The theoretical performance of different algorithms for shift estimation between SAR images

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Abstract

This paper compares several algorithms for shift estimation between SAR images, presenting their performance along with their respective merits and demerits. It shows that there is a parallelism between correlation-based estimators (coherent and incoherent) and two different implementations of split-spectrum methods (Delta-k), both in terms of robustness to interferometric phase variations within the estimation window and in terms of performance. The character of Delta-k estimators is regulated by the amount of multi-looking at interferogram level and is related to the statistical efficiency of the standard estimator for the interferometric phase in case of Gaussian speckle.

1 Introduction

Shift estimation, in range an azimuth directions, is a common operation in multi-image SAR processing, both for interferogram formation and for the extraction of geophysical signals. Despite being so fundamental, its performance has been characterized only relatively late, the first time specifically for SAR in [1], with analytical derivations limited to the coherent case (coherent cross-correlation or CCC). The performance of incoherent cross-correlation (ICC) has been derived more recently [2]. Split-spectrum (e.g. Delta-k) methods [3, 4, 5, 6, 7] are an alternative to cross-correlation in time-domain and offer some advantages (and disadvantages). In [8] it was shown that they can achieve very high efficiency, but this is valid only for a "orthodox" implementation, which is possibly not the preferred one in practice, like, e.g., the one suggested in [9]. "unorthodox" implementations might yield much degraded performance, as shown in [10].
In this paper we would like to clarify the various flavours of cross-correlation and Delta-k methods, comparing the relative merits, performances and efficiency. All through the paper, we assume for the scene a Gaussian-speckle model with constant coherence. As a consequence, this analysis does not apply to feature tracking or other methods which are purely incoherent. Despite the limiting assumption of Gaussianity, the results are nonetheless useful to illuminate areas of incomplete understanding, for instance, regarding the relationships among the different options.

2 Four popular algorithms

We will discuss four algorithms for shift estimation (see **Table 1**). We will classify them according to being based on time-domain correlation or split-spectrum and to being coherent or incoherent.

	correlation based	*split-spectrum*
coherent	CCC	early Delta-k
incoherent	ICC	late Delta-k

Table 1: Classification of four algorithms for shift estimation.

The maximum-likelihood shift estimator for Gaussian speckle is the maximization of the complex signal cross-correlation (CCC), which reaches asymptotically the Cramér-Rao bound:

$$\hat{t}_s = \text{argmax} \left| \sum_n y_m(t_n) y_s^*(t_n - t_s) \right|. \qquad (1)$$

A very common alternative is the cross-correlation of intensities (ICC), which has a reduced performance:

$$\hat{t}_s = \text{argmax} \sum_{n/2} |y_m(t_n)|^2 |y_s(t_n - t_s)|^2. \qquad (2)$$

The variables y_m and y_s represent the master and slave signals as a function of time t_n; we consider discrete signals and n is the sample index. The shift t_s is a continuous variable. Both methods (CCC & ICC) work on the full-spectrum signals, but the intensities need a doubled spectral support to avoid aliasing, hence the summation on "half indices". On the other hand, the simplest split-spectrum method is Delta-k. It requires the filtering of two sub-bands for both master and slave ($y_{m,1}, y_{m,2}, y_{s,1}, y_{s,2}$), separated by a frequency difference of Δf; then, we compute interferograms for the lower and the upper bands separately and finally the phase difference between the two. There are at least two possible implementations, depending on whether the averaging is performed at interferogram level (early multi-looking Delta-k) or after the final difference (late multi-looking Delta-k). The early-multi-looking alternative:

$$\hat{t}_s = \frac{1}{2\pi\Delta f} \arg\left(< y_{m,1}y^*_{s,1} >< y^*_{m,2}y_{s,2} >\right), \quad (3)$$

and the late multi-looking alternative:

$$\hat{t}_s = \frac{1}{2\pi\Delta f} \arg\left(< y_{m,1}y^*_{s,1}y^*_{m,2}y_{s,2} >\right). \quad (4)$$

