The Ambiguities related to Closure-Phase Model Inversion

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Abstract

This paper discusses the ambiguities that arise when trying to invert simple soil moisture models from closure phases in SAR interferometry. It shows that, under reasonable assumptions, the closure phase information is enough to sort acquisitions according to increasing moisture except for a special kind of ambiguity. The reconstructed moisture sequence can be represented on a circle where the most dry and most wet acquisitions lay side by side. The identification of this special pair of acquisitions solves the ambiguity and opens a way to constrained inversions reducing the number of multiple solutions.

1 Introduction

The influence of moisture on SAR interferometric phases and closure phases [1] is easily recognizable [2, 3], however the inversion of moisture sequences still seems today an impossible task. Interferometric phases might be contaminated by genuine motion and atmospheric delays and therefore are not reliable to base an inversion procedure. Models relying on closure phases are immune to motion or delay effects but are plagued by ambiguities: moisture series which differ a lot produce similar - if not identical - closure phases [2].

In this contribution we study the character of such ambiguities trying to provide a way forward towards ordering the acquisitions according to their moisture level. We show that, under some hypotheses, the problem boils down to identifying the acquisition with most or least moisture in a coherent set. After successful identification of the moisture order, one can move to a constrained inversion, which we have attempted in a preliminary study presented towards the end of this paper.

It is clear that closure phases have many degrees of freedom [1], to be precise $(N-1)(N-2)/2$ degrees of freedom in each averaging window for a set of N acquisitions and a temporal covariance matrix $N \times N$. For this reason a model is necessary: we have to suppose at least two scattering contributions with independent phase histories.

2 Phase closure approximation for simple models

The simplest model that contemplates closure phases deviating from zero is a two-scatterer model for the measured pixels:

$$y(n) = a + b\, e^{-j(\alpha - j\beta)m_n}. \qquad (1)$$

Here a and b are two statistically independent scatterers, with $E[|a|^2]$ dominanting over $E[|b|^2]$; m_n is the moisture level at acquisition n; α and β describe phase propagation and amplitude attenuation respectively. They are closely related to the dielectric constant. With such a model the expected interferogram will be:

$$I(n,k) = E[|a|^2] + E[|b|^2]\, e^{-j\alpha(m_n - m_k) - \beta(m_n + m_k)}. \qquad (2)$$

If $\beta = 0$ one finds that the resulting closure phase is approximately:

$$\begin{aligned} \Phi_{n,k,h} &= \angle I(n,k) I(k,h) I(h,n) & (3)\\ &\propto (m_n - m_k)(m_k - m_h)(m_h - m_n), & (4) \end{aligned}$$

i.e. the product of circular moisture differences. The approximation holds also for $\beta \neq 0$ provided that the ratio β/α is small enough. In physical terms this ratio corresponds broadly to the loss-tangent. Considering a loss tangent in the order of 1/10, the complex wavenumber has an imaginary- to real-part ratio of about 1/20.

Equation (4) is sufficiently general for our purposes: for instance, it predicts the correct sign of closure phases also for the model introduced in [3], though it is not particularly accurate for the magnitude. In the following section we will exploit only the closure phase signs.

3 Ambiguity structure

Adopting the above approximation (4), it is easy to recognize that the *sign* of closure phases gives some information on the sequence of moisture levels. **Figure 1** shows on the upper row the three different sequences that yield a positive closure, on the lower row the sequences that yield a negative closure. Assuming, for example, that we know that two acquisitions have increasing moisture, if the closure phase with a third acquisition is negative, we can for sure conclude that the moisture level of the third is intermediate between the first two; if it is positive we can only conclude that it is not intermediate - it might be higher or lower.

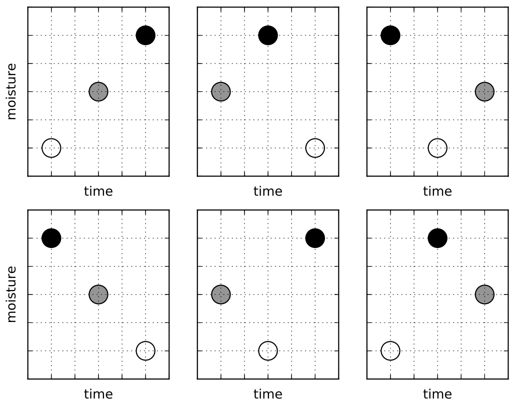

Figure 1: Moisture patterns related to positive (above) and negative (below) closure phase.

At this point one could have the impression that the closure phase information is really distant from the goal of ordering acquisitions according to their moisture levels. However things are not so bad as they seem. The reason is that the fist reconstruction mistake changes the landscape and prevents making additional ones. Consider, as an example, the moisture series on the left of **Figure 2** and a possible reconstruction on the right. It is clear that we have made a mistake at assigning acquisition 3 a moisture level higher than in acquisitions 1 and 2. The good news is that - after this mistake - we will be forced to assign acquisitions 4 and 5 at a position intermediate between acquisition 1 and 3. This is because the Φ_{134} is negative and we are assuming (wrongly) that the moisture is increasing going from acquisiton 1 to 3. This way acquisitions 4 and 5 must be assigned moisture levels between 1 and 3 and the series is not disrupted further.

