Bistatic-like Differential SAR Tomography

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Abstract—Motivated by prospective synthetic aperture radar (SAR) satellite missions, this paper addresses the problem of differential SAR tomography (D-TomoSAR) in urban areas using spaceborne bistatic or pursuit monostatic acquisitions. A bistatic or pursuit monostatic interferogram is not subject to significant temporal decorrelation or atmospheric phase screen and therefore ideal for elevation reconstruction. We propose a framework that incorporates this reconstructed elevation as deterministic prior into deformation estimation, which uses conventional repeat-pass interferograms generated from bistatic or pursuit monostatic pairs. By means of theoretical and empirical analyses, we show that this framework is, in the pursuit monostatic case, both statistically and computationally more efficient than standard D-TomoSAR. In the bistatic case, its theoretical bound is no worse by a factor of 2. We also show that reasonable results can be obtained by using merely 6 TanDEM-X pursuit monostatic pairs, if additional spatial prior is introduced. The proposed framework can be easily extended for multistatic configurations or external sources of scatterer’s elevation.

Index Terms—SAR tomography, Tandem-L, TanDEM-X, synthetic aperture radar (SAR).

I. INTRODUCTION

A. Motivation

Bistatic or multistatic configuration is a prominent feature of various future synthetic aperture radar (SAR) satellite missions. Some of these missions can be summarized as follows.

- Tandem-L, a German satellite mission concept whose primary goal is to observe the dynamic processes on earth’s surface in high resolution with an unprecedented accuracy [1]. It comprises two satellites (e.g., TL-1 and TL-2). Each of them will have on board a high-resolution wide-swath L-band SAR. Basically, these two satellites will fly in close formation and operate in bistatic mode. This mode utilizes either TL-1 or TL-2 as a transmitter to illuminate a common radar footprint, while both receive radar echoes from earth’s surface. In addition, a bidirectional radio frequency (RF) link is necessary for a highly accurate mutual time and phase referencing. This requirement will be easily fulfilled by means of the heritage of the TanDEM-X (TerraSAR-X add-on for Digital Elevation Measurements) mission [2]. Due to limited temporal decorrelation and atmospheric phase screen (APS), single-pass bistatic interferograms are characterized by better phase quality as compared to conventional repeat-pass ones and thus are more suitable for generating a global, consistent and high-resolution digital elevation model [3].

- SAOCOM-CS, a bistatic mission concept attaching to SAOCOM a passive companion SAR satellite operating in L-band [4].

- Sentinel-1 SAR Companion Multistatic Explorer (SESAME), a bistatic mission concept adding to Sentinel-1 two passive companion SAR satellites operating in C-band [5].

- Sentinel-1 “tandem”, i.e., one-day separation or bistatic mission concept involving the prospective Sentinel-1C and another satellite from the series [6].

- High Resolution Wide Swath (HRWS), the successor of TerraSAR-X comprising one or two SAR satellites operating in X-band [7], [8], and possibly several additional passive companion transponder satellites without bidirectional phase synchronization link (MirrorSAR) [9]–[11].

Above all, Tandem-L is the most intriguing mission to us, not only because it is the one and only concept that has already undergone very comprehensive and intensive studies (see for example [12]–[16] and the references therein), but it is also extremely promising for a huge variety of geophysical applications.

In this paper, we address the problem of spaceborne differential SAR tomography (D-TomoSAR, see for instance [17]–[23]) in urban areas using bistatic or pursuit monostatic data. The latter, on the contrary, requires two satellites in close formation to operate independently from each other [2]. It can be employed as a backup solution in case pulse or phase synchronization fails. Given a temporal baseline of a few seconds and a moderate wind speed, the temporal decorrelation is still small for most terrain types including vegetation and atmospheric path delays can be assumed to cancel each other out during interferometric processing [24]. Hereafter we refer to bistatic and pursuit monostatic collectively as “bistatic-like”.

