Model-Matching Design of Robust Vehicle Steer-by-Wire Systems

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Abstract— **The design and analysis of steer-by-wire systems is discussed. It is assumed that the target steer-by-wire dynamics is predefined and then the design is converted to a model-matching problem. The robustness analysis is performed in the space of physical uncertain parameters.**

I. Introduction

Vehicle steering technology is evolving by substituting the mechanical and hydraulic subsystems with electrical equivalents to boost performance and enhance safety. *Steer-by-wire* introduces a complex mechatronical steering technology consisting of computing units, sensors and actuators. Thereby the mechanical interface (steering column) between driver and vehicle is replaced by two electrical actuators which are algorithmically coupled by the steer-by-wire controller to provide the driver a desired steering feeling and the vehicle the desired steering response.

Steer-by-wire technology provides some essential advantages, such as, simplified construction and higher design flexibility at the price of redundancy and safety measures. However, the independent torque (power) assistance and active steering functionalities of some of today's steering technologies (so-called fully steer-by-wire functionality) motivates a part of automotive engineers for rejection of the necessity of steer-by-wire technology. The four decisive factors: safety, comfort, flexibility and costs are still relatively complex and unclear for a unique judgement so it is very difficult to predict when and if ever steer-by-wire vehicles will replace vehicles with steering column.

Steer-by-wire system is a typical master-slave system with neglectable time delay communication between the master and slave loop. Hence, it is natural to consider the transfer of the master-slave technology control know-how and know-what to the steer-by-wire one. In this paper it will be shown that yet new design challenges emerge since: (a) the so-called *intervening impedance* between the driver (operator) and the vehicle (environment) has a relatively complex dynamics and it may include static nonlinearities (power assisted steer-by-wire), and (b) the environment of steer-by-wire vehicles is non-passiv, thus putting some obstacles in working within the framework of *passivity*, which has been almost the traditional approach in master-slave system design.

The main contributions in this paper are: (a) model-matching solution of the design problem and (b) analysis of robust stability of a steer-by-wire system with respect to main parametric uncertainties of the environment (driver stiffness, vehicle speed and tyre-road adhesion coefficient). It is shown that the lateral vehicle impedance is not robust passive. The output passivity excess of the controller may not compensate for the passivity shortage of the lateral vehicle dynamics. Thus, for a given target dynamics, passivity requirements may not be achievable. However, the performance requirements may yet be kept without hurting the robust stability. The robustness analysis methods in the presence of static sector non-linearities (boost-curve) are provided based on solving for the bounds of positive-realness of transfer matrix functions in parameter space. Finally, these methods may be used for tuning of power assistance boost-curve, such that the robust absolute stability is guaranteed.

The paper is organized as follows. Section 2 discusses on the target dynamics of steer-by-wire systems. Section 3 presents the two basic control structures of a steer-by-wire system design: the admittance and hybrid one. Section 4 discusses the open-loop (i.e. actuation loops) of a steer-by-wire system. Section 5 presents the model-matching synthesis of a steer-by-wire controller. Section 6 collects simulation results and finally, Section 7 discusses thoroughly the analysis of robust stability of a steer-by-wire system in the space of physical uncertain parameters.

II. Target dynamics

The first step when designing the steer-by-wire steering dynamics is to set its reference, i.e. to give some answer in the natural question how a steer-by-wire system should feel like and how the vehicle should react on the driver steering command. In the sequel, such a desired system dynamics will be referred as *reference* or *target steering system.* Some straightforward target steering systems are, of course, the contemporary steering systems, s.a. Electrical (EPAS) or/and Hydraulic Power Assisted Steering (HPAS), which are schematically shown in Fig. 1.

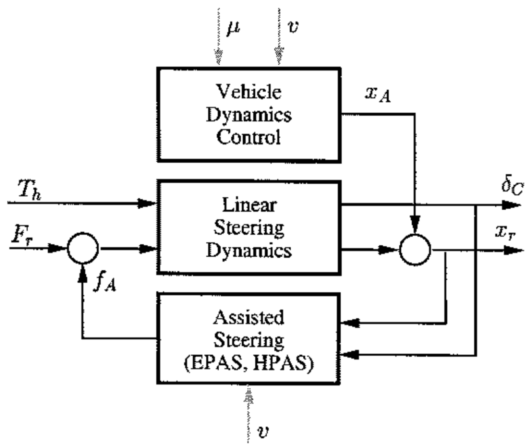


Fig. 1. Force-feedback in a steer-by-wire vehicle.

The notation used in Fig. 1 is as follows,

- T_h, Driver torque on the steering wheel
- f_A, Assisted steering torque
- F_r, Steering rack force
- v, Vehicle speed
- μ, Tyre-road adhesion coeffcient
- δ_C, Torsional angle on steering column
- x_r, Steering rack position
- x_A, Assisted steering angle.

Notice that steering systems comprehend assisting algorithms for independent force (for comfortable steering) and position assistance (for improved vehicle dynamics). Usually, the assisted torque steering filter is a nonlinear (parabolic) function of vehicle speed, v and steering column torsional angle, δ_C (the so-called *boost curve*). The steering angle assistance is introduced for improvement of lateral and/or vertical vehicle dynamics and variable steering transmission.

The linear steering dynamics in Fig. 1 includes the mechanical dynamics of the steering wheel, steering column, steering rack, power steering actuator, and that of the torque sensor on the steering column. Its dynamics can be represented by the following two-input two-output multivariable *admittance* system[1],

$$\begin{bmatrix} \dot{\delta}_h \\ \dot{x}_r \end{bmatrix} = \begin{bmatrix} y_{11} & y_{12} \\ y_{21} & y_{22} \end{bmatrix} \begin{bmatrix} T_h \\ F_r \end{bmatrix}, \qquad (1)$$

or by the equivalent *hybrid* representation,

$$\begin{bmatrix} \dot{\delta}_h \\ F_r \end{bmatrix} = \begin{bmatrix} y_{11} & h_{12} \\ h_{21} & z_{22} \end{bmatrix} \begin{bmatrix} T_h \\ \dot{x}_r \end{bmatrix}. \qquad (2)$$

Since the steering column dynamics is included in the linear part of the steering dynamics, it is clear that in order to match the steering dynamics of a given target steering dynamics (e.g. EPAS) with a steer-by-wire technology, exactly this dynamics has to be reproduced by a suitable steer-by-wire actuation. The code which provides the nonlinear power assisted steering can be reused as it is, thus reducing the design problem of a steer-by-wire system into a linear control problem.

