rangle &= \int_0^\infty \Gamma(1/k, x)\frac{\mathrm{d}x}{k\lambda} \\ &= \frac{1}{k\lambda}\Gamma\left(1+\frac{1}{k}\right) \\ &= \frac{1}{k^2\lambda}\Gamma\left(\frac{1}{k}\right) = \frac{\bar{\tau}}{k}.\end{aligned} \tag{2.17}$$

(Abramowitz and Stegun 1972, formula 6.5.37).

The result for the ACTA data is shown in Figure 7. The expected extension of contrail lifetime after it can no longer be observed increases with the actual duration of observation. For contrails that can only be observed for a period of less than about two hours, the expected extension after observation is even larger than the observation period. The overall mean is given in the plot as a symbol, ⊗. Its value for the ACTA data is 1.34 h, that is, a little bit longer than the average observation time.

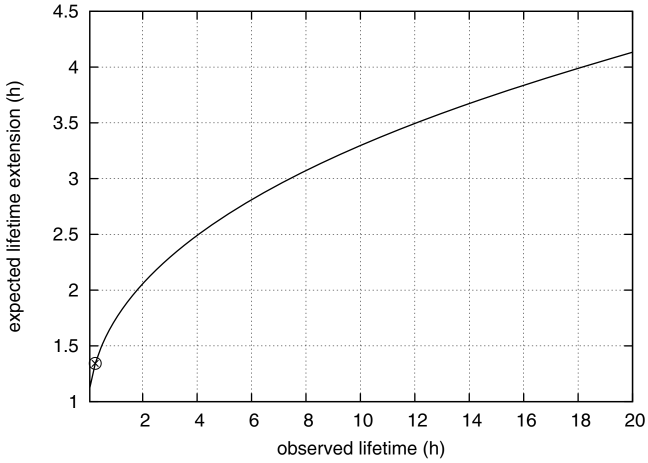

Figure 7: Expected lifetime extension of contrails in hours versus duration of observation. The small symbol (⊗) in the lower left corner marks the overall expectation of lifetime extension irrespective of the actual duration of observation.

So, taking all results of the analysis together, it can be seen that the actually observed lifetime of a contrail is sometimes the smallest part of its actual lifetime. At least the average values are thus: The average observed duration is about 0.9 h, but the mean age when they appear in the satellite imagery is about 1.5 h and the overall expected lifetime after the observation with SEVIRI is no longer possible is about 1.3 h. Thus the actual lifetime of a contrail can easily be 3 times larger than its observed lifetime.

2.4 Complete lifetime

In section 2.2 we had computed the cumulative distribution (and the probability density function) of the sum of the initial unobserved contrail spreading phase and the tracking period. It was possible to do this via a convolution exploiting the analytical expressions for the two pdfs at hand. Now we try to compute the distribution for the complete lifetime, i.e. the two phases just mentioned plus the unobserved extended lifetime after tracking. Although we have an analytical expression for the latter, we do not use a convolution here. Instead we employ a much more simple method, namely Monte Carlo simulation. We draw triples of random numbers. The first one represents the initial spreading phase and is drawn from a normal distribution with the parameters given above (very rare negative numbers are discarded). The second one represents the tracking period and is taken from a Weibull distribution as given above. For getting these random numbers we need the quantile function of the Weibull distribution which is

$$Q(p) = \lambda^{-1}\,[-\ln(1-p)]^{1/k}. \tag{2.18}$$

Here, p is produced by a random number generator for uniform deviates in the range $[0, 1)$, and the resulting Q is then Weibull distributed. The third random number