We propose an austere framework which 1) reconstructs the elevation dimension with only bistatic-like interferograms, and subsequently 2) uses this as deterministic prior to estimate deformation parameters with conventional repeat-pass interferograms generated from bistatic-like pairs. Note that 1) is essentially a non-differential TomoSAR subproblem. We will refer to 2) as the DefoSAR subproblem. For point-like scatterers, the advantages of this framework are at least two-
fold: a) the (almost) APS-free nature of bistatic-like interferograms leads to better elevation reconstruction, and in turn to more accurate deformation estimation; b) the dimension of the original problem is downscaled multiplicatively in each subproblem, which increases on the whole the algorithmic efficiency. For distributed scatterers, adaptive multilooking can be employed to increase their signal-to-noise ratio (SNR) to the level of point-like ones [25]–[27]. Therefore, the same arguments also apply. Besides, the elevation reconstruction of distributed scatterers relies less on the performance of adaptive multilooking, since their decorrelation is much less severe in bistatic-like interferograms than in conventional repeat-pass ones.

For the purpose of a practical demonstration, we use TanDEM-X—to date the sole civil spaceborne bistatic or multistatic mission—data in pursuit monostatic mode.

The proposed framework is envisioned to be incorporated into our Tandem-L processing chain. As one would expect, it is, with up to some minor adaptation, directly applicable to other prospective bi- or multistatic missions.

B. Notations and structure

We adopt the following mathematical notations throughout the whole paper. Scalars are denoted as lower- or uppercase letters, e.g., \( r, N, \lambda \). Vectors are denoted as bold lowercase letters, e.g., \( \mathbf{b}, \gamma \). Their elements are denoted as lowercase letters with subscript, e.g., the \( n \)-th entry of \( \mathbf{g} \) is denoted as \( g_n \). For vectors, \( \| \cdot \|_2 \) and \( \| \cdot \|_1 \) denote the \( \ell_2 \) and \( \ell_1 \) norm, respectively. The supports of any vector \( \mathbf{b} \), i.e., the index set of all nonzero entries of \( \mathbf{b} \), are denoted as \( \text{supp}(\mathbf{b}) \). Matrices and sets are denoted as bold uppercase letters, e.g., \( \mathbf{R}, \Omega \).

Single rows of matrices are denoted as bold lowercase letters with superscript, e.g., the \( n \)-th row of \( \mathbf{R} \) is denoted as \( \mathbf{r}^n \). For matrices, \( \| \cdot \|_F \) and \( \| \cdot \|_{1,2} \) denote the Frobenius and \( \ell_{1,2} \) norm, respectively. For any set \( \Omega \), \( |\Omega| \) denotes its cardinality and \( 2^\Omega \) its power set, i.e., the set of all subsets of \( \Omega \), including \( \Omega \) itself and the empty set \( \emptyset \). The sets of integers, real and complex numbers are denoted as \( \mathbb{Z}, \mathbb{R}, \mathbb{C} \), respectively. Their nonnegative subsets are denoted with the subscript \( + \), e.g., \( \mathbb{Z}_+ \) denotes the set of nonnegative integers.

The remainder of this paper is organized as follows. Sec. II introduces the aforementioned framework together with a theoretical analysis of its performance and complexity. This is followed by Sec. III where an empirical experiment with TanDEM-X pursuit monostatic data can be found. Sec. IV concludes this paper.

II. THE TOMO- AND DEFOSAR FRAMEWORK

As briefly mentioned in Sec. I, we divide the original differential TomoSAR problem using bistatic-like data sets into two ordered subproblems, namely (non-differential) Tomo- and DefoSAR. In the TomoSAR subproblem, the elevation dimension is reconstructed with only bistatic-like interferograms. Subsequently, the reconstructed elevation position of each scatterer is used as deterministic prior in the DefoSAR subproblem, where its deformation parameters are estimated with conventional repeat-pass interferograms. These two categories of interferometric combinations are illustrated in Fig. 1.

A. TomoSAR

Given \( N \) coregistered bistatic-like complex interferograms, we aim to reconstruct the reflectivity profile along elevation.