III. Steer-by-Wire Design Structure

A steer-by-wire system is basically a *master-slave* system, whereby the driver corresponds to the *operator*, the vehicle to the *environment*, the force-feedback actuator to the *master* and the front-wheel actuator to the *slave.*

From the control point of view, a steer-by-wire control system includes two actuation inner-loops, which are coupled by a suitable outer-loop controller for provision of a desired steering dynamics or/and driving feeling. While a force-feedback loop is needed at the steering wheel, to provide a haptic information to the driver, the road-wheel actuation may be designed as a position or a force control loop. Depending on the art of this actuation loop, one discriminates between two basic steer-by-wire topologies, the *admittance* and the *hybrid* topology, Fig. 2.

A. Admittance structure

In an admittance system description, (1), the steering wheel angle rate, $\dot{\delta}_h$ and the rack position rate, $\dot{x}_r$ are the measured/controlled variables. The controller structure in this case is,

$$\begin{bmatrix} \tau_m \\ f_s \end{bmatrix} = C_y \begin{bmatrix} \dot{\delta}_h \\ \dot{x}_r \end{bmatrix} + \begin{bmatrix} 0 \\ f_A \end{bmatrix}, \qquad (3)$$

whereby,

[1]Notice that this is a simplified model, whereby the nonlinear gearing friction is neglected.

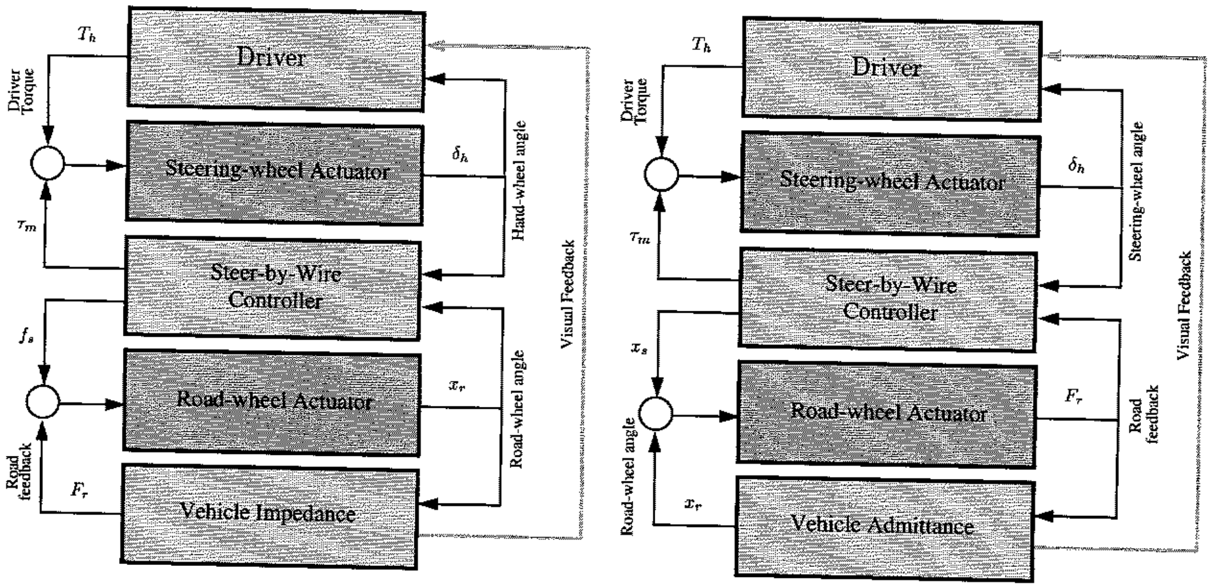


Fig. 2. Two steer-by-wire control topologies, the admittance (left) and the hybrid one (right).

f_A, Nonlinear boost force assistance, (1)
τ_m, Reference torque for the force-feedback loop
f_s, Reference force for road-wheel actuation loop.

The primary advantage of this structure is its simple sensory requirement (two common encoders). Notice that the admittance steer-by-wire structure is suitable for power assisted like steering systems.

B. Hybrid structure

In the hybrid steer-by-wire structure the rack force, F_r, is assumed to be known instead of the rack position rate, $\dot{x}_r$. Its controller structure has the following form,

$$\begin{bmatrix} \tau_m \\ \dot{x}_s \end{bmatrix} = C_h \begin{bmatrix} \dot{\delta}_h \\ F_r \end{bmatrix} + \begin{bmatrix} 0 \\ \dot{x}_A \end{bmatrix}. \qquad (4)$$

While mathematically both structures are equivalent, the hybrid one is perhaps more intuitive. It is to be expected that that the hybrid controller, C_h is of lower order than the admittance controller C_y, since the latter must reconstruct the dynamics of the force signal, F_r. However its drawback are sensory costs. An appealing alternative in this case is the input observation of F_r. Notice that, the hybrid structure is a more appropriate one for the integration of the vehicle dynamics algorithms.

IV. Actuation loops

As already noted a steer-by-wire control system includes two actuation inner-loops for torque and position control. The dynamics of the force-feedback loop can in general be described by the linear equation,

$$\dot{\delta}_h = y_h \left(T_h - \alpha_h \tau_m \right) \qquad (5)$$

This equation formally describes both, the closed-loop and open-loop force-feedback. Notice that the dynamics of the actuator, sensor and controller are lumped into the transfer functions y_h and α_h.

Similarly, in an admittance steer-by wire structure the road-wheel actuation loop is described by the equation,

$$\dot{x}_r = y_r \left(F_r - \alpha_r f_s \right), \qquad (6)$$

while in a hybrid steer-by-wire structure it is described by,

$$F_r = z_r \left(\dot{x}_r - \beta_r \dot{x}_s \right), \qquad (7)$$

Recall, the variables τ_m, f_s and $\dot{x}_s$ are the outputs of the steer-by-wire feedback controller, C_y i.e. C_h.