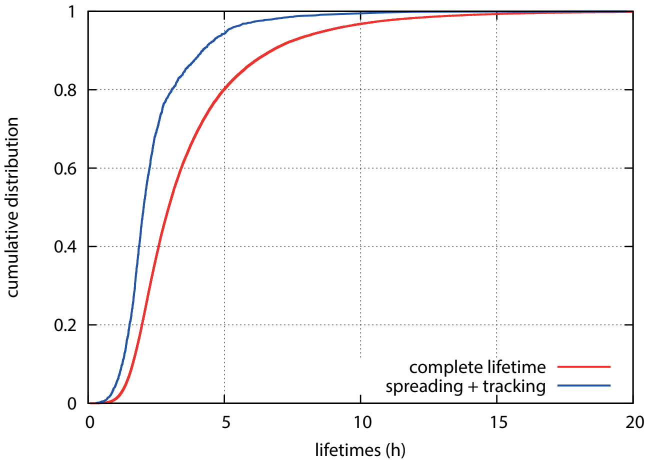

Figure 8: Cumulative distribution (red) of the complete lifetime, including initial spreading, tracking, and extended life periods, according to a Monte Carlo simulation. The blue curve is plotted for comparison; it is the cumulative distribution of the first two of the three periods (identical to the black curve in Figure 4).

depends on the one just determined in the same way the extended lifetime depends on the tracked lifetime. The needed quantile function in this case is

$$Q^*(q|Q) = \lambda^{-1}\{-\ln[(1-q)\exp(-(\lambda Q)^k)]\}^{1/k} - Q. \tag{2.19}$$

Again, q is uniformly distributed in $[0, 1)$ and Q^* has the desired distributional type, as derived above.

We draw 100000 triples of random numbers (from these 23 are discarded) and compute the complete lifetime as the sum of the three parts. The result is shown in Figure 8. The blue line repeats the black line of Figure 4 for comparison and the red line is the distribution of the complete lifetime according to the Monte Carlo simulation. The increasing horizontal distance between the blue and red curves may be noted; it reflects the monotonically increasing nature of the expected lifetime extension as function of the observation time (Figure 7). The Monte Carlo simulation yields a complete lifetime distribution of persistent contrails with mean 3.7 h and standard deviation 2.8 h. 80 % of all persistent contrails have a liftime of at most 5 h and only about 5 % have a lifetime exceeding 10 h. It is also noteworthy that the cdf curves are flat near zero, that is, very short lifetimes (say less than an hour) are very improbable for persistent contrails and the most probable value is between 2 and 3 h.

3 Discussion

3.1 The peculiar hazard rate behaviour

It is easy to understand that the hazard rate for old contrails decreases over time, that is, the probability to survive the next five minutes increases with contrail age. Indeed, this should be so because if a contrail gets old it is located in an ice-supersaturated airmass which in turn is often characterised by upward air motion (Gierens and Brinkop 2012). As the air cools, more and more humidity is available for condensation and the contrail total ice mass grows on and on. The longer the ice mass has grown (that is the older the contrail is) the longer it will take to sublimate the ice completely. In other words, the probability that the ice sublimates completely within the next five minutes decreases with the ice mass present. This explains the decreasing hazard rate function for old contrails.

The situation is less clear for the young contrails for which we found an increasing hazard rate. This finding is surely correct for those contrails that get sufficiently broad to be detectable by satellite imagery. But if we could take into account all those contrails that get not detected because they do not survive until they are broad enough, the picture could change essentially. The initial widths of the observed contrails are roughly normally distributed with a mean of 7.7 km. This is already quite large a width compared with the typical widths of contrails soon after their formation, which is typically a few 100 m (that is, a few times the wing span). As ice-supersaturation, the condition for contrail persistence, is given in perhaps 15 % of the flown distances (Gierens et al. 1999), while the thermodynamic criterion for contrail formation, the Schmidt-Appleman criterion (Schumann 1996) is fulfilled as soon as it is colder than about −40 °C, short non-persistent contrails are much more frequent than persistent ones that make it into satellite data. If these unobserved contrails would be taken into account in the statistics of initial widths, the normal distribution would probably turn into a distribution with a mode at a few hundred metres and with probability density decreasing with width. In other words, if the satellite instruments were so sensitive that they could see a contrail shortly after or even directly at formation, it can be conceived that the observation time statistics could again be modelled by a Weibull distribution with $k < 1$. In this case the hazard rate would decrease right from the beginning. In fact, all the non-persistent contrails in subsaturated air have a very high hazard rate because they disappear very quickly. Contrails in slightly supersaturated air survive longer and achieve then a lower hazard rate. It seems thus that a hazard rate function that starts with a high value at contrail formation and decreases more or less continuously with time would be consistent with expectations from physical considerations. The initially increasing hazard rate function is probably an artefact of the observation method.