For small \( N \), however, the distribution of cross-track perpendicular baselines could be one-sided (see for example Fig. 2a). In this case, we propose to flip the sign of some of the baselines in order to maximize the standard deviation of their distribution. The rationale is to achieve a more uniform sampling of elevation frequencies [28], as well as a better Cramer-Rao lower bound on the error of elevation estimates [29]. Let \( \mathbf{b} \in \mathbb{R}^N \) denote the vector of cross-track perpendicular baselines, the aforementioned problem can be formulated as

\[
\max_{\mathbf{z} \in \{-1,1\}^N} \sigma(\mathbf{b} \odot \mathbf{z}),
\]

where \( \sigma: \mathbb{R}^N \rightarrow \mathbb{R}_+ \) maps a vector to the sample standard deviation of its entries, and \( \odot \) denotes the Hadamard product. Problem (1) is equivalent to

\[
\min_{\mathbf{z} \in \{-1,1\}^N} -\|\mathbf{b} \odot \mathbf{z} - \mathbf{b}^T \mathbf{z} \mathbf{N} \|^2_2,
\]

which has two optimum points given unique entries of \( \mathbf{b} \): suppose \( z^* \) is one of them, then \(-z^* \) is the other. Since \( N \) is small, we solve (2) by exhaustive search. In the unlikely case of one-sided baseline distribution with large \( N \), the following heuristic can be adopted: sort baselines by their magnitude, choose a sign for the largest one in magnitude, set the second largest one to have the opposite sign, and so forth till all \( N \) baselines are exhausted. Accordingly, the signs of elevation frequency and interferometric phase are also flipped.

The optional sign flipping procedure is followed by layer-by-layer separation. By the first-order Born approximation, far-field diffraction is often modeled as the integration of a phase-modulated elevation-dependent reflectivity profile (see for example [19]). After discretizing the elevation dimension and replacing integration with finite sum, bistatic-like InSAR observations \( \mathbf{g} \in \mathbb{C}^N \) of a resolution cell can be approximated with the linear model \( \mathbf{g} \approx \mathbf{R}\gamma \), where \( \mathbf{R} \in \mathbb{C}^{N \times L} \) is the TomoSAR design matrix, and \( \gamma \in \mathbb{C}^L \) denotes the discrete reflectivity profile along elevation. The \( n \)-th entry \( g_n \) of \( \mathbf{g} \) is sampled at the elevation frequency \( \xi_n := 2b_n/(\lambda r) \), where \( b_n \) is the corresponding cross-track perpendicular baseline (after sign flipping), \( \lambda \) denotes the radar carrier wavelength, and \( r \) is the slant-range distance in the master acquisition. Let \( s \in \mathbb{R}^L \) denote the discretization of the elevation dimension, the \( n \)-th row of \( \mathbf{R} \) is defined as \( \mathbf{r}^n := \exp(-j2\pi \xi_n s) \), where \exp
where \( \tilde{\gamma} \) is sparse. A similar formulation of (3) can be found in [31]. Despite its super-resolution power and robustness in terms of layover separation, \( \ell_1 \)-regularization is prone to spurious spikes. For this reason, it is often concatenated with model order selection which we state as follows [28]

\[
\hat{\Omega} := \arg \min_{\Omega \subseteq \supp(\tilde{\gamma})} \frac{1}{2} \| \mathbf{R} \beta - \mathbf{g} \|_2^2 + C(\Omega),
\]

subject to \( \supp(\beta) = \Omega \subseteq \supp(\tilde{\gamma}) \),

where \( |\Omega| \) denotes the cardinality of the index set \( \Omega \), \( C : \mathbb{R}^+ \rightarrow \mathbb{R}^+ \) evaluates the model complexity according to, e.g., Bayesian or Akaike information criterion (see [28] and the references therein), and \( \supp(\beta) = \{ i \mid \beta_i \neq 0, i = 1, \ldots, L \} \)

i.e., it is the set of the indices of nonzero entries or supports in \( \beta \). The constraint in (4) renders the supports of the final reflectivity profile estimate a subset of those of \( \tilde{\gamma} \), and therefore allows outlier mitigation. Note that the underestimated amplitude is hereby debiased as a by-product.

In the next subsection, we introduce the DefoSAR subproblem which uses as deterministic prior the elevation estimates of single or multiple scatterers from TomoSAR reconstruction.

### B. DefoSAR

Given \( 2N - 2 \) coregistered conventional repeat-pass complex interferograms generated from \( N \) bistatic-like pairs (see Fig. 1), and the elevation estimates \( \hat{s} := s_\hat{\Omega} \) of a total number of \( K := |\hat{\Omega}| \) scatterers, our objective is to reconstruct their deformation by means of a composite model.

