Estimating the reference frame: a smooth twice-differentiable utility function for non-compensatory loss-averse decision-making

Francisco J. Bahamonde-Birke*

Institut für Verkehrsforschung
Deutsches Zentrum für Luft- und Raumfahrt (DLR)
and
Energy, Transportation and Environment Department
Deutsches Institut für Wirtschaftsforschung – Berlin
and
Technische Universität Berlin
e-mail: bahamondebirke@gmail.com

ABSTRACT

Since the introduction of prospect theory, reference-dependence and loss-aversion have become widely acknowledged as important elements affecting decision-making. Nevertheless, establishing and determining reference frames are not extensively analyzed in the literature; rather, in most applications, it is simply assumed that the reference frames can be represented through the status quo. This assumption, however, may lead to biased results, as not only the status quo affects reference frames, but also previous experiences or expectations, among many others.

Therefore, it would be more appropriate to estimate the reference frame directly as an unobserved latent variable. Unfortunately, current utility functions utilized to depict this kind of behavior are not useful for this purpose, as they are defined piecewise. This work proposes a smooth twice-differentiable utility function that indeed allows estimating reference frames. Further, this function satisfies all major properties of prospect theory. Finally, the approach is tested relying on three case studies. They show that in the context of semi-compensatory loss-averted decision-making reference frames may diverge from the status quo.

Keywords: loss-aversion, utility function, discrete choice modeling, non-compensatory models

1. INTRODUCTION

Since the introduction of prospect theory in 1979 (Kahneman and Tversky, 1979), it has gained numerous supporters, while concepts such as loss-aversion and reference dependence are now common in the behavioral economics literature (Barberis, 2012; Karle *et al.*, 2015). Moreover, it has laid the theoretical foundation for the development of non-compensatory models and theories (e.g. regret theory; Loomes and Sugden, 1982). challenging the classic economic assumptions about utility and its impact is felt in fields from marketing (Hardie *et al.*, 1993; Ho *et al.*, 2006) to labor economics (Dunn, 1996; Fehr *et al.*, 2009), as well as medicine (Rizzo and Zeckhauser, 2003; Sokol-Hessner *et al.*, 2012), safety (Flügel *et al.*, 2015), and transportation (de Borger and Fosgerau, 2008; Dixit *et al.*, 2015), among many others.

Even though prospect theory was originally developed for addressing choices under risk, it is straightforward to extend its principles to any kind of choice situation involving subjunctive valuations. Basically, prospect theory sustains the existence of reference frames and that the gains and losses relative to these reference points are valued differently by decision makers (Frijn *et al.*, 2008). Thus, in order to correctly assess the behavior, it would be necessary to know the referential frame.

The majority of the literature contributions dealing with loss aversion assumes that the referential points are known *a priori* (e.g. de Borger and Fosgerau, 2008; Thiemann, 2017), which allows for clearly differentiating gains from losses and using different functional forms to include them in the expected utility functions. Normally, the reference points are assumed to be equal to the *status quo* (Flügel *et al.*, 2015). At first glance, it appears to be a sound assumption, when it is possible to identify the current conditions (for instance, when dealing with repeated choices). Nevertheless, as several authors correctly point out references do not depend exclusively on the *status quo*, but also on the individuals' previous expectations (Kőszegi and Rabin, 2006), which, in turn, depend on the individuals characteristics and their previous experiences (as it is clearly shown by the value learning literature; Kingsley and Brown, 2010). Moreover, even if it were possible for the modeler to identify for certain these previous expectations (which it is not), the model would still exhibit shortcomings, as empirical evidence indicates that reference frames are also affected by the choice-sets offered to the individuals (Zeelenberg and Pieters, 2007), which is clearly illustrated by the well-known decoy-effect (Huber et al., 1982; Josiam and Hobson, 1995; Guevara and Fukushi, 2016).

A possible way to address the aforementioned issues, assuming loss-averted decision making, would be for the modeler to estimate directly the reference point as a part of the decision model, instead of assuming it *a priori*. This way, the reference would appear as a parameter of the model, representing a change or an inflection point in the utility provided by a given attribute of the decision. Nevertheless, under the usual assumptions for the utility functions - see Maggi (2004) for a good overview on S-shaped utility functions - this approach is not feasible, as they are defined piecewise (with the reference frame representing a discontinuity) and therefore, they are not twice differentiable around zero.

Therefore, it is not possible to estimate the position of the reference point, as no gradient (and hessian matrix) exists for it.[1]

This paper introduces an S-shaped utility function that is continuous and twice-differentiable around zero, while still satisfying the main properties of loss-averse utility functions and microeconomic theory. This paper presents an extensive analysis of its properties, such as non-satiation, decreasing marginal utilities, axial asymmetry (which can be calibrated), etc. Such a representation offers multiple possibilities when dealing with loss-averse decision-making as it not only allows estimating the reference frames, but also how frames are affected by the individuals' characteristics and/or the offered choice-sets. The function exhibits a simple structure, which allows for an easy implementation in discrete choice models. Finally, the proposed approach is tested with the help of three case studies that indicate that reference frames may indeed diverge from the *status quo*.

2. THEORETICAL FRAMEWORK

Decision-making is usually approached from a utilitarian perspective. It suggests that decision-makers q will opt for the alternative i, belonging to a given choice-set A_q, that maximize their expected utility U_{iq}. Random Utility Theory (Thurstone, 1927; McFadden, 1974) postulates that this utility can be represented as the sum of a representative component and an error term (ε). If we assume additive linearity, it leads to the following expression:

$$U = X \cdot \beta + \varepsilon \qquad [1],$$

where X is a multidimensional matrix, whose dimensions represent individuals, alternatives and the observed attributes and characteristics of the aforementioned alternatives and individuals, respectively. β is a matrix of parameters to be estimated (whose rows are associated with the different elements of X, while the columns represent the different alternatives in the choice-set) and U and ε are matrixes, whose rows and columns represent individuals and alternatives, respectively[2]. The error ε can follow any desired distribution, but it is customary assumed to be i.i.d. Extreme-Value Type 1 (EV1) distributed, which leads to the well-known Multinomial Logit model (Domencich and McFadden 1975; MNL). Finally, the choices by the decision-makers q are represented by the matrix Y (same dimensions as U), whose elements take the value of one if the alternative is selected, and zero otherwise.