3.2 Initial lateral expansion rate

The distribution of the initial unobserved times depends directly on the assumed lateral expansion rate of contrails. Of course, this rate is not a constant. Thus the uncertainty of the initial spreading time is larger than the 0.4 h computed from the data. Unfortunately, the ACTA width data themselves are not good enough to estimate the individual expansion rates, since the width is in ACTA defined as the area of a contrail divided by its length. If we try estimating the expansion rate via the final and initial widths, we often get negative results, probably since area and lenght values decrease considerably and get quite uncertain at the end of the observation, which is one reason to stop it. Freudenthaler et al. (1995), using a scanning lidar, found lateral expansion rates from 18 m/min to 140 m/min (about 1 km/h to 8.4 km/h). A probability distribution of these expansion rates is not known. For contrails older than a few minutes it is the vertical wind shear that drives lateral expansion. The probability distribution of vertical shear in flight levels is right-skewed, that is, cases with low shear predominate (Ernst 2012). It might thus be that the assumption of an average expansion speed of 5 km/h is too high. If so, this would mean that the unobserved inital phase of the ACTA contrails is on average even larger than the 1.5 h determined above. Furthermore, it can be assumed that narrow contrails experience a low expansion speed and broad contrails a high one. If this is generally true, the distribution of the initial unobserved times would be narrower than estimated from the assumption of Mannstein's rule.

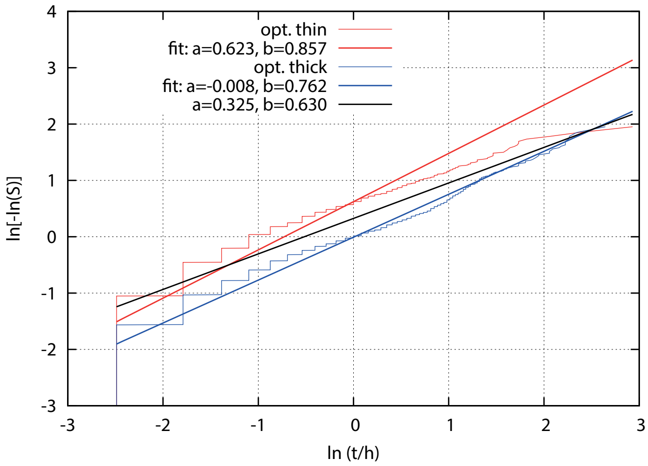

Figure 9: Survival functions in Weibull display for two subsets of ACTA contrails, optically thinner contrails which obtain a maximum optical thickness of less than 0.271 during their observation time (thin red), and contrails that reach a higher optical thickness (thin blue). The corresponding linear fits are presented in the respective colour as thick lines. The black line is for comparison. It is the linear fit obtained before (cf. Figure 2) for the whole data set.

3.3 Influence of the sensor characteristics

As argued above, the sensor characteristics must have an influence on the determination of the hazard function of young contrails. It is quite conceivable that it has an influence as well for the lifetime statistics of old contrails. A contrail can be observed the longer the more sensitive the satellite instrument is. Thus, if the instrument is more sensitive, we expect a longer mean duration of observation. Within the Weibull model this can be achieved with smaller (but positive) parameters λ and k, see Eq. (2.4). The effect of a decreasing k on the mean is very strong, such that it is rather λ than k which would be affected. A smaller value of λ would lead to an increased mean extended lifetime as well (Eq. (2.15)), which sounds paradoxical, but which is simply a consequence of a Weibull type decay of contrails with a parameter $k < 1$. Thus, neither k nor λ are probably characteristic of the atmospheric processes alone that carry contrails. Both parameters may depend on the sensor system used to observe contrails, but the rate parameter λ is probably more sensitive to the sensor characteristics than k.