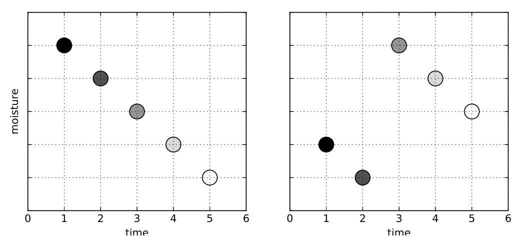

Figure 2: Two series of moisture with equivalent closure phase signs. Only one jump is possible.

One realizes soon that an algorithm based on closure-phase signs will order correctly the acquisitions according to their moisture level, provided that the result is put on a ring. Here the acquisition with lowest and highest moisture will lay side by side as in **Figure 3**. Therefore, the only extant difficulty resides in identifying where to cut the circle, distinguishing head from tail so to speak.

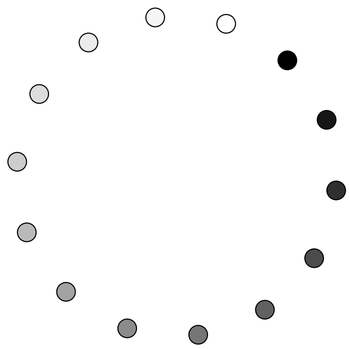

Figure 3: Ambiguity wheel: closure phases alone can not tell where to place the cut between the acquisition with most and least moisture.

4 Telling head from tail

Telling the most wet acquisition from the most dry will thus be crucial to enable ambiguity-free inversions. This cannot be accomplished by closure phases: interferometric coherence, amplitude information, external data or a-priori knowledge might help in this task.
Normally, moisture decays with time as the soil or vegetation dries in absence of precipitation and jumps suddenly up after rain precipitation events. This introduces some constrains on the temporal behavior of moisture and coherence.
For example a sudden loss of coherence (between two acquisitions close in time) is usually related to a rain event. Such an observation allows to identify a sector in the ambiguity wheel where the cut should *not* be placed in the ring in Fig. 3. Highly-coherent pairs are tipically dry and should belong to the dry end of the reconstructed moisture sequence.

5 Limitations

The proposed path suffers from several limitations.

- Necessity of sufficient temporal coherence between all images, so that all closure phases can be computed. In this respect longer wavelengths are preferable over shorter ones.

- Skewness conservation assumption: it is necessary that in a two-scatterer model the weaker scatterer stays weaker in all acquisitions. A switch in the brightness order implies a switch in closure phase signs.

- Dependence on a model: there is no guarantee of course that the two-scatterer model is correct. Real-world situations might be much more complicated with many layers and complex moisture profiles. However if the target is much more complicated (e.g. more than 2 independent scatterers) this will be revealed by the impossibility to construct a relative ordering as the one suggested in this paper.

6 Experiments with real data

We have experimented with real ALOS-2 SAR data acquired in HH polarization from March to August, 2016 over the area of Kumamoto, Japan. The observed closure phases reach several tens of degrees and from previous investigations we know that the baselines are too small for producing significant tomographic effects. The most probable explanation is therefore the presence of moisture variations.

The inversion procedure takes 7 images and generates 15 ($= 6 \times 5/2$) independent closure phases. We adopt the simple model of Eq. (4) and consider the mean square error as the figure of merit to be minimized. The minimizing procedure (a steepest descent) is started many times with different random inizializations, since we know that the landscape has many local minima. Each solution corresponds to a certain order of the acquisitions according to their inverted moisture level and solutions which are "circularly equivalent" are considered together. We select then the "correct" circular ordering based on the number of solutions that converged to it and the corresponding mean square error.

At this point we proceed to solve the circular ambiguity by selecting the acquisition with the most wet conditions looking at average brightness, which is also confirmed by precipitation records of the Japan Meteorological Agency. The algorithm then repeats the inversion adding the proper constrains to reflect the desired moisture order.

The solutions obtained by this algorithm are reasonable and spatially consistent (**Figure 4**). For a preliminary validation we follow two lines: an internal check and a comparison with external data from the SMAP mission. For the internal check, we have compared the mean square error of residuals (i.e. how well the model explains observed the closure phases) with the case of random noise as an input. The noise is scaled to have the same variance as real input data. The comparison indicates that the model can explain real data definitely better than random noise. The gap is expected to increase for longer datasets.

More relevant is the external validation. In **Figure 5** we present a first comparison with SMAP L4 mission data and ASCAT-derived products (EUMETSAT, H-SAF). It must be said that over Japan the SMAP mission is typically unable to produce L2 data from the radiometer because of radio-frequency interferences, so that the moisture L4 products are generated with external data (e.g. precipitation) and might have degraded quality. The product based on ASCAT needs to be scaled with the porosity, since it is originally given in degrees of saturation. We have scaled it to match the maximum value of the SMAP time series.