Obviously eq. [1] assumes a linear impact of the explanatory variables X over the utility function. This restriction, however, can be easily lifted by assuming that the elements of X represent, in fact, any possible transformation of the observed variables (e.g. an exponential or a Box-Cox transformation; Ortúzar and Willumsen, 2011). Hence, eq. [1] can be expressed in the following fashion.

---

[1] Nevertheless Kőszegi and Rabin (2006,2007) show that it is still possible to derive a functional model, although it still exhibits discontinuities and a degree of complexity that makes it impractical for most discrete choice modeling applications.

[2] The multidimensional matrix formulation differs from the standard notation considering individuals and alternatives as sub-indexes (Ben-Akiva and Lerman, 1985; Train 2009). Both specifications are, however, absolutely equivalent.

$$U = f(X) \cdot \beta + \varepsilon \qquad [2],$$

where *f(X)* is a matrix function of *X*. This representation is quite convenient to characterize real behavior as economic theory suggests that the marginal utilities of a given good are decreasing. Thus:

$$\frac{\partial^2 U}{\partial x^2} < 0 \qquad [3],$$

where x is a given element of X (attribute of the alternative) that can be considered to be a good in accordance to the Lancaster's principles (Lancaster, 1966). Therefore, it would be adequate to consider monotonically non-decreasing and concave functions for *f(X)* (obviously the function should have a negative sign if the considered attribute is an economic bad, such as the price).

Nevertheless, per definition, a discrete choice implies necessarily a trade-off. Thus, every time an individual makes a decision, they are giving assets and/or opportunities away. Therefore, it may be more convenient to represent utility functions in terms of changes relative to reference points representing resource and opportunity costs. In that case, giving more away than the reference would be considered as a loss (economic bad) while receiving more would be considered as gain. Hence, utility curves around the reference point for a given attribute should be S-shaped.

It is noteworthy, that discrete choice models *per se* are based on differences, but when considering linear specification, such as eq. [1], a reference value for a given attribute does not have any impact on the utility differences. When considering non-linear specifications, such as eq. [2], without references, it is implicitly assumed that the reference point is equal to zero.

Further, as proposed by Kahneman and Tversky (1979), individuals may also evaluate losses differently from gains (both relative to the reference frame). Hence, in accordance to prospect theory, utility functions may exhibit an S-shaped form of a type described in Figure 1:

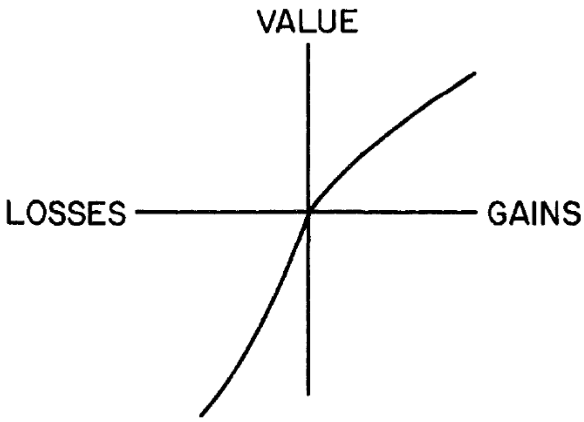

FIGURE 1 - Utility functions under prospect theory (Kahneman and Tversky, 1979)

Maggi (2004) summarizes the properties that utility functions should satisfy under prospect theory:

i) Function cuts the U-axis in the origin: $U(0)=0$

ii) Non-satiation - function is monotonically non-decreasing: $\dfrac{\partial U(x)}{\partial x}\geq 0 \quad \forall\, x$

iii) Axial asymmetry (loss aversion) - losses are valued more than gains: $\dfrac{\partial U(x)}{\partial x}<-\dfrac{\partial U(-x)}{\partial x} \quad \forall\, x>0$

iv) Decreasing marginal utilities - function is convex for x < 0 and concave for x > 0: $\dfrac{\partial^2 U(x)}{\partial x^2}\geq 0 \quad \forall\, x<0 \quad \wedge \quad \dfrac{\partial^2 U(x)}{\partial x^2}\leq 0 \quad \forall\, x>0$

Maggi (2004) also considers a fifth condition for the axial asymmetry implying that $U(x)<-U(-x)\,\forall\, x$, but it is straightforward to see that this condition will always be satisfied by continuous differentiable functions when the remaining properties are met.

Several specifications are proposed for loss-averse decision-making. They range from power S-shaped (Tversky and Kahneman, 1992; Benartzi and Thaler, 1995; among many others) to exponential functions (Schmidt and Zank, 2002; Köbberling and Wakker, 2005) over kinked lines (Thaler et al., 1997). Nevertheless, not one of these specifications is capable of satisfying all aforementioned properties over the whole domain of x (although the problems are minor and mostly limited to the close neighborhood of the reference point; see Maggi, 2004 for a good discussion).

The major inconvenience, however, is that all these functions are defined piecewise (hence, they are not twice differentiable around zero) and, thus, require the analyst to set the reference point *a priori*. While most applications simply consider the reference point to be equal to the status-quo (e.g. de Borger and Fosgerau, 2008; Flügel *et al.*, 2015), in many cases this information is not available. Further, it is established that reference frames do not only depend on the status-quo, but also on the individuals' expectations (Kőszegi and Rabin, 2006), previous experiences (Brown, 1995), the choice-set they face (Zeelenberg and Pieters, 2007), and, eventually, other circumstances. While models have been developed to take the influence of choice-sets into account (e.g. random regret minimization models; Chorus, 2012), these models fail to acknowledge that the extent to which an individual is susceptible to irrelevant alternatives depends on how familiar they are with a given choice-task (with individuals becoming less susceptible to choice-sets, as they get more experienced with the decision at hands).

