SIMULATION AND CONTROL OF SMART STRUCTURES IN MULTIBODY SYSTEMS

Ondřej Vaculín and Andreas Heckmann
DLR, German Aerospace Center, Institute of Robotics and Mechatronics, Vehicle System Dynamics, PO Box 1116, 82230 Wessling, Germany
{Ondrej.Vaculin|Andreas.Heckmann}@DLR.de

Abstract This paper presents a methodology for the simulation of smart structures with piezoceramic patches by means of multibody dynamics. A theoretical background is mentioned adapting a modal multifield approach. Then a methodology for the control design is proposed. The methodology includes the optimisation of actuator placement, which is based on a modal representation of the elasticity. An application example illustrates the implemented process chain. This procedure constitutes a complex development environment for the simulation, optimisation and control design of elastic structures with active materials.

1. Introduction

Adaptive or smart structures are mechatronic devices which allow vibration properties and responses of mechanical systems to be modified; they are particularly used to improve the performance of lightweight structures. Among the wide range of supposable physical effects and corresponding material compositions, thin piezoceramic patches integrated in the structures proved their potential as electromechanical and mechanoelectrical transducers, which can be simultaneously exploited as actuators and sensors to control the vibration of the elastic structures, [1]. The piezoceramic patches apply additional mechanical forces as actuators and generate electrical charges as sensors. The additional electrical and mechanical measures should be considered for simulating the behaviour of flexible bodies equipped with the piezopatches.

Since the smart structures are mechatronic devices, their design involves several engineering disciplines such as structural mechanics, electronics and control engineering. The optimisation of such a complex system is a challenging task which may be supported advantageously by multibody system (MBS) dynamics as a method of system dynamics. Moreover, the MBS approach enables an efficient simulation of complex systems composed of elastic and rigid bodies with large overall motion such as vehicles which can be equipped with piezopatches.

It is state-of-the-art of industrial MBS tools to incorporate the results of an appropriate finite element analysis to obtain the mechanical data of flexible bodies. This approach may not yet be applied to the data of smart structures. Although the finite element modelling of piezoelectric devices on shell elements is a field of active research, [2, 3], it is not yet introduced in an industrial finite element tool. Nevertheless, to enable the simulation of structures with shell elements, the following technique uses only purely mechanical data which are readily available.

Further, the multibody codes offer an excellent connection to computer aided control engineering (CACE) tools. These tools are brought into action during the controller design and simulation. The methods originating in control engineering and modal approach are then used to optimise the placement of the piezoceramic patches.

2. Theory Outline

Current industrial multibody tools are capable of describing the displacement field of elastic bodies based on their modal representation. A modal analysis of an elastic body yields discrete mode matrices for every node k, located at the position $\boldsymbol{r}_k \in \mathbb{R}^3$ which specify the displacements $\boldsymbol{\Phi}_{u,k} \in \mathbb{R}^{3,p}$ and rotations $\boldsymbol{\Psi}_{u,k} \in \mathbb{R}^{3,p}$ as functions of all p observed modes.

In order to simulate elastic structures with piezoceramic transducers, their electromechanical and mechanoelectrical behaviour have to be considered additionally. The constitutive equation, needed to base this multifield formulation, states the linearised relationship between the mechanical strain $\boldsymbol{S}$ and stress $\boldsymbol{T}$ and the electromechanical displacement $\boldsymbol{D}$ and electrical field strength $\boldsymbol{E}$ by defining appropriate material constants $\boldsymbol{c}$, $\boldsymbol{e}$ and $\boldsymbol{\varepsilon}$, [4]:

$$\begin{pmatrix} \boldsymbol{T} \\ \boldsymbol{D} \end{pmatrix} = \begin{pmatrix} \boldsymbol{c} & -\boldsymbol{e}^T \\ \boldsymbol{e} & \boldsymbol{\varepsilon} \end{pmatrix} \begin{pmatrix} \boldsymbol{S} \\ \boldsymbol{E} \end{pmatrix} . \tag{1}$$