Introduce the following definitions,

$$v_a = \begin{bmatrix} \dot{\delta}_h \\ \dot{x}_r \end{bmatrix}, \quad r_a = \begin{bmatrix} T_h \\ F_r \end{bmatrix}, \quad \tau_1 = \begin{bmatrix} \tau_m \\ f_s \end{bmatrix}, \qquad (8)$$

and,

$$v_h = \begin{bmatrix} \dot{\delta}_h \\ F_r \end{bmatrix}, \quad r_h = \begin{bmatrix} T_h \\ \dot{x}_r \end{bmatrix}, \quad \tau_2 = \begin{bmatrix} \tau_m \\ \dot{x}_s \end{bmatrix}. \qquad (9)$$

Now the dynamics of the open-loop admittance steer-by-wire structure can be described by the equation,

$$v_a = Y \left(r_a - A_1 \tau_1 \right), \qquad (10)$$

whereby,

$$Y = \begin{bmatrix} y_h & 0 \\ 0 & y_r \end{bmatrix}, \quad A_1 = \begin{bmatrix} \alpha_h & 0 \\ 0 & \alpha_r \end{bmatrix}. \qquad (11)$$

Similarly open-loop dynamics of the hybrid steer-by-wire structure may is modelled by the equation,

$$v_h = H\left(r_h - A_2 \tau_2\right), \qquad (12)$$

whereby,

$$H = \begin{bmatrix} y_h & 0 \\ 0 & z_r \end{bmatrix}, \quad A_2 = \begin{bmatrix} \alpha_h & 0 \\ 0 & \beta_r \end{bmatrix}. \qquad (13)$$

V. Model-matching based design

This section concentrates on the admittance steer-by-wire structure. Similar considerations apply for the hybrid structure.

By closing the loop in (10) and using (3) it follows,

$$v_a = Y\left(r_a - A_1 C_y v_a\right). \qquad (14)$$

Its solution v_a is,

$$v_a = (I + Y A_1 C_y)^{-1} Y r_a. \qquad (15)$$

The control problem may be formulated as follows: find the controller C_y such that the closed-loop response $r_a \to v_a$ resembles the steering dynamics described by (1), i.e.

$$\|(I + Y A_1 C_y)^{-1} Y - Y_d\|_\infty = \text{minimal}, \qquad (16)$$

whereby Y_d corresponds to the desired admittance matrix in (1).

In order to solve this problem the controller C_y is parameterized using Youla parameterization[1], [2],

$$C_y = (B - MQ)(A - NQ)^{-1}, \qquad (17)$$

whereby matrices M and N coprime-factorize the product YA_1,

$$YA_1 = NM^{-1} = \tilde{M}^{-1}\tilde{N}, \qquad (18)$$

and

$$\begin{bmatrix} \tilde{A} & -\tilde{B} \\ -\tilde{N} & \tilde{M} \end{bmatrix} \begin{bmatrix} M & B \\ N & A \end{bmatrix} = I. \qquad (19)$$

Q belongs to Hardy space, $\mathcal{RH}_\infty$, of all proper and real rational stable transfer matrices and represents the free parameter. Using the last three equations, after some algebraic operations, (16) may be transformed to the *model-matching problem*,

$$\|T_1 - T_2 Q T_3\|_\infty = \text{minimal}, \qquad (20)$$

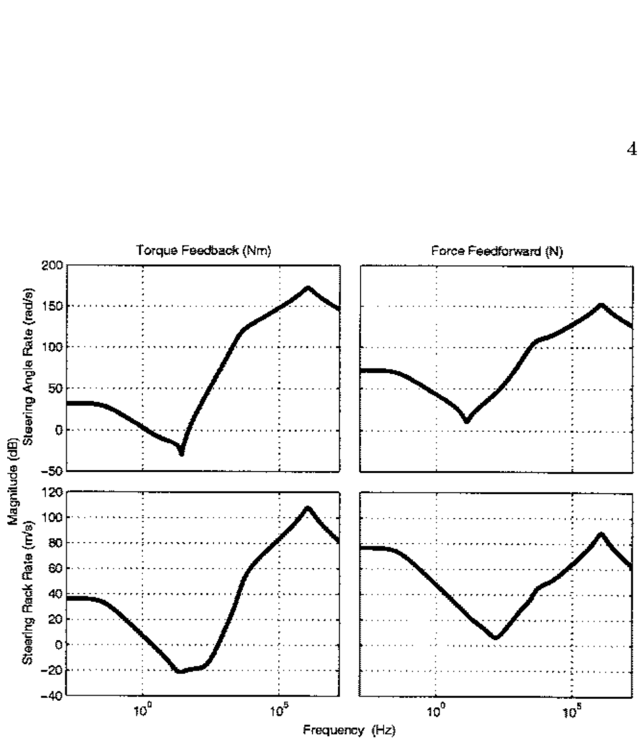


Fig. 3. Bode magnitude of model-matching controller.

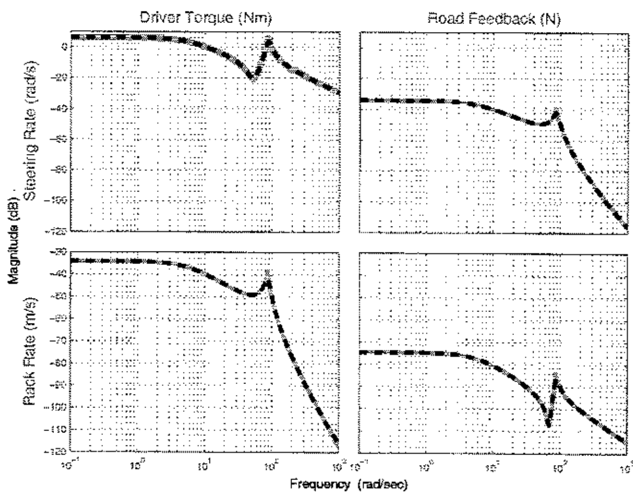
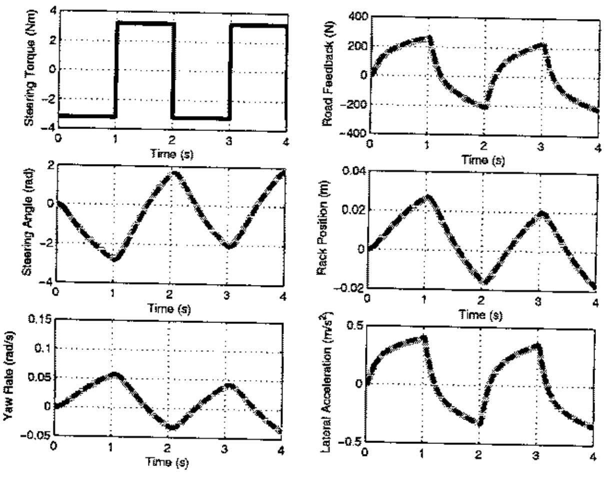


Fig. 4. Comparison of desired steer-by-wire (solid line) dynamics and closed-loop dynamics (dashed line).

