Uncertainty Dispersion Analysis of Atmospheric Re-entry using the Stochastic Liouville Equation

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Outline

- Analyzing re-entry onto Earth is subjected to uncertainties
- Prediction of impact zone and its dispersion necessary

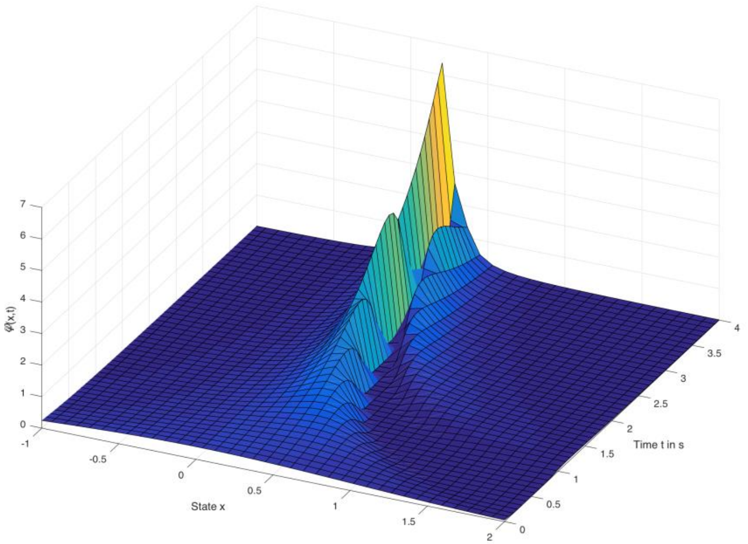
- Evolution of uncertainties in the initial conditions throughout the re-entry

- Stochastic Liouville Equation as alternative method for the Monte-Carlo Analysis
- Utilizing probability density functions (PDF)
- Solve PDF transport equation directly

- Comparison of results with reference dispersion generated by operational tool of GSOC (Monte-Carlo)

The Stochastic Liouville Equation

- Examine the development of the probability density of system dynamics over space and time
- Consider uncertainties in states and parameters
- From the resulting probability density function (PDF) all stochastic moments (e.g. covariance) can be deduced
- Non-linear model usable, no linearization needed



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The Stochastic Liouville Equation

- Consider a nonlinear state space model as following

$$\dot{X} = F(X), \text{where } X = (x, p)^T \in \mathbb{R}^{n_x + n_p}$$

- The temporal evolution of probabilistic uncertainty is governed by

$$\boxed{\frac{\partial \varphi(X,t)}{\partial t} + \sum_{i=1}^{n_x} \frac{\partial}{\partial X_i} [\varphi(X,t) F_i(X)] = 0}$$

Stochastic Liouville Equation

- Quasi linear partial differential equation (PDE) depending on the joint PDF

The Stochastic Liouville Equation

- the PDE can be reduced to an ordinary differential equation (ODE)

$$\frac{d\varphi(X,t)}{dt} = -\varphi(X,t)\sum_{i=1}^{n_x}\frac{\partial F_i}{\partial X_i}$$

which yields the solution

$$\boxed{\varphi(X,t) = \varphi_0 \exp\left(-\int_0^t \sum_{i=1}^{n_x}\frac{\partial F_i}{\partial X_i}\,\mathrm{d}X\right)}$$

- with the initial state and parametric uncertainties specified in terms of a joint PDF