The symbol $< \cdot >$ represents the operation of spatial averaging.

	correlation-based	*split-spectrum*
coherent	$\frac{3}{2N}\frac{1-\gamma^2}{\pi^2\gamma^2}$	$\frac{27}{16N}\frac{1-\gamma^2}{\pi^2\gamma^2}$
incoherent	$\frac{3}{10N}\frac{(1-\gamma^2)(2+7\gamma^2)}{\pi^2\gamma^4}$	$\frac{9}{16N}\frac{(1-\gamma^2)(1+4\gamma^2)}{\pi^2\gamma^4}$

Table 2: Performance of four algorithms for shift estimation as standard deviation of the error normalized to the resolution element.

The performances for all these algorithms are derived in [1, 8, 2, 10] and are reported in **Table 2** for a synoptic view. They depend on the number of independent samples N and the interferometric coherence γ. The expressions are valid for large N and are normalized to the resolution element. The classification of "late Delta-k" as incoherent might sound arbitrary but it will be justified later. As one can see from the graph in **Figure 1**, early Delta-k is substantially equivalent to CCC, having an efficiency of $8/9$. Late Delta-k is instead equivalent to ICC: the two curves are indistinguishable for all practical purposes. It is possible to show that this equivalence is not limited to having a similar performance, but it is deeper.

2.1 Link between ICC and late Delta-k

The correlation of ICC and "late Delta-k" estimates is pretty high [10], meaning that, fed with the same inputs, they will output very similar shift estimates. Late Delta-k is almost the perfect translation of ICC to the frequency domain. Note that this is a result that has a validity beyond the strict hypothesis of Gaussian speckle. To prove this, let us express the intensity signals as a function of the sub-bands of the original complex signal:

$$\begin{aligned} |y_m|^2 &= |y_{m,1} + y_{m,2}|^2 \quad (5)\\ &= y_{m,1}y^*_{m,2} + |y_{m,1}|^2 + |y_{m,2}|^2 + y_{m,2}y^*_{m,1}. \end{aligned}$$

Here we have assumed that the two sub-bands cover the full spectrum. The same expansion can be done for the slave intensity signal. Consider in the above equation that the central terms constitute the low-pass component of the intensity signal, whereas the cross-products constitute respectively the lower and upper frequencies, with trivial Hermitian symmetry. Looking for a delay between master and slave intensities is the same as finding a relative slope in the frequency domain, which is substantially equivalent to scaling the phase between $y_{m,2}y^*_{m,1}$ and $y_{s,2}y^*_{s,1}$: The other cross-term can be safely ignored thanks to the Hermitian symmetry. Including the spatial averaging and the scaling we get:

$$\begin{aligned} \hat{t}_s &\approx \frac{1}{2\pi\Delta f} \arg\left(< y_{s,2}y^*_{s,1}(y_{m,2}y^*_{m,1})^* >\right) \quad (6)\\ &= \frac{1}{2\pi\Delta f} \arg\left(< y_{s,2}y^*_{s,1}y^*_{m,2}y_{m,1} >\right). \quad (7) \end{aligned}$$

This expression is identical to (4) and shows clearly the link between incoherent cross-correlation and late multi-looking. In other words: we have derived "late Delta-k" from ICC, with some small approximations.

2.2 Coherent vs. incoherent methods

Generally speaking, coherent methods have a better performance, provided that the phase variations within the estimation window are compensated before cross-correlation or interferogram averaging (for early Delta-k). One should use coherent methods insofar it is possible to remove the mentioned interferometric fringes. Otherwise, incoherent methods provide a robust alternative. The robustness to phase variation does not come for free: incoherent methods perform much more poorly than coherent ones, with efficiency ranging from just over 1/2 for high coherence down to zero for low coherence's.