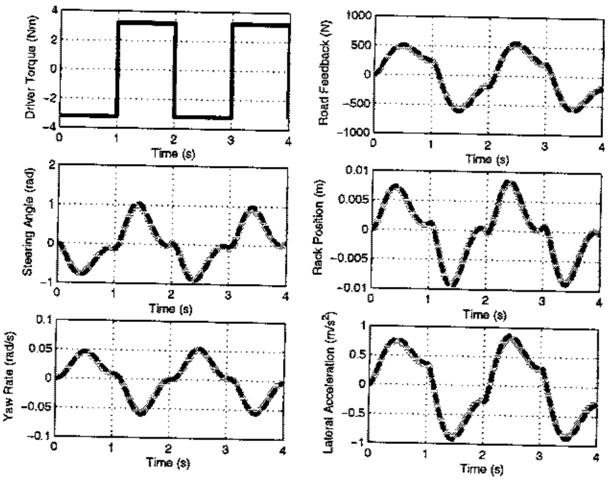
whereby,

$$\begin{aligned} T_1 &= A\tilde{M}Y - Y_d & (21) \\ T_2 &= N\tilde{M} & (22) \\ T_3 &= \tilde{M}Y. & (23) \end{aligned}$$

The model-matching problem may be solved by using LMI tools or some other H_∞ optimization tool. Note that up to a certain frequency ($\sim 10^3$ Hz) the controller is a coupled $PD-$like structure with respect to position measurements, δ_h and x_r. In Fig. 4 the frequency response of the linear reference steer-by-wire system and the closed-loop dynamics are compared.

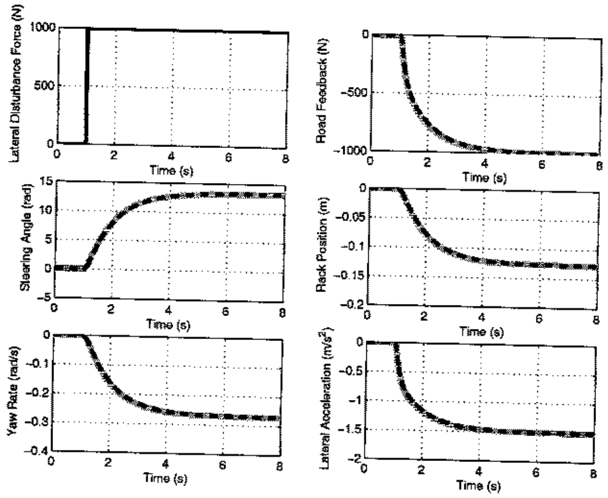


(a) $v = 20$ km/h, $\mu = 1$

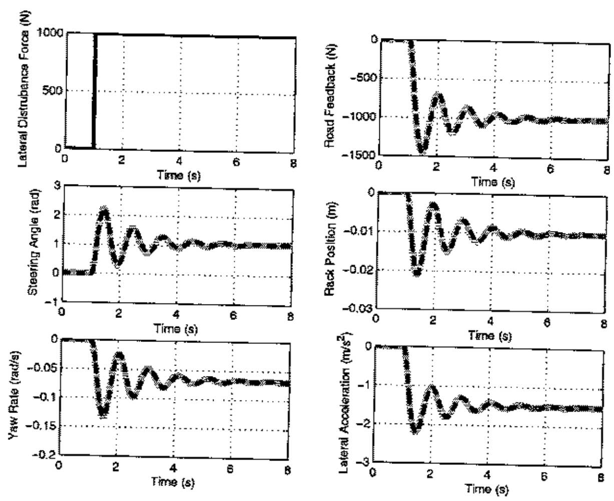


(b) $v = 80$ km/h, $\mu = 0.8$

Fig. 5. Responses of linear steer-by-wire vehicle (dashed line) and conventional vehicle (solid line) to rectangular periodic driver input torque.



(a) $v = 20$ km/h, $\mu = 1$

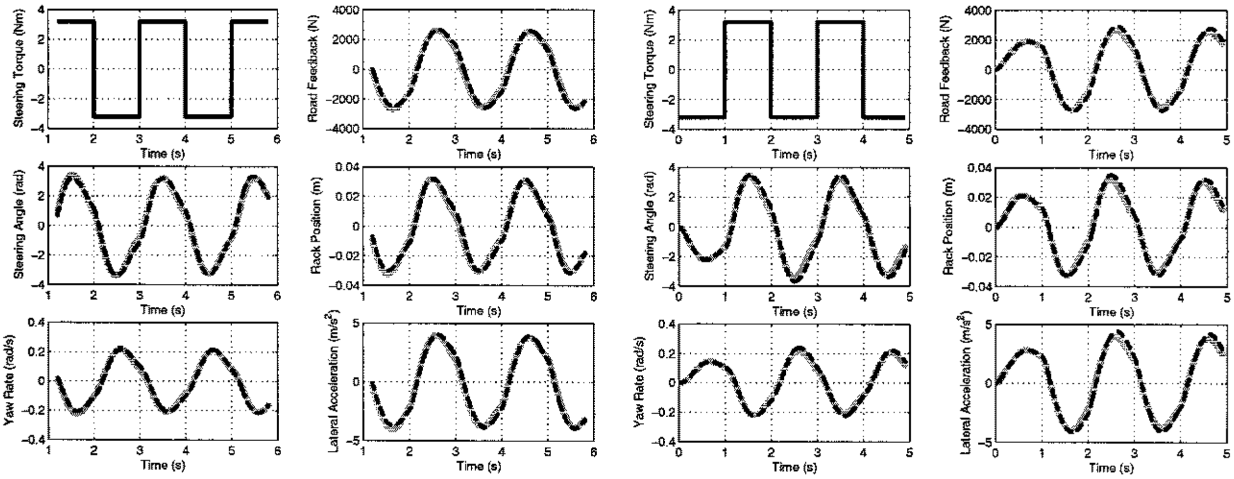


(b) $v = 80$ km/h, $\mu = 0.8$

Fig. 6. Responses of linear steer-by-wire vehicle (dashed line) and conventional vehicle (solid line) to a disturbance force on steering link.