For single point-like scatterers (i.e., no layover effect), the elevation estimates can be straightforwardly converted into topographic phase and compensated in the conventional repeat-pass interferograms. A similar approach for distributed scatterers can be found in [32].

In a more general sense, \( \hat{s} \) can be considered as deterministic prior. Let \( \Psi := \{ \psi_{m} \} \) denote a set of \( M \) basis functions which are parametrized by the temporal baseline \( t_n \) and employed to model scatterer’s deformation, and \( c_m \in \mathbb{R}^{1 \times |c_m|} \) be the discretization of the unknown coefficient of \( \psi_{m} \), we can construct the DefoSAR design matrix \( \mathbf{R}(s, \Psi) \in \mathbb{C}^{(2N-2) \times L} \), where \( L := K \prod_{m=1}^{M} |c_m| \). Its \( n \)-th row can be expressed as \( \mathbf{r}_n := \exp(-j 2\pi \xi_n \hat{s}) \otimes \exp(-j (4\pi / \lambda) \psi_{1}(t_n) c_{1}) \otimes \cdots \otimes \exp(-j (4\pi / \lambda) \psi_{M}(t_n) c_{M}) \), where \( \xi_n \) is the elevation frequency of the \( n \)-th conventional repeat-pass interferogram with cross-track perpendicular baseline \( \hat{b}_n \), and \( \otimes \) denotes the Kronecker product. Likewise, the repeat-pass InSAR observations \( \tilde{g} \in \mathbb{C}^{2N-2} \) of the same resolution cell can be approximated by \( \tilde{g} \approx \mathbf{R}(\hat{s}, \Psi) \hat{\gamma} \), where \( \hat{\gamma} \in \mathbb{C}^L \) denotes the discrete reflectivity profile along elevation and deformation. The coefficients of deformation basis functions can be estimated with a variant of non-linear least squares [22] which additionally constrains \( \hat{\gamma} \) to have exactly one nonzero entry at each elevation position in \( \hat{s} \). In order to avoid overfitting, we propose furthermore to perform deformation model order selection. Let \( 2^\Psi \) be the power set of \( \Psi \), i.e., all possible combinations of deformation basis functions including the non-differential case represented by the null set \( \emptyset \). The deformation model order selection problem can be cast as

\[
\hat{\Theta} := \arg \min_{\Theta \subseteq 2^\Psi} \frac{1}{2} \| \mathbf{R}(\hat{s}, \Theta) \hat{\beta} - \mathbf{g} \|_2^2 + C(|\Theta|)
\]

subject to \( |\supp(\hat{\beta})| = |\supp(\Theta)| = |\hat{\Omega}| \),

where \( \Theta : \mathbb{C}^L \rightarrow \mathbb{C}^K \) integrates over each deformation coefficient. The constraint in (5) enforces that the discrete reflectivity profile in the elevation-deformation domain, and the one in the (integrated) elevation domain share the same number of supports, which leads to the previously mentioned desired effect. Again, we solve this subproblem by exhaustive search. In the case of a highly complex composite model, we can proceed in a greedy manner: choose from the remaining scatterers the one with the strongest power, rebuild the DefoSAR design matrix, find the best fit in terms of penalized likelihood (5), and subtract it from the residues of \( \tilde{g} \), etc.

Assuming that the elevation estimate of a single scatterer is perfect, the Cramér-Rao lower bound (CRLB) on the error of the coefficient estimate \( \hat{c} \) of a single basis function \( \psi \) is

\[
\sigma_\psi := \frac{\lambda}{4\pi \sqrt{2N - 2\sqrt{2SNR} \sigma_\psi}},
\]

where \( \sigma_\psi \) is the standard deviation of \( \psi \) evaluated at different \( t_n \), i.e., \( \psi(t_1), \ldots, \psi(t_{2N-2}) \). A proof of (6) is given in the appendix.

In a nutshell, our proposed framework can be summarized as follows. A simple theoretical analysis is provided in the next subsection.

### C. Tomo- and DefoSAR vs. D-TomoSAR: a theoretical analysis

Now we analyze the performance and complexity of the proposed framework from a theoretical point of view.

We start with a proof that Case 1 (pursuit monostatic): the proposed framework has a tighter theoretical bound, and...
Case 2 (bistatic): its CRLB is no worse by a factor of 2.