Therefore, it appears much more appropriate for the analyst to attempt a direct estimation of the reference frames underlying a given decision. With that purpose in mind, the utility functions can be defined in the following manner:[3]

[3] Note that from a pure economic standpoint, it would be more appropriate to represent the marginal utilities (β) as a function of X. However, it would lead to more complex mathematical specifications departing from the usual representation of non-linearity in both prospect theory and discrete choice modeling. Thus, for comparison purposes, this specification is preferred.

$$U = f(X-\omega)\cdot\beta+\varepsilon \quad [4],$$

where *f(X)* is a matrix function satisfying prospect theory assumptions and ω is a matrix of reference points to be calibrated. Notwithstanding, the reference frames can also be a function of the characteristics of the individuals and attributes of the alternatives as well as of non-modeled variables (not considered or ignored by the analyst) that can be represented through error terms. Hence, ω can be depicted in the following fashion:

$$\omega = g(Z)\cdot\alpha+\zeta \quad [5],$$

where Z is a multidimensional matrix of characteristics of the individuals and attributes of the alternatives that may or may not be also contained in X (provided the model is identified), α is a matrix of parameters to be estimated and ζ is a multidimensional matrix of error terms. *g(Z)* is a matrix function, which for the purpose of this work would be assumed to be linear (making $g(Z)\bullet\alpha = Y\bullet\alpha'$) . Hence, replacing eq. [5] in eq. [4] results in the general specification for the utility being given by:

$$U = f(X-g(Z)\cdot\alpha+\zeta)\cdot\beta+\varepsilon = f(X-Z\cdot\alpha'+\zeta)\cdot\beta+\varepsilon \quad [6],$$

which allows for estimating the reference frames. Nevertheless, such a specification requires for *f(X)* to be twice differentiable around zero. Otherwise the model would not be able to estimate the parameters associated with the reference frame (as previously mentioned a non-twice differentiable utility function results in the non-existence of a gradient, making the estimation unfeasible).

3. A SMOOTH TWICE-DIFFERENTIABLE UTILITY FUNCTION FOR NON-COMPENSATORY LOSS-AVERSE DECISION-MAKING

As can be intuited, the core of the problem is defining a smooth twice-differentiable utility function satisfying the properties of prospect theory. Here, the problem is that most S-shaped functions are bounded (e.g. arctan(x) or the logistic function) and are therefore unable to satisfy the property iii).

Let's define the following function:

$$f(x) = \ln(1+\mathrm{e}^{a\cdot x})^c - \ln(1+\mathrm{e}^{-b\cdot x})^c \quad [7],$$

Where $a \le b$ and $0 < c < 1$ are parameters to be calibrated, and assure that eq. [7] satisfy the properties of prospect theory. Here, the left side of eq. [7] represents changes of utility associated with gains in the good *x*. Similarly to regret functions (Chorus, 2012), the left side of eq. [7] increases monotonically and unboundedly for $x>0$ and is bounded between 0 and 1 for $x<0$. Conversely, the right side of eq. [7] stands for utility changes associated with losses; thus, it increases monotonically for $x<0$ and is bounded between -1 and 0 for $x>0$. As a consequence, the left side dominates in the gains domain ($x>0$), while the right side dominates in the domain of the losses. Such a function would exhibit a shape similar to the one depicted in the following figure (where $a = 2; b = 3; c = 0.6$):

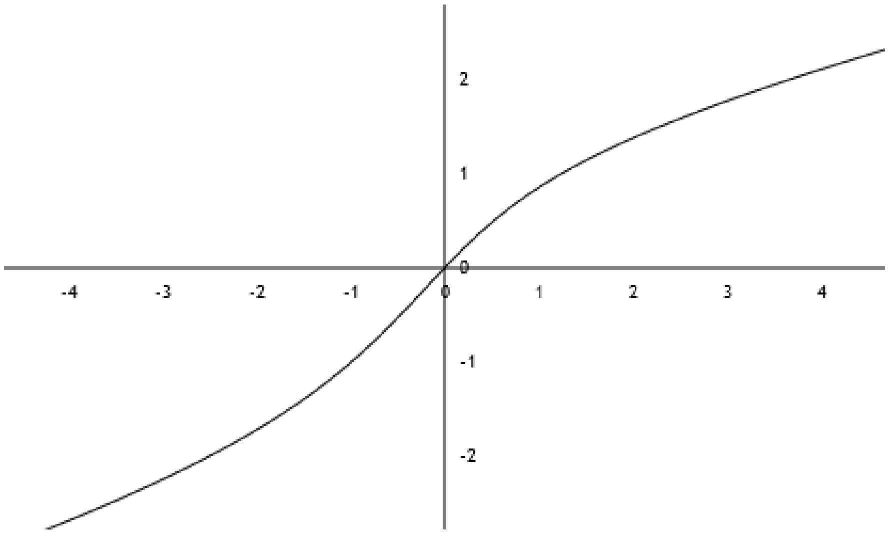

FIGURE 2 - A smooth twice-differentiable utility function

As can be easily seen, *f(x)* is defined for all real numbers. Further, it is twice differentiable around zero, satisfying the main conditions suggested in the previous section.

The analysis regarding the properties of loss-averse decision-making leads to the following conclusions:

i) Function cuts the U-axis in the origin:

$$f(0) = \ln(1+\mathrm{e}^{a\cdot 0})^{c} - \ln(1+\mathrm{e}^{-b\cdot 0})^{c} = 0 \qquad [8],$$

Hence, the property is satisfied.

ii) Non-satiation: The first derivative of eq. [7] is calculated:

$$\frac{\partial f(x)}{\partial x} = \frac{\ln(1+\mathrm{e}^{a\cdot x})^{c-1} ca\,\mathrm{e}^{a\cdot x}}{(1+\mathrm{e}^{a\cdot x})} + \frac{\ln(1+\mathrm{e}^{-b\cdot x})^{c-1} cb\,\mathrm{e}^{-b\cdot x}}{(1+\mathrm{e}^{-b\cdot x})} \qquad [9],$$

It is straightforward to see that $\frac{\partial U(x)}{\partial x} \geq 0 \quad \forall\, x$, as all elements are non-negative. Thus, this property is also satisfied.

iii) Axial asymmetry (loss aversion): Here, $a \leq b$ guarantees that the impact on the utility of losses in good x be larger than similar gains.

Even though the analysis of the second and third derivatives of eq. [7] after x (required for analyzing this property) does not lead to appealing closed-form expressions, it can be numerically shown that for $a \leq b$ this property will be always satisfied, as long as $c \geq [1 - \ln(2)]$[4].

In any other case, the condition will not be met in the neighborhood of zero (the reference point) but it will be asymptotically satisfied (when $x -> \infty$). The length of the interval for which the property will not be satisfied increases as c approaches zero.

Obviously, if the analyst is not considering an economic good but an economic bad (such as costs or travel time), loss aversion would imply that $b \leq a$, but the aforementioned condition still holds.

iv) Decreasing marginal utilities: Again, this property requires the analysis of the highly-involved second and third derivatives after of eq. [7]. It can be numerically shown that this property can only be satisfied if $c = [1 - \ln(2)]$. If $c < [1 - \ln(2)]$ the change of convexity will occur slightly after zero and *vice versa* (with $a \leq b$; again, the analyst is considering an economic bad $b \leq a$ and the previous statement is inverted). Regardless, it does not appear to have major implications, as it only affects the close neighborhood of the reference point and the effects of the decreasing marginal utilities are only significant as *x* increases.[5]

If *c* approaches one, the curvature of *f(x)* tends to zero. In that case, *f(x)* would simply represent a continuous kinked line.