As a test we have divided the dataset into 1150 contrails which obtained during their observation period a maximum optical thickness at 352 nm below 0.271 ("thin" contrails) and 1155 contrails with maximum optical thickness exceeding this value. The method for deriving the optical thickness from the IR channels is explained in detail in Kox et al. (2014) and has been applied to the ACTA dataset in Vázquez-Navarro et al. (2015). The survival functions of these two subsets are plotted in Figure 9, again displayed as Weibull plots (red and blue thin lines). The optimum linear fits are shown es well (red and blue thick lines). Obviously the Weibull distribution is not a good model for the subset of thin contrails. Perhaps this is partly due to the finite size of the data sets. At the high end, where both optical thickness classes have only one surviving member, the survival functions are nearly equal, 1/1150 and 1/1155, respectively. Thus the curves must approach each other. Apart from this, comparison with the black line (characterising the whole data set as before) shows that both parameters (a and b) differ between the subsets themselves and also between any subset and the whole set. The λ values for the two subsets are 2.07 h^{-1} and 0.99 h^{-1} for the thin and thick contrails, respectively. Not surprisingly, the generalised decay rate is larger for the thin than for the thick contrails, λ varies by a factor of two between these two subsets. The values of k vary as well, but less strongly, and they are both below unity. The general property of a hazard rate that decreases with contrail age, signified by $k < 1$, is valid for both subsets and this should be so according to the physical explanation of this behaviour given above. If one could take into account contrails that are already initially too optically thin for detection, one would probably get an even larger decay rate than for the optically thin subset from above, but still an exponent $k < 1$, because the latter is controlled by the physics of contrails more than by sensor characteristics.

The tracking of contrails in ACTA is actually initialised with data from MODIS, the Moderate Resolution Imaging Spectroradiometer on board the Terra satellite, which is a polar orbiting satellite that gives better spatial resolution than SEVIRI and which is thus used for contrail detection (Vázquez-Navarro et al. 2010). As an orbiting satellite, Terra overpasses a region with contrails in relatively short time, and only contrails present at the satellite overpass times are detectable. The question therefore arises whether this can produce a bias in the analysis. We don't think so. To produce a such a bias because of the discrete overpass times would require that contrail evolution is controlled by processes with a diurnal cycle which is not the case. Instead, contrail lifetimes depend on microphysical processes as crystal growth followed by sedimentation. These in turn depend on the amount of ice supersaturation, a synoptic feature. Thus contrail lifetimes are related to synoptic features, for instance wind speed and direction in comparison to the movement of ice supersaturated regions. The coverage of persistent contrails does not follow a diurnal cycle but rather it varies on longer (synoptic) time scales. This fact is evident for everybody and it makes it very plausible that the special mode of contrail detection in ACTA with polar orbiting satellites does not give rise to biases in the statistical results.

3.4 Considerations on the residual life after observation

The mean extended lifetime function can formally be derived when a survival function is determined. In the expression $f(\delta|\tau)$, that is, the probability density for the hypothetical unobserved residual lifetime δ given a concretely observed lifetime τ, the latter has usually not any special meaning apart from that it is in the concrete case known that the lifetime is at least τ. But in our application τ has the special meaning of being the end time of the observation period. Or, in other words, while τ in the theory is un-systematically chosen, here it appears systematic. Does this pose a problem for the application?