The field equations are formulated by means of Jordain's principle of virtual power, [5]:

$$\begin{aligned} \int \delta \boldsymbol{v}^T \varrho \boldsymbol{a} + \delta \dot{\boldsymbol{S}}^T \underbrace{(\boldsymbol{c}\boldsymbol{S} - \boldsymbol{e}^T \boldsymbol{E})}_{\boldsymbol{T}} - \delta \dot{\boldsymbol{E}}^T \underbrace{(\boldsymbol{e}\boldsymbol{S} + \boldsymbol{\varepsilon}\boldsymbol{E})}_{\boldsymbol{D}} \, \mathrm{d}V = \\ \int \delta \boldsymbol{v}^T \boldsymbol{f}_V \, \mathrm{d}V + \int \delta \boldsymbol{v}^T \boldsymbol{f}_B - \delta \dot{\varphi} Q_\varphi \, \mathrm{d}B \, . \end{aligned} \tag{2}$$

The right hand side of equation (2) represents all external force and charge loads acting on volumes or boundaries. The variables $\boldsymbol{v}$ and $\boldsymbol{a}$ denote the absolute velocity and acceleration of a volume element; Q_φ and φ are used to name the applied charges and their electric potential. Furthermore, the dependent variables $\boldsymbol{T}$ and $\boldsymbol{D}$ are eliminated, pointing out the coupling of mechanical and electrical fields by the material description in (1).

A floating frame of reference formulation, [6], enables the superimposition of nonlinearly described, large overall motion, later on denoted by the subscript $_R$, with linearised, small elastic deformations $\boldsymbol{u}_u$. Based on the Ritz approximation, separating $\boldsymbol{u}_u(\boldsymbol{r}, t)$ in only space dependent mode shapes $\boldsymbol{\Phi}(\boldsymbol{r})$ and time dependent variables $\boldsymbol{q}(t)$, the strain tensor $\boldsymbol{S}$ can be evaluated by applying the differential displacement-strain-operator $\mathcal{L}$:

$$\boldsymbol{u}_u(\boldsymbol{r}, t) = \boldsymbol{\Phi}_u(\boldsymbol{r})\boldsymbol{z}_u(t) , \qquad \boldsymbol{S} = (\mathcal{L}\boldsymbol{\Phi}_u)\, \boldsymbol{z}_u = \boldsymbol{B}_u \boldsymbol{z}_u . \tag{3}$$

The electric field vector $\boldsymbol{E}$ is evaluated analogously by an approximation of the scalar electric potential field φ, defining the electric mode shapes $\boldsymbol{\Phi}_\varphi$ and the patch electrode voltages $\boldsymbol{z}_\varphi$ and the negative gradient operation:

$$\varphi(\boldsymbol{r}, t) = \boldsymbol{\Phi}_\varphi(\boldsymbol{r})\boldsymbol{z}_\varphi(t) , \qquad \boldsymbol{E} = (-\nabla\boldsymbol{\Phi}_\varphi)\, \boldsymbol{z}_\varphi = \boldsymbol{B}_\varphi \boldsymbol{z}_\varphi . \tag{4}$$

Further, the electromechanical coupling matrix $\boldsymbol{K}_{u\varphi} = \boldsymbol{K}_{\varphi u}^T$, the electric capacity matrix $\boldsymbol{K}_{\varphi\varphi}$ and the mechanical stiffness matrix $\boldsymbol{K}_{uu}$ are presentable as only volume dependent integrals:

$$\begin{aligned} \boldsymbol{K}_{uu} &= \int \boldsymbol{B}_u^T \boldsymbol{c} \boldsymbol{B}_u \, \mathrm{d}V , \\ \boldsymbol{K}_{u\varphi} &= \int \boldsymbol{B}_u^T \boldsymbol{e}^T \boldsymbol{B}_\varphi \, \mathrm{d}V , \\ \boldsymbol{K}_{\varphi\varphi} &= \int \boldsymbol{B}_\varphi^T \boldsymbol{\varepsilon} \boldsymbol{B}_\varphi \, \mathrm{d}V . \end{aligned} \tag{5}$$