As discussed, the proposed curve satisfies all properties of prospect theory for $c = [1 - \ln(2)]$ and exhibits only minor, barely relevant, problems for $[1 - \ln(2)] < c < 1$. Given that the obvious advantages of considering a continuous smooth twice-differentiable utility function clearly outweigh the small inconveniences, *f(x)* is considered to be appropriate to characterize loss-averse decision-making.

As final remarks, it may be advisable to fix either *a* or *b* at one, as they may be highly correlated with the parameters of β (also to be estimated; see eq. [6]). Also, the ratio *b/a* could be interpreted as a loss-aversion-index. Finally, the analyst may decide to consider different exponents *c* for both parts of eq. [7]. In that case, it cannot be guaranteed that the function will satisfy the properties of prospect theory. Nevertheless empirical evidence (Kahneman and Tversky, 1979; Flügel *et al.*, 2015; among others) shows no statistically

---

[4] Normally, the right side of eq. [7] (and consequentially of eq. [9]) dominates on the losses domain (x<0), while the left side dominates on the domain of the gains. However, if *c* is sufficiently small (c<[1-ln(2)]), the maximum of the right side of the first derivative of eq. [7] (the right side of eq. [9]) would occur for *x* slightly larger than 0 (c=[1-ln(2)] guarantees that the optimum occurs at x=0). Therefore, in the neighborhood of zero, the right side of eq. [9] would be larger for x>0 than for x<0. Nonetheless, as the right side of eq. [9] rapidly decreases to zero as *x* continues to increase, the phenomenon would be constrained to close neighborhood of zero. While the opposite would occur for the left side of eq. [9], *b* being larger than *a* implies that the optimum of the right side of eq. [9] will be larger than the optimum of the left side, causing the phenomenon.

[5] As explained in the previous case, for $a \neq b$ the optimum of eq. [9] would occur for x=0 , if and only if c=[1-ln(2)]. In any other case, eq. [9] would be maximized slightly before or after zero. $0 < c < 1$ guarantees that the second derivative of eq. [7] be negative for $x \to \infty$ and positive for $x \to -\infty$.

significant differences between both exponents (they, however, considered utility functions defined piecewise and established the reference frames *a priori*).

4. CASE STUDIES

To show the extent of the proposed approach as well as the advantages of estimating reference frames instead of relying on the *status quo*, three case studies are considered.

4.1 Monte Carlo Simulation

First and foremost, it is important to analyze the model's capability of recovering the true population parameters. For this purpose, I rely on a synthetic dataset, as it allows the analyst to know for certain the target values. Furthermore, it allows considering the effects of neglecting the true nature of the data.

Given the aforementioned purposes, the structure of the synthetic dataset is quite simple and it considers only one explanatory variable. Thus, a dataset of individuals behaving compensatorily in a Binomial Logit framework is generated in accordance with the following equation:

$$U = \beta_0 + \beta_x \cdot f(X_1) - \beta_x \cdot f(X_2) + \varepsilon \qquad [10],$$

where the first alternative is selected if $U>0$ and the second otherwise. The error term ε follows a Logistic distribution with mean zero and scale parameter one. Both X_1 and X_2 were randomly generated following a Normal distribution with mean 10 and standard deviation 5. It is also assumed that both X_1 and X_2 were subject of loss-aversion in accordance with the following equations:

$$f(X_i) = \left[\ln(1+\mathrm{e}^{a\cdot(X_i-\delta)})^c - \ln(1+\mathrm{e}^{-b\cdot(X_i-\delta)})^c \right] \qquad [11],$$

where δ represents the reference frame. Hence, if X_i is larger than δ, X_i would be perceived as a gain; otherwise it would be perceived a loss. For the purposes of this simulation exercise, the reference frame δ was set at 5, while *a*, *b*, and *c* were assumed to be equal to 1, 2 and 0.7, respectively. Finally β_0 and β_x were set at 1 and 2. In total, 25,000 pseudo-individuals were generated, observing that 13,874 opted for alternative one, while 11,126 opted for the second alternative.

Using this synthetic database, three different models were estimated. The first model considers the proposed approach, following the specification used in the generation of the dataset. The second model also considers the proposed approach but fixes the reference frame arbitrarily (in this case at 2). Finally, the last model presents the results associated with considering a linear impact of X_i on the utility function. The results are presented in Table 1.

As it can be observed, the first model recovers the true parameters used in the generation of the dataset without major problems. Both, the second and the third model fail to recover the target values biasing the results. Also, the second model shows that imposing an incorrect reference frame *a priori* does not only lead to the estimation of an incorrect

β_x parameter, but also suggest the non-existence of loss aversion (*b* is not statistically different from *a*) and a substantially larger degree of decrease for the marginal utilities (*c* is significantly smaller than the target value). Finally, the adjustment of the model considering the proper specification is widely superior to the goodness-of-fit of the alternative models, as can be easily shown by conducting a likelihood ratio test (LRT), as the second and third models are special cases of the first. The Akaike Information Criterion (AIC) and the Bayesian Information Criterion (BIC) exhibit similar results.

Table 1 – Parameter estimates. Simulation Exercise

Variable	*Target Value*	*Loss-averted model* **Estimate** *(st. dev.)*		*Loss-averted model* (δ=2) **Estimate** *(st. dev.)*		*Linear model* **Estimate** *(st. dev.)*	
β_0	*1*	**1.03**	*(0.03)*	**1.02**	*(0.03)*	**0.984**	*(0.029)*
β_x	*2*	**1.95**	*(0.193)*	**4.38**	*(0.316)*	**0.89**	*(0.013)*
a	*1*	**1**	*(fixed)*	**1**	*(fixed)*	**1**	*(fixed)*
b	*2*	**2.15**	*(0.193)*	**0.924**	*(0.107)*	**1**	*(fixed)*
c	*0.7*	**0.702**	*(0.035)*	**0.518**	*(0.019)*	**1**	*(fixed)*
δ	*5*	**5.19**	*(0.289)*	**2**	*(fixed)*	**0**	*(fixed)*
Log-likelihood		**-4,673.6**		**-4,690.1**		**-4,921.7**	
# of parameters		**5**		**4**		**2**	
LRT		**-**		**33**	*(p<0.0001)*	**496.2**	*(p<0.0001)*
AIC		**9,357.2**		**9,388.2**		**9,847.4**	
BIC		**9,397.8**		**9,420.7**		**9,863.7**	

Summarizing, the results of the simulation show that the proposed framework is capable of recovering the true population parameters and that imposing a reference frame *a priori* (as many researchers do when dealing with prospect theory, when assuming that the reference frame is equal to the *status quo*) may lead to widely biased results.