2.3 Correlation-based vs. split-spectrum methods

Split-band spectrum methods have the advantage that it is not necessary to oversample the correlation peak to achieve sub-pixel performances. They also avoid a border effects (a bias) that affect cross-correlation if no special measures are taken. However they have also some disadvantages. They suffer from ambiguities (this is mitigated in a multi-band implementation) and are problematic in case of spectral shift. The effect of spectral shift on the coherence depends on the bandwidth and has a bigger impact on the sub-bands than on the full-band signals. If one image is demodulated w.r.t. the other before the formation of the sub-bands to maximize coherence, the spectral shift problem disappears but then one has to carefully consider the change in carrier separation between the two sub-bands.

In the azimuth case, split-spectrum methods have a nice geometrical interpretation. Each sub-band corresponds to a different (range) line of sight and azimuth shifts appear as differential range shifts in the two lines of sight [5, 11].

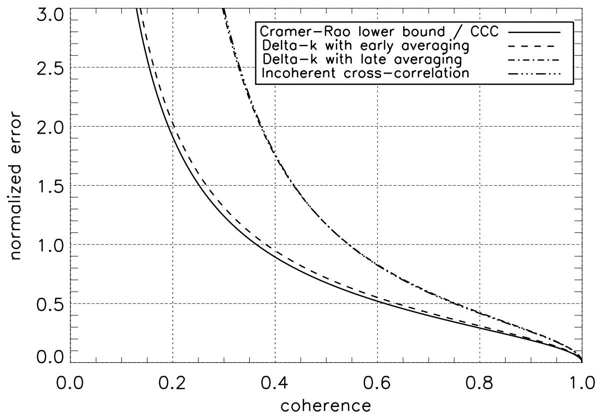

Figure 1: The normalized performance ($\sigma\sqrt{N}$) of different estimation algorithms and the Cramér-Rao bound. Note that the curves for Delta-k with late averaging and ICC are indistinguishable.

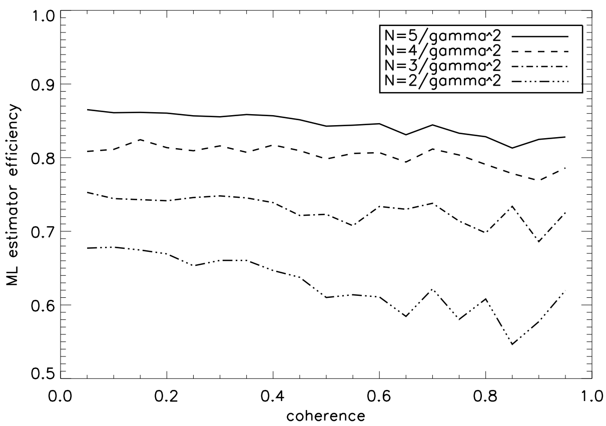

Figure 2: The efficiency of the maximum-likelihood estimator of the interferometric phase with $N(\gamma) = N_0/\gamma^2$ independent samples, for different N_0.

3 Efficiency of interferometric estimators

The peculiarity of Delta-k is that it can oscillate between a coherent and an incoherent estimate, depending on the amount of averaging performed at interferogram level. Its efficiency is directly related to the statistical efficiency (defined as closeness to the Cramér-Rao bound) of the underlying phase estimator operating on the upper and lower interferograms. In turn this efficiency depends, to a first approximation, on the quantity $N_0 = \gamma^2 N$. **Figure 2** illustrates this point, for different choices of N_0 (roughly speaking, the larger N_0, the higher the efficiency). In practice on can see that in many cases a small amount of early average will suffice. For instance, for $\gamma = 0.5$, 16 independent looks will allow reaching an efficiency of 0.8. If such averaging is not possible - because of large phase variations withing the averaging set - some degradation is inevitable. The oscillations in the graph are a consequence of the rounding N, especially visible for small N_0 and large γ.

4 Conclusions

This contribution has investigated the performances and relationships between different four popular algorithms for shift estimation, trying to clarify similarities and differences, both in the performance and in the advantages/disadvantages each implementation carries. It has shown that Delta-k approaches can be close to either coherent or incoherent cross-correlation, depending on the implementation details (early or late multi-looking). We have also investigated, with simulations, the efficiency of phase estimators for interferometry and shown that it depends on the quantity $\gamma^2 N$.

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