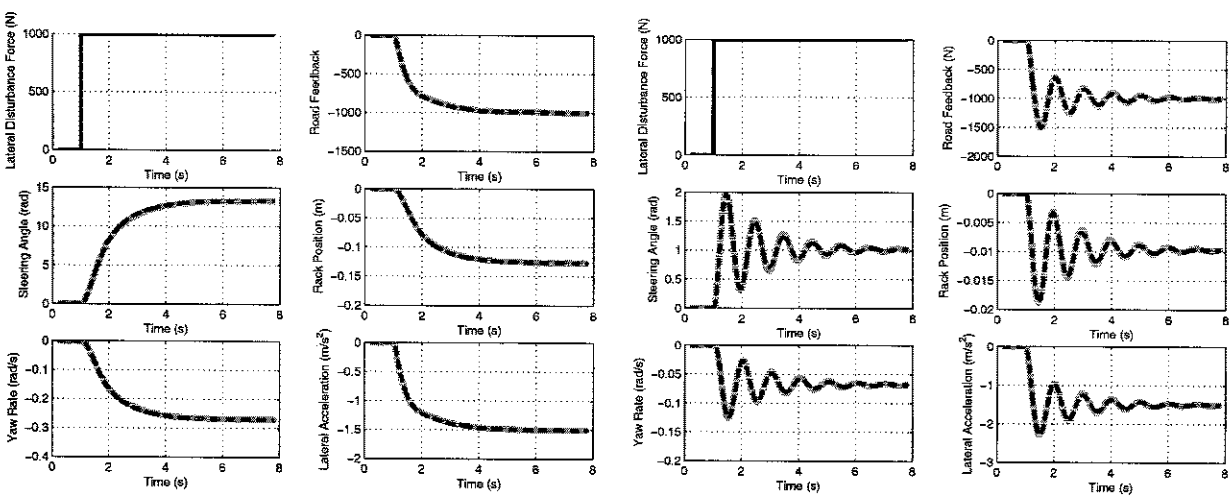
VI. Simulation results

This section shows simulation results of a steer-by-wire system with the linear model-matching controller, whereby its dynamics is coupled to the standard single-track vehicle model (Section 5) for vehicle dynamics simulation. Basically the responses of the closed-loop steer-by-wire system and target steering systems with respect to two input scenarios are compared: (a) the driver torque on the steering wheel is a periodic rectangular signal of frequency 0.5 Hz and amplitude 3.2 Nm, and (b) a step-wise disturbance force of amplitude 1 kN is applied on steering link. Comparisons for both: linear (no force assistance) and non-linear (with force assistance) systems are done in terms of responses of the steering wheel angle δ_h, steering rack position x_r, road-feedback, F_r, and vehicle dynamics variables: yaw rate, r, and lateral front-wheel acceleration, a_{yf}.



(a) $v = 20$ km/h, $\mu = 1$ (b) $v = 80$ km/h, $\mu = 0.8$

Fig. 7. Responses of non-linear steer-by-wire vehicle (dashed line) and EPAS vehicle (solid line) to rectangular periodic driver input torque.



(a) $v = 20$ km/h, $\mu = 1$ (b) $v = 80$ km/h, $\mu = 0.8$

Fig. 8. Responses of non-linear steer-by-wire vehicle (dashed line) and EPAS vehicle (solid line) to a disturbance force on steering link.

VII. Robustness Analysis

A. *Passivity approach*

Passivity provides the basic framework of state of the art methods used for design and analysis of teleoperation systems. Thus, it is important to investigate its usability for steer-by-wire.

A.1 Basic definitions

This subsection recalls briefly some very basic definitions on passivity, which will be used on later discussions, [8], [5], [9]. Consider a system H with an input vector u and output vector y of the same dimension m, whereby $u : \mathbb{R}^+ \mapsto \mathbb{R}^m$ is bounded. Assume further that for the system H the *supply rate* $w : \mathbb{R}^m \times \mathbb{R}^m \mapsto \mathbb{R}$ may be defined such that $\int_{t_o}^{t_1} |w(u(t), y(t))| \mathrm{d}t < \infty,\ \forall t_o \leq t_1$. The system H

is said to be *dissipative* with the supply rate $w(u,y)$ if there exists a function of states $x(t)$, $S(x) \geq 0$, $S(0) = 0$, which is called *storage function* such that

$$S(x(T)) - S(x(0)) \leq \int_0^T w(u(t), y(t))\mathrm{d}t \qquad (24)$$

$\forall T \geq 0$. Further, H is said to be *passive* if it is dissipative with supply rate $w(u,y) = u^T y$.

Assume a system may be divided in a set of subsystems, such that its storage function is the sum of the storage functions of the subsystems. The whole system may still be passive, even if some its subsystem is not active. In other words, one can assign to a system the notions *shortage* and *excess* of passivity. The system H is said to be *output feedback passive* if it is dissipative with respect to $w(u,y) = u^T y - \rho y^T y$ for some $\rho \in \mathbb{R}$. Analogously, H is said to be *input feedback passive* if it is dissipative with respect to $w(u,y) = u^T y - \eta u^T u$ for some $\eta \in \mathbb{R}$. Accordingly, positive sign of ρ and η means that the system has an excess of passivity, and conversely, negative sign of ρ and η means that the system has a shortage of passivity.

A.2 Robust stability

Fig. 9 represents a master-slave observation of a steer-by-wire system. Z_h represents the impedance of the arm of the driver (*master*) and Z_v the lateral vehicle impedance (*environment*) , i.e. the friction force of the tyre-road friction due to the rack rate $\dot{x}_r$. The two impedances interact via the steer-by-wire actuation and control algorithm. Notice that the forces generated at the driver arm muscles and lateral vehicle disturbances are ignored.

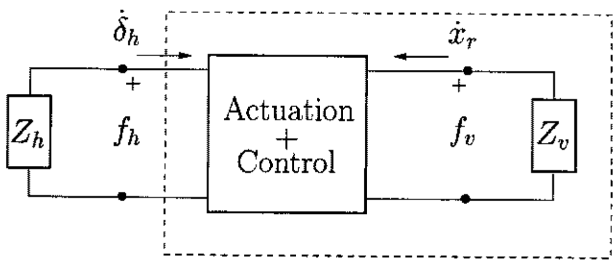


Fig. 9. Master-slave representation of steer-by-wire interaction.

A well-known passivity-based theorem on the robust stability of such a master-slave structure is: given the passivity of the master and environment impedance, the whole master-slave system is robust stable iff the block *Actuation + Control* is passive. Notice that no other assumptions regarding the master and environment dynamics are assumed besides being passive.

A.3 Passivity bounds in parameter space

This work aims to analyze the stability and/or passivity regions in the space of physical parameters of a steer-by-wire system, such as driver arm stiffness, c_o, vehicle speed, v, and tyre-road adhesion coefficient μ. To this end, the approach of mapping of specifications in parameter space will be used. This section provides a brief introduction to the method used therefore.