4.2 Real Dataset 1

The data for the second case study comes from an SP experiment on interurban modal choice conducted in Germany during 2014. The experiment was carried out in three waves (January 2014, March 2014 and April/May 2014), contacting students and employees of two universities in Berlin (Technische Universität Berlin and Humboldt-Universität zu Berlin), as well as employees of member institutions of the Leibniz-Gemeinschaft (for further details see Bahamonde-Birke *et al.*, 2015). Here, the respondents were asked to choose between different interurban public transport alternatives in Germany (regional and intercity trains, and interurban coaches; the interurban coach market had just been liberalized in 2013; Bahamonde-Birke *et al.*, 2014). Respondents were required to choose between a first pivotal alternative, representing a trip previously described, and a new one. Alternatives were described in terms of their travel time (TT in minutes), fare (P in €), number of transfers (NT), mode of transport - regional trains (RE; dummy variable), intercity trains (FVZ; dummy variable) and coaches (LB; dummy variable) - and a safety level.

In this case, the first alternative would obviously represent the *status quo* (as it is associated with a previously described trip). The survey also included the gathering of socioeconomic information as well as perceptual and attitudinal indicators, but for the illustrative purposes of this case study, only the attributes of the alternatives will be considered. Also for illustrative purposes, the dataset was limited to only consider trips that originally cost between 50€ and 80€[6]. After reduction, the dataset consisted of 2,446 observations.

To address the reference frames, the following variable is defined:

$$\omega_i = P_i - P_1 + \delta \qquad [8],$$

where P_i and P_1 represent the fare of the considered alternative and of the alternative 1, respectively. The fare associated with the first alternative also represents the *status quo*. δ, in turn, represents a gap indicating that the reference frame may diverge from the *status quo*. A positive value for δ would imply that the reference frame is set lower than the *status quo*; hence changes would be considered as losses even at price levels below P_1. Conversely, a negative value would imply a higher reference frame (above the *status quo*). All remaining variables (travel time, transportation mode and number of transfers) are considered linearly. Hence, the utility function is defined in the following fashion:

$$U_i = ASC_i + \beta_{TT} \cdot TT_i + \beta_{FVZ} \cdot FVZ + \beta_{RE} \cdot RE + \beta_{LB} \cdot LB + \beta_{NT} \cdot NT + \beta_P \cdot f(\varpi) \qquad [9],$$

where $f(\omega)$ exhibits a functional form as described in eq. [7]. To avoid collinearity problems, a is fixed at one and it was allowed for b and c to be estimated. Obviously, loss aversion would require for b to be smaller than one, as travel expenses are an economic bad. The ASC of the second (new alterative) was fixed at zero, so that the ASC of the pivotal alternative can be considered as an inertia variable. Similarly to the previous case, three models are considered. The first model considers the proposed framework, while the second model imposes that the reference frame be equal to the *status quo*. Then, the third model presents the linear treatment. For comparison purposes, a fourth model considering a piecewise treatment around the status quo is also considered (considering different marginal utilities if the price is below - $\beta_{P\ (Gain)}$ – or above - $\beta_{P\ (Loss)}$ – the *status quo*). The results of the model estimation are presented in Table 2.[7]

As it can be observed, the model estimated consistent with the proposed framework (first model) clearly outperforms the model considering loss aversion but assuming a reference frame equal to the status quo ($\delta=0$;second model), the linear model (third model), and the piecewise linear model (fourth model). While the AIC (confirming these results) and the BIC (yielding inconclusive results between the first and the second model) provide some

[6] As the original survey considered trips ranging between 4€ and 300€, considering the whole dataset would require addressing heteroscedasticity issues while addressing the reference frames. Similarly, as the data was collected in the context of an SP-off-RP experiment, it would require correcting for potential endogeneity (Train and Wilson, 2008). While, it is possible to correct for these issues, doing so would substantially increase the complexity of the illustrative examples. Therefore, the price spread was reduced, as it effectively contributes to minimize the aforementioned issues.

[7] This time, instead of the standard deviations (which were necessary to compare the estimators with the target values in the first case study), the table includes the t-test against zero for all parameters, as it eases the assessment of statistical significance.

insights, the results are based on the more appropriate LRT (again as the second and the third models are special cases of the first, the LRT is applicable), as it is a proper statistical test and not a heuristic information criterion. The LRT allows rejecting the null hypothesis of model equivalence at the 1% significance level. While a comparison of the first and fourth model is not possible on the basis of the LRT (as the latter is not a special case of the former) the heuristics information criteria suggest a much better performance of the former. Furthermore, it must be pointed out that the piecewise linear model closely resembles the proposed approach, when $\delta = 0$ and $c = 1$ (in fact both models yield almost identical goodness of-fit and parameters); in that case a LRT is possible, and it allows rejecting the simpler specification at the 1% significance level (LRT=28).

Table 2 – Parameter estimates. Real Dataset 1.