A comparison of (2) with the classical equation of motion of unconstrained flexible multibody systems, e.g. in [6], yields the following:

$$\underbrace{\begin{pmatrix} \boldsymbol{M}_{aa} & \boldsymbol{M}_{a\alpha} & \boldsymbol{M}_{au} \\ & \boldsymbol{M}_{\alpha\alpha} & \boldsymbol{M}_{\alpha u} \\ \text{symm.} & & \boldsymbol{M}_{uu} \end{pmatrix}}_{\boldsymbol{M}} \underbrace{\begin{pmatrix} \boldsymbol{a}_R \\ \boldsymbol{\alpha}_R \\ \ddot{\boldsymbol{z}}_u \end{pmatrix}}_{\ddot{\boldsymbol{z}}} = \begin{pmatrix} \boldsymbol{h}_a \\ \boldsymbol{h}_\alpha \\ \boldsymbol{h}_u \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ -\boldsymbol{K}_{uu}\boldsymbol{z}_u + \boldsymbol{K}_{u\varphi}\boldsymbol{z}_\varphi \end{pmatrix} \tag{6}$$

and further, the sensor equation can be written as follows:

$$\boldsymbol{Q}_\varphi = \boldsymbol{K}_{u\varphi}^T \boldsymbol{z}_u + \boldsymbol{K}_{\varphi\varphi} \boldsymbol{z}_\varphi . \tag{7}$$

The mass matrix $\boldsymbol{M}$ on the left hand side of (6) is formulated as 3×3 block matrix to specify the inertia coupling between translational, angular and elastic motion acceleration terms $\boldsymbol{a}_R$, $\boldsymbol{\alpha}_R$ and $\ddot{\boldsymbol{z}}_u$. Further, $\boldsymbol{h}_a$, $\boldsymbol{h}_\alpha$ and $\boldsymbol{h}_u$ summarise all time and state dependent inertia, damping and external forces. The added product $\boldsymbol{K}_{u\varphi}\boldsymbol{z}_\varphi$ demonstrates the use of the piezopatches as structural actuators. The sensor equation (7) is needed to calculate the electric quantities, e.g. the electric charges $\boldsymbol{Q}_\varphi$, if the piezoelements are used as sensors or are parts of arbitrary electric circuits.

3. Control of Smart Structures

The controller design problem for the control of the vibration is connected with the selection of the patches. The proposed controller design methodology is based on the modal description of the elastic body and the placement of the patches results from the controller gains.

The supposed goal for the controller design is to control vibration of one node of the elastic structure.

3.1 Transformation to the State Space Form

The transformation of the description of the elastic body with piezoelements to a state space form needed for the controller design results in a multi-input multi-output (MIMO) system:

$$\begin{aligned} \dot{\boldsymbol{x}} &= \boldsymbol{A}\boldsymbol{x} + \boldsymbol{B}\boldsymbol{u} \quad , \\ \boldsymbol{y} &= \boldsymbol{C}\boldsymbol{x} + \boldsymbol{D}\boldsymbol{u} \quad , \end{aligned} \tag{8}$$

where $\boldsymbol{x}$ is the state, $\boldsymbol{u}$ the input and $\boldsymbol{y}$ the output vector and $\boldsymbol{A}, \boldsymbol{B}, \boldsymbol{C}, \boldsymbol{D}$ the system matrices as follows:

$$\begin{aligned} \boldsymbol{A} &= \begin{pmatrix} \boldsymbol{O} & \boldsymbol{I} \\ -\boldsymbol{M}_{uu}^{-1}\boldsymbol{K}_{uu} & -\boldsymbol{M}_{uu}^{-1}\boldsymbol{D}_{uu} \end{pmatrix}, \quad & \boldsymbol{C} &= \begin{pmatrix} \boldsymbol{K}_{u\varphi}^T & \boldsymbol{O} \end{pmatrix}, \\ \boldsymbol{B} &= \begin{pmatrix} \boldsymbol{O} \\ -\boldsymbol{M}_{uu}^{-1}\boldsymbol{K}_{u\varphi} \end{pmatrix}, & \boldsymbol{D} &= \begin{pmatrix} \boldsymbol{K}_{\varphi\varphi}^T \end{pmatrix}. \end{aligned} \tag{9}$$

The matrices $\boldsymbol{M}_{uu}$, $\boldsymbol{K}_{uu}$, $\boldsymbol{K}_{u\varphi}$ and $\boldsymbol{K}_{\varphi\varphi}$ are defined in (5) and (6), matrix $\boldsymbol{D}_{uu}$ represents the structural damping of the elastic body, matrix $\boldsymbol{I}$ is the identity matrix and matrix $\boldsymbol{O}$ is the zero matrix. The number of inputs r and outputs m in (8) corresponds to the number of piezoelements and the number of states n is the double of the elastic degrees of freedom.