In order to simplify the argument, suppose without loss of generality that $N$ bistatic-like pairs are coregistered with a redundant master scene that is not used in tomographic processing. Thereby $2N$ (instead of $2N - 1$ if we count the interferogram in the middle of Fig. 1 as a repeat-pass one with zero temporal baseline) conventional repeat-pass interferograms are generated from these pairs. Note that this assumption certainly favors the D-TomoSAR approach.

For TomoSAR using $N$ bistatic-like pairs, the CRLB on the elevation estimate $\hat{s}$ of a single scatterer is [29]

$$\sigma_\hat{s} := \frac{\lambda_r}{4\pi \sqrt{2N} \sqrt{SNR} \sigma_b},$$

where $\sigma_b$ is the standard deviation of the perpendicular baselines $\{b_n\}$ of the $N$ bistatic-like pairs. On the other hand, D-TomoSAR uses as inputs $2N$ conventional repeat-pass interferograms that are generated from the $N$ bistatic-like pairs. For a single scatterer, suppose that its deformation time is slow and therefore subject to layover. The APS was compensated temporarily put into practice from October 2014 to February 2015 during the TomoSAR Science Phase [24]. In order to avoid RF interference between radar signals, the along-track distance was set to approximately 76 km, which corresponds to a temporal baseline of circa 10 seconds. During this five-month period, 12 staring spotlight scenes of the City of Las Vegas were acquired. Out of these, 6 pursuit monostatic interferograms were generated and their baselines are plotted in Fig. 2a.

As can be observed, relatively large values in magnitude are available, whereas in the usual cases of TSX and TDX the baselines are bounded between ±250 m. As a matter of fact, in order to favor TomoSAR and other applications in polar regions, cross-track perpendicular baselines were programmed to slowly drift (in magnitude) from 0 to 750 m [24]. Since all baselines but one are negative, we applied the sign flipping procedure that was introduced in Sec. II-A. The baselines after sign flipping are plotted in Fig. 2b. The sign was indeed flipped for two baselines and the standard deviation increased from approximately 286.7 to 308.3 m. As a consequence, the CRLB was improved by 7.5%.

As a practical demonstration of the proposed framework, we focus on a small area which contains a high-rise building and is therefore subject to layover. The APS was compensated by subtracting the phase of a nearby ground reference point in each interferogram. This step is also known as phase calibration [35], [36]. Given a sufficiently large number of bistatic-like pairs (for example $N \geq 11$), a stack of $2N - 1$ repeat-pass interferograms can be generated. Subsequently, a standard persistent scatterer interferometry (PSI) approach

\[^1\] For the sake of simplicity, we count each complex addition or multiplication as one flop.

\[^2\] In this context we refer to the two satellites in the TanDEM-X mission as TSX and TDX.
A. Joint sparsity reconstruction for extremely small $N$

Although the pursuit monostatic interferograms in Fig. 3 are mostly unaffected by APS or temporal decorrelation, the number of elevation frequencies (i.e., 6) is extremely small. Zhu et al. reported that, for $N = 6$, not exploiting special signal structure can lead to results that are subject to outliers [41]. With the objective of achieving high-quality elevation reconstruction, we introduced spatial prior in the form of iso-height line segments along range on the building facade. The iso-height line segments were derived from freely available geospatial data containing building footprints. All resolution cells in a given line segment form an iso-height cluster, which was jointly reconstructed. In other words, we solve instead the $\ell_{1,2}$-regularized least squares problem

$$
\hat{\Gamma} := \arg\min_{\Gamma} \frac{1}{2} \| \mathbf{R} \Gamma - \mathbf{G} \|_F^2 + \lambda \| \Gamma \|_{1,2},
$$

where the $p$-th column of $\Gamma \in \mathbb{C}^{L \times P}$ represents the discrete reflectivity profile in the $p$-th resolution cell (also known as snapshot) along the iso-height line segment, the $p$-th column of $\mathbf{G} \in \mathbb{C}^{N \times P}$ contains the InSAR observations of the $p$-th resolution cell, $\| \cdot \|_F$ denotes the Frobenius norm, and $\| \cdot \|_{1,2}$ denotes $\ell_{1,2}$ norm, i.e., $\| \Gamma \|_{1,2} := \sum_{i=1}^{L} \| \gamma_i \|_2$. A treatise on this algorithm can be found in [41], where it was shown empirically that solving the $\ell_{1,2}$-regularized least squares problem (10) with $N$ interferograms and $P$ snapshots achieves almost the same performance, in terms of elevation estimate error, as solving the $\ell_1$-regularized least squares problem (3) with $NP$ interferograms. Similar approaches using multiple snapshots can be found in, e.g., [42], [43]. Subsequently, the model order selections (4) and (5) were performed individually for each resolution cell.