Variable	*Equation*	***Loss-averted model*** ***Estimate***	*(t-test)*	***Loss-averted model (status quo)*** ***Estimate***	*(t-test)*	***Linear model*** ***Estimate***	*(t-test)*	***Piecewise linear model*** ***Estimate***	*(t-test)*
$\beta_{Inertia}$	***Utility Alternative 1***	**0.878**	*(9.15)*	**0.826**	*(7.66)*	**0.477**	*(7.2)*	**0.762**	*(7.66)*
β_{TT}	***Utility Alternative 1 and 2***	**-0.0111**	*(-10.33)*	**-0.0116**	*(-11.02)*	**-0.0111**	*(-10.66)*	**-0.0113**	*(-10.82)*
β_{FVZ}	***Utility Alternative 1 and 2***	**0**	*(fixed)*	**0**	*(fixed)*	**0**	*(fixed)*	**0**	*(fixed)*
β_{LB}	***Utility Alternative 1 and 2***	**-1.13**	*(-6.55)*	**-1.19**	*(-6.85)*	**-1.07**	*(-6.42)*	**-1.21**	*(-7.04)*
β_{RE}	***Utility Alternative 1 and 2***	**-0.435**	*(-4.11)*	**-0.438**	*(-4.15)*	**-0.343**	*(-3.31)*	**-0.386**	*(-3.68)*
β_{NT}	***Utility Alternative 1 and 2***	**-0.302**	*(-5.05)*	**-0.307**	*(-5.14)*	**-0.280**	*(-4.75)*	**-0.309**	*(-5.19)*
β_{P}	***Utility Alternative 1 and 2***	**-0.865**	*(-3.62)*	**-0.222**	*(-2.69)*	**-0.0614**	*(-11.6)*	-	-
a	***Utility Alternative 1 and 2***	**1**	*(fixed)*	**1**	*(fixed)*	**1**	*(fixed)*	-	-
b	***Utility Alternative 1 and 2***	**0.0203**	*(0.25)*	**3.47**	*(1.63)*	**1**	*(fixed)*	-	-
c	***Utility Alternative 1 and 2***	**0.344**	*(3.22)*	**0.489**	*(4.31)*	**1**	*(fixed)*	-	-
δ	***Utility Alternative 1 and 2***	**8.42**	*(3.98)*	**0**	*(fixed)*	**0**	*(fixed)*	-	-
$\beta_{P\ (Gain)}$	***Utility Alternative 1 and 2***	-	-	-	-	-	-	**-0.0882**	*(-10.88)*
$\beta_{P\ (Loss)}$	***Utility Alternative 1 and 2***	-	-	-	-	-	-	**-0.0426**	*(-6.59)*
Log-likelihood		**-1,417.2**		**-1,421**		**-1,444.5**		**-1,431.2**	
# of parameters		**9**		**8**		**6**		**7**	
LRT		-		**7.6**	*(p=0.006)*	**54.6**	*(p<0.001)*	(not defined)	
AIC		**2,852.4**		**2,858**		**2,901**		**2,876.4**	
BIC		**2,904.6**		**2,904.4**		**2,935.8**		**2,917**	

Focusing on the first model, all estimated parameters exhibit the expected signs and, other than *b*, are statistically different from zero. A closer examination of the parameters associated with $f(\omega)$ reveals a strong loss-aversion, as *b* is not statistically different from zero. It means that fare reductions below the reference frame are not associated with statistically significant increases in utility. Conversely, price increases above the reference frame imply a significant reduction of the utility ascribed to the alternative. It contrasts with the results associated with the second model, which suggest that *b* is larger than 1 meaning that losses would have a smaller effects than gains (as the fare is an economic bad), which contradicts the theory (however, *b* is not statistically different from 0). Similar contra-intuitive results are observed for the piecewise model (with gains having a larger impact that losses), which also assumes that that the reference level is given by the status quo. Further, the analysis reveals the existence of decreasing marginal utilities (*c* is statistically different from both zero and one), which is in line with our expectations. Finally, the parameter δ has a positive value and it is statistically significantly different from zero. It implies that the reference frame associated with the travel fare is indeed 8.42€ below the *status quo* and that current prices are also being interpreted as a loss. Therefore, in this case it would not be appropriate to rely on the *status quo* to depict the reference and doing so would results in biased estimators (see second model). A possible explanation for the phenomenon is that references are not solely based on the current situation but also on previous experiences and personal appreciations.

Figure 3 presents the differences of utility associated with changes in the travel fare.

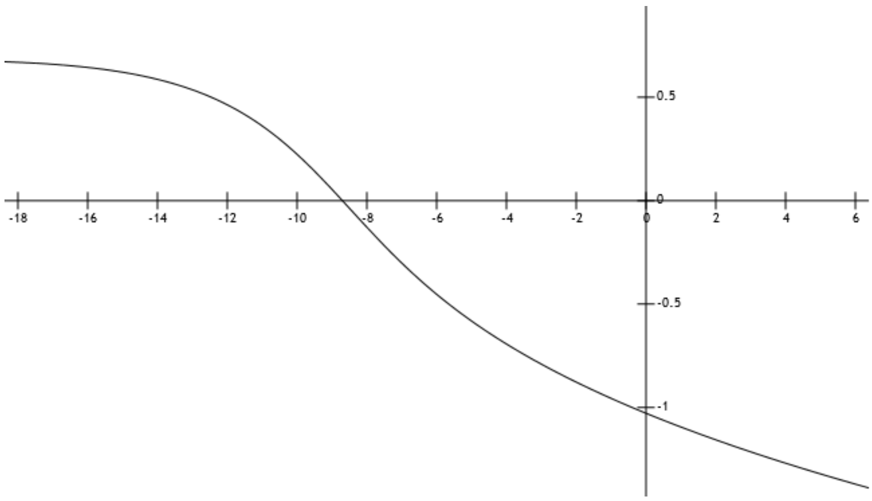

FIGURE 3 - ΔU associated with changes in the travel fare.

Here, the x-axis represents the changes in the travel fare as compared with the *status quo*, while the y-axis represents the changes in utility. Obviously, increases in the fare are associated with utility reductions, but the interesting fact is that all changes above -8.42€ (the estimated reference frame) are perceived as losses and are, therefore, hardly penalized.

4.3 Real Dataset 2

The data for the last case study arises from the same dataset considered in the former case study. This time, however, the dataset was limited to consider only trips that originally costed between 10€ and 15€. This allows addressing short interurban trips, for which both the trade-off between different attributes as well as the reference frames may be different. This way, the considered dataset consisted of 2,268 observations.

The treatment of the data was analogous to the previous case study and the results are presented in Table 3.

As it can be observed, and opposite to the previous case study, this time neither *b* nor *c* are statistically different from one, while the reference frame is not statistically different from the *status quo* (δ=0). As a consequence, both the first and the second model are not statistically superior to the linear model, and therefore, the latter should be preferred. Same applies to the fourth model ($\beta_{P\ (Gain)}$ and $\beta_{P\ (Loss)}$ are not statistically different).

These results suggest that in this case neither loss aversion nor decreasing marginal utilities exist, while the reference frame is adequately represented by the *status quo*. Possible explanations for the phenomenon are that the lower price level of short interurban trips has a lesser effect on the personal budget, while the higher frequency of these trips contributes to individuals associating their references with current prices.

Table 3 – Parameter estimates. Real Dataset 2.