$$\varphi_0 = \varphi(X,t_0)$$

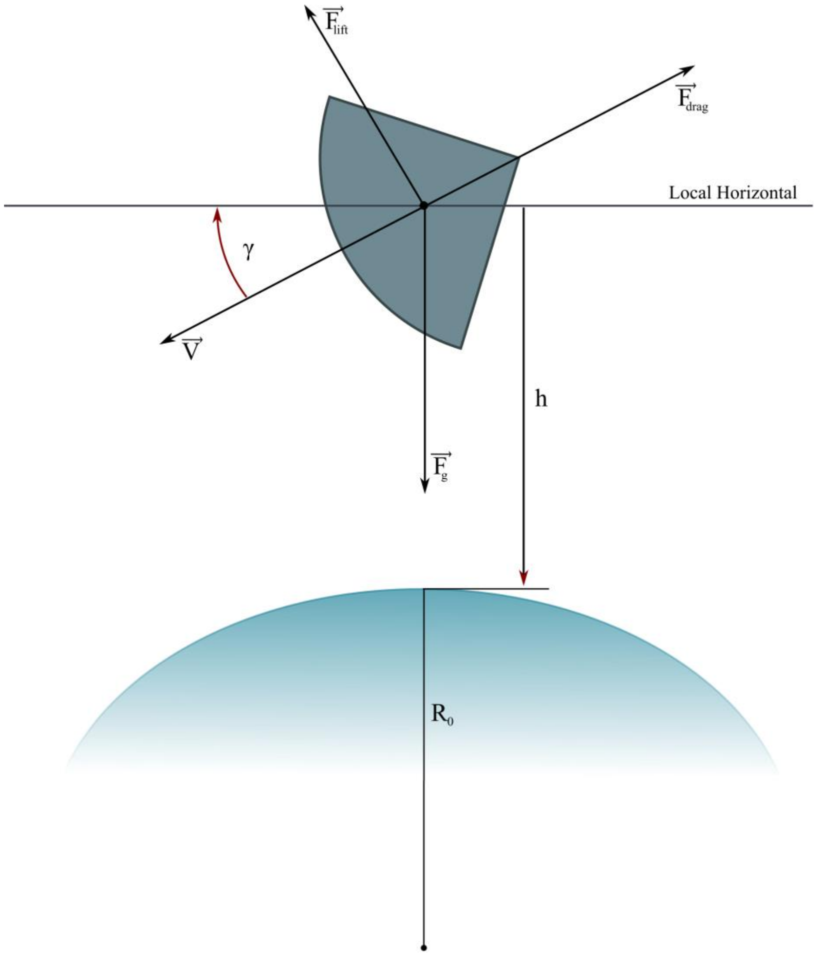
Uncertainty Dispersion of Space Debris Re-Entry

Consider the four state model
in case of non-rotating and spherical Earth

$$\dot{h} = v \sin\gamma$$
$$\dot{v} = -\frac{\rho}{2B_c}v^2 - g\sin\gamma$$
$$\dot{\gamma} = \frac{\rho}{2B_c}\frac{C_L}{C_D}v + \cos\gamma\left(\frac{v}{R_0+h} - \frac{g}{v}\right)$$
$$\dot{\theta} = \frac{v\cos\gamma}{R_0+h}$$

with

h	Altitude	g	Gravitational field
v	Velocity	ρ	Atmosphere density
γ	Flight Path Angle (FPA)	B_c	Ballistic coefficient
θ	Longitude	$\frac{C_L}{C_D}$	Lift-to-Drag Ratio



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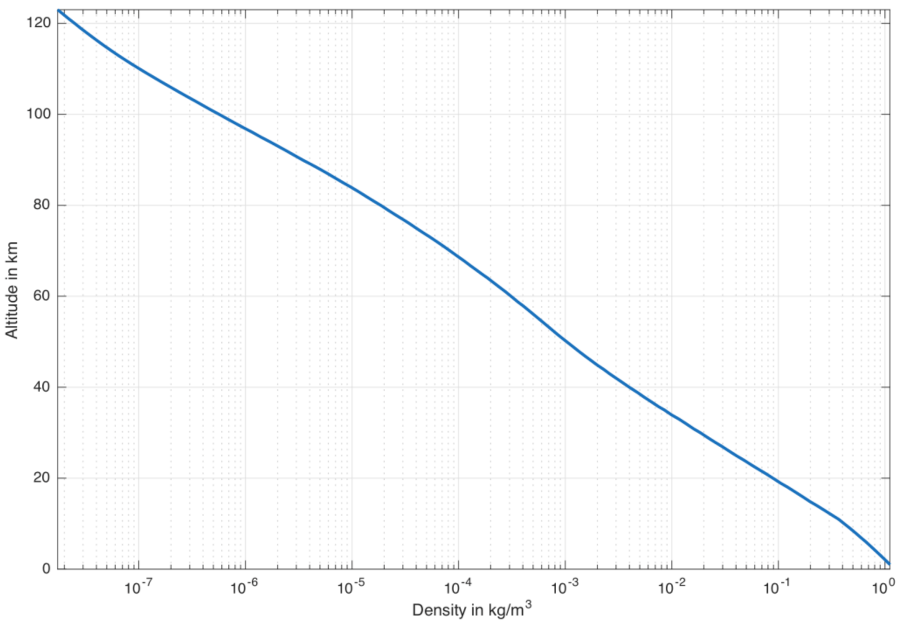
Example Re-Entry Trajectory