3.2 Controller Design

Traditional state feedback LQR control is proposed to be applied for the controller design of the MIMO system (8). In the first phase it is supposed that every shell element of the elastic structure is equipped with one piezopatch on every side. Every piezopatch serves simultaneously as an actuator and a sensor. The output vector $\boldsymbol{y}$ includes output charges of the piezopatches instead of states, which are needed for the LQR design. However, one can construct a state estimate $\hat{\boldsymbol{x}}$ such that the control law retains similar closed-loop properties, [4].

The first step in the control design process is the selection of parameters of the weighting matrix $\boldsymbol{Q}$ in the LQR design cost function:

$$J = \int_0^\infty \left(\boldsymbol{x}^T \boldsymbol{Q} \boldsymbol{x} + \boldsymbol{u}^T \boldsymbol{R} \boldsymbol{u}\right) dt \ . \tag{10}$$

Dependent on the design goal, the $\boldsymbol{Q}$ matrix is proposed to have the block structure:

$$\boldsymbol{Q} = k_Q \begin{pmatrix} \boldsymbol{Q}_{11} & \boldsymbol{O} \\ \boldsymbol{O} & \boldsymbol{O} \end{pmatrix} \ , \tag{11}$$

where k_Q is a scalar parameter and $\boldsymbol{Q}_{11}$ is a diagonal matrix. The elements $\varphi_{u,k,i,j}$ of modal matrix $\boldsymbol{\Phi}_{u,k}$ identify the contribution of the eigenmodes i on the motion of the selected node r in the direction j. This information about the system is used for the definition of the $\boldsymbol{Q}_{11}$ matrix, which has diagonal elements, e.g. for the z-direction:

$$q_{ii} = \left(\varphi_{u,k,i,z}\omega_i\right)^h \ \ , \ \ 1 \le i \le p \ , \tag{12}$$

where p is the number of modelled eigenmodes, ω_i denotes the corresponding eigenfrequency of the i-th eigenmode and h is the power factor. The expression $x_i q_{ii} x_i$ from (10) corresponds for $h = 2$ to the local potential energy of the eigenmode i, [7].

3.3 Selection of Patches

An important feature is the efficient selection of the piezoelectric patches, which will be used for the controller of the flexible body. In the previous paper, [8], a design-by-simulation method was applied to select the important patches. Instead of that a new selection criterion is applied, which is directly based on the feedback gain $\boldsymbol{K}$ of the LQR controller:

$$\boldsymbol{u} = -\boldsymbol{K}\boldsymbol{x} \ . \tag{13}$$

The matrix $\boldsymbol{K}$ is a r-by-n matrix, where r is the number of inputs and n is the number of states of the controlled system. Since the inputs represent the voltages applied on the piezopatches, the most important patches should have the largest norm ζ_i of the corresponding column vector in the matrix $\boldsymbol{K}$, e.g. 2-Norm:

$$\zeta_i = \left(\sum_{j=1}^{n} |k_{i,j}|^2\right)^{1/2} \ \ , \ \ 1 \le i \le r \ . \tag{14}$$

In the last step, after selection of the reduced set of patches, a new LQR and observer design should be performed and the parameter k_Q from (11) should be tuned in order to exploit the patches as efficiently as possible, i.e. the controller should use the whole linear range of the piezoelement for the expected disturbances.

4. Simulation Environment

In order to implement the theory outlined above in a multibody computational environment, a developer version of the multibody simulation tool SIMPACK has been chosen, [9]. The process chain begins with a finite element analysis of the considered elastic structure. The standard FE-SIMPACK interface FEMBS uses the results of a modal analysis in order to create the modal multibody representation of a flexible structure. But because the electric data are not yet available in industrial finite element tools, the capacity and coupling matrices were additionally calculated based on the purely mechanical mode shape information. The developer version of SIMPACK is extended to deal with the electromechanical and mechanoelectrical coupling terms, which are indicated in equations (6) and (7).