Variable	*Equation*	***Loss-averted model*** **Estimate** *(t-test)*		***Loss-averted model (status quo)*** **Estimate** *(t-test)*		***Linear model*** **Estimate** *(t-test)*		***Piecewise linear model*** **Estimate** *(t-test)*	
$\beta_{Inertia}$	***Utility Alternative 1***	**0.125**	*(1.35)*	**0.125**	*(1.35)*	**0.0867**	*(1.19)*	**0.125**	*(1.32)*
β_{TT}	***Utility Alternative 1 and 2***	**-0.0215**	*(-9.88)*	**-0.0215**	*(-9.88)*	**-0.0215**	*(-10.11)*	**-0.0218**	*(-10.06)*
β_{FVZ}	***Utility Alternative 1 and 2***	**0**	*(fixed)*	**0**	*(fixed)*	**0**	*(fixed)*	**0**	*(fixed)*
β_{LB}	***Utility Alternative 1 and 2***	**-2.22**	*(-13.99)*	**-2.22**	*(-14)*	**-2.21**	*(-13.99)*	**-2.22**	*(-13.98)*
β_{RE}	***Utility Alternative 1 and 2***	**-0.415**	*(-3.8)*	**-0.415**	*(-3.81)*	**-0.408**	*(-3.82)*	**-0.422**	*(-3.88)*
β_{NT}	***Utility Alternative 1 and 2***	**-0.692**	*(-11.33)*	**-0.693**	*(-11.4)*	**-0.69**	*(-11.5)*	**-0.688**	*(-11.45)*
β_{P}	***Utility Alternative 1 and 2***	**-0.261**	*(-3.21)*	**-0.26**	*(-3.38)*	**-0.311**	*(-13.79)*	-	-
a	***Utility Alternative 1 and 2***	**1**	*(fixed)*	**1**	*(fixed)*	**1**	*(fixed)*	-	-
b	***Utility Alternative 1 and 2***	**1.17**	*(3.86)*	**1.16**	*(5.59)*	**1**	*(fixed)*	-	-
c	***Utility Alternative 1 and 2***	**1.08**	*(6.66)*	**1.08**	*(7.32)*	**1**	*(fixed)*	-	-
δ	***Utility Alternative 1 and 2***	**0.114**	*(0.04)*	**0**	*(0)*	**0**	*(fixed)*	-	-
$\beta_{P\ (Gain)}$	***Utility Alternative 1 and 2***	-	-	-	-	-	-	**-0.0341**	*(-6.64)*
$\beta_{P\ (Loss)}$	***Utility Alternative 1 and 2***	-	-	-	-	-	-	**-0.0304**	*(-12.10)*
Log-likelihood		**-1228.6**		**-1228.6**		**-1228.9**		**-1,228.7**	
# of parameters		**9**		**8**		**6**		**7**	
LRT		**-**		**0.02**	*(p=089)*	**0.6**	*(p=09)*	(not defined)	
AIC		**2,475.2**		**2,473.2**		**2,469.8**		**2,471.4**	
BIC		**2,526.7**		**2,519**		**2,504.2**		**2,511**	

5. CONCLUSIONS

It is a well-established fact in behavioral economics that loss-aversion and reference dependence significantly impact the decision-making process (Kahneman and Tversky, 1979; Barberis, 2012; Karle et al., 2015). Nevertheless, determining the reference frames is not easy as they may be affected by several reasons, including the *status quo,* previous experiences, personal appreciation, choice-sets, etc. (Kőszegi and Rabin, 2006; Zeelenberg and Pieters, 2007). Therefore, estimating and calibrating the reference frame is a significant issue for modeling in presence of non-compensatory loss-averting behavior.

Notwithstanding, the large majority of literature simply associates references frames with the *status quo* (de Borger and Fosgerau, 2008; Flügel et al., 2015). The reason is that loss-averted utility functions are defined piecewise (Maggi, 2004) and, thus, it is necessary to know the reference frame *a priori*. This work proposes that reference frames should indeed be estimated and introduces a smooth twice-differentiable utility function, which allows for their estimation. This function satisfies all major properties of prospect theory, including non-satiation, loss-aversion, and decreasing marginal utilities (even though minor problems can be identified in the close neighborhood of the reference point).

Three case studies exhibiting the properties of the proposed approach are considered. First, a Monte Carlo simulation shows the model's capability to recover the true parameters used in the generation of the dataset. It also shows that neglecting the true nature of the decision-making process (e.g. imposing an incorrect reference frame) may lead to biased results. A second case study shows that, for real data, in the context of semi-compensatory lost-averted decision-making, reference frames may diverge from the *status quo*. In fact, the case study shows that the reference point for travel expenses was 8.42€ below the *status quo* and, thus, even paying the current fare would be perceived as a loss. Previous experiences as well as personal appreciations provide plausible explanations for the phenomenon. Finally, a third case study shows that in a different context, the decision-making process may not be subject of loss aversion and that reference frames may be properly represented by the *status quo*.

The aforementioned case studies highlight the importance of adequately depicting the actual underlying decision-making process, and that simpler representations or assuming the reference frame to be equal to the *status quo* may lead to biased results. Therefore, it is important to test for non-compensatory loss-averted decision-making. However, while the proposed framework allows addressing these issues, significant need for further research can be identified. It includes defining necessary and sufficient conditions for identification, empirical identifiability, required samples sizes, and considering the inclusion of random error components inside the proposed structure (as it is fundamental to address issues such as endogeneity and heterogeneity). Along the same lines, the proposed framework opens several opportunities regarding the analysis of reference frames' formation and the factors affecting it (characteristics of the individuals, previous experiences, choice-sets, etc.)

REFERENCES

Bahamonde-Birke, F., Kunert, U., Link, H. and Ortúzar, J. de D. (2014). Bewertung der Angebotsmerkmale des Personenfernverkehrs vor dem Hintergrund der Liberalisierung des Fernbusmarktes. *Zeitschrift für Verkehrswissenschaft* **85(2)**, 107-123.

Bahamonde-Birke, F.J., Kunert, U., Link, H. and Ortúzar, J. de D. (2015). About attitudes and perceptions – finding the proper way to consider latent variables in discrete choice models. *Transportation* **44(3)**, 475-493.

Barberis, N. C. (2012). Thirty years of prospect theory in economics: A review and assessment. *Journal of Economic Perspectives* **27(1)**, 173–196.

Ben-Akiva, M. E., and Lerman, S. R. (1985). *Discrete choice analysis: theory and application to travel demand* (Vol. 9). MIT press.