Assume $Y(s, \boldsymbol{q})$ is a given transfer function matrix, which depends on some parameters $\boldsymbol{q}$, which may include uncertain physical parameters or/and controller parameters. Transfer function matrix $Y(s, \boldsymbol{q})$ is passive at some $\boldsymbol{q}$ iff the hermitian matrix, [9],

$$H(j\omega, \boldsymbol{q}) = Y(j\omega, \boldsymbol{q}) + Y^T(-j\omega, \boldsymbol{q}) \qquad (25)$$

is positive-definite for each ω at $\boldsymbol{q}$.

The following theorem is standard in linear algebra, [10].

Theorem 1: A hermitian matrix $H(j\omega) = (h_{ij})$ is positive definite iff,

$$\Delta_k^{(H)}(j\omega) > 0, \ \forall \omega > 0, \ k = 1, 2, \ldots, m$$

with

$$\Delta_k^{(H)}(j\omega) = \begin{vmatrix} h_{11}(j\omega) & \cdots & h_{1i}(j\omega) \\ \vdots & & \vdots \\ h_{i1}(j\omega) & \cdots & h_{ii}(j\omega) \end{vmatrix}.$$

Now define

$$\boldsymbol{Q}_\pi^{(k)} = \left\{ \boldsymbol{q} : \ e_k(\omega, \boldsymbol{q}) \doteq \Delta_k^{(H)}(\omega, \boldsymbol{q}) \geq 0 \right\}.$$

Then the parameter region which drives $Y(j\omega, \boldsymbol{q})$ passive is the intersection,

$$\boldsymbol{Q}_\pi = \bigcap_k \boldsymbol{Q}_\pi^{(k)}.$$

Thus, finding the passivity bounds of the whole matrix is reduced to solving a system of $m-$inequalities. It can be shown that the bounds of the set $\boldsymbol{Q}_\pi^{(k)}$ are defined by the solution of the two following parametric nonlinear equations,

$$e_k(\omega, \boldsymbol{q}) = 0, \quad \frac{\partial e_k}{\partial \omega}(\omega, \boldsymbol{q}) = 0. \qquad (26)$$

Notice that, this methodology is convenient if the vector $\boldsymbol{q}$ includes few parameters.

B. System uncertainties

B.1 Bio-mechanical human arm impedance

The impedance of the human arm describes the force/velocity relationship $F = Z(v)$, whereby $Z(v)$ is some nonlinear function, [6]. The exact modelling of human arm is yet a complex problem; nonlinear modelling does not really provide essentially better results than the linear one. The muscle actuator of the driver arm may be roughly modelled by a linear equation as follows,

$$\Delta x = g_{m\alpha}\Delta f_\alpha + g_{mf} f_m, \tag{27}$$

whereby

x,	Internal muscle length
Δf_α,	Nerve excitation of the α-neuron
f_m,	Force acting on the muscle
$g_{m\alpha}$,	Feedforward control of muscle length
g_{mf},	Driving point admittance.

The transfer functions in this equation depend on muscle stiffness, muscle damping and the combined mass of the limb and interface element (i.e. steering wheel). The feedforward transfer function affects the muscle length through changes of afferent neural activation Δf_α and the adaption of arm impedance and control of arm position is based on feedback of neural activation. It has been shown that the impedance adaption feedback loop is a low bandwidth one, 1.7 Hz, [4].

The muscular actuator driven by $\alpha-$efference is certainly an active system since, as any actuator, the muscle supplies energy. However, different experiments (citation) suggest that the driving point impedance of the arm shows passive behavior. A common modelling approach of the driving point impedance of the human arm has been accepted the variable spring-like behavior, whereby its stiffness is modified by neural feedback. Measurement suggest, [3], minimal incremental elbow stiffness of $2Nm/rad$ and maximal of $400Nm/rad$. On the other hand, the damping was estimated to be around 5.5 N s/m, thus indicating that the human arm is lightly damped.

B.2 Lateral vehicle impedance

The car model which has been used for the investigations in this paper is the classical linearized single track model, [7], as illustrated in Fig. 10.

Its basic variables and geometric parameters are[2]

[2]The parameter values of the linearized single track model assumed in this paper are l_f=1.25 m, l_r=1.32 m, m=1296 kg, J=1750 kg m^2, c_{f0}=9244 N/rad and c_{r0}=105750 N/rad.

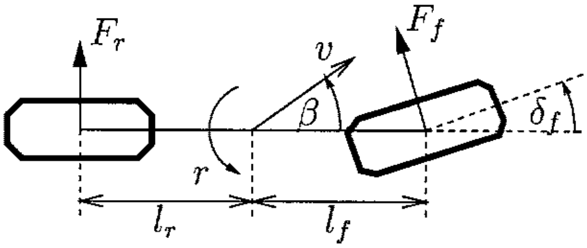


Fig. 10. Single-track model

$F_f(F_r)$	Lateral wheel force at front (rear) wheel
r	Yaw rate
β	Chassis side slip angle at center of gravity (CG)
v	Magnitude of velocity vector at CG ($v > 0,\ \dot{v} = 0$)
$\ell_f(\ell_r)$	Distance from front (rear) axle to CG
δ_f	Front wheel steering angle

The mass of the vehicle is m and J is the moment of inertia with respect to a vertical axis through the CG. For small steering angle δ_f and small side slip angle β, the linearized equations of motion are

$$mv(\dot{\beta} + r) = F_f + F_r \tag{28}$$

$$m\ell_f\ell_r\dot{r} = F_f\ell_f - F_r\ell_r \tag{29}$$

The tire force characteristics are linearized as

$$F_f(\alpha_f) = \mu c_{f0}\alpha_f, \quad F_r(\alpha_r) = \mu c_{r0}\alpha_r \tag{30}$$

with the tire cornering stiffness c_{f0}, c_{r0}, the road adhesion factor μ, and the tire side slip angles

$$\alpha_f = \delta_f - \left(\beta + \frac{\ell_f}{v} r\right) \tag{31}$$

$$\alpha_r = -\left(\beta - \frac{\ell_r}{v} r\right) \tag{32}$$

A variable of great importance in lateral vehicle dynamics is the lateral acceleration at the front axle, $a_{y,f}$. It can be easily shown that its computing equation is,

$$a_{y,f} = v\left(\dot{\beta} + r\right) + \ell_f\dot{r}. \tag{33}$$

For steer-by-wire control the essential characteristic of vehicle dynamics is its *lateral impedance*, which will be defined as the ratio between the front lateral reaction force, F_f and the front wheel steering angle, δ_f, $Z_v = F_f/\delta_f$. In this paper, for the sake of simplicity, the single-track model will be used to compute it. By combining the equations (29), (30), (31) and (32) it can be shown that

$$Z_v(s) = \frac{a_2 s^2 + a_1 s + a_0}{s(b_2 s^2 + b_1 s + b_0)} \tag{34}$$

whereby,

$$\begin{aligned} a_2 &= c_f \mu v^2 m J \\ a_1 &= \mu^2 c_f c_r v \left(J + m\ell_r^2\right) \\ a_0 &= c_f c_r \ell_r \mu^2 v^2 m \\ b_2 &= m v^2 J \\ b_1 &= \mu\left((c_f + c_r)J + m(c_f \ell_f^2 + c_r \ell_r^2)\right) \\ b_0 &= \mu\left(\mu c_r c_f (\ell_f + \ell_r)^2 - m v^2 (c_f \ell_f - c_r \ell_r)\right). \end{aligned}$$

Notice that, $Z_v(s, \mu, v)$ is uncertain because of the uncertainties in physical parameters μ and v.