Fig. 4 shows the mean intensity map of the building of interest and several exemplary iso-height line segments. The height estimates of single and layover scatterers are plotted in Fig. 5. Roof interacts with facade and ground in the near range, while facade and ground are subject to layover in the far range. The smooth color transition from near to far range indicates a good quality of height estimates. Nevertheless, there are indeed a few outliers in the far range. These outliers, which we managed to reproduce with simulated data sets, are presumably due to the yet nonuniform distribution of the extremely small number of baselines. The height profile, generated via averaging within each iso-height cluster, can be found in Fig. 6, where roof and facade are clearly identifiable. In order to assess the relative accuracy of height estimates, we extracted the point cloud segment corresponding to facade by thresholding of point density [44] and fitted a vertical plane with $\ell_1$-loss (see Fig. 7). From the bird’s-eye view, all scatterers appear to be evenly distributed w.r.t. the fitted facade plane. We calculated the elevation distance of each scatterer’s estimated position to the facade plane, and projected it into the vertical direction. We refer to this vertical component as the height estimate error relative to the fitted vertical plane. Its histogram resembles a zero-mean normal distribution (see Fig. 8). The relative vertical accuracy, which is defined in this context as the median absolute deviation (MAD) of height estimate error, was estimated to be approximately 0.29 m.
Fig. 4. Mean intensity map (left) and overlaying exemplary iso-height line segments (right).

Fig. 5. Height estimates of single (left) and layover scatterers (right). In the case of layover, the height estimate of the highest scatterer is shown.

Note that this can be interpreted as an upper bound on the true relative vertical accuracy, since the building facade is not entirely flat.

As explained in Sec. II-B, these height estimates can be used as deterministic prior for repeat-pass interferometry. For single scatterers, the topographic phase was compensated by using the height-to-phase conversion factor (see Fig. 9). The residual phase is presumably mainly due to scatterer’s motion and already reveals a pattern of it. Note that every pair of repeat-pass interferograms sharing the same temporal baseline (in days) appear almost identical after topographic phase compensation. This provides a compelling argument for limited temporal decorrelation and APS within a pursuit monostatic pair. Needless to say, an increase in the coherence of prospective Tandem-L repeat-pass interferograms can be expected. Even for distributed scatterers, L-band signal is known to maintain a certain degree of coherence after more than two years of time [45], [46]. This would undoubtedly lead to a greater coverage of retrievable information. Given the span of temporal baselines of 132 days, the motion was assumed to consist primarily of thermal contraction and expansion due to temperature change (see for example [47]). To this end, a sinusoidal model was employed. By solving the DefoSAR subproblem (5), we obtained the amplitude estimates of periodical deformation for single and layover scatterers, which are shown in Fig. 10. In general, the amplitude of periodical deformation is positively correlated with height (see the scatter plot in Fig. 11) and relatively large in magnitude at the top of the building as well as at the side. This pattern accords with that of repeat-pass interferograms of single scatterers after topographic phase compensation in Fig. 9b, which partially validates our results.

A preliminary comparison with D-TomoSAR is provided in the next subsection.
Fig. 9. Repeat-pass interferometric phase of single scatterers before (9a) and after topographic phase compensation (9b), annotated with their cross-track perpendicular or temporal baselines.
Fig. 10. Periodical deformation amplitude estimates of single (left) and layover scatterers (right). In the case of layover, the amplitude estimate of the highest scatterer is shown.

Fig. 11. Scatter plot of averaged height and periodical deformation amplitude.