The question is then *how* systematic τ actually is. We stress that there are various circumstances that may lead to termination of the observation. As already noted, a contrail might get too optically thin for satellite detection. Of course, at the end of their lifetime all contrails get optically thin, but there are several processes that affect the optical thickness of contrails. The rate at which contrails get optically thin is not a constant. In a cloudy sky or when multiple contrails are present as in regions of heavy air traffic the observation of an individual contrail can get impossible when the contrast between the contrail and its background scene gets too weak. Obviously there are many ways this can happen, e.g. the surface type may change when the contrail is advected, background clouds may appear or disappear, neighbouring contrails may spread and overlap with the contrail under investigation. Hence neither in this case there is a constant rate by which the contrast is reduced. If we assume, for the sake of argument, that contrails vanish from the SEVIRI image always when a certain threshold of optical thickness or contrast is undercut, then it is evident from the variety of possible reasons for this that it would happen at different times and with contrails of otherwise (that is apart from optical thickness or contrast) different properties. Furthermore, it is not probable that the end of the observation of an individual contrail only depends on the contrail and its backgound. It surely depends as well on the viewing angle between the satellite and the contrail and thus on contrail location. Contrails can also disappear from the field of view by advection and observation ceases when the situation gets too involved, e.g. when a contrail gets surrounded by too many other contrails. This all justifies the assumption that the end of a contrail observation is hardly a systematic point in time; it has much more random than systematic components. For these reasons one can also assume that the contrail "process", if it does not actually stop before the next SEVIRI scan, proceeds with the same parameters than before, that is, the same λ and k, so that the mathematics can be used as shown. But even if some contrails actually stop to exist, this is included in the theory: $f(\delta|\tau)$ is defined for all non-negative real values of δ.

Contrail cirrus in the global model of Newinger and Burkhardt (2012) have a considerable larger mean (total) lifetime (say, 8 h) than the contrails that have been analysed by ACTA. This is further evidence that contrails may exist still after their tracking is over. Additionally the model contrails comprise specimen that are too optically thin to be detected at all with a satellite instrument. For the model this poses no problem, but here it points to the fact that satellite data inevitably underlie selection biases.

3.5 Are there day vs. night differences?

The ACTA data set contains a day/night flag for each record which is important for a correct interpretation of the radiative flux data in the set. Here we use it to see whether there are lifetime differences between contrails at night and contrails during the day.

The data set contains 1515 contrails that exist only during daytime, 729 contrails that exist only during night, and 61 contrails whose lifetime extends from night to day or from day to night. These numbers indicate that most contrails do not experience a day/night or night/day transition during their lifetime, but this impression could be aroused or amplified by the mentioned selection bias.

Figure 10 shows the survival functions for the three groups of contrails, daytime contrails (blue), nighttime contrails (black), and the remaining contrails (red). The survival functions for daytime and nighttime contrails show similar characteristics as before, a steep decay at small times and a less steep decay later. A fit with the same straight line ($f(t) = 0.325 + 0.63t$) as before would not look bad for both curves although it would in neither case be optimal. From the data we find the following means and standard deviations of their tracked lifetimes: daytime $\bar{t} = 0.88 \pm 1.26$ h, nighttime $\bar{t} = 0.70 \pm 0.99$ h. A Kolmogorov-Smirnov test (Press et al. 1989) indicates that the survival functions differ significantly (max. distance 0.126, $p < 3 \cdot 10^{-7}$). Also the mean values differ in a statistically significant way, as a Wilcoxon rank sum test shows ($p = 0.0044$). May this be as it

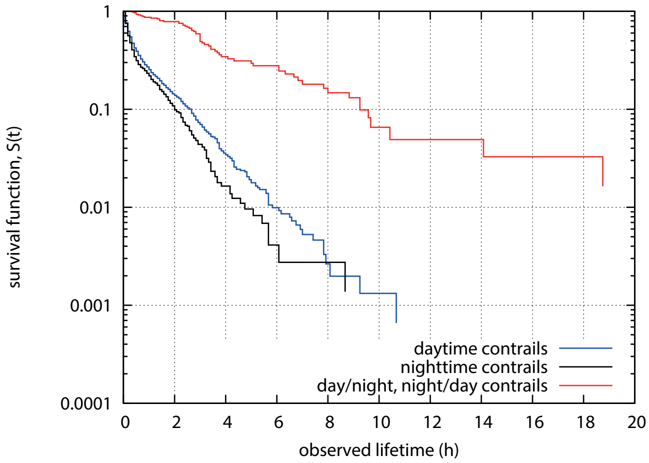

Figure 10: As Figure 1, but with contrails separated for observed lifetime during day only (blue), during night only (black), or mixed (red).

is, it does however not show that these small lifetime differences are physically important.