The outlined control approach is implemented in MATLAB/Simulink. The final system is then simulated in two packages; SIMPACK and MATLAB/Simulink are connected via an inter-process communication interface, [10].

5. Control of a metal sheet

A metal sheet from Figure 1 equipped with piezoelements to control the vibration is presented as an example in order to demonstrate the feasibility of the proposed methodology for the modelling of piezoelements in multibody systems. The displacements on the four corners are constrained to be zero.

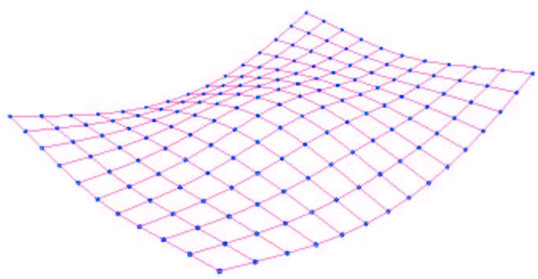

Figure 1. Simulation model of the metal sheet

5.1 Model Description and Simulation Scenario

The model of an elastic metal sheet the size of 1 m $\times$ 1.3 m and width 0.9$\cdot$10^{-3} m is studied. The model considers 14 eigenvalues ranging from 2 to 20 Hz. The structural damping is set at 0.01. The piezoelements, which are 0.4 mm in width, are attached on both sides of 140 finite elements visualised by the mesh in Figure 1. Such piezoelements provide approximately linear behaviour up to the voltage of 400 V. If higher voltages are applied, the piezoelements behave nonlinearly and expose hysteresis effects.

The elastic metal sheet is excited at the time 0.1 s with a force impact at the center position. The force impact is characterised by the amplitude of 20 N and length 0.01 s. The goal is to minimise the acceleration at the center of the metal sheet.

5.2 Controller Design for the Metal Sheet

The proposed control approach has been applied to design a controller for the metal sheet. In its initial version the model has 280 piezopatches, which serve as actuators and as sensors, i.e. the system has 280 inputs and 280 outputs. Since the model contains 14 modes, the state space model has 28 states. According to the matrix $\boldsymbol{\Phi}_{u,k}$ the modes 1, 4, 7, 11 and 14 (see Table 1) contribute to the motion in the z-direction (perpendicular to the metal sheet). The other elements in z-direction of the matrix $\boldsymbol{\Phi}_{u,k}$ for the center of the sheet are zero. Since the system is symmetric w.r.t. two main axes and the patches are located on both sides of the metal sheet (collocated patches), the final number of patches will be a multiple of eight. The contribution of the patches to the control of the sheet's center point is illustrated in Figure 2. According to the results presented in Figure 2, the most important patches are selected, see Fig-

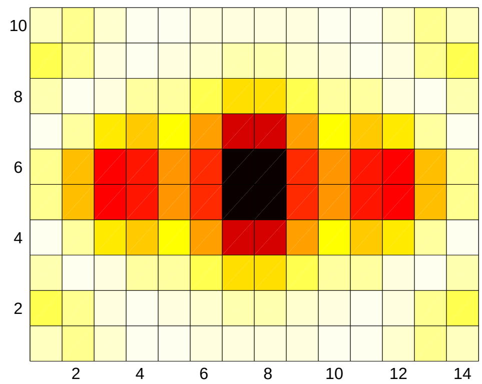

Figure 2. Mesh with piezopatches, grayscaled according to their importance for control

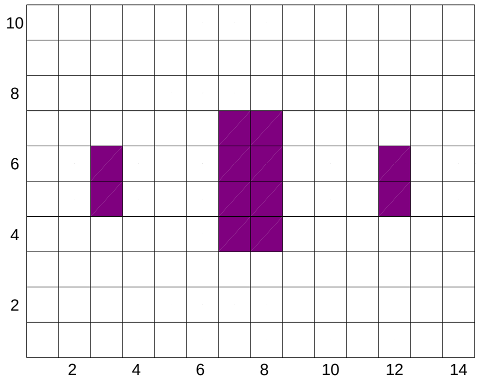

Figure 3. Patches selected for control

ure 3. Because of feasibility, the final configuration has 24 patches, 12 patches on each side of the plate.