GSOC Re-entry model

- Gravity model
    - point mass
      (degree and order set to zero)

- Sun and moon forces considered

- Atmosphere model
    - Jacchia-Gill ($\mathrm{h} \geq 90\mathrm{km}$)
    - US Standard Atmosphere of 1976
      (USSA76, $h < 90\mathrm{km}$)



- For the use within the SLE, data of the Jacchia-Gill and USSA76 was interpolated

Example Re-Entry Trajectory

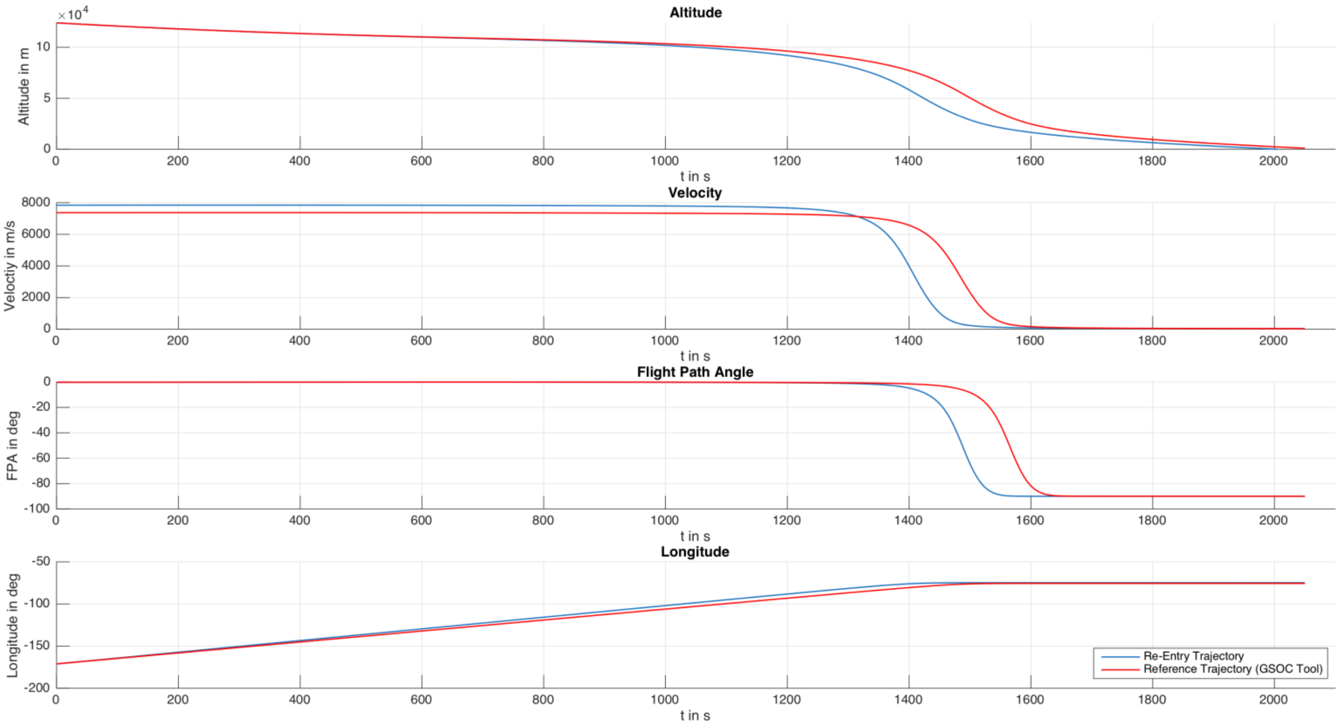
The three state model was extended by the Longitude $\boxed{\dot{\theta} = \frac{v \cos \gamma}{R_0 + h}}$ to determine the dispersion along track.