In contrast, the 61 contrails that experience a day-night (or vice versa) transition have a lifetime distribution that differs very much from that of the other contrails. This subset contains the contrails with the longest lifetimes. The maximum tracked lifetimes of day- and nighttime contrails in the ACTA data set are 10.67 and 8.67 h, respectively, while that of the mixed class is 18.75 h. This is not surprising: the longer a contrail exists the larger is its chance to exist during the sunlit and the dark hours of the day. For a contrail that exists longer than 12 h this can hardly be avoided outside the polar circle. The mean tracked lifetime of these contrails, $\bar{t} = 4.17 \pm 3.54$ h, is accordingly much larger than that of the pure day- or nighttime contrails.

4 Summary and conclusions

The lifetime data of 2305 contrails in the ACTA data set have been analysed statistically in the present paper. It is argued that the recorded lifetimes are only a part of the actual lifetimes of contrails because the satellite-borne detectors miss the formation and early expansion phase of contrails and the observation may terminate if a contrail gets too optically thin or if it loses contrast with its background scene. The three components of contrail lifetimes are the initial expansion phase, the observed phase, and the potential phase after observation (extended residual lifetime).

The observed lifetimes can fairly be modelled statistically using a Weibull distribution. The most important property of the Weibull distribution is its slope parameter k and the fact that $k < 1$. This means that the probability that a certain contrail survives the next 5 min increases with the contrail's age. This property can easily be explained from the fact that old persistent contrails are located in ice-supersaturated regions in mostly uplifting air. The mean observation time of the ACTA contrails is about 0.9 h.

The initial expansion phase of contrails can only indirectly be evaluated using the recorded initial widths as proxy of the expansion times. Assuming an average expansion speed of 5 km/h the contrails expand on average for 1.5 h before they get detected by a satellite. This expansion time may be even larger if the average expansion rate is smaller than the assumed 5 km/h. The initial widths are approximately normally distributed with a mean of 7.7 km for contrails that live long enough to become part of the ACTA data. For contrails in general, including the large number of non-persistent contrails, the initial widths and expansion times are probably strongly right-skewed. The Weibull model with $k < 1$ that works for the observed lifetimes could then work also if younger and narrower contrails could be observed as well. The data indicate independence of the initial widths and the observed lifetimes of contrails.

After the observation of a contrail ceases it can still exist for a while. The property $k < 1$ of the Weibull distribution of tracked lifetimes implies that the expected lifetime after observation increases with the actual duration of the observation period. A contrail that has been observed for two hours can be expected to exist for another two hours, but a contrail that has been observed for 18 hours can be expected to exist another four hours. The average over all observation durations of this extended lifetime after observation is about 1.3 h for the ACTA data set.

All in all, the actually observed contrail lifetime can be the smallest of the three lifetime parts. Actual contrail lifetimes can easily be at least three times as large as the lifetime measured by satellites. In a Monte Carlo simulation we can sum the three phases of life together to derive the distribution function of the complete lifetime. The mean value (and standard deviation) of it amounts to 3.7 ± 2.8 h. 80 % of all persistent contrails have lifetimes up to 5 h and only about 5 % have lifetimes exceeding 10 h.

Nightime contrails have on average a slightly shorter lifetime than daytime contrails in the ACTA data set. The differences are statistically significant, but whether they have important physical consequences is at least doubtful.

Finally we remark that "lifetime" is an ill-defined notion for clouds and contrails. It makes no sense to strive for exact values. Eventually we are interested in the climate impact of contrails. In regions as those observed by SEVIRI, that is the north Atlantic and Europe, air traffic is dense and it is not the lifetime of individual contrails that is important. It is rather the lifetime of the ice-supersaturated regions (Gierens et al. 2012) that is important since these carry persistent contrails. Studies of the latter topic are ongoing.

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