The Simulink block diagram of the control loop including the state estimator is presented in Figure 4. In order to drive the piezoelements within their linear range, the saturation block is proposed.

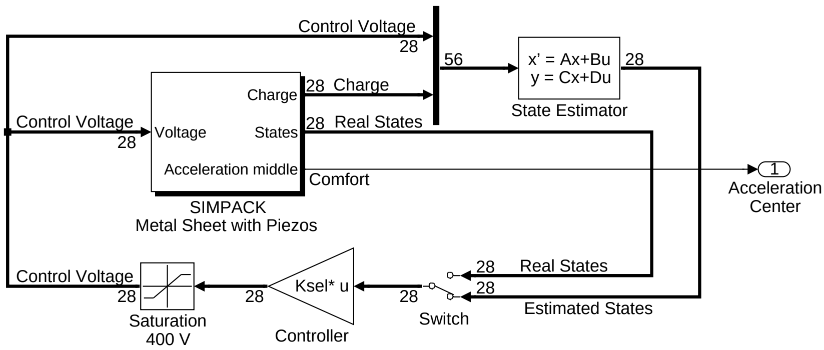

Figure 4. Simulink block diagram of the control loop

Table 1. Eigenfrequencies of the metal sheet

Mode i	1	4	7	11	14
Frequency f_i [Hz]	1.136	3.613	7.825	14.24	19.82

5.3 Simulation Results

A comparison of the accelerations in the center of the metal sheet is presented in Figure 5. The thin, light grey line shows the response of the sheet without any controller. The only existing damping is structural damping. The thick line represents the controlled systems from the Figure 4, which contains a state estimator and LQR feedback controller. The power spectral density of the output acceleration is compared in Figure 6.

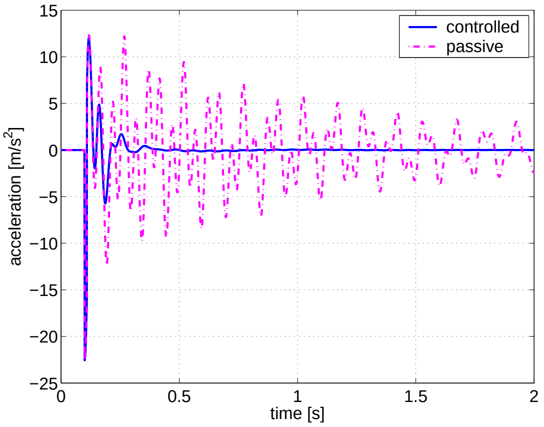

Figure 5. Acceleration in the center of the metal sheet

The corresponding voltages applied to the piezopatches are presented in Figure 7. The controller would remain within the linear range of the piezoelements

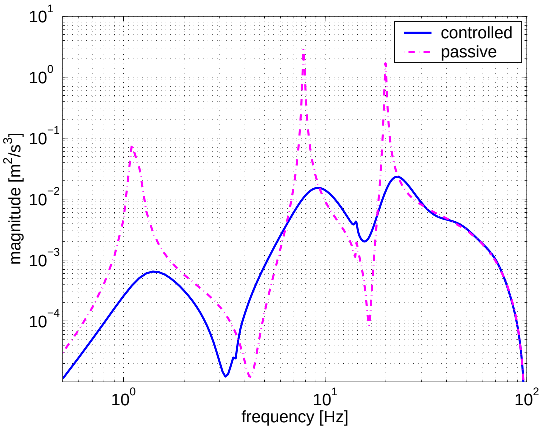

Figure 6. Power spectral density of the acceleration in the center of the metal sheet