- S/C parameter:

Mass:	*100 kg*
C_D:	*2.3*
C_L:	*0.0*
Area:	*1.0* m^2

- Initial conditions:

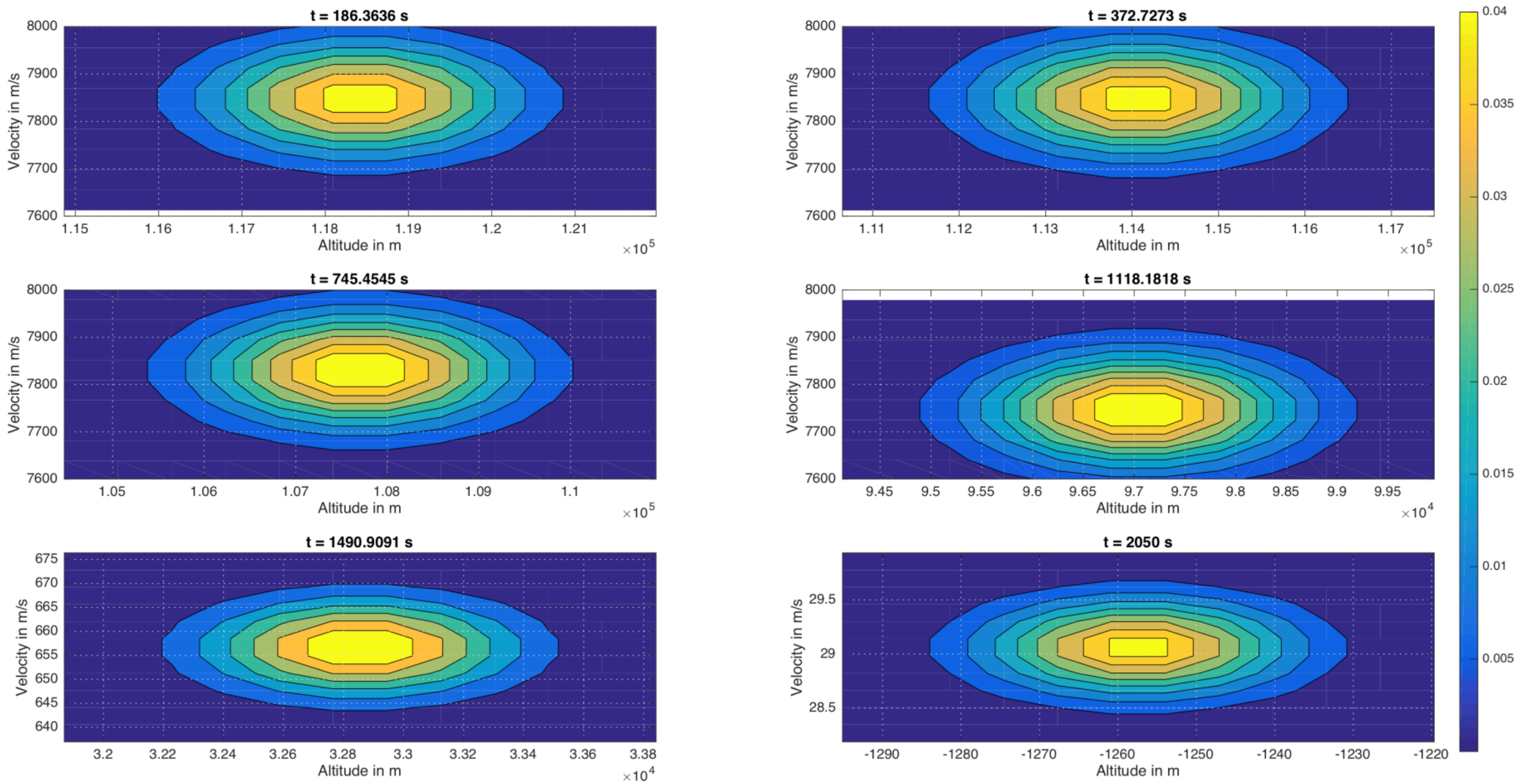
h_0	*123.996 km*
v_0	*7.8442 km/s*
γ_0	*-0.243°*
θ_0	*-171.11°*