Benartzi, S and Thaler, R. (1995), Myopic Loss Aversion and the Equity Premium Puzzle, *The Quarterly Journal of Economics* **110**, 74—92.

de Borger, B. and Fosgerau, M. (2008). The trade-off between money and travel time: A test of the theory of reference-dependent preferences. *Journal of Urban Economics* **64(1)**, 101-115.

Brown, P. M. (1995). Learning from experience, reference points, and decision costs. *Journal of Economic Behavior & Organization* **27(3)**, 381-399.

Chorus, C. (2012). Random regret minimization: an overview of model properties and empirical evidence. *Transport Reviews* **32(1)**, 75-92.

Dixit, V. V., Harb, R. C., Martínez-Correa, J. and Rutström, E. E. (2015). Measuring risk aversion to guide transportation policy: Contexts, incentives, and respondents. *Transportation Research Part A: Policy and Practice* **80**, 15-34.

Domencich, T.A. and McFadden, D. (1975). *Urban travel demand - A behavioral analysis*. Amsterdam, Netherlands.

Dunn, L. F. (1996). Loss aversion and adaptation in the labor market: Empirical indifference functions and labor supply. *The Review of Economics and Statistics* **78(3)**, 441-450.

Fehr, E., Goette, L. and Zehnder, C. (2009). A behavioral account of the labor market: The role of fairness concerns. *Annu. Rev. Econ.* **1(1)**, 355-384.

Flügel, S., Elvik, R., Veisten, K., Rizzi, L. I., Meyer, S. F., Ramjerdi, F. and Ortúzar, J. de D. (2015). Asymmetric preferences for road safety: Evidence from a stated choice experiment among car drivers. *Transportation Research Part F: Traffic Psychology and Behaviour* **31**, 112-123.

Frijns, B., Koellen, E., and Lehnert, T. (2008). On the determinants of portfolio choice. *Journal of Economic Behavior & Organization* **66(2)**, 373-386.

Guevara, C. A., and Fukushi, M. (2016). Modeling the decoy effect with context-RUM Models: Diagrammatic analysis and empirical evidence from route choice SP and mode choice RP case studies. *Transportation Research Part B: Methodological* **93**, 318-337.

Hardie, B. G., Johnson, E. J. and Fader, P. S. (1993). Modeling loss aversion and reference dependence effects on brand choice. *Marketing science* **12(4)**, 378-394.

Ho, T. H., Lim, N. and Camerer, C. F. (2006). Modeling the psychology of consumer and firm behavior with behavioral economics. *Journal of marketing Research* **43(3)**, 307-331.

Huber, J., Payne, J. W., and Puto, C. (1982). Adding asymmetrically dominated alternatives: Violations of regularity and the similarity hypothesis. *Journal of consumer research* **9(1)**, 90-98.

Josiam, B. M., and Hobson, J. P. (1995). Consumer choice in context: the decoy effect in travel and tourism. *Journal of Travel Research* **34(1)**, 45-50.

Kahneman, D. and Tversky, A. (1979). Prospect theory: An analysis of decision under risk. *Econometrica: Journal of the Econometric Society* **47**, 263-291.

Karle, H., Kirchsteiger, G. and Peitz, M. (2015). Loss Aversion and Consumption Choice: Theory and Experimental Evidence. *American Economic Journal: Microeconomics* **7(2)**, 101-120.

Kingsley, D. C., and Brown, T. C. (2010). Preference uncertainty, preference learning, and paired comparison experiments. *Land Economics* **86(3)**, 530-544.

Köbberling, V. and Wakker, P.P. (2005). An index of loss aversion. *Journal of Economic Theory* **122(1)**, 119-131.

Kőszegi, B. and Rabin, M. (2006). A model of reference-dependent preferences. *The Quarterly Journal of Economics* **121(4)**, 1133-1165.

Kőszegi, B. and Rabin, M. (2007). Reference-dependent risk attitudes. *The American Economic Review* **97(4)**, 1047-1073.

Lancaster, K. J. (1966). A new approach to consumer theory. *The journal of political economy* **74**, 132-157.

Loomes, G., and Sugden, R. (1982). Regret theory: An alternative theory of rational choice under uncertainty. *The economic journal* **92(368)**, 805-824.

Maggi, M. A. (2004). A characterization of s-shaped utility functions displaying loss aversion. Quaderni di Dipartimento **165**, *EPMQ, Università degli Studi di Pavia*.

McFadden, D. (1974). Conditional logit analysis of qualitative choice behaviour. In Zarembka, P. (ed.), *Frontiers in Econometrics* 105-142. Academic Press, New York.

Ortúzar, J. de D. and Willumsen, L.G. (2011). *Modelling Transport*. Fourth Edition, John Wiley and Sons, Chichester.

Rizzo, J. A. and Zeckhauser, R. J. (2003). Reference incomes, loss aversion, and physician behavior. *Review of Economics and Statistics* **85(4)**, 909-922.

Schmidt, U. and Zank, H. (2002). Strong risk aversion in cumulative prospect theory. Working Paper, The University of Manchester, UK.

Sokol-Hessner, P., Camerer, C. F. and Phelps, E. A. (2012). Emotion regulation reduces loss aversion and decreases amygdala responses to losses. *Social cognitive and affective neuroscience* **8**, 341-350.

Thaler, R. H., Tversky, A., Kahneman, D. and Schwartz, A. (1997). The effect of myopia and loss aversion on risk taking: An experimental test. *The Quarterly Journal of Economics* **112**, 647-661.

Thiemann, K. (2017). Ability tracking or comprehensive schooling? A theory on peer effects in competitive and non-competitive cultures. *Journal of Economic Behavior & Organization* **137**, 214-231.

Thurstone, L.L. (1927). A law of comparative judgment. *Psychological Review* **34**, 273 286.

Train, K., and Wilson, W. W. (2008). Estimation on stated-preference experiments constructed from revealed-preference choices. *Transportation Research Part B: Methodological* **42(3)**, 191-203.

Train, K. (2009). *Discrete choice with simulation*. Cambridge: University Press.

Tversky, A. and Kahneman, D. (1992). Advances in prospect theory: Cumulative representation of uncertainty. *Journal of Risk and uncertainty* **5(4)**, 297-323.

Zeelenberg, M. and Pieters, R. (2007). A theory of regret regulation 1.0. *Journal of Consumer psychology* **17(1)**, 3-18.