B. Tomo- and DefoSAR vs. D-TomoSAR: an empirical analysis

In Sec. III-A, we introduced additional spatial prior in order to boost the sparse reconstruction for extremely small $N$. The joint sparsity reconstruction method (10) is, however, only applicable to bistatic-like data sets. Therefore, a direct comparison with the results generated by the same method using conventional repeat-pass interferograms is ruled out. For the sake of fairness, we employed the pixel-wise sparse reconstruction method (3) with identical parameter quantization for both TomoSAR and D-TomoSAR processing using 6 pursuit monostatic and 11 repeat-pass interferograms, respectively. As might be expected, the former was followed by DefoSAR processing for deformation parameter estimation. Tab. I summarizes the overall runtime on a desktop with a quad-core Intel processor at 3.40 GHz and 16 GB RAM. The Tomo- and DefoSAR framework was computationally approximately 6.4 times as efficient.

As listed in Tab. II, the standard deviation $\sigma_b$ of the cross-track perpendicular baselines $\{b_n\}$ of the 6 pursuit monostatic interferograms is approximately 1.4 times as high as the one $\sigma_{\tilde{b}_n}$ of the 11 repeat-pass interferograms. Note that our assumption in Sec. II-C implies that $\sigma_b = \sqrt{2}\sigma_{\tilde{b}_n}$. The correlation between $\tilde{b}_n$ and the deformation basis function $\psi_n$ leads to $\sqrt{1-\rho^2} \approx 0.92$. This can be interpreted as a degradation of $\sigma_{\tilde{b}}$ by 8% at the expense of taking deformation into account. Given a single scatterer with an SNR of 5 dB, the CRLB of height estimates for the proposed framework is approximately 0.48 m, which is 1.1 times as low.

Same as in Sec. III-A, we extracted the point cloud segment corresponding to building facade by thresholding of the 2-D point density. This process also eliminated false alarms due to the extremely small number of interferograms. As shown in Fig. 12, both facade segments appear quite similar, except that the uppermost part of the facade is incomplete in the D-TomoSAR result. A possible explanation could be that the already complex short-distance roof-facade layover of point-like scatterers is furthermore complicated by their deformation behavior. The facade segment produced by D-TomoSAR has slightly more scatterers (see Tab. III), but we consider this difference to be insignificant. In order to access the quality of the point cloud, we followed the same approach that was introduced in the last subsection, namely to fit a vertical plane into each facade segment, project the distance of each point to the fitted plane into the vertical axis, and interpret it as the height estimate error relative to the fitted plane. The normalized histograms are shown in Fig. 13. While both histograms are centered around zero, the one of TomoSAR has less deviation. The MAD is in fact approximately 1.16 as low for TomoSAR (cf. 1.10 as predicted in Tab. II for an average SNR of 5 dB).

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**TABLE I**

<table>
<thead>
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<th>Tomo- and DefoSAR</th>
<th>D-TomoSAR</th>
<th>Ratio</th>
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<td>Runtime [h]</td>
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<td>3.94</td>
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**TABLE II**

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<th>TomoSAR</th>
<th>D-TomoSAR</th>
<th>Ratio</th>
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<tr>
<td>Baseline $\sigma$ [m]</td>
<td>308.31</td>
<td>226.33</td>
</tr>
<tr>
<td>$\sqrt{1-\rho^2}$</td>
<td>n.a.</td>
<td>0.92</td>
</tr>
<tr>
<td>CRLB of height [m]</td>
<td>0.48</td>
<td>0.52</td>
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**TABLE III**

<table>
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<th>TomoSAR</th>
<th>D-TomoSAR</th>
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<td>No. of scatterers</td>
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<td>91063</td>
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<td>Median [m]</td>
<td>0.00</td>
<td>0.00</td>
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<tr>
<td>Mean [m]</td>
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<td>0.00</td>
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<tr>
<td>MAD [m]</td>
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<tr>
<td>Standard Deviation [m]</td>
<td>0.50</td>
<td>0.58</td>
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TomoSAR using more statistics.

Fig. 12. Reconstructed facade segments (color-coded by ellipsoidal height).