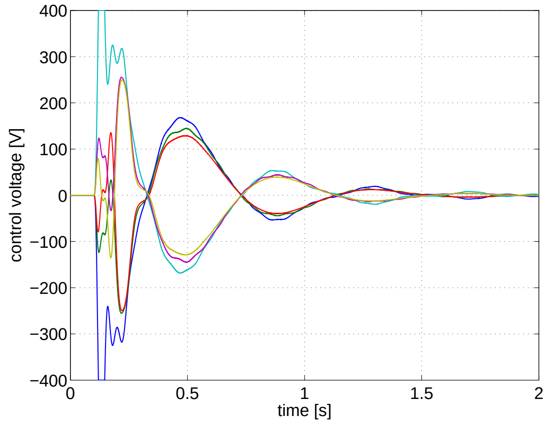

Figure 7. Voltage applied to the patches

except for the maximum values during the force impact, in which the saturation block limits the voltage to 400 V.

6. Conclusions and Open Problems

The presented methodology extends the classical modal description of elastic bodies in multibody systems with the effects of piezoelectricity and provides a tool which enables the development of design concepts with smart structures. In this way, the mechatronic approach may be evaluated from the very beginning of the design phase. Anyway, the performance appraisal of adaptive elements and their feasibility must be evaluated taking risks, costs, weights, complexity etc. into account. This evaluation is a challenging task which the outlined methodology is intended to support.

The presented controller design is based on the modal description of the elasticity; the finite element discretisation determines the size of the patches. The selection of the patches depends on the controller parameters in order to use the patches as efficiently as possible.

The feasibility of the proposed methodology is demonstrated by a simple example, a metal sheet. The simulation results indicate visible improvement of the response of the metal sheet to the force impact.

Future work will be focused on the extension of the methodology to control more than one node and on more complicated examples with large overall motion.

Acknowledgments

The authors would like to acknowledge the collaboration and the kind support provided by Dr. Stefan Dietz and Dr. Wolfgang Rulka from INTEC GmbH. Furthermore, the authors appreciate the cooperativeness of the Adaptronics Section, in particular Dr. Michael Rose and its former head Prof. Delf Sachau, DLR Institute of Structural Mechanics in Brunswick. Last but not least the authors appreciate the scientific and personal support of Prof. Martin Arnold from Martin Luther University in Halle.

References

[1] R.L. Clark, W.R. Saunders, and G.P. Gibbs. *Adaptive Structures - Dynamics and Control*. John Wiley & Sons, New York, 1998.

[2] V. Piefort. *Finite Element Modelling of Piezoelectric Active Structures*. PhD thesis, Université Libre de Bruxelles, 2001.

[3] J. Lefèvre and U. Gabbert. Finite element simulation of smart lightweight structures for active vibration and interior acoustic control. *Technische Mechanik*, 23(1):59 – 69, 2003.

[4] A. Preumont. *Vibration Control of Active Structures*. Academic Publishers, Dordrecht, second edition, 2002.

[5] M. Rose and D. Sachau. Multibody Simulation of Mechanism with Distributed Actuators on Lightweight Components. In *Proceedings of the SPIE's 8th Annual International Symposium on Smart Structures and Materials, Newport Beach*, 2001.

[6] R. Schwertassek and O. Wallrapp. *Dynamik flexibler Mehrkörpersysteme*. Vieweg Verlag, Braunschweig, 1999.

[7] A. Hać and L. Liu. Sensor and actuator location in motion control of flexible structures. *Journal of Sound and Vibration*, 162(2):239 – 261, 1993.

[8] A. Heckmann and O. Vaculín. Multibody simulation of actively controlled carbody flexibility. In P. Sas and B. van Hal, editors, *Proc. of International Conference on Noise and Vibration Engineering (ISMA 2002)*, pages 1123 – 1132, Leuven, Belgium, 2002.

[9] W. Kortüm, W. Schiehlen, and M. Arnold. Software tools: From multibody system analysis to vehicle system dynamics. In *Proc. of International Congress on Theoretical and Applied Mechanics, ICTAM 2000*, pages 225 – 238, Chicago, 2000.

[10] O. Vaculín, W.-R. Krüger, and M. Spieck. Coupling of multibody and control simulation tools for the design of mechatronic systems. In *ASME 2001 Design Engineering Technical Conferences*, number DETC2001/VIB-21323, Pittsburgh, 2001.