Using the theoretical notes described in the previous subsection, passivity properties of the lateral vehicle impedance are mapped into the parameter plane (μ, v) in Fig. 11.

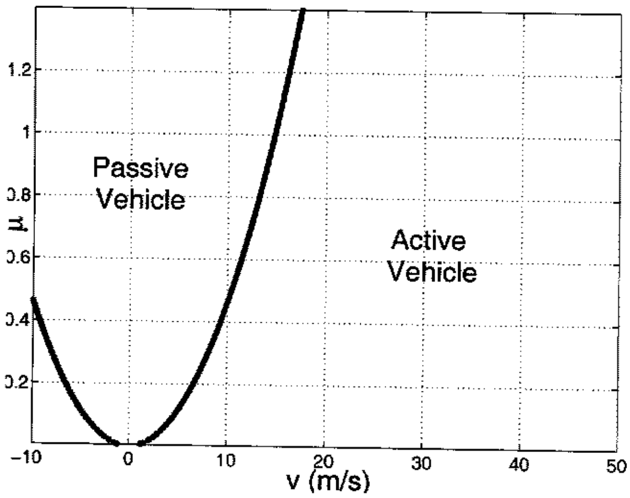


Fig. 11. Passivity bounds of the lateral vehicle impedance, Z_v.

Note that, unlike the usual tele-operation applications, steer-by-wire system comprehends non-passive environment, which complicates to some extent the robust stability considerations, since the conditions of the cited theorem in Section A.2 are not fulfilled. In fact, physically this is not a strange condition, since for velocities $v \neq 0$, when steering, an amount of longitudinal kinetic energy is sent to the lateral dynamics. (Notice, for $v = 0$ the vehicle is passive, which is intuitive.) For a given friction coefficient, μ, it can be shown that the critical vehicle speed when passivity properties switch is,

$$v_{cr} = \sqrt{\frac{\mu c_r (\ell_f + \ell_r)(J + \ell_r^2 m)}{m(J + \ell_f \ell_r m)}}. \tag{35}$$

C. Robust passivity

C.1 Linear steer-by-wire system

Assume that the steer-by-wire controller is passive. The model-matching controller designed in this paper, indeed shows output passivity excess. Hence, it is interesting to investigate the impact of such an excess into the passivity property of the feedback loop designated by the dashed line in Fig. 9. Because of the passivity excess of the controller, it is to be expected that the passivity region of the feedback loop will increase compared to that shown in Fig. 11. In Fig. 12 passivity bounds of feedback loop are computed. Notice that the passivity excess of the controller can compensate for the activity of the vehicle in the region of small μ. Nevertheless, passivity in the whole operating domain of a vehicle is not provided. Hence, for robust passivity a new controller has to be designed (probably) at the price of performance, i.e. of model-matching.

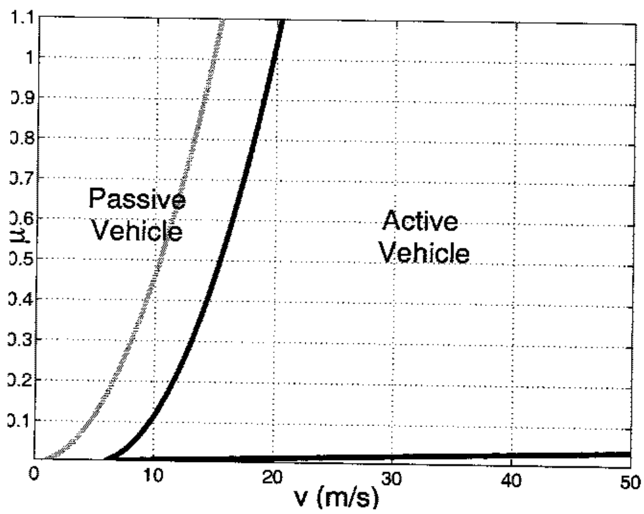


Fig. 12. Passivity bounds of the coupling between steer-by-wire steering system and lateral vehicle dynamics.

C.2 Non-linear steer-by-wire system

Consider the non-linear steer-by-wire vehicle with force assistance, Fig. 13[3]. Such a structure provides

[3]In the figure the torque assistance is plotted. This signal is transformed to a linear force assistance through the rack-and-pinion gearing transmission coefficient. Both of the terms, force and torque transmissions are used in this paper.

an additional power source, so it is intuitively clear that passivity shortage of the feedback structure will be further increased. According to the previous dis-

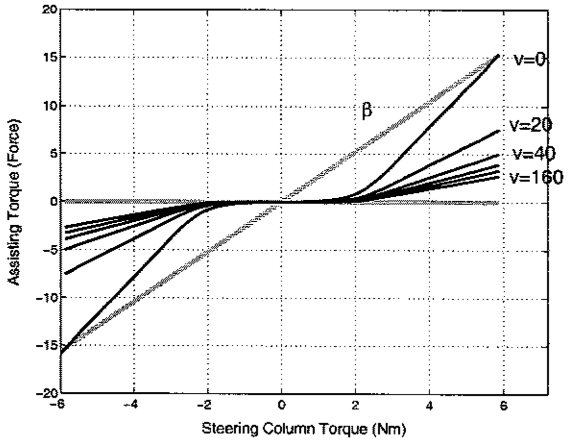


Fig. 13. Boost curve and its dependence on vehicle speed.

cussion, the linear steer-by-wire system has already output passivity shortage, i.e.,

$$\int_0^T \left(u^T y + \rho y^T y\right) \mathrm{d}t \geq S(x(T)) - S(x(0)) \quad (36)$$

whereby $\rho > 0$. Since the dynamics of the boost-curve, Fig. 13, is void, the storage function of the feedback loop is equal to that of the open-loop, $S(x)$. Substitution of

$$u = r + \psi(y) \quad (37)$$

in (36), whereby, $\beta|y| > |\psi(y)|, \forall y$, Fig. 13, yields the condition of passivity of the closed-loop,

$$\int_0^T \left(r^T y - \left(-\psi(y)^T y - \rho y^T y\right)\right) \mathrm{d}t \geq S(x(T)) - S(x(0)), \quad (38)$$

which further requires an impossible condition,

$$-\beta - \rho > 0. \quad (39)$$

Thus, it is to be concluded that passivity tools fail to prove robust stability of steer-by-wire systems.