Uncertainty Dispersion of Space Debris Re-Entry

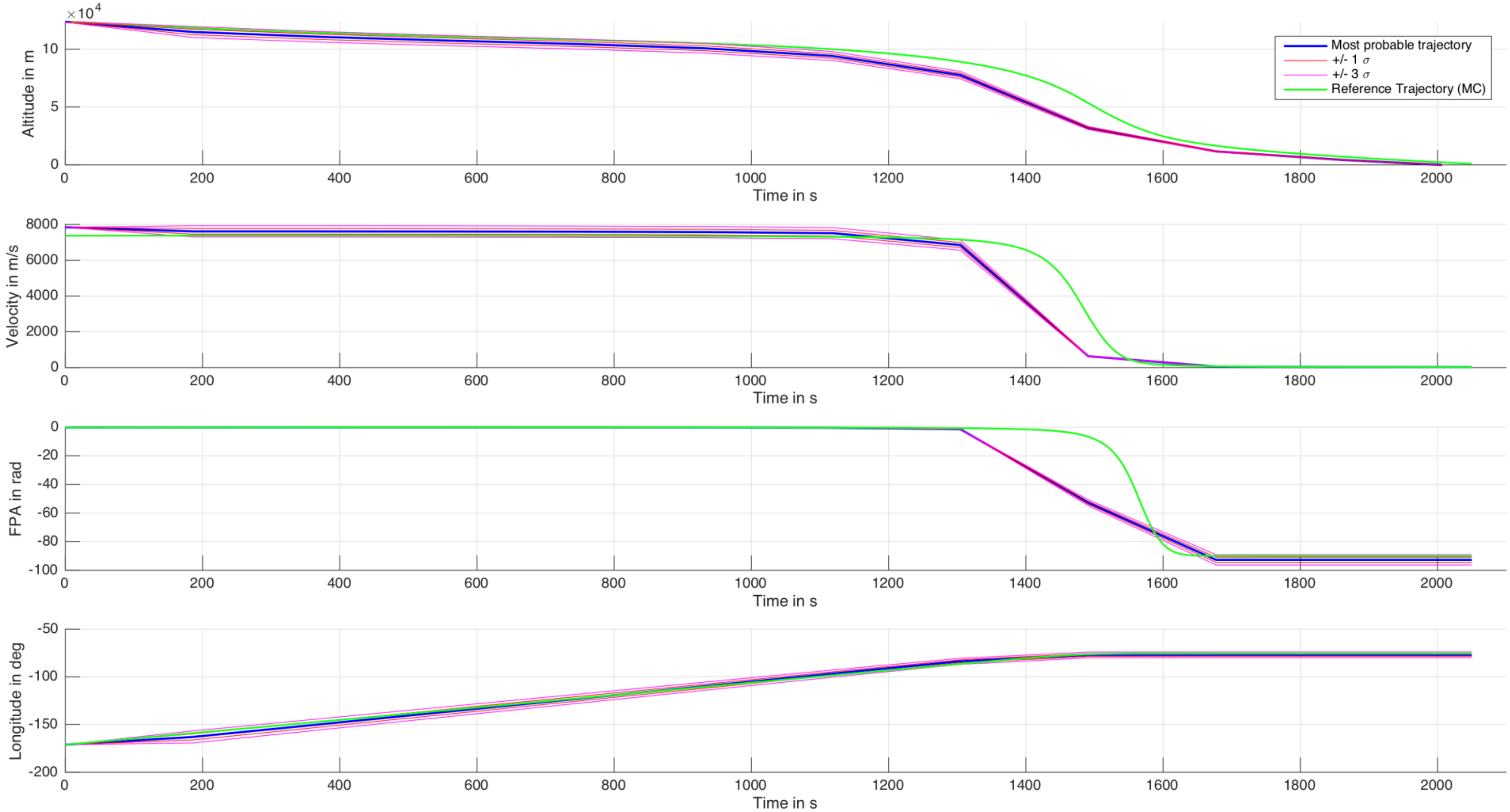
- Exemplary evolution of the two dimensional PDFs $\varphi(h, v)$
- The initial dispersions were chosen as:

$$
\begin{array}{llll}
h_0 & \pm 100m & \gamma_0 & \pm 0.001° \\
v_0 & \pm 0.1\ m/s & \theta_0 & \pm 0.001°
\end{array}
$$



Uncertainty Dispersion of Space Debris Re-Entry

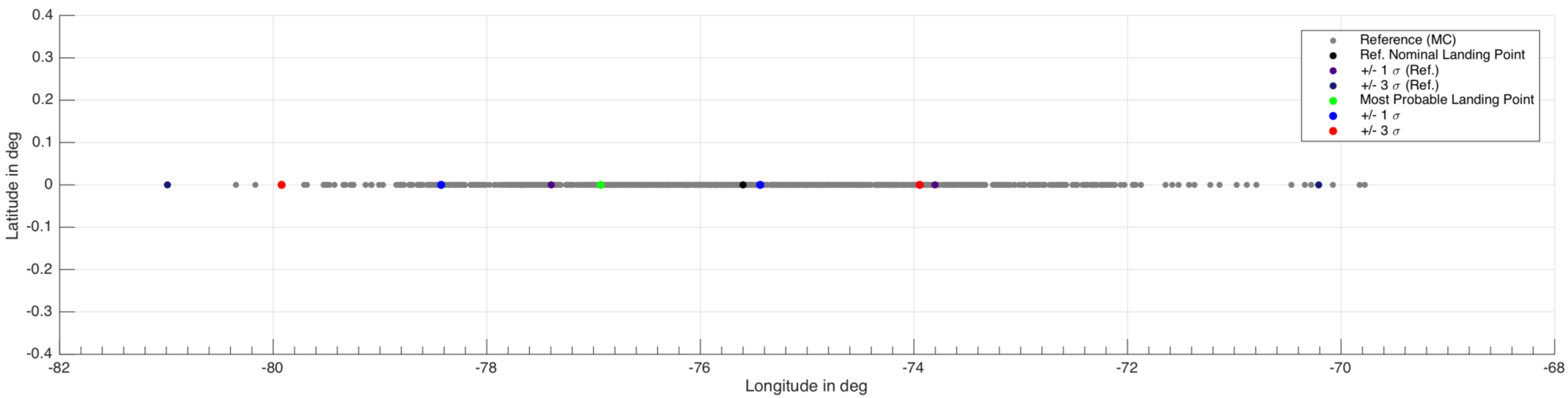
- Most probable trajectory generated from the PDFs in each time step in comparison with the reference trajectory



Uncertainty Dispersion of Space Debris Re-Entry

Landing Dispersion of Longitude in comparison with the landing dispersion of the reference run generated using the Monte-Carlo approach

	Results Stochastic Liouville Equation		Reference Monte-Carlo Analysis (GSOC)	
nominal	-76.93°		-75.6°	
σ	1.494		1.79	
$\pm 1\sigma$	-	-75.44°	-77.4°	-73.8°
$\pm 3\sigma$	-	-73.94°	-80.9°	-70.21°



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Summary and Conclusion

- The Stochastic Liouville Equation presents a computational efficient alternative for the Monte-Carlo Analysis
- Derive landing dispersions and footprints directly from PDFs

- The presented method was part of my master thesis at the University of Bremen in collaboration with DLR Institute of Space Systems in Bremen
- The shown example and reference trajectory/ dispersion was developed by DLR Space Flight Technology and Astronaut Training/ German Space Operation Center (GSOC)
- Thanks to Dr. Marco Scharringhausen (DLR RY-LET) and Dr. Michael Kirschner (DLR RB-RFT)

- Any questions? Contact me via E-Mail: maren.huelsmann@dlr.de

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