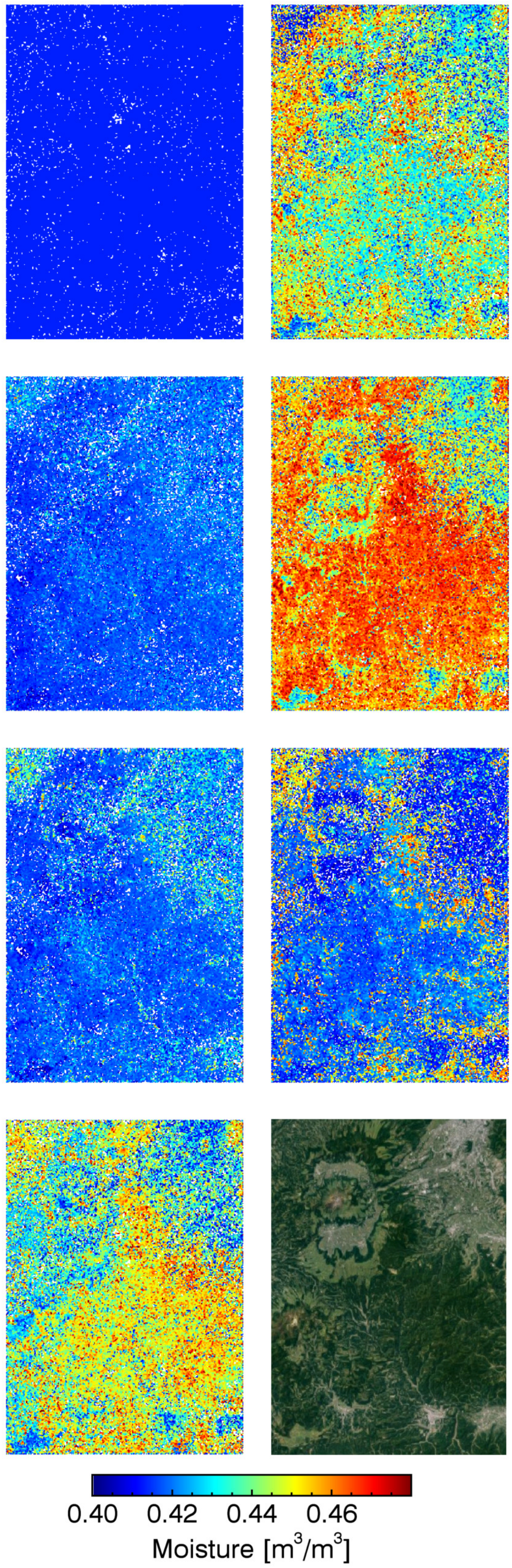

Figure 4: Inversion results for each image, ordered vertically. Bottom right: optical image of the area.

Similarly, the inversion results are linearly scaled for comparison purposes since our simple model in Eq. (4)

lacks a scale factor and is blind to a moisture offset. In general, a part from the issue of scaling, we observe good correlation between our results and both SMAP and ASCAT products. Contrary to the expectations, the inversion seems to work on forested areas.

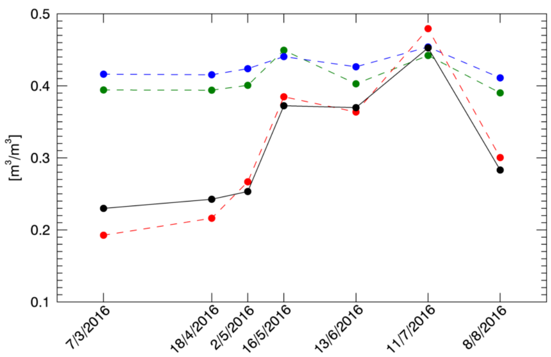

Figure 5: Blue: SMAP L4 / root zone; Green SMAP L4 / surface; Red: ASCAT scaled to match SMAP maximum; model inversion, scaled for best overlap with ASCAT.

7 Conclusions

Our results show that the proposed solution for the ambiguities allows inverting closure phases in a consistent way. The results correlate well with moisture products derived from the SMAP and ASCAT missions. An offset calibration of the retrieved moisture seems to be unavoidable, since the closure phase model is to be rather insensitive to moisture offsets. However it is not clear whether a scaling is also necessary or not. For this is probably necessary to compare with testsites where the SMAP mission was able to generate L2 products or to compare directly to moisture probes on the ground. More work is also needed to validate the retrieval concept over different sites, frequencies and land covers.

References

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[2] Zwieback, S.; Hensley, S. and Hajnsek, I., *Soil Moisture Estimation Using Differential Radar Interferometry: Toward Separating Soil Moisture and Displacements*, IEEE Transactions on Geoscience and Remote Sensing, vol. 55, n. 9, 2017.

[3] De Zan, F.; Parizzi, A.; Prats-Iraola, P.; Lopez-Dekker, P., *A SAR interferometric model for soil moisture*, IEEE Transactions on Geoscience and Remote Sensing, vol. 52, n. 1, 2014.