D. Robust stability

This section presents an alternative approach for analysis of robust stability of steer-by-wire systems with respect to parameter uncertainties. Again the aim is to find the stability regions in the space of uncertain parameters.

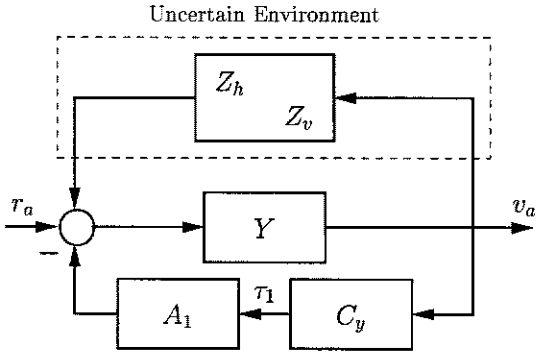


Fig. 14. Linear steer-by-wire system in feedback with the uncertain environment.

D.1 Linear steer-by-wire system

Consider the linear system shown in Fig. 14 with the feedback loop containing the uncertainties of a steer-by-wire system. The main uncertain parameters are vehicle speed, v, friction coefficient, μ, ($Z_v = Z_v(s, v, \mu)$), and driver stiffness, c_h ($Z_h = c_h/s$). A straightforward method is to compute the characteristic polynomial of the system, which describes the eigenvalues of the steer-by-wire system in dependence of uncertain parameters. By doing this, and separating the characteristic equation into its real and imaginary part, once again is met the system of two parametric nonlinear equations,

$$h(\omega, v, \mu, c_h) = 0, \quad g(\omega, v, \mu, c_h) = 0, \quad (40)$$

whereby $0 < \omega < \infty$ stands for the Hurwitz frequency and represents the gridding parameter of the equations. Fig. 15 shows its solution (40). An interesting fact is that for a given vehicle dynamics operating point, i.e. $v = const$ and $\mu = const$, the stability radius increases with increasing driver stiffness, c_h. The curves in Fig. 15 represent the stability bounds for a linear model, which neglects the non-linearities (e.g. gearing friction) in the system. However they provide insight on the stability robustness with respect to parameters uncertainties.

D.2 Nonlinear steer-by-wire system

Now consider Fig. 16 with the boost-curve feedback. In nonlinear control the global asymptotic stability of this structure is denoted as absolute stability of H, with respect to the sector $(0, \beta)$ static nonlinearity. For its investigation in parameter space, in this paper the *circle criterion*, [8], [5], will be used, since it fits well within the analysis framework developed in

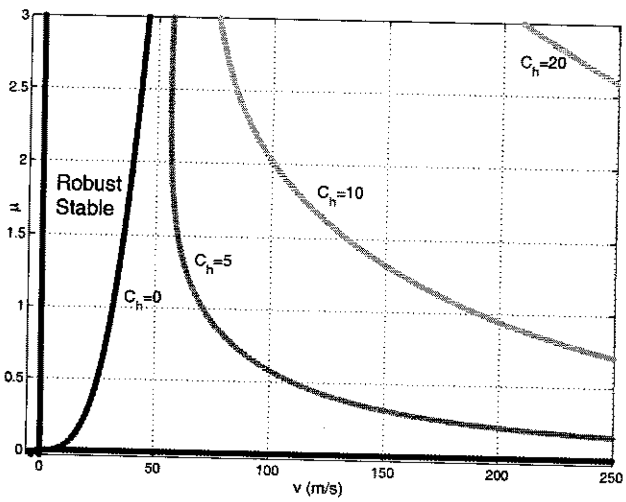


Fig. 15. Robust stability of linear steer-by-wire system.

Section A.3 of mapping of positive realness bounds in parameter space. According to the circle criterion, the feedback loop of a linear system $H(s)$ with any static nonlinearity of the sector $(0, \beta)$ is global asymptotic stable, if the transfer function

$$\tilde{H}(s) = H(s) - \frac{1}{\beta} \qquad (41)$$

is positive real ($\beta > 0$). In the case of steer-by-wire system, the linear part is further uncertain, thus extending the requirement for *robust absolute stability* with respect to uncertainties v, μ and c_h. Based on the discussions in Section A.3, the mapping equations, which correspond to circle criterion are directly derived to be,

$$e(\omega, v, \mu, c_h) = 1/\beta, \quad \frac{\partial e}{\partial \omega}(\omega, v, \mu, c_h) = 0. \qquad (42)$$

Thus, by tuning β, bounds of robust absolute stability

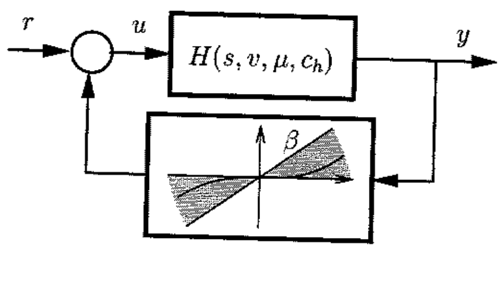


Fig. 16. Absolute stability of non-linear steer-by-wire system with boost-curve feedback.

in the space of parameters (v, μ, c_h) are gained. Notice that this approach provides good insights also for the practical design of the boost-curve, since the required stability radius for a given β may be clearly read in charts similar to that shown in Fig. 15.

VIII. Conclusions

Methods for design and analysis of steer-by-wire systems are presented. Model-matching approach is shown to be appropriate method, once the desired steering dynamics is known. The design method is illustrated for the admittance steer-by-wire structure, but it can be applied equally well for the hybrid structure. Further, methods for the analysis of robust stability of a given steer-by-wire system with respect to the uncertain physical parameters are introduced. They apply for both, linear and nonlinear steer-by-wire systems with static nonlinearities. It is shown that though possible, the design within the framework of passivity, which has been the traditional method for the design of master-slave system, is rather conservative.

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