IV. CONCLUSION

In the preceding sections, we tackled the problem of differential TomoSAR in urban areas using bistatic-like data sets, which will be delivered by the prospective Tandem-L mission as operational products. We proposed a framework which divides the original problem into two subsequent subproblems. The first subproblem is essentially non-differential TomoSAR with bistatic-like interferograms and can be dealt with using already existing methods. In the second subproblem, elevation estimates are incorporated as deterministic prior into the DefoSAR design matrix in order to estimate the coefficients of deformation basis functions with conventional repeat-pass interferograms. We showed via theoretical and empirical analyses that this framework, when applied to pursuit monostatic data, not only outperforms standard D-TomoSAR but is also less expensive. In an extreme case, we applied our framework to merely 6 TanDEM-X pursuit monostatic pairs and achieved reasonable results for both elevation and deformation estimates. The relative vertical accuracy of the resulted point cloud was estimated to be approximately 0.29 m.

Although we proposed to estimate each scatterer’s elevation position using bistatic-like interferograms, it could indeed stem from other sources such as ray-tracing simulation with an external 3-D building model [48] or with one reconstructed from a single SAR intensity image [49]. The corresponding minor adaptation would extend the applicability of the proposed framework to interferometric stacks composed of nothing but conventional repeat-pass acquisitions and thereby allow precise object-based infrastructure monitoring.

APPENDIX

PROOF OF (6)

First we state the following result from [50].
Suppose that $x \in \mathbb{R}^I$ and $y \in \mathbb{C}^N$ are the parameter and data vectors, respectively, and $y$ is the random Gaussian observation of the deterministic signal vector $u(x) \in \mathbb{C}^N$ with covariance matrix $C_y(x).$ The likelihood function is

$$f(y \mid x) = \frac{1}{\pi^N \det(C_y(x))} \exp\left(-\frac{(y - u(x))^H C_y^{-1}(x) (y - u(x))}{2}\right).$$

It can be shown that the Fisher information matrix $I(x)$ is given by

$$I(x)_{kl} := \text{tr}\left(C_y^{-1}(x) \frac{\partial C_y(x)}{\partial x_k} C_y^{-1}(x) \frac{\partial C_y(x)}{\partial x_l}\right) + 2 \text{Re} \left( \frac{\partial u^H(x)}{\partial x_k} C_y^{-1}(x) \frac{\partial u(x)}{\partial x_l} \right),$$ \hspace{1cm} (11)

$k, l = 1, \ldots, L$.

Now let us consider the DefoSAR data model

$$\tilde{y}_n = \tilde{\epsilon}_n \exp(-j2\pi \xi_n s) \exp(-j(4\pi/\lambda)\psi_n c) + \tilde{\epsilon}_n, n = 1, \ldots, 2N-2,$$

where $\tilde{\epsilon}_n$ is complex white Gaussian noise with variance $\sigma_\tilde{\epsilon}^2$, and $\psi_n := \psi(t_n).$ Here we assume that the elevation estimate is perfect, i.e., $\tilde{\epsilon} = s.$ By replacing $\tilde{\gamma}$ by $a \exp(j\phi)$ where $a, \phi \in \mathbb{R},$ we define the new real parameter vector as $x := (a \ c \ \phi)^T.$ The signal vector is given by $u_n(x) := a \exp(j(\phi - 2\pi \xi_n s - (4\pi/\lambda)\psi_n c)).$

Straightforward computations using (11) yield the Fisher information matrix

$$I(x) = 2 \sigma_\tilde{\epsilon}^2 \begin{pmatrix} 2N - 2 & 0 & 0 \\ 0 & \frac{(4\pi)^2 a^2}{\lambda^2} \sum_n \psi_n^2 & -\frac{4\pi^2 a^2}{\lambda} \sum_n \psi_n \\ 0 & -\frac{4\pi a^2}{\lambda} \sum_n \psi_n & (2N - 2) \gamma^2 \end{pmatrix}.$$ 

The CRLB for $\tilde{c}$ is found to be

$$\sigma_{\tilde{c}}^2 := [I^{-1}(x)]_{22} = \frac{\lambda^2 \sigma_\tilde{\epsilon}^2}{(4\pi)^2 (2N - 2) 2a^2 \sigma_\psi^2}.$$ 

By defining $SNR := a^2 / \sigma_\tilde{\epsilon}^2$, this reduces to

$$\sigma_{\tilde{c}} = \frac{\lambda}{4\pi \sqrt{2N - 2} \sqrt{2SNR} \sigma_\psi},$$

which completes the proof.
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